A conceptual partial equilibrium model of global agricultural land-use

David K. Bryngelsson and Kristian Lindgren

We introduce a conceptual partial equilibrium model of global agricultural land use, based on heterogenous land quality—an area that has received less theoretical attention than location theory. The model is based on maximization of land rent at each parcel through choice of crop and input intensity. Mechanisms of land rent and land-use competition are illustrated in a transparent way, which can be used for e.g. policy testing and for improved understanding of results from larger land-use models. A strength with this approach is that the model to a large extent can be analytically explored. We show how different crops are optimally distributed on land according to their respective area-dependent cost, i.e. costs paid per area regardless of yield. Crops with high such costs are grown on more productive land and crops with low such costs are grown on less productive land, in equilibrium. The equilibrium solution of the model is unique. Further we show how prices are connected between crops that compete for land.

Key words: agricultural land, bioenergy, food price, land rent, land use, land-use competition, partial equilibrium model, price effects, Ricardian rent.

Land has a long history in economic theory, starting in ancient times (Hubacek and van den Bergh 2006) when agriculture and access to land dominated the economy, and has through time been treated by many of the most influential economists, such as Adam Smith, David Ricardo (e.g. Buchanan 1929; Ricardo 1821; Stigler 1952), Johann Heinrich von Thünen (e.g. Beckmann 1972; Heijman and Schipper 2010) and Karl Marx (Haila 1990). These theories have later been further developed in the second half of the 20$^{\text{th}}$ century, see e.g. Hardie and Parks (1997) Lichtenberg (1985), Lichtenberg (1989), Palmquist (1989), Weinschenk, Henrichsmeyer, and Aldinger (1968). See Haila (1990) and Hubacek and van den Bergh (2006) fore a more thorough description of how land has been treated in economic theory over time.

Historically, discussions about how land rent works have shown up in response to urgent social issues concerning land and land use (Haila 1990). Examples of such are Ricardo's theory in response to a controversial corn duty in the early 19$^{\text{th}}$ century, and a revived discussion in the 1970s as a result of rapidly increasing land and housing prices in the U.S. (Haila 1990).

The question of how land rent works is once again hugely relevant, as energy security issues in combination with climate change mitigation efforts, rapidly have increased the demand for bioenergy. This rapid increase in demand for bioenergy has put pressure on agricultural markets and subsequently affected global food prices via competition for land.

Focus of the research has—until recently—been on building theory and explanatory models, that can be used to get analytical answers or at least understandings of causal links. This theory building has, however, mainly worked with the location rent of von Thünen, whereas the productivity rent of Ricardo has received far less attention (Lichtenberg 1985). Location rent was, of course, more important in a world where transport was carried out with horse and carriage and distances were not long enough to allow for significantly different productivity levels of land. This can be assumed to have changed, however, with greatly improved infrastructure and cheaper transports, in combination with trade on global markets where goods produced on separate continents compete. Differences in productivity of land are, of course, huge on a global scale that cover all different climatic zones.

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We have found two tracks in the literature of the last decades that have dealt with productivity rent of land. The first has been with simple explanatory models, see e.g. Hardie and Parks (1997), Lichtenberg (1985, 1989), James (2010), and Swinton et al. (2011). These models are, however, very generalized and do not explicitly try to explain the mechanisms in neither analytical nor quantitative manners.

The other track for treatment of productivity of land has been in larger and more detailed models. Availability of ever more powerful computers and increasingly accessible data from around the world, have helped push the development of large quantitative models of global land use and trade flows. These models have been used for questions regarding land-use changes and land-use competition in near-future, as well ass long-term, settings (see e.g. Gillingham, Smith, and Sands 2008; Havlík et al. 2011; Johansson and Azar 2007; Melillo et al. 2009; Schneider, McCarl, and Schmid 2007; Searchinger et al. 2008).

Such models are much richer in detail (e.g. in terms of crop types, producer and consumer regions, and user sectors), than the explanatory models above, and they give quantitative results on rents and price effects. However, the drawback of this approach is that greater detail decreases the transparency of the models and therefore makes it more difficult to understand which mechanisms lead to which outcomes or how robust the results are regarding parameter values and assumptions made.

There has, however, not been much work done in the intermediate between the two.

