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# Optimized Iterative (Turbo) Reception for QAM OFDM with CFO over Unknown Double-Selective Channels

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**Abstract**—A novel iterative (turbo) receiver is introduced, suitable for orthogonal frequency division multiplexing (OFDM) employing quadrature amplitude modulation (QAM) and receiver diversity. The system operates over a double-selective channel and includes a carrier frequency offset (CFO). We propose a maximum a posteriori probability expectation-maximization (MAP-EM) receiver with a different EM parameter division than standard methods. In such standard MAP-EM receivers, the E-step parameters correspond to the channel, while the M-step parameters correspond to the CFO and data symbols. This standard receiver parameter division results into a highly complex receiver for QAM, due to the large modulated symbol alphabet size, and the non-constant constellation symbol amplitude. In this paper, a new receiver framework introduces a different parameter division that leads to reduced complexity turbo receivers for QAM signaling, while still achieving close to optimal system performance. The new approach adapts the sum-product algorithm (SPA) parameter framework to the MAP-EM receiver. Thus, in the new receiver framework, the E-step parameters are data symbols, while the M-step parameters are the channel and the CFO. We evaluate the performance of the proposed receiver with and without automatic repeat request (ARQ), where in the former case packet combining applies to further improve performance.

## I. INTRODUCTION

Iterative receiver design [1], [2], in which the detector and the channel decoder are optimized jointly to offer better performance, represents an area of much current wireless communication research effort, especially within the context of OFDM for fourth generation (4G) cellular wireless systems. The most popular iterative receiver design methodologies are based on: a) the sum product algorithm (SPA) [2], and b) The expectation maximization (EM) algorithm [3], both of which minimize a Kullback-Leibler metric, under different modeling assumptions. Both design methodologies can be seen as optimized designs since both converge to the MAP (or maximum-likelihood (ML)) receiver, depending on the specific receiver setup. Recently, [4] has proposed an optimized iterative receiver that jointly estimates channel coefficients, compensates for additional distortions (CFO, or non-linearities), and decodes the message. However, existing turbo

receivers for OFDM are only practical for  $M$ -ary phase shift keying (PSK) modulation, due to complexity considerations. As OFDM systems are expected to operate with a variety of modulation formats, there is an urgent need to develop optimized turbo receivers, suitable for QAM and other non-constant amplitude modulation formats.

In this paper, a novel optimized MAP-EM receiver is presented that is suitable for QAM-type signaling. The receiver achieves the same performance as the previously presented receivers [1], [4], but due to a new parameter setup, at a significant lower complexity, thus allowing for large signaling alphabet, including non-constant amplitude modulation formats such as  $M$ -QAM. The receiver as presented herein jointly estimates the channel, compensates for the CFO, and demodulates the data message. Several examples are presented to show the performance of the new receiver.

The paper is organized as follows: Section II presents the System Description. Section III presents the new receiver. Section IV includes results on the performance of the proposed receiver. Finally, Section V presents our conclusions.

*Notational Conventions:* Vectors are denoted in boldfaced small letters, (as in  $\mathbf{v}$ );  $\|\mathbf{v}\|$  denotes the magnitude of  $\mathbf{v}$ , while the  $l$ -th element of  $\mathbf{v}$  is denoted by  $v_l$ . Matrices are denoted in boldfaced capital letters, (as in  $\mathbf{A}$ );  $\mathbf{A}^H$ ,  $\mathbf{A}^T$ ,  $\mathbf{A}^*$ ,  $\mathbf{A}[\text{row } m]$ ,  $\mathbf{A}[\text{col } m]$ , and  $\text{trace}(\mathbf{A})$  denote the Hermitian, the transpose, the conjugate, the  $m$ -th row, the  $m$ -th column, and (for square matrices) the trace of  $\mathbf{A}$ , respectively. The diagonal matrix resulting from  $\mathbf{v}$  is denoted as  $\text{diag}(\mathbf{v})$  or  $\tilde{\mathbf{V}}$ . ;  $\mathbf{W}$  denotes the discrete Fourier transform (DFT) matrix with row  $m$ , column  $n$  element ( $1 \leq m, n \leq Q$ ),  $\mathbf{W}_{m,n} = \exp(-j2\pi(m-1)(n-1)/Q)$ .  $\mathbf{I}_K$ , and  $\mathbf{0}_K$  denote the identity matrix of size  $K$ , and the zero vector with  $K$  elements, respectively. Sets are represented in calligraphic typeface, e.g.,  $\mathcal{S}$ . The received vector over antenna  $d$ , transmission  $t$ , including all subcarriers is denoted by  $\mathbf{y}_{d,t}$ , while  $y_{q,d,t}$  is the complex scalar corresponding to the observation over subcarrier  $q$ , antenna  $d$ , transmission  $t$ .

