BRIEF PAPER *Special Section on Microwave and Millimeter-Wave Technology*

Fast Converging Measurement of MRC Diversity Gain in Reverberation Chamber Using Covariance-Eigenvalue Approach

Xiaoming CHEN[†a)], ***Nonmember***, **Per-Simon KILDAL**[†], ***Member***, ***and*** **Jan CARLSSON**[††], ***Nonmember***

SUMMARY In this paper, we show that the covariance-eigenvalue approach converges much faster than using cumulative distribution function (CDF) for determining diversity gain from channel measurements in reverberation chamber. The covariance-eigenvalue approach can be used for arbitrary multi-port antennas, but it is limited to Maximum Ratio Combining (MRC).
key words: *MRC, diversity gain, eigenvalue, reverberation chamber*

1. Introduction

Diversity technology offers an effective leverage to mitigate fading in wireless multipath environment. There are mainly three types of diversity techniques: frequency diversity, time diversity and antenna (or spatial) diversity [1]. For antenna diversity, there are three main diversity combining techniques: selection combining (SC), equal gain combining (EGC) and maximum ratio combining (MRC) [1], [2]. MRC offers the best performance at the expense of complexity. As the technology has developed over the past decades, MRC has become more feasible and popular [1]–[4]. In this paper, we will focus on MRC diversity gain.

Previously, SC diversity gains of multi-port antennas have been measured in reverberation chamber [5], [6] as follows: The SC signal is derived based on the measured power levels of each branch, and the three cumulative distribution functions (CDF) is formed of the received power for each of the diversity branches and the SC signal. Diversity gain is then determined based on the CDF improvement at 1% outage probability, [2], [5]. However, it is shown in [7] that CDF at 1% outage probability requires huge amount of channel samples in order to converge. This is, in practice, both difficult and time-consuming to achieve during reverberation chamber measurement. On the other hand, embedded radiation efficiencies and correlations converge after receiving fewer channel samples. Therefore, it is judicious to determine diversity gain from these fast converging parameters.

Diversity gain for power-balanced multi-port antenna is well known in early literature [2]. SC and MRC diversity gains for power-unbalanced two-port antenna were devised in [8]. Using the compact formulas devised in [8], which depends only on efficiencies and correlation between the two branches, diversity gains converge after receiving much fewer channel samples than when CDFs are used. In this paper, we use the method explained in [9]–[11] to determine the MRC diversity gain; and we call the method covariance-eigenvalue approach. The approach is based on generating theoretical CDFs from the eigenvalues of the covariance matrix of the measured signals at the diversity antenna. Since the covariance matrix depends only on efficiencies and correlations, the CDF (at 1% outage probability) obtained in this way will converge equally fast as efficiencies and correlations do. And the corresponding MRC diversity gain, determined based on this CDF, converges much faster than that derived from CDFs.

2. Theory

Assuming an N-port diversity antenna in complex Gaussian (Rayleigh) fading environment, the covariance matrix of the received signals be the receive diversity antenna is

$$\mathbf{R} = E[\mathbf{v}\mathbf{v}^H] \tag{1}$$

where $\mathbf{v}$ is $N \times 1$ vector containing the received signals for each receive branches, superscript H is Hermitian operator, and E is mathematical expectation. In practice, $\mathbf{R}$ is usually nonsingular. The joint probability density function (PDF) of $\mathbf{v}$ is [12],

$$p(\mathbf{v}) = \frac{1}{(2\pi)^N \det(\mathbf{R})^{1/2}} \exp\left(-\frac{1}{2}\mathbf{v}^H\mathbf{R}^{-1}\mathbf{v}\right) \tag{2}$$

The MRC output power is

$$P_{MRC} = \frac{1}{2}\mathbf{v}^H\mathbf{v} \tag{3}$$

Instead of using the generating function of P_{MRC} as in [9], [11], we find it easier to use the characteristic function of it like in [10], which gives a closed-form CDF formula. The characteristic function of the MRC output is

$$\phi(z) = E[\exp(jzP_{MRC})] \tag{4}$$

Equation (4) can be derived as [13]

$$\phi(z) = \frac{1}{\det(\mathbf{I} + z\mathbf{R})} = \prod_{i=1}^{N} \frac{1}{1 + z\lambda_i(\mathbf{R})} \tag{5}$$

where $\lambda_i(\mathbf{R})$ denotes the eigenvalues of $\mathbf{R}$. The PDF of P_{MRC}