In this article we present a model framework that builds on the work by Lichtenberg (1985, 1989) and Palmquist (1989), but with an explicit class of cost functions for agricultural production. This allows us to derive analytic results of the characteristics of the model behavior, such as the mechanisms for land rent, crop distribution on land and price connections between crops. This approach can fill an important gap in knowledge where studies based on large models present results without any attempts to explain how the results depend on the underlying mechanisms.

The model developed in this article is used as a basis for analysis of the equilibrium outcome of several bioenergy scenarios in Bryngelsson and Lindgren (2013) and as a foundation for the development of an agent based model (ABM) in which the system can be studied out of equilibrium (forthcoming).

A conceptual model for agricultural land use

We construct a model in order to determine the optimal distribution of different crops over land of varying quality. For simplicity we assume that different characteristics of the land quality can be aggregated into a main aspect determining the normalized productivity potential on each parcel of land, ranging from 1 for the best land (realizing the full potential for each crop) to 0 for the worst (no yields). If we arrange all parcels of land in a declining order with respect to this productivity, we get a declining function $Y(a) \in [1, 0]$, which states that an area a of the best land has a productivity of $Y(a)$ or more. This is illustrated in figure 1. Lichtenberg (1989) and Palmquist (1989) use similar approaches.

We consider n different crops that can be produced on the land. The yield[1] y_i for each crop i on a land parcel at a is given by the productivity of land times a crop specific yield parameter η_i, i.e., $y_i(a) = \eta_i Y(a)$. All land owners are assumed to maximize profits and decide to produce the crop that gives the highest profit, which also involves a decision on levels of inputs to use, i.e. cost, that should be put into the production. Land owners can always be assumed to decide what is produced on the land, since they either rent out the land to the highest bidder, or "rent" the land to themselves by choosing the crop with highest return (Hardie and Parks 1997; Palmquist 1989).

We assume that there are two types of costs involved (not including land rent, which is endogenously determined in the model). The first, α_i, is a cost per unit area of land used (area dependent cost), and this is constant for each crop, regardless the land productivity where it is grown. Area dependent costs are connected to the area under production, such as tillage and equipment capital. The second cost, β_i, is a cost per unit of produced crop i (harvest dependent cost), and it is a variable that the farmer can decide on as part of the profit maximization. Harvest dependent costs are connected to the quantities produced, such as pesticides, fer-

[1] We assume that yield is measured in units of energy per area, GJ/ha. The same unit is used for η_i while $Y(a)$ is dimensionless.

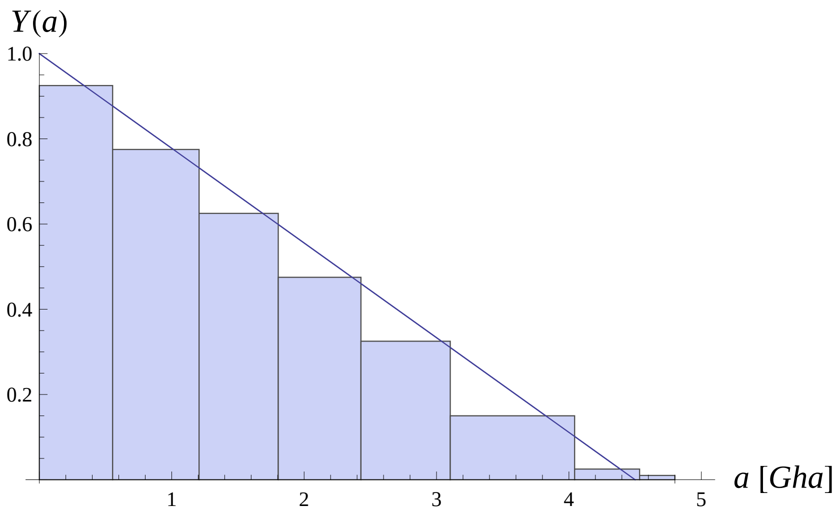

Figure 1. Representation of global agricultural land with diminishing productivity. The bars represent data for *Suitability for rain-fed crops* from plate 46 when forested areas have been deducted (plate 55), developed by Fisher et al. (2002) and the curve represents the continuous approximation (*Y(a)*) used in this model.

tilizer and transport. This means that the yield directly depends on β_i via the yield parameter $\eta_i = \eta_i(\beta_i)$. It is assumed that $\eta_i(0) = 0$, $\eta_i'(\beta_i) > 0$, and $\eta_i''(\beta_i) < 0$ and that $\eta_i(\beta_i)$ is bounded.

