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Electron geodesic acoustic modes in electron temperature gradient mode turbulence

Johan Anderson,[1,a)] Hans Nordman,[1] Raghvendra Singh,[2] and Predhiman Kaw[2]
[1]Department of Earth and Space Sciences, Chalmers University of Technology, SE-412 96 Göteborg, Sweden
[2]Institute for Plasma Research, Bhat, Gandhinagar, Gujarat 382428, India

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In this work, the first demonstration of an electron branch of the geodesic acoustic mode (el-GAM) driven by electron temperature gradient (ETG) modes is presented. The work is based on a fluid description of the ETG mode retaining non-adiabatic ions and the dispersion relation for el-GAMs driven nonlinearly by ETG modes is derived. A new saturation mechanism for ETG turbulence through the interaction with el-GAMs is found, resulting in a significantly enhanced ETG turbulence saturation level compared to the mixing length estimate. *© 2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4742321]

I. INTRODUCTION

There has been overwhelming evidence that coherent structures such as vortices, streamers, and zonal flows ($m = n = 0$, where m and n are the poloidal and toroidal modenumbers, respectively) play a critical role in determining the overall transport in magnetically confined plasmas.[1,2] Some of these coherent structures, so called streamers, are radially elongated structures that cause intermittent, bursty events, which can mediate significant transport of heat and particles, for instance, imposing a large heat load on container walls. Zonal flows on the other hand may impede transport by shear decorrelation.[1,2] The geodesic acoustic mode (GAM)[3–12] is the oscillatory counterpart of the zonal flow ($m = n = 0$ in the potential perturbation, $m = 1$, $n = 0$ in the perturbations in density, temperature and parallel velocity) and thus a much weaker effect on turbulence is expected. Nevertheless, experimental studies suggest that GAMs are related to the L-H transition and transport barriers. The GAMs are weakly damped by Landau resonances and moreover this damping effect is weaker at the edge suggesting that GAMs are more prominent in the region where transport barriers are expected.[9]

The electron-temperature-gradient (ETG) mode driven by a combination of electron temperature gradients and field line curvature effects is a likely candidate for driving electron heat transport.[13–17,22] The ETG driven electron heat transport is determined by short scale fluctuations that do not influence ion heat transport and is largely unaffected by the large scale flows stabilizing ion-temperature-gradient (ITG) modes.

In this work, the first demonstration of an electron branch of the geodesic acoustic mode (el-GAM) driven by ETG modes is presented. The frequency of the el-GAM is higher compared to the ion GAM by the square root of the ion-to-electron mass ratio ($\Omega_q(electron)/\Omega_q(ion) \approx \sqrt{m_i/m_e}$, where $\Omega_q(electron)$ and $\Omega_q(ion)$ are the real frequencies of the electron and ion GAMs, respectively.). We have utilized a fluid model for the ETG mode based on the Braginskii equations with non-adiabatic ions including impurities and finite β-effects.[16,17] A new saturation mechanism for ETG turbulence through the interaction with el-GAMs, balanced by Landau damping, is found, resulting in a significantly enhanced ETG turbulence saturation level compared to the mixing length estimate.

The remainder of the paper is organized as follows: In Sec. II, the linear ETG mode including the ion impurity dynamics is presented. The linear el-GAM is presented and the non-linear effects are discussed in Sec. III, whereas the saturation mechanism for the ETG turbulence is treated in Sec. IV. The paper is concluded in Sec. V.

II. THE LINEAR ELECTRON TEMPERATURE GRADIENT MODE

In this section, we will describe the preliminaries of the ETG mode which we consider under the following restrictions on real frequency and wave length: $\Omega_i \leq \omega \sim \omega_\star \ll \Omega_e$, $k_\perp c_i > \omega > k_\parallel c_e$. Here, Ω_j are the respective cyclotron frequencies, ρ_j the Larmor radii, and $c_j = \sqrt{T_j/m_j}$ the thermal velocities. The diamagnetic frequency is $\omega_\star \sim k_\theta \rho_e c_e / L_n$, $k_\perp$ and $k_\parallel$ are the perpendicular and the parallel wavevectors. The ETG model consists of a combination of an ion and electron fluid dynamics coupled through the quasineutrality including finite β-effects.[16,17]

A. Ion and impurity dynamics

In this section, we will start by describing the ion fluid dynamics in the ETG mode description. In the limit $\omega > k_\parallel c_e$ the ions are stationary along the mean magnetic field $\vec{B}$ (where $\vec{B} = B_0 \hat{e}_\parallel$) whereas in the limit $k_\perp c_i \gg \omega$, $k_\perp \rho_i \gg 1$ the ions are unmagnetized. We note that the adiabatic ion response follows from the perpendicular ion momentum equation by balancing the linear parts of

$$-e n_i \nabla \phi = T_i \nabla n_i, \quad (1)$$

and we find

$$\tilde{n}_i = -\tau \tilde{\phi}. \quad (2)$$

a)Electronic mail: anderson.johan@gmail.com.

