An ML optimal CDMA multiuser receiver

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A maximum likelihood optimum detector with the asymptotic complexity per user $O(1.5^K)$ has been derived for the synchronous DS/CDMA channel. The detector employs a local descent algorithm through the Voronoi regions for the equivalent hypothesis detector.

Introduction: In the last decade there has been much work concerning multiuser detection for Direct-Sequence Code Division Multiple Access (DS/CDMA) systems. The maximum likelihood (ML) optimum receiver for a synchronous CDMA system is a K-ary hypothesis detector, where K is the number of users in the system. The drawback though is that the asymptotic complexity per user for this receiver is $O(2^K)$. Thus much attention has been directed to suboptimal detectors [1]. Many of these multiuser receivers are sensitive to near-far effects, that is, when the received power from one user is much lower than the received power from another user. For example, the conventional receiver will only be near-far resistant if orthogonal codes are used. However, it has been shown that the decorrelator and multistage receivers are near-far resistant and that the ML optimal detector is optimal in this sense too [2, 3]. Hence, an ML optimum low complexity receiver would be very attractive. In this paper we present an ML optimum receiver with asymptotic complexity $O(1.5^K)$.

System model: Assume a synchronous DS/CDMA system with K users. User k transmit at a given time with power w_k the BPSK modulated bit $b_k \in \{\pm 1\}$. Then it can be shown that the matched filter outputs can be written as $\mathbf{y} = \mathbf{RWb} + \mathbf{z}$, where $\mathbf{b} = (b_1, \cdots, b_K)^T$ and $\mathbf{W} = \mathrm{diag}(w_1, \cdots, w_K)$ [1]. The crosscorrelation matrix $\mathbf{R}$ consists of the periodical (even) crosscorrelations between all codes of the users. The vector $\mathbf{z}$ is additive coloured Gaussian noise with zero mean and covariance matrix $\sigma^2\mathbf{R}$. To decide on $\mathbf{b}$ given an observed vector $\mathbf{y}$, the Maximum Likelihood criterion is usually expressed as

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}\in\{-1,1\}^K} \left\{ 2\mathbf{y}^T\mathbf{W}\mathbf{b} - \mathbf{b}^T\mathbf{W}\mathbf{R}\mathbf{W}\mathbf{b} \right\} \tag{1}$$

To whiten the noise (assuming $\mathbf{R}$ to be positive definite), we transform the matched filter outputs $\mathbf{y}$ into $\mathbf{u} = \mathbf{R}^{-1/2}\mathbf{y} = \mathbf{T}\mathbf{b} + \mathbf{n}$, where $\mathbf{T} = \mathbf{R}^{1/2}\mathbf{W}$ and the covariance of the Gaussian noise $\mathbf{n}$ is $\sigma^2\mathbf{I}$.

If vectors are regarded as points in K-dimensional space, then the 2^K vectors $\mathbf{b}$ constitute the vertices of a hypercube. Similarly, the constellation

$$\mathcal{X} = \{\mathbf{T}\mathbf{b}\}_{\mathbf{b}\in\{-1,1\}^K}$$

spans a K-dimensional parallellotope, that is, a linear transform of a hypercube, where $\mathbf{T}$ is the transformation matrix. Given an observation vector $\mathbf{u}$, the most likely point in $\mathcal{X}$ is given by the minimum Euclidean distance to $\mathbf{u}$, i.e.

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}\in\{-1,1\}^K} \| \mathbf{u} - \mathbf{T}\mathbf{b} \|$$

This detection rule, which is equivalent to (1), partitions space into 2^K convex polytopes, called *Voronoi regions*,

$$\mathcal{V}(\mathbf{x}) = \left\{ \mathbf{u} \in \mathrm{R}^K : \ \| \mathbf{u} - \mathbf{x} \| \le \| \mathbf{u} - \mathbf{x}' \|; \ \forall \mathbf{x}' \in \mathcal{X} \right\} \tag{2}$$

one for each $\mathbf{x} \in \mathcal{X}$. The ML decision on $\mathbf{b}$ given $\mathbf{u}$ is thus to find $\hat{\mathbf{b}}$ such that $\mathbf{u} \in \mathcal{V}(\mathbf{T}\hat{\mathbf{b}})$.