Assuming a market price p_i for each crop i, the corresponding profits $\pi_i(\beta_i, a)$ and production intensities β_i on a land parcel at a can be written

$$\pi_i(\beta_i, a) = (p_i - \beta_i)\eta_i(\beta_i)Y(a) - \alpha_i \,. \tag{1}$$

The crop that yields the maximum profit at each parcel of land defines the land rent $r(a)$. The land rent for a parcel of land at a, or the maximum willingness to pay for the land, can thus be expressed as

$$r(a) = \max_{i,\,\beta_i} \pi_i(\beta_i, a) \tag{2}$$

This maximization problem thus determines, for each parcel of land, the decisions made by the landowners on which crop i to grow and with which intensity β_i. The decision actually taken by landowners at each parcel of land is thus $(i(a), \beta_i^*(a)) = \arg\max_{j,\,\beta_j} \pi_j(a)$, and in aggregate form this leads to the equilibrium land rents in equation (2).

The produced quantity q_i is given by the integral over all land parcels Γ_i for which π_i is the maximum profit, $\Gamma_i = \{a : r(a) = \pi_i(\beta_i, a)\}$. The quantity q_i can then be written

$$q_i = \int_{\Gamma_i} \eta_i(\beta_i)Y(a)\, da \,. \tag{3}$$

The market price, $p_i = D_i(q_i)$, for each crop i is endogenously determined by the aggregate quantity supplied, q_i, and an elastic inverse demand function (D_i) that is unbounded for small quantities, i.e. $p_i \to \infty$ as $q_i \to 0$. It is assumed that $D_i(q_i)$ is decreasing with q_i, i.e. $D_i' < 0$. The model can easily be extended to handle exogenous prices or inverse demand functions that are not unbounded at $q_i \to 0$.

Equations (1)–(3) and the demand function (D_i) define the equilibrium problem that is the basis for this article.

In the following, we argue for why there must exist an equilibrium solution and we derive some general properties of the solution.

Existence of land rent equilibrium

Since there is a limit on the total production of crops, there is also, for the purpose of maximization, an upper limit of the total surplus, i.e. the sum of combined consumer and producer surplus. Thus, a maximum must exist for the total surplus. At a maximum of total surplus, one finds that the equilibrium criteria, equations (1)–(2), for the land rent problem must hold by the following arguments. First, at a small parcel of land da at a where crop i is grown, the farmer's choice of β_i must be the one that maximizes profit. Otherwise a change to the optimal value would directly lead to increased producer surplus, as the profit would increase for the farmer at a. The resulting change in total production of crop i would also lead to a change in crop price, dp_i, but to first order in da that would only transfer surplus between producers and consumers without a net change. Thus, a change from a suboptimal to an optimal β_i would lead to an increase in total surplus. Second, the same reasoning can be done for the choice of crop i at a, leading to the conclusion that to maximize surplus, the profit maximizing crop is always chosen. Thus, since there is a maximum in total surplus, there must also exist an equivalent equilibrium for the land rent problem. See Macmillan (1982), O'Kelly (1988), O'Kelly and Bryan (1996) and O'Sullivan and Ralston (1980) for a discussion on the equivalence between the maximum surplus problem and land rent equilibrium in the case of von Thünen location rent.

Unbounded prices for the inverse demand functions at $q_i \to 0$ guarantees that all crops are grown on some land in equilibrium.

The optimal intensity β

First we look at the choice of intensity β_i. Assume that the system is in equilibrium, characterized by crop prices p_i^*. For each crop i that the farmer considers, the maximum profit is given by $d\pi_i/d\beta_i = 0$, see equation (1), which results in

$$\beta_i^* = p_i^* - \frac{\eta_i(\beta_i^*)}{\eta_i'(\beta_i^*)}\,, \tag{4}$$

which is an equation that is independent of a. It is straightforward to show that the solution β_i^* to this equation is unique. The optimum intensity β_i^* for each crop i is thus global and we also have a global yield parameter η_i^*.