In this paper, we will use the non-adabatic responses in the limits $\omega < k_\perp c_I < k_\perp c_i$, $c_I = \sqrt{\frac{T_I}{m_I}}$ and assume that $\Omega_i < \omega < \Omega_e$ are fulfilled for the ions and impurities. In the ETG mode description, we can utilize the ion and impurity continuity and momentum equations of the form

$$\frac{\partial n_j}{\partial t} + n_j \nabla \cdot \vec{v}_j = 0, \qquad (3)$$

and

$$m_j n_j \frac{\partial \vec{v}_j}{\partial t} + e n_j \nabla \phi + T_j \nabla n_j = 0, \qquad (4)$$

where $j = i$ for ions and $j = I$ for impurities. Now, we derive the non-adiabatic ion response with $\tau_i = T_e/T_i$ and impurity response with $\tau_I = T_e/T_I$, respectively. We have thus for the ions

$$\tilde{n}_i = -\left(\frac{\tau_i}{1 - \omega^2/(k_\perp^2 c_i^2)}\right)\tilde{\phi}, \qquad (5)$$

and similarly we find for the impurities

$$\tilde{n}_I = -\left(\frac{z\tau_I}{1 - \omega^2/(k_\perp^2 c_I^2)}\right)\tilde{\phi}. \qquad (6)$$

Here, T_j and n_j are the mean temperature and density of species ($j = e,\ i,\ I$), where $\tilde{n}_i = \delta n/n_i$, $\tilde{n}_I = \delta n_I/n_I$, and $\tilde{\phi} = e\delta\phi/T_e$ are the normalized ion density, impurity density, and potential fluctuations. Next, we present the electron dynamics and the linear dispersion relation.

B. The electron model

The electron dynamics for the toroidal ETG mode are governed by the continuity, parallel momentum, and energy equations adapted from the Braginskii's fluid equations. The electron equations are analogous to the ion fluid equations used for the toroidal ITG mode

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_E + n_e \vec{v}_{\star e}) + \nabla \cdot (n_e \vec{v}_{pe} + n_e \vec{v}_{\pi e}) + \nabla \cdot (n_e \vec{v}_{\| e}) = 0 \qquad (7)$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} + n_e T_e \nabla \cdot \vec{v}_e + \nabla \cdot \vec{q}_e = 0. \qquad (8)$$

Here, we used the definitions $\vec{q}_e = -(5p_e/2m_e\Omega_e)e_\| \times \nabla T_e$ as the diamagnetic heat flux, $\vec{v}_E$ is the $\vec{E} \times \vec{B}$ drift, $\vec{v}_{\star e}$ is the electron diamagnetic drift velocity, $\vec{v}_{Pe}$ is the polarization drift velocity, $\vec{v}_\pi$ is the stress tensor drift velocity, and the derivative is defined as $d/dt = \partial/\partial t + \rho_e c_e \hat{e} \times \nabla\phi \cdot \nabla$. A relation between the parallel current density and the parallel component of the vector potential ($J_\|$) can be found using Ampère's law

$$\nabla_\perp^2 \tilde{A}_\| = -\frac{4\pi}{c}\tilde{J}_\|. \qquad (9)$$

Taking into account the diamagnetic cancellations[14] in the continuity and energy equations, the Eqs. (7), (8), and (9) can be simplified and written in normalized form as

$$-\frac{\partial \tilde{n}_e}{\partial t} - \nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi} - \left(1 + (1+\eta_e)\nabla_\perp^2\right)\nabla_\theta \tilde{\phi} - \nabla_\| \nabla_\perp^2 \tilde{A}_\| + \epsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \sin\theta \frac{\partial}{\partial r}\right)(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) = 0, \qquad (10)$$

$$\left((\beta_e/2 - \nabla_\perp^2)\frac{\partial}{\partial t} + (1+\eta_e)(\beta_e/2)\nabla_\theta\right)\tilde{A}_\| + \nabla_\|(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) = 0, \qquad (11)$$

$$\frac{\partial}{\partial t}\tilde{T}_e + \frac{5}{3}\epsilon_n\left(\cos\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \sin\theta\frac{\partial}{\partial r}\right)\frac{1}{r}\frac{\partial}{\partial\theta}\tilde{T}_e + \left(\eta_e - \frac{2}{3}\right)\frac{1}{r}\frac{\partial}{\partial\theta}\tilde{\phi} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_e = 0. \qquad (12)$$

Note that similar equations have been used previously estimating the zonal flow generation in ETG turbulence[18] and have been shown to give good agreement with linear gyrokinetic calculations.[17] Extended fluid models treating the gyroviscous cancellations by including the higher order moments in the Braginskii's gyroviscous tensor have been presented in Refs. 19–21. The variables are normalized according to

$$(\tilde{\phi}, \tilde{n}, \tilde{T}_e) = (L_n/\rho_e)(e\delta\phi/T_{eo}, \delta n_e/n_0, \delta T_e/T_{e0}), \qquad (13)$$

$$\tilde{A}_\| = (2c_e L_n/\beta_e c \rho_e) e A_\| / T_{e0}, \qquad (14)$$

$$\beta_e = 8\pi n T_e/B_0^2. \qquad (15)$$

Using the Poisson equation in combination with Eqs. (5) and (6), we then find

$$\tilde{n}_e = -\left(\frac{\tau_i n_i/n_e}{1 - \omega^2/k_\perp^2 c_i^2} + \frac{(z^2 n_I/n_e)\tau_I}{1 - \omega^2/(k_\perp^2 c_I^2)} + k_\perp^2 \lambda_{De}^2\right)\tilde{\phi}. \qquad (16)$$