Manuscript received January 17, 2011.
Manuscript revised March 26, 2011.
[†]The authors are with Chalmers University of Technology, Gothenburg, Sweden.
[††]The author is with SP Technical Research Institute of Sweden, Boras, Sweden.
a) E-mail: xiaoming.chen@chalmers.se
DOI: 10.1587/transele.E94.C.1657

is inverse Fourier transform of $\phi(z)$ [10], [11],

$$p(P_{MRC}) = \frac{1}{\prod_i \lambda_i} \sum_i \frac{\exp(-P_{MRC}/\lambda_i)}{\prod_{k\neq i}(1/\lambda_k - 1/\lambda_i)} \tag{6}$$

The CDF of P_{MRC} can be readily derived as [10],

$$P(P_{MRC}) = 1 - \sum_{i=1}^{N} \frac{\lambda_i^{N-1}\exp(-P_{MRC}/\lambda_i)}{\prod_{k\neq i}^{N}(\lambda_i - \lambda_k)} \tag{7}$$

For practical multi-port antennas (with nonzero correlations between different branches) and imperfect estimation of the covariance matrices, the eigenvalues will not be equal to each other, therefore (6) and (7) hold in practice. However, for a special ideal case, where a multi-port antenna is power-balanced and has zero correlations among all branches, and with a further assumption of perfect estimation of covariance matrix, all eigen values are equal to each other, and (6) or (7) is not valid anymore. Nevertheless, this special ideal case will never happen in reality. First of all, most practical multi-port antenna will have nonzero correlations between its branches; and the perfect estimation of covariance matrix of a multi-port antenna can never be measured with finite number of independent samples. As long as there are finite correlations or imperfect covariance matrix estimation, the eigenvalues of the covariance matrix will not be identical.

The effective diversity gain is defined as the output of a diversity antenna improvement compared with that of a single ideal antenna at certain outage probability level [5], [14]. The MRC effective diversity gain (EDG) is,

$$EDG = \left.\frac{P_{MRC}}{P_{ref}}\right|_{1\%} \tag{8}$$

A compact closed-form formula for two-port MRC diversity gain is given by [8],

$$ADG = \sqrt{\left(1+\frac{e_{emb,\min}}{e_{emb,\max}}\right)^2 + (14.78^2-4)\frac{e_{emb,\min}}{e_{emb,\max}}(1-|\rho|^2)} \tag{9}$$

$$EDG = e_{emb,\max} ADG \tag{9a}$$

where ADG is apparent diversity gain, ρ denotes correlation, $e_{emb,\max}$ and $e_{emb,\min}$ are maximum and minimum embedded radiation efficiencies respectively. A fast converging ADG can be obtained with statistical samples using (9). EDG can then be determined using (9a). Thus, formula (9) offers a simple way to calculate diversity gain with fast convergence. Implementing it for reverberation chamber measurement will result in better accuracy. However, it applies only for two-port antenna with arbitrary branch efficiencies and correlation.

The method presented by [9], [10], given by (1), (7) and (8) works for any number of antenna branches with arbitrary efficiencies and correlations. And, it has almost the same convergent rate as (9), as will be shown later. Implementing it in reverberation chamber, diversity gain for any multi-port antenna can be measured. Compared with directly measured CDF, the new method offers better accuracy with the same finite number of samples. However, it can only be used for MRC.