The optimal distribution of crops

The question on how the crops are distributed over land of different quality is resolved by the following proposition.

PROPOSITION 1. *In equilibrium, if there is a shift in crop produced at a certain parcel of land a^* (sweeping over a), then the crop i with the higher area dependent cost $\alpha_i > \alpha_j$ is produced on the more productive land, so that crop i is produced on $a < a^*$ while j is produced on $a > a^*$.*

See Appendix for proof of proposition 1.

The implication of proposition 1 is that each crop i (with $i = 1, 2, 3, ..., n$) is produced in a certain range of land productivity, i.e., crop i is produced at land parcels a for which $Y(a_{i-1}) < Y(a) < Y(a_i)$. The land parcels at a_{i-1} and a_i thus correspond to land productivity levels at which there is a shift in choice of produced crop. It then follows that the crops are distributed over the land variable a with the most productive land being used for the the crop with the highest area dependent cost α followed by the next crop etc. Note that, at land parcels a_i, where there is a shift in optimal crop, the profit for the two involved crops is identical, which means that the curves for the willingness to pay functions of a (equation 1) intersect at these points. Land rents are thus continuous across the intersections between crops. (Note that, even in the case of general inverse demand functions or exogenous prices, the proposition holds, but the arrangement of the crops only applies to the crops that are present in the equilibrium solution.)

A straightforward argument for this arrangement of the crops is that an agricultural system with high area dependent costs (that have to be payed per hectare regardless of yield) has much to benefit from access to highly productive land. Such a system can support high land rent payments in order to reduce the required area. An agricultural system that pays little in area dependent costs (e.g., cattle grazing) has, however, smaller such benefits and tend to prefer larger areas of lower productivity in exchange for lower land rent payments.

There is a special case that can arise, namely $\alpha_i = \alpha_{i+1}$. In this case, the land owners become indifferent in the choice between these two crops and they can be distributed in any way on land of productivity between a_{i-1} and a_{i+1}. However, the total quantity of each of these crops will be determined by there respective inverse demand functions.

Example with three generic crops

Here we construct an example with three generic crops, numbered $i = 1, 2, 3$, ordered so that $\alpha_1 > \alpha_2 > \alpha_3$, to show how the model can be solved. Crop 1 supports the highest land rent on the most productive land and is produced there, while crops 2 and 3 are consecutively grown on land of decreasing productivity. At certain values of a there is a shift from one crop to the next, defined by equal values of the willingness to pay at these points. In addition, if the productivity is too low, no crop is produced, and the land rent beyond that point is zero. This can be summarized in the following equations,

$$(5) \qquad r(a_1) = \pi_1(\beta_1^*, a_1) = \pi_2(\beta_2^*, a_1)$$

$$(6) \qquad r(a_2) = \pi_2(\beta_2^*, a_2) = \pi_3(\beta_3^*, a_2)$$

$$(7) \qquad r(a_3) = \pi_3(\beta_3^*, a_3) = 0\,.$$

If these continuity conditions are not fulfilled, there are parcels of land for which there is arbitrage to be made, i.e. the land is not utilized by the crop that can support the highest rent and land owners would change crops until equilibrium is reached and conditions (5)–(7) are fulfilled.

These different functions for willingness to pay at equilibrium are schematically depicted in Figure 2.

Equations (5)–(7) in combination with equations (3) and the demand function(s) give the solution to the model.

Price relations between crops

There are two potential connections between prices for separate crops. The first is as substitutes (or complements) on the market, where the crops compete (help each other) for the same market. Their prices are thus connected via some cross-price elasticity. The second connection goes via scarcity of production factors, i.e. limited supply of land and/or labor or other inputs.

In the example we show in this article, we have selected three generic crops that are supposed to not compete in any way on the market, which means that the first connection is not existing, or in other words, the cross-price elasticity is zero. The second connection, however, is not zero and depends on limitation of land and the competition thereof. This connection vía land-use competition is schematically described in e.g. James (2010) and Swinton et al. (2011). Here we continue on their description, but make it analytically explicit.