First, we will consider the linear dynamical equations (10), (11), and (12) and utilizing Eq. (8) in the same manner as in Refs. 16 and 17 and we find a semi-local dispersion relation as follows

$$\left[\omega^2\left(\Lambda_e + \frac{\beta_e}{2}(1+\Lambda_e)\right) + (1 - \bar{\epsilon}_n(1+\Lambda_e))\omega_\star + k_\perp^2\rho_e^2(\omega - (1+\eta_e)\omega_\star)\right]\left(\omega - \frac{5}{3}\bar{\epsilon}_n\omega_\star\right) + \left(\bar{\epsilon}_n\omega_\star - \frac{\beta_e}{2}\omega\right)\left(\left(\eta_e - \frac{2}{3}\right)\omega_\star + \frac{2}{3}\omega\Lambda_e\right) = c_e^2 k_\|^2 k_\perp^2 \rho_e^2 \left(\frac{(1+\Lambda_e)\left(\omega - \frac{5}{3}\bar{\epsilon}_n\omega_\star\right) - \left(\eta_e - \frac{2}{3}\right)\omega_\star - \frac{2}{3}\omega\Lambda_e}{\omega\left(\frac{\beta_e}{2} + k_\perp^2\rho_e^2\right) - \frac{\beta_e}{2}(1+\eta_e)\omega_\star}\right). \qquad (17)$$

In the following, we will use the notation $\Lambda_e = \tau_i(n_i/n_e)/(1 - \omega^2/k_\perp^2 c_i^2) + \tau_I(z_{eff} n_I/n_e)/(1 - \omega^2/k_\perp^2 c_I^2) + k_\perp^2\lambda_{De}^2$. We also define $z_{eff} \approx z^2 n_I/n_e$. Note that in the limit $T_i = T_e$, $\omega < k_\perp c_i$, $k_\perp \lambda_{De} < k_\perp \rho_e \leq 1$ and in the absence of impurity ions, $\Lambda_e \approx 1$ and the ions follow the Boltzmann relation in the standard ETG mode dynamics. Here, $\lambda_{De} = \sqrt{T_c/(4\pi n_e e^2)}$ is the Debye length, the Debye shielding effect is important for $\lambda_{De}/\rho_e > 1$.[16] The dispersion relation Eq. (17) is analogous to the toroidal ion-temperature-gradient mode dispersion relation except that the ion quantities are exchanged to their electron counterparts. Equation (17) is derived by using the ballooning mode transform equations for the wave number and the curvature operator

$$\nabla_\perp^2 \tilde{f} = -k_\perp^2 \tilde{f} = -k_\theta^2 \left[1 + (s\theta - \alpha \sin\theta)^2\right]\tilde{f}, \tag{18}$$

$$\nabla_{\|}\tilde{f} = ik_{\|}\tilde{f} \approx \frac{i}{qR}\frac{\partial \tilde{f}}{\partial \theta}, \tag{19}$$

$$\tilde{\epsilon}_n \tilde{f} = \epsilon_n[\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta]\tilde{f}. \tag{20}$$

The geometrical quantities will be determined using a semi-local analysis by assuming an approximate eigenfunction while averaging the geometry dependent quantities along the field line. The form of the eigenfunction is assumed to be[23]

$$\Psi(\theta) = \frac{1}{\sqrt{3\pi}}(1 + \cos\theta) \quad \text{with} \quad |\theta| < \pi. \tag{21}$$

In the dispersion relation, we will replace $k_{\|} = \langle k_{\|}\rangle$, $k_\perp = \langle k_\perp \rangle$, and $\omega_D = \langle \omega_D \rangle$ by the averages defined through the integrals

$$\langle k_\perp^2 \rangle = \frac{1}{N(\Psi)}\int_{-\pi}^{\pi} d\theta \Psi k_\perp^2 \Psi = k_\theta^2\left(1 + \frac{s^2}{3}(\pi^2 - 7.5) - \frac{10}{9}s\alpha + \frac{5}{12}\alpha^2\right), \tag{22}$$

$$\langle k_{\|}^2 \rangle = \frac{1}{N(\Psi)}\int_{-\pi}^{\pi} d\theta \Psi k_{\|}^2 \Psi = \frac{1}{3q^2R^2}, \tag{23}$$

$$\langle \omega_D \rangle = \frac{1}{N(\Psi)}\int_{-\pi}^{\pi} d\theta \Psi \omega_D \Psi = \epsilon_n \omega_\star \left(\frac{2}{3} + \frac{5}{9}s - \frac{5}{12}\alpha\right), \tag{24}$$

$$\langle k_{\|} k_\perp^2 k_{\|}\rangle = \frac{1}{N(\Psi)}\int_{-\pi}^{\pi} d\theta \Psi k_{\|} k_\perp^2 k_{\|} \Psi = \frac{k_\theta^2}{3(qR)^2}\left(1 + s^2\left(\frac{\pi^2}{3} - 0.5\right) - \frac{8}{3}s\alpha + \frac{3}{4}\alpha^2\right), \tag{25}$$