3. Simulation

We first use a numerical simulation to illustrate the convergence of the different methods. We consider a two-port antenna with embedded radiation efficiencies of −3 dB and −6 dB for best and worst branches. The correlation between the two branches is assumed to be 0.7. Based on these assumptions, the covariance matrix is

$$\mathbf{R} = \begin{bmatrix} 0.5 & 0.7\sqrt{0.5\times 0.25} \\ 0.7\sqrt{0.5\times 0.25} & 0.25 \end{bmatrix} \tag{10}$$

We generate two uncorrelated independent and identically distributed complex Gaussian signals, represented by a 2×1 vector $\mathbf{v}_w$, with Frobenius norm $E[\|\mathbf{v}_w\|_F^2] = 2$. The signals received by the diversity antenna can then be expressed as

$$\mathbf{v} = \mathbf{R}^{1/2}\mathbf{v}_w \tag{11}$$

where $\mathbf{R}^{1/2}$ is the Cholesky decomposition of $\mathbf{R}$. The MRC output power P_{MRC} can now be calculated using (3), and the effective diversity gain can be determined using above mentioned methods: from CDF based on measured power, from compact formula (9), and from theoretical CDF based on the eigenvalues of the covariance matrix. The corresponding effective diversity gains as a function of number of realizations is shown in Fig. 1. As expected, the effective diversity gain based on CDF from measured power has the worst convergence, due to the extra uncertainty from reading the CDFs at 1% level; and it does not give any results before gathering 100 samples. The other two methods have almost the same convergence and they almost overlap. Moreover, it can be seen that all the three methods give the same mean value of the effective MRC diversity gain.

4. Measurements

The reverberation chamber used in the paper is Bluetest HP reverberation chamber [16]. A drawing of it is shown in Fig. 2. It has a size of $1.75\times 1.25\times 1.8\,\text{m}^3$ and is provided with two plate stirrers, and platform and polarization stirring [16]. In the measurements, the platform was moved to 20 positions spaced by 18°, and for each platform position each of the two plates move simultaneously to 10 positions, evenly distributed along the total distance they can move. At each stirrer position and for each of the three wall antennas, a full frequency sweep (with a frequency step of 1 MHz) was performed by the vector network analyzer. A frequency stirring over 10 MHz is used to increase measurement accuracy [16]. Therefore, in total there are equivalently 6000 samples. A well stirred reverberation chamber emulates a rich isotropic multipath environment [16].

In order to confirm the convergence of the three methods by measurement, we use two identical monopole antennas working at 900 MHz (with a length of 8.3 cm) as in [5]. The two monopoles are mounted 2 cm away from each other on the circular ground-plane with radius of 14 cm. One of the monopole is connected with a 6-dB attenuator to make a power-unbalanced two-port antenna.

Using the all the 6000 samples, the measured embedded efficiencies for the best and worst branches are −3.7 and −9.7 respectively; and the complex correlation is 0.67 + j0.54. To show the convergence rates of different methods, we calculate EDG as a function of measured samples, as can be seen in Fig. 3. As expected, for the same number of samples of reverberation chamber measurement, the diversity gains determined using covariance-eigenvalue approach (from theoretical CDFs based on eigenvalues of covariance matrix of the channels) or compact formula (9) has much smaller statistical variations (better accuracy) than the traditional method (based on CDF from measured power). The EDGs obtained by traditional method, compact formula (9) and covariance-eigenvalue approach using all the 6000 samples are 2.5 dB, 2.2 dB and 2.3 dB respectively.

To estimate the stand deviation (STD) of the measured diversity gain rigidly, we need to repeat the measurement many times which is tedious and impractically time-time consuming. Therefore, we use Monte Carlo simulation (like in section 3) using the embedded radiation efficiencies and correlation estimated with 6000 samples. Repeat the simulation 50 times at each realization number to estimate the STD. The calculated STD of linear-valued EDG is shown in Fig. 4. From both Figs. 3 and 4, it can be seen that the covariance-eigenvalue approach and compact formula (9) converge equally fast and almost overlap, and both methods converge much faster than the traditional method, which is in agreement with the simulation in Fig. 1. Note that unlike the compact formula (9), the covariance-eigenvalue approach is valid for arbitrary antenna with any number of antenna ports. However, due to page limitation, and in order to compare it against the compact formula (9), we only use