An increase in demand (and therefore the quantity supplied in equilibrium) for a crop directly increases equilibrium prices for crops grown on adjacent land. An increase in demand of, for example, the second crop, leads to increased ability to pay land rent at all levels of a, i.e. $\pi_2(a)$ increases. There would be a shift upwards of the red curve in Figure 2. This holds also for the land parcel at a_1 and a_2, and continuity of land rent then implies a higher land rent also for crop 1 and crop 3. For crop 1, the available responses to increased land-use competition are to either decrease production (a_1 moves left) and/or pay more land rent ($\pi_1(a)$ increases). The latter is accomplished by an increased market price. The magnitudes of these different effects depend on the demand function for crop 1. For crop 3 the response is a combination of higher land rent and prices, as well as a move towards land of lower productivity. The latter effect also implies more costly production, with higher prices and lower quantity at equilibrium. The price effect can trickle down in a chain reaction in the case of more crops.

From the willingness to pay for land equation (1) and the continuity conditions equations (5)–(7) it is straight forward to deduct a price relationship between adjacent crops at equilibrium, here exemplified by $p_1(p_2^*)$,

$$(8) \qquad p_1(p_2^*) = (p_2^* - \beta_2^*)\frac{\eta_2^*}{\eta_1^*} + \beta_1^* + \frac{\alpha_1 - \alpha_2}{\eta_1^* Y(a_1^*)}\,.$$

The other relationships are analogous, of course.

A key question here is to what extent changed conditions for one crop affect prices for other crops. This could e.g. be driven by an exogenous change in demand for one crop, such as increased bioenergy demand. The dominating factor influencing the crops' price relationship is the ratio η_i^*/η_j^*, but to find the exact effect from an externally driven price change of one crop, the new market clearing prices etc. need to be calculated and to do this the inverse demand functions need to be exactly known. The effect is, however, bounded upwards by the ratio η_i^*/η_j^* and bounded downwards by zero. The upper limit applies for $\varepsilon_1 = 0$ and the second for $\varepsilon_1 = \infty$.

These price responses apply at equilibrium, i.e. when the market has settled and found the new optimum.

Uniqueness of land rent equilibrium

PROPOSITION 2. *The land rent equilibrium given by equations. (1)–(3), is unique.*

For proof of proposition 2, see Appendix. However, the logic is that for a given set of market prices, p_i, there is a given set of optimal input levels, β_i^*, and corresponding willingness-to-pay for land functions, $\pi_i(\beta_i^*, a)$. In order

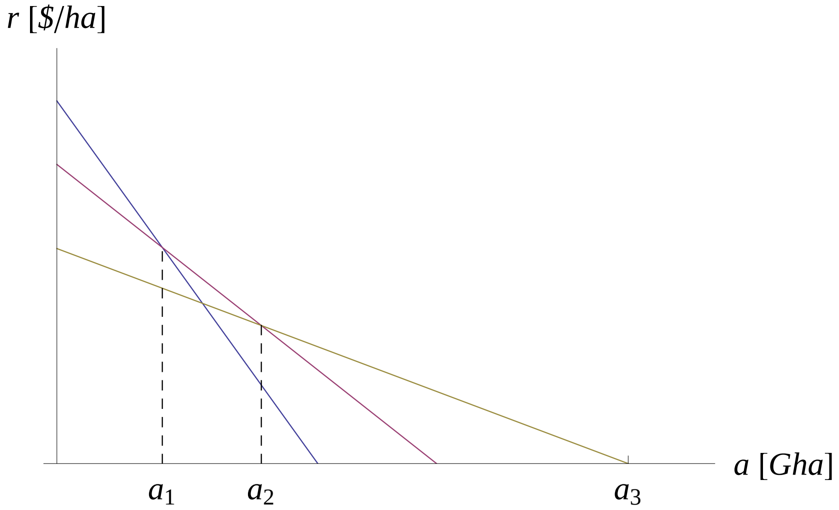

Figure 2. Depiction of willingness to pay curves for three different crops. Blue curve is crop 1, red curve crop 2 and yellow curve crop 3 from equations (5)–(7).

for another equilibrium to exist there must be a combination with higher prices, with corresponding higher input levels and willingness-to-pay functions. For prices to be higher there must be smaller quantities, q_i supplied on the market (due to demand elasticity). For quantities to be smaller there must be a shift towards the left for each willingness-to-pay-equilibrium point, a_i, see figure 2 and equations (5)–(7). Higher prices and thus higher willingness-to-pay functions are incompatible with a shift towards the left for the point, a_n where the last crop n has a zero willingness to pay land rent, $\pi_n(a) = 0$. The argument is analogous in the opposite direction. There can thus only be one equilibrium.