$$N(\Psi) = \int_{-\pi}^{\pi} d\theta \Psi^2. \tag{26}$$

Here, we have from the equilibrium $\alpha = \beta q^2 R(1 + \eta_e + (1 + \eta_i))/(2L_n)$ and $\beta = 8\pi n_o(T_e + T_i)/B^2$ is the plasma β, q is the safety factor, and $s = rq'/q$ is the magnetic shear. The α-dependent term above (Eq. (22)) represents the effects of Shafranov shift. In the limit, low-beta ($\beta \to 0$), no impurity ions $\Lambda_e \approx 1$, while neglecting parallel motion, we find approximate solutions to the dispersion relation as

$$\omega_r \approx -\frac{\omega_\star}{2(1 + k_\perp^2\rho_e^2)}\left(1 - \epsilon_n\left(\tau + \frac{10}{3}\tau\right) - k_\perp^2\rho_e^2\left(1 + \eta_e + \frac{5}{3}\epsilon_n\right)\right) \tag{27}$$

$$\gamma \approx \frac{\omega_\star \sqrt{\epsilon_n}}{2(1 + k_\perp^2\rho_e^2)}\sqrt{\eta_e - \eta_{eth}} \tag{28}$$

$$\eta_{eth} \approx \frac{2}{3} - \frac{\tau}{2} + \epsilon_n\left(\frac{\tau}{4} + \frac{10\tau}{9}\right) + \frac{\tau}{4\epsilon_n}. \tag{29}$$

III. MODELING ELECTRON GEODESIC ACOUSTIC MODES

The geodesic acoustic modes are the $m = n = 0$, $k_r \neq 0$ perturbation of the potential field and the $n = 0$, $m = 1$, $k_r \neq 0$ perturbation in the density, temperatures, and the magnetic field perturbations. The el-GAM (q, Ω_q) induced by ETG modes (k, ω) is considered under the conditions when the ETG mode real frequency satisfies $\Omega_e > \omega > \Omega_i$ at the scale $k_\perp \rho_e < 1$ and the real frequency of the GAM fulfills $\Omega_q \sim c_e/R$ at the scale $q_r < k_r$.

A. Linear electron geodesic acoustic modes

We start by deriving the linear electron GAM dispersion relation, by writing the $m = 1$ equations for the density, parallel component of the vector potential, temperature and the $m = 0$ of the electrostatic potential

$$-\tau_i \frac{\partial \tilde{n}_{eG}^{(1)}}{\partial t} + \epsilon_n \sin\theta \frac{\partial}{\partial r}\tilde{\phi}_G^{(0)} - \nabla_{\|}\nabla_\perp^2 \tilde{A}_{\|G}^{(1)} = 0, \tag{30}$$

$$\left((\beta_e/2 - \nabla_\perp^2)\frac{\partial}{\partial t} + (1 + \eta_e)(\beta_e/2)\nabla_\theta\right)\tilde{A}_{\|G}^{(1)} - \nabla_{\|}(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}) = 0, \tag{31}$$

$$\frac{\partial}{\partial t}\tilde{T}_{eG}^{(1)} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_{eG}^{(1)} = 0, \tag{32}$$

$$-\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi}_G^{(0)} - \epsilon_n \sin\theta \frac{\partial}{\partial r}\left(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}\right) = 0. \tag{33}$$

First, we will derive the linear GAM frequency assuming electrostatic GAMs ($\beta_e \to 0$) this yields a relation between the parallel component of the vector potential and the density and electron perturbations using Eq. (31) as

$$-\nabla_\perp^2 \frac{\partial \tilde{A}_{\|G}^{(1)}}{\partial t} - \nabla_{\|}\left(\tilde{n}_G^{(1)} + \tilde{T}_{eG}^{(1)}\right) = 0. \tag{34}$$

The $m = 1$ component of the electron density can be eliminated by taking a time derivative of Eq. (33) and using Eq. (30) and we get

$$\rho_e^2 \frac{\partial^2}{\partial t^2}\nabla_\perp^2 \tilde{\phi}^{(0)} + \epsilon_n v_\star \left\langle \sin\theta \frac{\partial}{\partial r}\left(\epsilon_n v_\star \sin\theta \frac{\partial \tilde{\phi}^{(0)}}{\partial r} + \nabla_{\|}\frac{J_{\|}^{(1)}}{en_0}\right)\right\rangle = 0. \tag{35}$$

Here, $\langle \cdots \rangle$ is the average over the poloidal angle θ. In the simplest case, this leads to the dispersion relation

$$\Omega_q^2 = \frac{5}{3}\frac{c_e^2}{R^2}\left(2 + \frac{1}{q^2}\right). \tag{36}$$

Note that the linear electron GAM is purely oscillating analogous to its ion counterpart c.f. Refs. 6 and 24. In this section, we computed the linear dispersion relation for the GAM, now we will study the non-linear contributions through a modulational instability analysis.