The new maximum likelihood receiver: We employ an iterative approach, called *neighbour descent* (ND), to locate the Voronoi region that the observation belongs to [4]. It utilises the fact that out of the $2^K - 1$ nontrivial inequalities in (2), some may be redundant, so that the same region can be described as

$$\mathcal{V}(\mathbf{x}) = \left\{ \mathbf{u} \in \mathrm{R}^K : \ \| \mathbf{u} - \mathbf{x} \| \le \| \mathbf{u} - \mathbf{x}' \|; \ \forall \mathbf{x}' \in \mathcal{N}(\mathbf{x}) \right\} \tag{3}$$

The minimal set $\mathcal{N}(\mathbf{x})$ for which this is true is called the set of *neighbours* of $\mathbf{x}$. Geometrically, each element of $\mathcal{N}(\mathbf{x})$ corresponds to one facet ($K-1$-

dimensional face) of the polytope $\mathscr{V}(\mathbf{x})$. Thus, to check if a given vector $\mathbf{u}$ belongs to the Voronoi region $\mathscr{V}(\mathbf{x})$, it is sufficient to compute $|\mathscr{N}(\mathbf{x})|$ distances, a number that is often considerably smaller than $2^K - 1$. We compute these distances sequentially, and if a point $\mathbf{x}' \in \mathscr{N}(\mathbf{x})$ is found for which $\|\mathbf{u}-\mathbf{x}'\| < \|\mathbf{u}-\mathbf{x}\|$, we immediately terminate the examination of $\mathscr{N}(\mathbf{x})$, replace $\mathbf{x}$ with $\mathbf{x}'$ as the best found vector, and restart. After a finite number of steps, the algorithm terminates at a vector $\check{\mathbf{x}} = \mathbf{T}\check{\mathbf{b}}$ with the property that none of its neighbours is better. That $\mathbf{u} \in \mathscr{V}(\check{\mathbf{x}})$ now follows from (3), and (2) completes the proof that $\check{\mathbf{x}}$ is indeed the global optimum. Thus, the ML decoded bit vector is $\check{\mathbf{b}}$.

The algorithm assumes that the neighbours $\mathscr{N}(\mathbf{x})$ for all $\mathbf{x} \in \mathscr{X}$ have been precomputed and stored in memory. To identify the neighbour pairs, the following statement is useful.

> ***Theorem:*** *If every pair of points in $\mathscr{X}$ is joined by a line, the neighbours are given by the lines that are only intersected by longer lines.*

This theorem can be proved using the theory of *indecomposable vectors* by Verdú [5].

The lines between points in $\mathscr{X}$ can be regarded as diagonals in a parallellotope. They intersect each other at the points

$$\mathscr{I} = \{\mathbf{Tb}\}_{\mathbf{b}\in\{-1,0,1\}^K} \setminus \mathscr{X}$$

and nowhere else. Since at most one pair of neighbours can meet at each intersection point, the total number of neighbour pairs in the point set is upperbounded by $|\mathscr{I}| = 3^K - 2^K$. In other words, an average point in $\mathscr{X}$ has fewer than $2(3/2)^K$ neighbours. This number sets the complexity for the ND receiver, since most of the time is used for verification, that is, comparing the final point $\check{\mathbf{x}}$ to its neighbours [4].

In any parallellotope, there are several congruent faces. Because the lines passing through any point in $\mathscr{I}$ have the same lengths as the lines through one of the points in the subset

$$\mathscr{I}' = \{\mathbf{Tb}\}_{\mathbf{b}\in\{0,1\}^K \setminus \{1\}^K}$$

a full list of neighbours is obtained by examining these $2^K - 1$ intersection points. The same property can be used to reduce the amount of memory needed for storage of the neighbours [6].

Numerical results: The ND algorithm described above was tested on a system using Gold-sequences of period 7. For comparison, we also simulated the conventional detector (matched filters followed by signum decisions), the decorrelator [1], and the multistage detector with two stages and the conventional detector as the first stage [3]. The average bit error rate for these detectors versus the signal-to-noise ratio for $K = 6$ users is shown in Fig. 1. Considerable performance gain is achieved at high signal-to-noise ratios. For a system with either more users or worse correlation properties, the performance gain is even greater. Further, the performance gain for the ML receiver is, as stated above, higher in near-far situations.