Application of the model to a bioenergy scenario

Here we illustrate the model with some plausible parameter settings in two "scenarios", one with no bioenergy whatsoever and one with 100 EJ bioenergy, and compare the two to get some indications of how equilibrium prices may be affected by such an increased demand. The supply level of 100EJ is in the lower middle of the range of global bioenergy potentials estimated in the meta analysis by Berndes, Hoogwijk, and van den Broek (2003), and in the recent report by IPCC working group 3 (Chum et al. 2011).

Consider three generic crops: intensively produced edible-type and forage crops (IP), extensively produced permanent pasture and forage crops (EP), and bioenergy crops (BE).

Table 1 presents the parameter values we used to illustrate how the model works.

Table 1. Parameter values for application of the model.

	η	α	β	q_0	p_0	ε
IP	85	500	4	60	12	-0.5
EP	60	50	1	95	3.6	- 1
BE	200	300	3	100	-	0

Note 1: Units of parameters: η: [GJ ha^{-1} yr^{-1}], α: [US$ ha^{-1} yr^{-1}], β: [US$ GJ^{-1}], q_0: [EJ yr^{-1}], p_0: [US$ GJ^{-1}]
Note 2: Values are based on data from Berndes, Hoogwijk, and van den Broek (2003), Chum et al. (2011), FAOSTAT, Huang and Lin (2000), Regmi et al. (2001, p. 21), Wirsenius (2000, p. 108), Wirsenius, Hedenus, and Mohlin (2010), and personal communication with Christel Cederberg.

Exactly what the land productivity function $Y(a)$ looks like in the real world is difficult to know, but based on the data from IIASA presented in figure 1, the function $Y(a) = 1 - a/A$ offers a relatively good fit, where A is the total amount of productive land available, here set at 4.4 Gha (forested areas are deducted).

Isoelastic inverse demand functions are assumed for both the intensively produced crops and extensively produced crops, but, of course, with different elasticities. The bioenergy demand is perfectly inelastic.

Results

In equilibrium the crops are placed in the order (intensive crops, bioenergy, extensive grazing) according to their area specific production costs. In figure 3 the optimal land-rent curves for the three crops are shown for the situation with and without bioenergy production, respectively.

Adding a crop with a large demand (bioenergy, BE) to the system significantly increases land rents at all levels (see figure 3), from more than 60 % increase in land rent for the most productive land, to a multifold increase in rent for land of lower productivity. The area used for intensive production decreases somewhat (and thus also the quantity by 6 %) in response to the competition and its production price increases by 13 %. Extensive production is "forced" onto less productive land by the energy crops and thus the quantity produced decreases by 27 % at the same time as its price increases by over 35 %. Total land area under cultivation increases by over 10 % (from 3.2 Gha to 3.6 Gha), as can be seen in figure 3.

Discussion and conclusion

The main contribution with this article is the presentation of a conceptual partial equilibrium model for the global agricultural land-use system, which is simple enough to be analytically explored to a large extent, but detailed enough to represent main mechanisms and to capture several important features of the system. This model works as a transparent tool to illustrate important mechanisms of productivity based land rent, land-use competition and crop distribution over land (in optimum). It can also be used to check how different policies (regarding land, agriculture, food systems etc.) affect important system behaviors, such as food prices and land rent. This is done by deriving explicit land rent functions from yield functions applied in a conceptual framework.

In this article we have shown that

(*i*) area-dependent costs determine the optimal distribution of crops on land of different productivity,

(*ii*) the land-rent equilibrium problem has a unique solution that is identical to the maximization of the combined consumer and producer surplus,

(*iii*) analytical price connections between crops from the competition for land are easily derived.