B. The non-linearly driven geodesic acoustic modes

We will now study the system including the non-linear terms and derive the electron GAM growth rate. The non-linear extension to the evolution equations presented previously in Eqs. (10)–(12) is in the electrostatic limit ($\beta_e \to 0$)

$$-\frac{\partial \tilde{n}_e}{\partial t} - \nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi} - \left(1 + (1+\eta_e)\nabla_\perp^2\right)\nabla_\theta \tilde{\phi} - \nabla_\| \nabla_\perp^2 \tilde{A}_\| + \epsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \sin\theta \frac{\partial}{\partial r}\right)\left(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e\right) = \left[\tilde{\phi}, \nabla^2 \tilde{\phi}\right], \quad (37)$$

$$\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{A}_\| + \nabla_\| \left(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e\right) = \left[\tilde{\phi}, \nabla_\perp^2 \tilde{A}_\|\right], \quad (38)$$

$$\frac{\partial}{\partial t}\tilde{T}_e + \frac{5}{3}\epsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \sin\theta\frac{\partial}{\partial r}\right)\frac{1}{r}\frac{\partial}{\partial\theta}\tilde{T}_e + \left(\eta_e - \frac{2}{3}\right)\frac{1}{r}\frac{\partial}{\partial\theta}\tilde{\phi} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_e = -[\tilde{\phi}, \tilde{T}_e]. \quad (39)$$

In order to find the relevant equations for the electron GAM dynamics, we consider the $m=1$ component of Eqs. (37)–(39),

$$-\frac{\partial \tilde{n}_{eG}^{(1)}}{\partial t} + \epsilon_n \sin\theta \frac{\partial}{\partial r}\tilde{\phi}_G^{(0)} - \nabla_\| \nabla_\perp^2 \tilde{A}_{\|G}^{(1)} = \langle[\tilde{\phi}_k, \nabla^2 \tilde{\phi}_k]\rangle^{(1)} = 0, \quad (40)$$

$$\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{A}_\|^{(1)} - \nabla_\| \left(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}\right) = \left\langle \left[\tilde{\phi}_k, \nabla_\perp^2 \tilde{A}_{\|k}\right]\right\rangle^{(1)} = 0, \quad (41)$$

$$\frac{\partial}{\partial t}\tilde{T}_{eG}^{(1)} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_{eG}^{(1)} = -\langle[\tilde{\phi}_k, \tilde{T}_{ek}]\rangle^{(1)} = N_1^{(1)}, \quad (42)$$

where superscript (1) over the fluctuating quantities denotes the $m=1$ poloidal mode number and $\langle\cdots\rangle$ is the average over the fast time and spatial scale of the ETG turbulence and that non-linear terms associated with parallel dynamics are small since $\frac{1}{q^2} \ll 1$. We now study the $m=0$ potential perturbations

$$-\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi}_G^{(0)} - \epsilon_n \sin\theta \frac{\partial}{\partial r}\left(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}\right) = \left\langle \left[\tilde{\phi}_k, \nabla^2 \tilde{\phi}_k\right]\right\rangle^{(0)} = N_2^{(0)}. \quad (43)$$

Here, we have defined the non-linear term on the RHS in Eqs. (40)–(43) as $N_2^{(0)} = \rho_e^3 c_e \hat{z} \times \nabla\tilde{\phi} \cdot \nabla\nabla_\perp^2 \tilde{\phi}$. This can be written $\tilde{T}_e = \frac{2}{3}\tilde{n}_e^{(1)} + N_2^1$, where the $m=1$ component is determined by an integral of the convective non-linear term as $N_1^1 = -\int dt \rho_s c_s \hat{z} \times \nabla\tilde{\phi}^{(0)} \cdot \nabla\tilde{T}_e^{(1)}$. This leads to a relation between the $m=1$ component of the density and temperature fluctuations modified by a non-linear term. Here, the non-linear terms can be written in the form

$$N_1^{(1)} = \sum_k k_\theta^2 \frac{\eta_e \gamma}{|\omega|^2}\nabla_r |\tilde{\phi}_k|^2 \quad (44)$$

$$N_2^{(0)} = q_r^2 \sum_k k_r k_\theta |\tilde{\phi}_k|^2. \quad (45)$$

We continue by considering the Eqs. (40) and (43) for the $m=1$ component and $m=0$ component, respectively,

$$\frac{\partial \tilde{n}_{eG}^{(1)}}{\partial t} - \frac{\nabla_\| \tilde{J}_\|^{(1)}}{e n_0} - \epsilon_n \sin\theta \frac{\partial \tilde{\phi}_G^{(0)}}{\partial r} = N_1^{(1)}, \quad (46)$$

$$\frac{\partial}{\partial t}\nabla_\perp^2 \tilde{\phi}_G^{(0)} + \epsilon_n \left\langle \sin\theta \frac{\partial}{\partial r}\left(\frac{5}{3}\tilde{n}_{eG}^{(1)}\right)\right\rangle = N_2^{(0)}. \quad (47)$$

We keep the $N_1^{(1)}$ non-linear term in order to quantify the effects of the convective non-linearity. Similar to the operations performed to find the linear electron GAM frequency, we eliminate the $m=1$ component of the electron density by taking a time derivative of Eq. (47) this yields

$$\frac{\partial^2}{\partial t^2}\nabla_\perp^2 \tilde{\phi}_G^{(0)} + \epsilon_n \left\langle \sin\theta \frac{\partial}{\partial r}\left(\epsilon_n \sin\theta \frac{\partial \tilde{\phi}_G^{(0)}}{\partial r} + N_1^{(1)} + \frac{\partial}{\partial t}N_1^{(1)}\right)\right\rangle = \frac{\partial}{\partial t}N_2^{(0)}. \quad (48)$$

Note that the el-GAM wave equation will be modified by the effects of the parallel current density ($\tilde{J}_\|$) and the $m=1$ non-linear terms in the general case, however, we see by inspection that on average the term $N_1^{(1)}$ does not contribute whereas the $N_2^{(0)}$ non-linearity may drive the GAM unstable.