## II. SYSTEM DESCRIPTION

### A. System Model

The OFDM system uses  $Q$  subcarriers from the set  $\mathcal{Q}$ . Let  $L_p$  denote the number of pilot subcarriers employed. Then,  $\mathcal{Q}$  can be partitioned into two subsets:  $\mathcal{Q}_P$ , the subcarriers transmitting pilot symbols ( $L_p$  in number), and  $\mathcal{Q}_D$ , the subcarriers transmitting data symbols ( $Q - L_p$  in number). There is one transmit antenna and  $D$  uncorrelated receive antennas, resulting in independent channels. The channel between the transmitting and a receiving antenna is a frequency selective channel [5], therefore it presents multipath fading. The overall channel can be modeled by a tapped delay line (TDL) [5]. Each tap in the TDL model is associated with a Rayleigh fading process, which at baseband may be represented by a zero-mean complex Gaussian process. In addition, the time correlation function of each tap follows the Clarke/Jakes model [6] with normalized Doppler frequency  $F_{D,NOR} = T_{OFDM,CP} F_D$  where  $F_D$  is the Doppler frequency and  $T_{OFDM,CP}$  is the OFDM word duration, including the cyclic prefix (CP) inserted in the beginning of each word, i.e.,  $T_{OFDM,CP} = T_S(Q + L - 1)$  where  $L$  is the number of available time-domain channel paths (taps), assumed the same for all antennas and transmissions,  $T_S$  is the OFDM sampling period, i.e.,  $T_S = 1/(Q\Delta f_{sc})$ , with  $\Delta f_{sc}$  denoting the OFDM subcarrier separation. The system is assumed to be perfectly time-synchronized. The channel realizations are assumed to be independent from one transmission of a packet to any retransmissions, due to the high normalized Doppler used.

Let  $d$  ( $1 \leq d \leq D$ ) indicate a receiver antenna, and  $t$  ( $1 \leq t \leq N$ ) the ARQ transmission attempt, where  $N$  is the number of multiple transmissions currently available at the receiver. The time-domain channel for antenna  $d$ , transmission  $t$  is denoted by  $\mathbf{h}_{d,t} = [h_{0,d,t}, h_{1,d,t}, \dots, h_{L-1,d,t}]^T$ , where each element is a complex Gaussian random variable (r.v.). In addition, let  $\mathbf{h}_t = [\mathbf{h}_{1,t}^T, \dots, \mathbf{h}_{D,t}^T]^T$  be the aggregated channel for transmission  $t$ . The channel components are independent identically distributed (i.i.d.) complex Gaussian r.v. of zero mean and variance equal to the corresponding channel's power profile component.<sup>1</sup> The channels are normalized, so that  $\mathbb{E}(\|\mathbf{h}_{d,t}\|^2) = 1$  for all antennas and transmissions.

Forward error correction (FEC) is used to further enhance performance through turbo processing. In this paper, convolutional codes of different constraint lengths are employed for FEC. For modulation, QAM type modulation in conjunction with Gray mapping is considered. Let  $\mathbf{c} = [c_0, \dots, c_{Q-1}]^T$  be the normalized<sup>2</sup>  $Q$ -length QAM symbol vector in frequency, where each  $c_q$  ( $0 \leq q \leq Q - 1$ ) is a r.v. over the employed QAM constellation  $\mathcal{C}_{QAM}$ . Then  $\mathbf{c}$  is converted to time domain symbols in a particular OFDM slot (packet), after appropriate inverse Fourier transform (IDFT) takes place.

<sup>1</sup>Baseband description of the system is assumed herein.

<sup>2</sup>This normalization is performed to ensure that  $c_q$  have unit average power, i.e.  $\mathbb{E}(|c_q|^2) = 1$ .

Finally, we employ an ARQ protocol. In particular, we consider a selective repeat ARQ (SRARQ) system based on type-I hybrid ARQ, in which only failed packets are retransmitted. In this paper, it is assumed that a packet comprises a single OFDM word and that the feedback channel is perfectly accurate. Perfect packet error detection is also assumed. Packets are dropped (rejected) from the system if their reception fails after  $N_R$  total retransmissions (or  $N_R + 1$  total transmissions).