Result (*i*) implies that production of each crop gets located on land of similar productivity (and not spread out), which is in line with the results of Lichtenberg (1989). The crop with the highest area-dependent cost is placed on the most productive land and the other crops in consecutive order with the crop with the lowest such cost placed on the least productive land that is brought under cultivation. This can be understood as crops with high area-dependent costs have more to gain from reducing their area under cultivation, than crops with low such costs. The latter "prefer" to expand production on less productive land, as compared to paying high rent for access to more productive land.

Analytically deduced price connections between crops (result *iii*) show that changes in demand for one crop affect prices for other crops mainly in proportion to their respective yields, but the connections also depend on various other parameters. This result can have relevant implications for cases when some crops act as price takers in another system (e.g. bioenergy in the energy system) and thus have exogenous prices, while other crops compete for the same land (in this case food crops and grazing).

The model developed in this article is applied to several policy scenarios for large scale bioenergy introduction in Bryngelsson and Lindgren (2013), where the model is also characterized in an extensive parameter analysis. The model framework is also used as a basis for an agent based model, in which the system can be studied out of equilibrium, in transitions between equilibria and how equilibria form and land rents are set (forthcoming).

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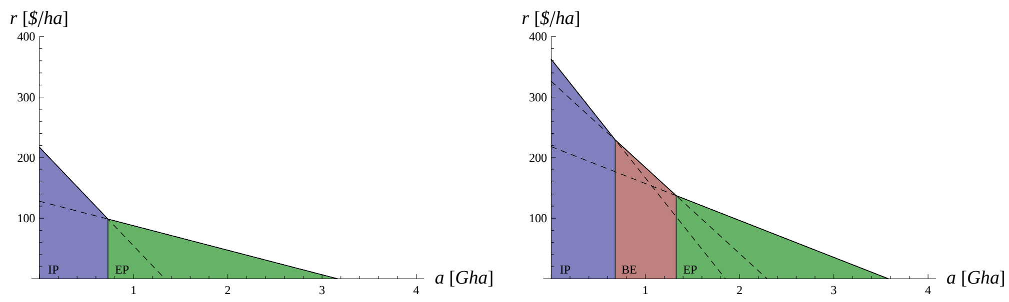

Figure 3. In optimum, the crops are placed in the order (intensive crops, bioenergy, extensive grazing). The optimal land-rent curves are depicted in the left panel with no bioenergy production and in the right panel with 100EJ/yr bioenergy production.

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Appendix

Proof of proposition 1

Proof. In equilibrium, the profit in equation (1) for a crop i can be written

$$\begin{aligned}\pi_i(a) &= (p_i^* - \beta_i^*)\eta_i^*(\beta_i^*)Y(a) - \alpha_i \\ &= \phi_i Y(a) - \alpha_i\end{aligned} \tag{A.1}$$

where we have introduced the crop-specific parameter $\phi_i = (p_i^* - \beta_i^*)\eta_i^*$. If there always is a positive demand for a crop, regardless of its price, all crops are grown and $\phi_i > 0$. Each crop is thus characterized by a profit function (or a willingness to pay for land), and the crop that gives the highest profit at each a will therefore be grown there. How the crops are distributed on the land is simply determined by their respective profit functions.

For a pair of adjacent crops, say i and j, let us assume that $\alpha_i > \alpha_j$. The profit functions intersect at a certain a_i^*, given by $Y(a_i^*) = (\alpha_i - \alpha_j)/(\phi_i - \phi_j)$ (implying that the crops must have different ϕ-values). Since $Y(a_i^*) > 0$, we also know that $\phi_i - \phi_j > 0$. At the point a_i^* the profits are the same, but moving to less productive land (higher a) leads to a decrease in the profit that is different for the two crops,

$$\begin{aligned}&\frac{d}{da}\left(\pi_i(a) - \pi_j(a)\right) = \\ &= (\phi_i - \phi_j)\frac{dY(a)}{da} < 0\,.\end{aligned} \tag{A.2}$$

Since π_i decreases faster than π_j, crop i is grown on the more productive land while crop j is grown on the less productive land. The assumption was that the area specific cost was higher for crop i than for crop j, and the result does not then explicitly depend on the market prices for the crops. Instead a higher market price for crop i is a result of the crops' competition for land. ∎

Uniqueness of land rent equilibrium

Proof. The land rent equilibrium given by equations (1)–(3), is unique. This follows from the following arguments. Assume that an equilibrium is characterized by $(a_1, a_2, ..., a_n)$, i.e., the land parcels at which there is a switch from one crop i to the next crop $i+1$. From Proposition 1, we know that crop 1 with the highest α is produced on all land with quality higher that $Y(a_1)$, followed by the other crops, so that $\alpha_1 > \alpha_2 > ... > \alpha_n$.