We will use the wave kinetic equation[1,6,8,25–30] to describe the background short scale ETG turbulence for $(\Omega_q, \vec{q}) < (\omega, \vec{k})$, where the action density $N_k = E_k/|\omega_r| \approx \epsilon_0 |\phi_k|^2/\omega_r$. Here, $\epsilon_0 |\phi_k|^2$ is the total energy in the ETG mode with mode number k, where $\epsilon_0 = \tau + k_\perp^2 + \frac{\eta_e^2 k_\theta^2}{|\omega|^2}$. In describing the large scale plasma flow dynamics, it is assumed that there is a sufficient spectral gap between the small scale ETG turbulent fluctuations and the large scale GAM flow. The electrostatic potential is represented as a sum of fluctuating and mean quantities

$$\phi(\vec{X}, \vec{x}, T, t) = \Phi(\vec{X}, T) + \tilde{\phi}(\vec{x}, t), \quad (49)$$

where $\Phi(\vec{X}, T)$ is the mean flow potential. The coordinates $(\vec{X}, T)$, $(\vec{x}, t)$ are the spatial and time coordinates for the mean flows and small scale fluctuations, respectively. The wave kinetic equation can be written as

$$\frac{\partial}{\partial t}N_k(r,t) + \frac{\partial}{\partial k_r}(\omega_k + \vec{k}\cdot\vec{v}_g)\frac{\partial N_k(r,t)}{\partial r} - \frac{\partial}{\partial r}(\vec{k}\cdot\vec{v}_g) \times \frac{\partial N_k(r,t)}{\partial k_r} = \gamma_k N_k(r,t) - \Delta\omega N_k(r,t)^2. \quad (50)$$

We will solve Eq. (50) by assuming a small perturbation (δN_k) driven by a slow variation for the GAM compared to the mean (N_{k0}) such that $N_k = N_{k0} + \delta N_k$. The relevant non-linear terms can be approximated in the following form:[16]

$$\left\langle \left[\tilde{\phi}_k, \nabla_\perp^2 \tilde{\phi}_k \right] \right\rangle \approx q_r^2 \sum_k k_r k_\theta \frac{|\omega_r|}{\epsilon_0} \delta N_k(\vec{q}, \Omega_q), \quad (51)$$

$$\left\langle \left[\tilde{\phi}_k, \tilde{T}_{ek} \right] \right\rangle \approx -i q_r \eta_e \sum_k \frac{k_\theta}{\gamma_k} \frac{|\omega_r|}{\epsilon_0} \delta N_k(\vec{q}, \Omega_q). \quad (52)$$

For all GAMs, we have $q_r > q_\theta$, with the following relation between δN_k and $\partial N_{k0}/\partial k_r$

$$\delta N_k = -i q_r^2 k_\theta \phi_G^0 R \frac{\partial N_{0k}}{\partial k_r} + \frac{k_\theta q_r \tilde{T}_{eG}^{(1)} N_{0k}}{\tau_i \sqrt{(\eta_e - \eta_{eth})}}, \quad (53)$$

where we have used $\delta\omega_q = k \cdot v_{Eq} \approx i(k_\theta q_r - k_r q_\theta)\phi_G^{(0)}$ in the wave kinetic equation and the definition $R = \frac{1}{\Omega_q - q_r v_{gr} + i\gamma_k}$. Furthermore, in the present work, we will shortly consider the effects of the modulation terms of ω_r and γ using the approximate analytical solutions found in Eqs. (27) and (28) for

$$\delta\tilde{\omega}_r = \frac{5}{3} k_\theta \frac{\rho_e}{L_n} \epsilon_n \tilde{T}_{eG}^{(1)} + k_\theta \nabla_r \tilde{\phi}_G^{(0)}, \quad (54)$$

$$\nabla_r \delta\tilde{\omega}_r = \frac{5}{3} k_\theta \frac{\rho_e}{L_n} \tilde{T}_{eG}^{(1)} + k_\theta \nabla_r^2 \tilde{\phi}_G^{(0)}, \quad (55)$$

$$\delta\tilde{\gamma} = \frac{k_\theta}{\tau\sqrt{\eta_e - \eta_{eth}}} (-\nabla_r \tilde{T}_{eG}^{(1)}). \quad (56)$$

The modulation will enter in the perturbation of the kinetic invariant

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \vec{v}_g \cdot \nabla + \gamma \right) \delta N_k &= \frac{\partial}{\partial \vec{r}} \delta\omega \frac{\partial N_{0k}}{\partial k} + \delta\gamma N_{0k}, \\ &= k_\theta \nabla_r^2 \tilde{\phi}_G^{(0)} \frac{\partial N_{0k}}{\partial k_r} \\ &\quad - \frac{k_\theta}{\tau\sqrt{\eta_e - \eta_{eth}}} \nabla_r \tilde{T}_{eG}^{(1)} N_{0k} \\ &\quad - k_r \nabla_\theta^2 \tilde{\phi}_G^{(0)} \frac{\partial N_{0k}}{\partial k_\theta}, \end{aligned} \quad (57)$$

where the last term can be neglected since the contribution from ∇_θ^2 vanishes

$$\delta N_k = -i k_\theta \left(q_x^2 \tilde{\phi}_G^{(0)} \right) R \frac{\partial N_{0k}}{\partial k_r} + \frac{k_\theta}{\tau\sqrt{\eta_e - \eta_{eth}}} (q_x \tilde{T}_{eG}^{(1)}) N_{0k}. \quad (58)$$

In this last expression, the last term comes from the modulation of the growth rate.