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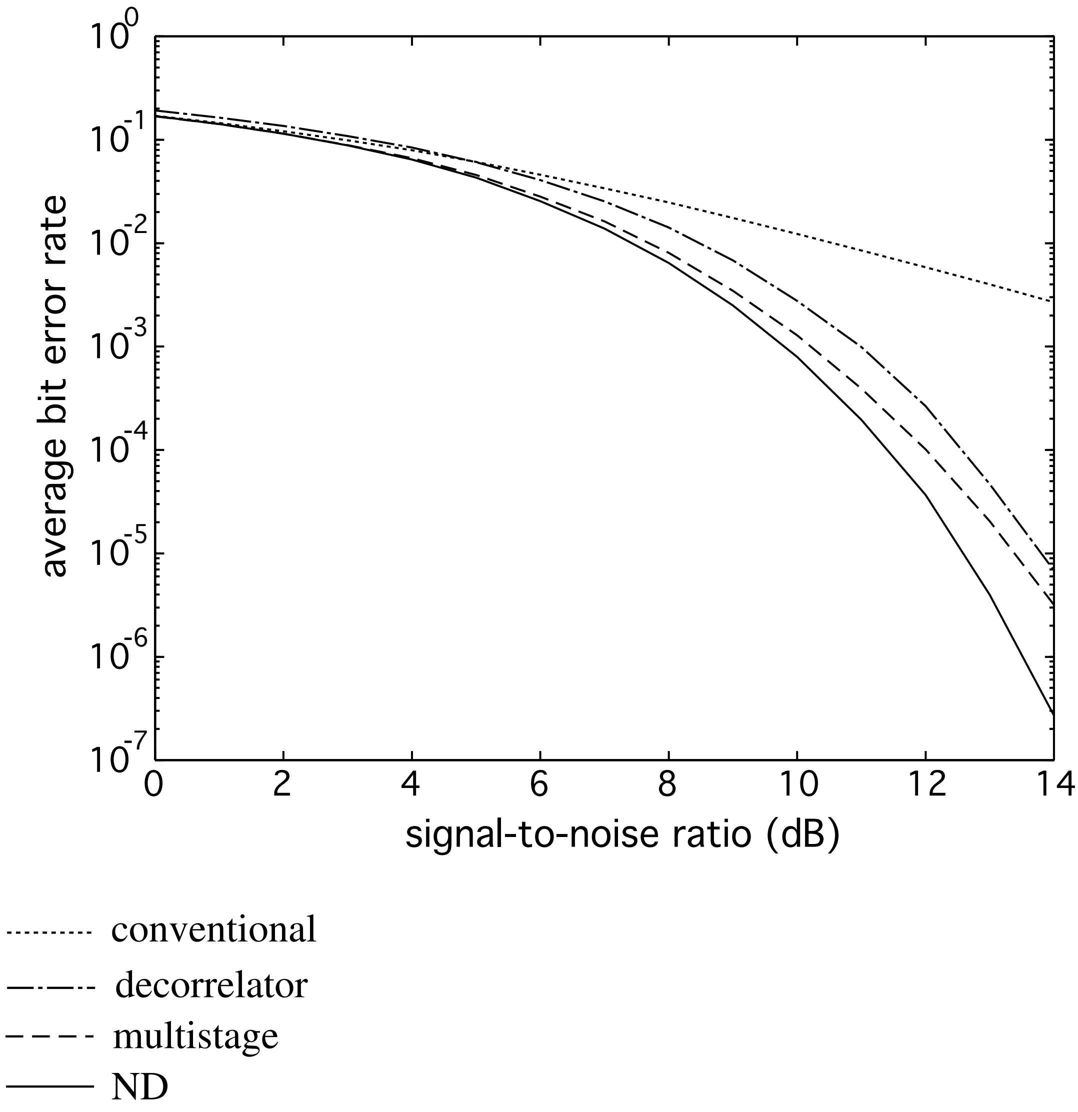

Fig. 1 *Bit error rate versus signal-to-noise ratio for* $K = 6$ *users.*