Mathematical modelling of a real flexible job shop in aero engine component manufacturing

Karin Thörnblad (Speaker) * Michael Patriksson †
Ann-Brith Strömberg ‡ Torgny Almgren §

1 Introduction

We formulate two mixed integer programming models stemming from a real flexible job shop problem with a total of ten resources and five main processing multipurpose machines. The models are compared w.r.t. memory usage, computation times, and accuracy.

The job-shop, called the multitask cell, is supposed to carry out a large variety of jobs since five of the cell's resources are multi-purpose machines that are able to process three types of operations: milling, turning, and drilling. Typically, each product visits the multitask cell multiple times on its way to completion. Inside the multitask cell each part follows a specific routing consisting of three to five operations, starting and ending respectively with the mounting and removing of fixtures at one of the three set-up stations. The second operation is always processed in one of the multitask machines. Some parts need manual and/or robot deburring. Figure 1 shows a part's possible path in the cell drawn with dashed lines.

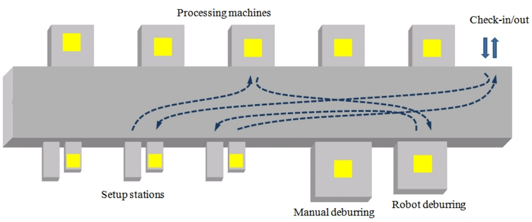

Figure 1: A schematic overview of the production cell.

*karint@chalmers.se. Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, 412 96 Göteborg, Sweden.

†mipat@chalmers.se. Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, 412 96 Göteborg, Sweden.

‡anstr@chalmers.se. Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, 412 96 Göteborg, Sweden.

§torgny.almgren@volvo.com. Department of Logistics Development, Volvo Aero Corporation, 461 81 Trollhättan, Sweden.

2 The time-indexed formulation

Due to initial problems with high computation times, the model has been decomposed into two to be solved in sequence. The first model, called the *machining model*, finds an optimal sequence of operations for each of the five processing machines. The second model, henceworth called the *feasibility model*, generates a feasible schedule for all ten resources, with the optimal sequence for the five processing machines as input data. The loss of accuracy from this decomposition is very small since the workload of the processing resources are much higher than for the other resources. In [1] we present the first developed machining model, here called the *engineer's model*. The feasibility model is based on the same logic. These two models work well, but the computation times still are far too long for practical usage.

Therefore, we have developed a new model using discrete time steps, inspired by [2] and [3], in order to find the optimal sequences of operations for the processing resources. The discretization approximates all data in multiples of the length of the discrete time interval chosen, and hence the final schedule may alter from the result of the engineer's model.

The main decision variables are defined as

$$x_{jku} = \begin{cases} 1, & \text{if job } j \text{ starts at resource } k \text{ at the beginning of time interval } u, \\ 0, & \text{otherwise.} \end{cases}$$

The discrete time machining model minimizes the sum of the total tardiness (h_j) and completion time (s_j) for all jobs $j \in \mathcal{J}$ and is formulated as to

$$\text{minimize} \quad \sum_{j\in\mathcal{J}} (s_j + h_j), \tag{1a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{T}} x_{jku} = 1, \qquad j \in \mathcal{J}, \tag{1b}$$

$$\sum_{u\in\mathcal{T}} x_{jku} \leq \lambda_{jk}, \quad j \in \mathcal{J}, k \in \mathcal{K} \tag{1c}$$

$$\sum_{j\in\mathcal{J}} \sum_{\nu=(u-p_j+1)_+}^{u} x_{jk\nu} \leq 1, \qquad k \in \mathcal{K}, u = 0, \ldots, T, \tag{1d}$$

$$\sum_{k\in\mathcal{K}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+v_{jq}^{\mathrm{pm}}} x_{qk\nu} \right) \geq 0, \qquad (j,q) \in \mathcal{Q}, u = 0, \ldots, T - v_{jq}^{\mathrm{pm}}, \tag{1e}$$

$$x_{jku} = 0, \qquad (j,q) \in \mathcal{Q},\ k \in \mathcal{K},\ u = T - v_{jq}^{\mathrm{pm}}, \ldots, T, \tag{1f}$$

$$\sum_{k\in\mathcal{K}} \sum_{u\in\mathcal{T}} u x_{jku} + p_j + p_j^{\mathrm{pm}} = s_j, \qquad j \in \mathcal{J}, \tag{1g}$$

$$s_j - h_j \leq d_j, \qquad j \in \mathcal{J}, \tag{1h}$$

$$h_j \geq 0, \qquad j \in \mathcal{J}, \tag{1i}$$

$$x_{jku} = 0, \qquad j \in \mathcal{J},\ k \in \mathcal{K},\ u = 0, 1, \ldots, r_j, \tag{1j}$$

$$x_{jku} = 0, \qquad j \in \mathcal{J},\ k \in \mathcal{K},\ u = 0, 1, \ldots, a_k, \tag{1k}$$

$$x_{jku} \in \{0,1\}, j \in \mathcal{J},\ k \in \mathcal{K},\ u \in \mathcal{T}, \tag{1l}$$

where $(u)_+ := \max\{0, u\}$ for all $u \in \mathcal{T}$, r_j and d_j denotes realease and due dates respectively, v_{jq}^{pm} is the planned lead time between preceding machining operations, p_j is the processing time, and λ_{jk} is 1 if a job is allowed to be processed on a resource, and 0 otherwise.

3 Computational results

Presently, the multitask cell is processing about 30 different machining operations on eight different products. Results from the engineer's model and the discrete time model with varying discrete time intervals were compared for six real data scenarios collected from the cell during the autumn of 2010. For each scenario, we created up to ten different instances, comprising the first $5m$ jobs in the current queue to be scheduled, where $m \in \{1, \dots, 10\}$.

The starting and completion times obtained from the optimal solution of the discrete machining model are given in terms of multiples of the length of the discrete time interval ℓ, and the optimal value of the objective function differs from that of the engineer's model. Therefore, the completion times s_j are recalculated using the original non-discrete data while retaining the ordering of operations on each processing machine received from the discrete time model. The optimal value after this postprocessing is then compared with the optimal value from the engineer's model.

Although the smallest processing time for the multipurpose machines was 0.6h (the largest was 22.4h), the optimal value of the discrete time maching model with a discrete interval length of at most 1h were identical with that of the engineer's model for all instances except one with 20 jobs, that we were able to solve using this model.

The computations were carried out using AMPL-CPLEX12 as optimization software on a computer with two 2.66GHz Intel Xeon 5650, each with six cores. The total memory was 48 Gbyte. Optimum was reached for all problem sizes with at most 15 jobs for the engineer's model, and with at most 45 jobs for the discrete time model. No results were obtained for four of the instances with 20 jobs and more for the engineer's model, since CPLEX ran out of memory, and the same happened for instances with 50 jobs and more, for the discrete time model with time interval length of 1h. This should however, not cause any problem for the real application, since the maximum amount of storage in the multitask cell is around 35 parts.

References

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