Using the results from the wave-kinetic treatment, we can compute the non-linear contributions to be of the form

$$\begin{aligned} \left\langle \tilde{\phi}, \nabla_\perp^2 \tilde{\phi} \right\rangle &= -i q_r^4 \sum k_r k_\theta^2 \frac{|\omega_r|}{\epsilon_0} R \frac{\partial N_k}{\partial k_r} \tilde{\phi}_G^{(0)} \\ &\quad + \frac{2}{3} q_r^3 \sum k_r k_\theta \frac{|\omega_r|}{\epsilon_0} \frac{R N_0}{\tau(\eta_e - \eta_{the})^{1/2}} \tilde{n}_{eG}^{(1)}, \end{aligned} \quad (59)$$

$$\begin{aligned} \left\langle \tilde{\phi}, \tilde{T}_e \right\rangle &= q_r^3 \sum k_\theta^2 \frac{\eta_e \gamma}{|\omega_r|^2} \frac{|\omega_r|}{\epsilon_0} R \frac{\partial N_0}{\partial k_r} \tilde{\phi}_G^{(0)} \\ &\quad + i q_r^2 \sum \frac{2}{3} \frac{k_\theta^3 \eta_e \gamma}{|\omega_r|^2} \frac{|\omega_r|}{\epsilon_0} \frac{R N_0}{\tau(\eta_e - \eta_{the})} \tilde{n}_{eG}^{(1)}. \end{aligned} \quad (60)$$

In order to find the non-linear growth rate of the electron GAM, we need to find relations between the variables $\tilde{n}_{eG}^{(1)}$, $\tilde{T}_{eG}^{(1)}$, and $\tilde{\phi}_G^{(0)}$

$$\tilde{n}_{eG}^{(1)} = -\frac{\epsilon_n q_r \sin\theta}{\Omega_q - \frac{5 q_\parallel^2}{3\Omega_q}} \tilde{\phi}_G^{(0}, \quad (61)$$

$$\tilde{T}_{eG}^{(1)} = \frac{2}{3} \tilde{n}_{eG}^{(1)} - \frac{2}{3} q_r^2 \sum \frac{k_\theta^3 \eta_e \gamma}{|\omega|^2} \frac{|\omega_r|}{\epsilon_0} \frac{R N_0}{\tau(\eta_e - \eta_{the})^{1/2}} \tilde{n}_{eG}^{(1)}. \quad (62)$$

Using Eqs. (61) and (62) in the Fourier representation of Eq. (48) resulting in

$$\Omega_q q_r^2 \tilde{\phi}_G^{(0)} + \epsilon_n q_r \sin\theta (\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}) = i \langle \tilde{\phi}, \nabla_\perp^2 \tilde{\phi} \rangle^{(0)}, \quad (63)$$

and we finally find

$$\begin{aligned} \Omega_q^2 - \frac{5}{3} q_\parallel^2 - \frac{5}{6} \epsilon_n^2 &= -\frac{1}{3} \epsilon_n^2 q_r^2 \sum \frac{k_\theta^3 \eta_e \gamma}{|\omega|} \frac{|\omega_r|}{\epsilon_0} \frac{R N_0}{\tau(\eta - \eta_{the})^{1/2}} \\ &\quad + \left(\frac{\Omega_q^2 - \frac{5}{3} q_\parallel^2}{\Omega_q} \right) q_r^2 \sum k_r k_\theta^2 \frac{|\omega_r|}{\epsilon_0} R \frac{\partial N_k}{\partial k_r}. \end{aligned} \quad (64)$$

Equation (64) is the sought dispersion relation for the electron GAM and we solve it perturbatively by assuming $\Omega_q = \Omega_0 + \Omega_1$, where Ω_0 is the solution to the linear part c.f. Eq. (36). Now, we find the perturbation $\Omega_1 = i\gamma_q$ which will determine the growth rate of the GAM as

$$\begin{aligned} \frac{\gamma_q}{c_e/L_n} &= i \frac{\epsilon_n^2}{6\Omega_0} q_r^2 \rho_e^2 \sum \frac{k_\theta^3 \rho_e^3 \eta_e \gamma}{|\omega|} \frac{|\omega_r|}{\epsilon_0} \frac{N_0}{i\gamma} \frac{1}{(\eta_e - \eta_{the})^{1/2}} \\ &\quad - i \frac{5}{12} \frac{\epsilon_n^2}{\Omega_0^2} q_r^2 \rho_e^2 \sum k_r k_\theta^2 \rho_e^3 \frac{|\omega_r|}{\epsilon_0} \frac{1}{i\gamma} \left| \frac{\partial N_0}{\partial k_r} \right| \\ &\approx \frac{1}{2} \frac{q_r^2 \rho_e^2 k_\theta \rho_e}{\sqrt{\epsilon_n(\eta_e)}} \frac{1}{1 + 1/2q^2} \left| \tilde{\phi}_k \frac{L_n}{\rho_e} \right|^2. \end{aligned} \quad (65)$$

Here, the main contribution to the non-linear generation of el-GAMs originates from the Reynolds stress term. In the last expression, we have assumed that the GAM frequency (Ω_0) is given by Eq. (36). The non-linearly driven electron GAM is unstable with a growth rate depending on the saturation level $|\tilde{\phi}_k|^2$ of the ETG mode turbulence.