If there is another equilibrium $(a'_1, a'_2, ..., a'_n)$, we can assume that $a'_1 < a_1$. (If $a'_1 = a_1$, we go on to the crop for which there is a difference, and start the reasoning at that point. If $a'_1 > a_1$, we would instead consider the equilibrium given by a'_1 as the starting point.) First, we note that in a given interval $[0, a]$, there is only one equilibrium for crop 1 characterized by equation (4), which we can re-write as

$$\text{(A.3)} \quad \beta_1 = D_1(q_1) - \frac{\eta_1(\beta_1)}{\eta'_1(\beta_1)}\,,$$

where we have replaced the price p_1 with the inverse demand function $D(q_1)$ for the produced quantity q_1 of crop 1. Since all terms are continuous functions of β_1, and in the limit of $\beta_1 \to 0$, the left-hand side is 0 and increases linearly with β_1, while the right-hand side is larger than 0 but decreases, as $d/d\beta_1$ applied to the right-hand side is $D'(q_1)dq_1/d\beta_1 - 1 + \eta_1\eta''_1/(\eta'_1)^2 < 0$. This implies that there is a unique solution to the equation. Thus there is not only a unique equilibrium for β_1 locally (for each farmer), as is expressed by equation (4), but also for the aggregate production in the entire interval for crop 1, $[0, a'_1]$.

When we move from a_1 to a'_1 the equilibrium characteristics for crop 1 changes as follows: By applying d/da to equation (A.3), we see that

$$\text{(A.4)} \quad \frac{d\beta_1}{da}\left(2 - \eta\frac{\eta''}{(\eta')^2}\right) = D'_1(q_1)\eta_1 Y(a_1)\,.$$

With $\eta'' < 0$ and $D'_1(q_1) < 0$, we conclude that $d\beta_1/da < 0$. So, at the point $a'_1 < a_1$ we have an equilibrium with higher intensity $\beta'_1 > \beta_1$. With this also follows a higher price, $p'_1 > p_1$, since $p_1 = \beta_1 + \eta_1/\eta'_1$ implies

$$\text{(A.5)} \quad \frac{dp_1}{d\beta_1} = 2 - \frac{\eta\eta''}{(\eta')^2} > 0\,.$$

This also means that the produced quantity is lower, $q'_1 < q_1$.

At the point a'_1 between crop 1 and 2, the profit for crop 1 has increased, since at equilibrium,

$$\text{(A.6)} \quad \pi_1 = (p_1 - \beta_1)\eta_1 Y(a'_1) - \alpha_1 = \\ = \frac{\eta_1^2}{\eta'_1} Y(a'_1) - \alpha_1$$

and both η_1 and $1/\eta'_1$ increases with price. We also know that, if a'_1 is a transition point between crop 1 and crop 2, the profit for both crops are equal, which means that the price for crop 2 must increase as well (since the profit for crop 2 was lower than for crop 1 at that point when the transition point was at a_1). It then follows that also for crop 2 the intensity increases, $\beta'_2 > \beta_2$, and the produced quantity decreases, $q'_2 < q_2$. With higher β and access to higher quality land this means that the transition point between crop 2 and crop 3 also must move to the left, $a'_2 < a_2$.

This scheme then repeats all the way to the last crop n, for which we also conclude that $p'_n > p_n$ and $q'_n < q_n$. The last point a_n must also move to the left, $a'_n < a_n$, but in equilibrium the new point must be characterized by zero profit. This is not possible since

$$\text{(A.7)} \quad \pi_n(a'_n) = \frac{\eta_1^2}{\eta'_1} Y(a'_n) - \alpha_1$$

and since the factor η_1^2/η'_1 increases with intensity (and price) so that $\pi'_n(a'_n) > \pi_n(a'_n) > 0$. Thus we conclude that the equilibrium is unique. ■