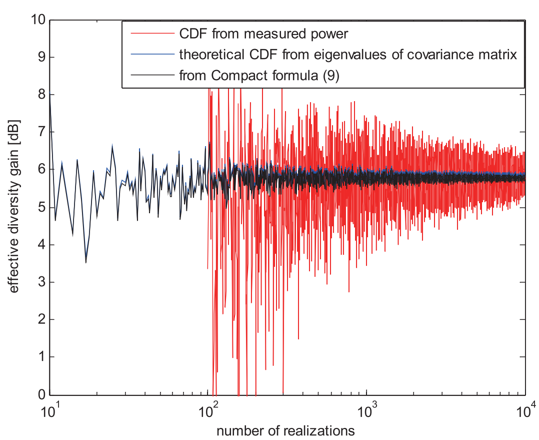

Fig. 1 Comparison of numerically simulated effective diversity gain by different methods as a function of number of realizations ($e_{emb,\max} = -3$ dB, $e_{emb,\min} = -6$ dB and $\rho = 0.7$).

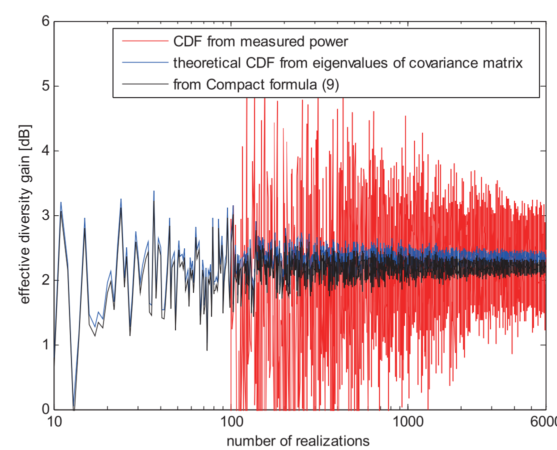

Fig. 3 Comparison of measured effective diversity gain by different methods as a function of number of measured samples ($e_{emb,\max} = -3.7$ dB, $e_{emb,\min} = -9.7$ dB and $\rho = 0.67 + j0.54$ estimated using all 6000 samples).

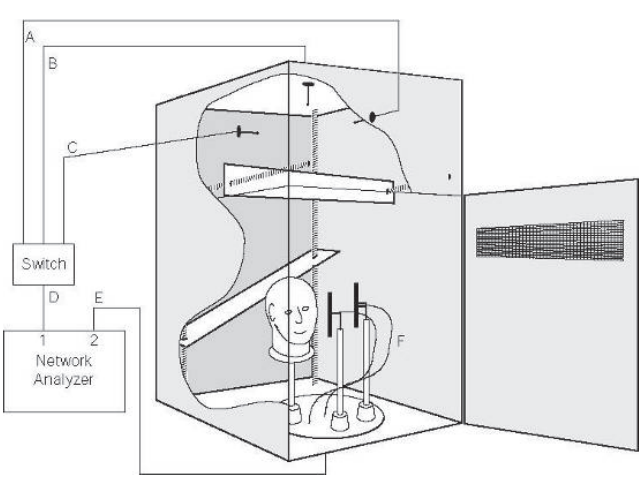

Fig. 2 Drawing of Bluetest reverberation chamber with two mechanical plate stirrers, platform, three wall antennas and example of a two-port antenna under test.

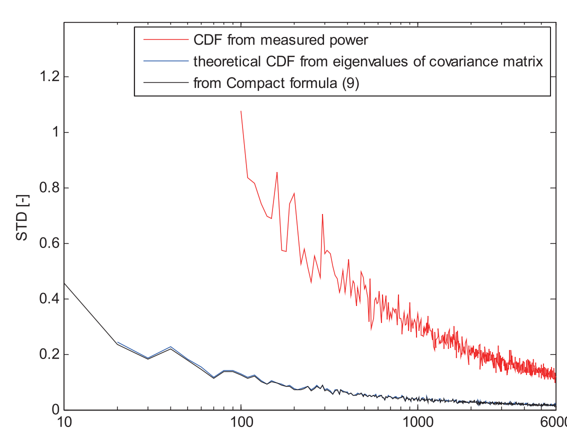

Fig. 4 STD of linear-valued EDG as function of sample number for different methods.

two-port antenna for measurement in the present paper.