IV. SATURATION MECHANISM

In this section, we will estimate a new saturation level for the ETG turbulent electrostatic potential ($\tilde{\phi}_k$) by balancing the Landau damping in competition with the non-linear growth rate of the GAM in a constant background of ETG mode turbulence. For simplicity, the non-linear transfer from

$\langle p_i \sin\theta \rangle$ to turbulence is neglected. This effect may impact on the saturation level.[32–35] According to the well known predator-prey models used,[31] c.f. Eq. (4) in Ref. 5 and as well as Ref. 7 we have

$$\frac{\partial N_k}{\partial t} = \gamma_k N_k - \Delta\omega N_k^2 - \gamma_1 U_G N_k \quad (66)$$

$$\frac{\partial U_G}{\partial t} = \gamma_q U_G - \gamma_L U_G - \nu^\star U_G. \quad (67)$$

Here, we have represented the ETG mode turbulence as $N_k = |\phi_k|^2 \frac{L_n^2}{\rho_e^2}$ and $U_G = \langle \frac{e\phi_G^{(0)}}{T_e} \frac{L_n}{\rho_e} \sin\theta \rangle$ with the following parameters: γ is the ETG mode growth rate, γ_1 is the coupling between the ETG mode and the GAM. The Landau damping rate $\left(\gamma_L = \frac{4\sqrt{2}}{3\sqrt{\pi}} \frac{c_e}{qR}\right)$ is assumed to be balanced by GAM growth rate Eq. (65) modified by the neoclassical damping in stationary state $\frac{\partial N}{\partial t} \rightarrow 0$ and $\frac{\partial U_G}{\partial t} \rightarrow 0$. In steady state find the saturation level for the ETG turbulent intensity as $(\gamma_q = \gamma_L + \nu^\star)$,

$$\left|\frac{e\phi_k}{T_e}\frac{L_n}{\rho_e}\right|^2 \approx \frac{2L_n}{qR}\left(1 + \frac{1}{2q^2}\right)\sqrt{\epsilon_n \eta_e}\left(\frac{4}{3}\sqrt{\frac{2}{\pi}} + \nu^\star\right) \times \left(\frac{k_\theta}{q_r}\right)^2 \left(\frac{1}{k_\theta \rho_e}\right)^3. \quad (68)$$

Here, the saturation level is modified by the neoclassical damping $\nu^\star = \nu_e \frac{qR}{v_{eth}}$ and the $\frac{k_\theta}{q_r}$ factor arises due to the spatial extension of the GAM and we obtain

$$\left|\frac{e\phi_k}{T_e}\frac{L_n}{\rho_e}\right| \sim 30 - 40. \quad (69)$$

Note that the result found using a mixing length estimate with $\left|\frac{e\phi}{T_e}\frac{L_n}{\rho_e}\right| \sim 1$ is significantly smaller. Here, in this estimation, we have used $L_n = 0.05$, $q = 3.0$, $R = 4$, $\epsilon_n = 0.025$, $1/q_r \sim (\rho_e^2 L_T)^{1/3}$, $k_\theta \rho_e = 0.3$, where $k_\theta/q_r \approx 4$ and $\eta_e \sim 1$.

V. CONCLUSION

In this paper, we have presented the first derivation of an el-GAM. The linear dispersion relation of the el-GAM showed that the new branch is purely oscillatory with a frequency $\Omega_q \sim \frac{c_e}{R}$. Note that, the frequency of the el-GAM is higher compared to the ion GAM by the square root of the ion-to-electron mass ratio ($\Omega_q(electron)/\Omega_q(ion) \approx \sqrt{m_i/m_e}$, where $\Omega_q(electron)$ and $\Omega_q(ion)$ are the real frequencies of the electron and ion GAMs, respectively). To estimate the GAM growth rate, a non-linear treatment based on the wave-kinetic approach was applied. The resulting non-linear dispersion relation showed that the el-GAM is excited in the presence of ETG modes with a growth rate depending on the fluctuation level of the ETG mode turbulence. An analytical expression for the resulting GAM growth rate was obtained. To estimate the ETG mode fluctuation level and GAM growth, a predator-prey model was used to describe the coupling between the GAMs and small scale ETG turbulence. The stationary point of the coupled system implies that the ETG turbulent saturation level $\tilde{\phi}_k$ can be drastically enhanced by a new saturation mechanism, stemming from a balance between the Landau damping and the GAM growth rate. This may result in highly elevated particle and electron heat transport, relevant for the edge pedestal region of H-mode plasmas.

The present work was based on a fluid description of ETG mode turbulence, including finite beta electromagnetic effects and retaining non-adiabatic ions. A more accurate treatment based on quasi-linear and non-linear gyrokinetic simulations is left for future work.

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