### B. Reception under CFO

Under CFO, the received vectors per antenna, and transmission are given as follows [7]

$$\mathbf{y}_{d,t} = \frac{1}{Q} \mathbf{W} \mathbf{T} \mathbf{W}^H \tilde{\mathbf{C}} \mathbf{W}_L \mathbf{h}_{d,t} + \mathbf{z}_{d,t}, \quad (1)$$

where the CFO-related matrix  $\mathbf{\Gamma} = \text{diag}[1 \ \mu \cdots \mu^{Q-1}]$  with  $\mu = \exp(-j2\pi\epsilon/Q)$  and  $\epsilon = \Delta f_c / \Delta f_{sc}$  (the normalized CFO [7]). In (1),  $\mathbf{z}_{d,t}$  is a complex white noise vector with zero mean and covariance per component equal to  $\sigma_z^2 = (\text{SNR}_{\text{symbol}})^{-1}$ , where  $\text{SNR}_{\text{symbol}} = \bar{E}_s / N_0$  is the (average) QAM symbol signal-to-noise ratio (SNR) at the receiver input, and  $N_0$  is the power spectral density (PSD) of the thermal noise (double-sided). This reception model assumes that the CFO remains constant over long periods of time and thus it remains fixed between an initial packet transmission and its retransmissions.

## III. PROPOSED OPTIMIZED QAM RECEIVER

In this section, the proposed optimized receiver is presented for a general number of  $N$  received copies of a packet, where  $N \leq N_R + 1$ . Thus,  $N_R$  becomes a design parameter, representing a tradeoff between allowable delay and the packet rejection probability  $P_{\text{REJ}}$ . We first present the initialization phase of the receiver, involving data-aided channel and CFO estimation. Then, we describe the different receiver components and their interface. These components are the EM estimator and the SPA detector. The subsequent two subsection describe the EM estimator and the SPA detector.

### A. Initial Channel and CFO Estimation

The iterative nature of the detector used in this paper requires initial estimates of the per antenna and per transmission channels  $\mathbf{h}_{d,t}$ , and the CFO distortion  $\epsilon$ . The embedded pilot scheme used is assumed to be of the equally spaced type, due to its optimality in performance [8]. For proper estimation,  $L_P \geq L$  pilot tones are required. Ones are used for the pilot symbols, same for all packet transmissions. Further details on pilot-based least squares estimate (LSE) that we have used can be found in [9] for the ideal case.<sup>3</sup> If virtual pilots are also employed, the assumption of equally-spaced pilots can still be valid [10]. However, for the optimized receiver presented here, inclusion of virtual pilots will result in a throughput decrease.

<sup>3</sup>LSE is employed as the initial estimate, due to the fact that the receiver might not have an estimate of the received signal (average) SNR available during initial channel estimation. From a theoretical standpoint, an MMSE could also be applied for initial channel estimation, if the received SNR is known, with some additional gain in performance.

Initial CFO estimation is performed using two identical, subsequent CFO estimation OFDM words as per [7]. This initial CFO estimation ( $\hat{\epsilon}^{(0)}$ ) is then used to compensate for the CFO before LSE takes place. After compensating with the estimated CFO matrix, the ‘cleaned’ received vector becomes

$$\mathbf{y}_{\text{mod},d,t}^{(0)} = \frac{1}{Q} \mathbf{W}(\hat{\Gamma}^{(0)})^H \mathbf{W}^H \mathbf{y}_{d,t}. \quad (2)$$

where  $\hat{\Gamma}^{(0)}$  is the CFO-related matrix,  $\Gamma$  with  $\epsilon = \hat{\epsilon}^{(0)}$ . Thus, the LSE of the channel can be expressed as

$$\hat{\mathbf{h}}_{\text{lse},\text{cfo},d,t} = L_p^{-1} \mathbf{W}_{\mathcal{Q}_P,L}^H \mathbf{y}_{\text{mod},\mathcal{Q}_P,d,t}^{(0)} \quad (3)$$

Note that due to ARQ protocol, initial estimates of the channel need to be computed for every (re)transmission.