5. Conclusion

In this paper, we study the convergence of the covariance-eigenvalue approach to determine MRC diversity gain from measured channels in reverberation chamber. The method involves calculation of theoretical CDF from the eigenvalues of the covariance matrix of the channels. The method clearly outperforms the alternative way of determining diversity gain from CDFs based on measured power. For two-port antenna, the compact formula (9) may be easier to use (with almost the same convergent rate as the new method), yet it works only for two-port antennas. The covariance-eigenvalue method used in this paper, however, is general and applicable to any multi-port antenna with any number of ports.

References

[1] A. Goldsmith, Wireless Communications, Cambridge University Press, 2005.
[2] M. Schwartz, W.R. Bennet, and S. Stein, Communication Systems and Techniques, IEEE Press, 1996.
[3] J. Luo, J.R. Zeidler, and S. McLaughlin, "Performance analysis of compact antenna arrays with MRC in correlated Nakagami fading channels," IEEE Trans. Veh. Technol., vol.50, no.1, pp.267–277, Jan. 2001.
[4] A. Paulraj, R. Nabar, and D. Gore, Introduction to space-time wireless communications, Cambridge University Press, 2006.
[5] K. Rosengren and P.-S. Kildal, "Radiation efficiency, correlation, diversity gain and capacity of a six monopole antenna array for a MIMO system: Theory, simulation and measurement in reverberation chamber," Proc. IEE, Microwaves, Optics and Antennas, vol.152, no.1, pp.7–16, Feb. 2005.
[6] J. Yang, S. Pivnenko, T. Laitinen, J. Carlsson, and X. Chen, "Measurements of diversity gain and radiation efficiency of the eleven antenna by sing different measurement techniques," EuCAP 2010, Barcelona, Spain, 2010.
[7] A. Hussian, U. Carlberg, J. Carlsson, and P.-S. Kildal, "Analysis of statistical uncertainties involved in estimating ergodic MIMO capacity and diversity gain in Rayleigh fading environment," 20th Internatial Conference on Applied Electromagnetics and Communication, Dubrovnik, Croatia, Sept. 2010.
[8] N. Jamaly, P.-S. Kildal, and J. Carlsson, "Compact formulas for diversity gain of two-port antennas," IEEE Antennas and Wireless Propag. Lett., vol.9, no.9, pp.970–973, Aug. 2010.
[9] J.N. Pierce and S. Stein, "Multiple diversity with nonindependent fading," Proc. IRE, pp.89–104, 1960.
[10] W.C.Y. Lee, Mobile Communications Engineering, 2nd ed., McGraw-Hill, New York, 1997.
[11] O. Norklit, P.D. Teal, and G. Vaughan, "Measurement and evaluation of multi-antenna handsets in indoor mobile communication," IEEE Trans. Antennas Propag., vol.49, no.3, pp.429–437, March 2001.
[12] S.M. Kay, Fundamentals of statistical signal processing: Estimation theory, volumn I: estimation theory, Prentice Hall, 1993.
[13] G.L. Turin, "The characteristics function of Hermitian quadratic forms in complex normal variables," Biometrika, vol.47, pp.199–201, 1960.
[14] P.-S. Kildal, K. Rosengren, J. Byun, and J. Lee, "Definition of effective diversity gain and how to measure it in a reverberation chamber," Microw. Opt. Technol. Lett., vol.34, no.1, pp.56–59, 2002.
[15] B. Vucetic and J. Yuan, Space-time coding, Wiley, 2003.
[16] P.-S. Kildal and K. Rosengren, "Correlation and capacity of MIMO systems and mutual coupling, radiation efficiency and diversity gain of their antennas: Simulations and measurements in reverberation chamber," IEEE Commun. Mag., vol.42, no.12, pp.102–112, Dec. 2004.