### B. Optimized MAP ARQ Turbo Processing Receiver

Here a description of the operation of the optimized receiver is presented. The general block diagram of a MAP-EM receiver is shown in Fig. 1. The model for non-ARQ is similar, but with  $N = 1$ . The receiver employs input  $\mathcal{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_N\}$ , where  $\mathbf{y}_n = \{\mathbf{y}_{1,n}, \dots, \mathbf{y}_{D,n}\}$ , the LSE channel estimates  $\hat{\mathcal{D}}_{\text{ch}}^{(0)} = \{\hat{\mathbf{h}}_1^{(0)}, \dots, \hat{\mathbf{h}}_N^{(0)}\}$ , and the initial CFO estimate,  $\hat{\mathcal{D}}_o^{(0)} = \hat{\epsilon}^{(0)}$ . Within each APP (outer) decoding iteration,  $I_{\text{EM}}$  inner EM iterations performed. If a total of  $I_{\text{TURBO}}$  APP decoding iterations are used, also called receiver cycles, then the total number of EM iterations used in the complete receiver process is  $I_{\text{EM}}I_{\text{TURBO}}$ . Thus, the detector iteration number,  $i$ , takes values in  $\{1, 2, \dots, I_{\text{EM}}I_{\text{TURBO}}\}$ . The decoder loop iteration number,  $k$ , satisfies  $1 \leq k \leq I_{\text{TURBO}}$ , and it equals  $k = \text{quot}(i - 1, I_{\text{EM}}) + 1$ , where  $\text{quot}(x, y)$  denotes the quotient of the division of  $x$  by  $y$ . The initial intrinsic LLR vector, when the receiver initializes, is set to the all-zero vector, i.e.  $\lambda_{2,N}^{(0)} = \theta_{N_b}$ , where  $N_b$  denotes the number of coded bits per OFDM word.  $\lambda_{1,N}^{(k)}$  is appropriately deinterleaved, then the resulting  $\lambda_{1,N,\text{DIL}}^{(k)}$  is fed to the APP decoder for the  $k$ -th iteration. The output of the APP decoder  $\lambda_{2,N,\text{DIL}}^{(k)}$ , after interleaving, is the intrinsic LLR vector  $\lambda_{2,N}^{(k)}$  used in the next receiver cycle (iteration  $k + 1$ ). Finally, after  $I_{\text{TURBO}}$  APP iterations, the estimated information bit sequence is estimated from the information LLR output of the APP decoder,  $\lambda_{B,N,\text{DIL}}^{(k)}$ .

### C. EM Processor Framework

The EM algorithm [3] is able to determine maximum likelihood estimates in the presence of nuisance parameters. The EM algorithm consists of two steps, the E-step and the M-step, which are executed iteratively. When estimating a parameter  $a$  from an observation  $y$  in the presence of a nuisance parameter  $b$ . Starting from an initial estimate  $\hat{a}^{(0)}$ , the E-step at iteration  $i$  involves determining an objective function

$$Q^{(i)}(a) = \mathbb{E}_{b|\mathbf{r}, \hat{a}^{(i-1)}} \{\log \Pr(y|a, b)\}. \quad (4)$$

The M-step at iteration  $i$  involves maximizing the objective function w.r.t.  $a$ :

$$\hat{a}^{(i)} = \arg \max_a Q^{(i)}(a). \quad (5)$$

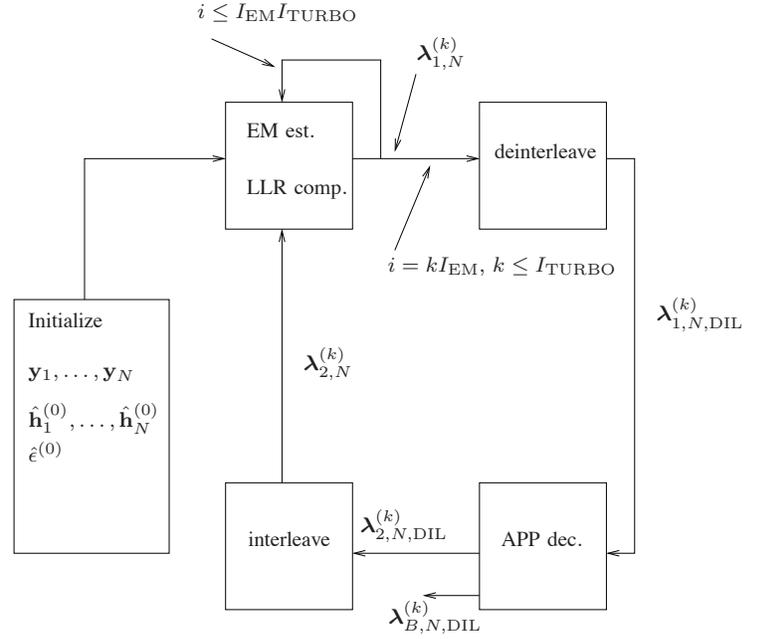


Fig. 1. Block diagram of the optimized OFDM ARQ iterative receiver based on MAP-EM algorithm.

The EM-estimates have the property that their likelihood is non-decreasing. In addition, provided the initial estimate  $\hat{a}^{(0)}$  is sufficiently good, the EM estimates are guaranteed to converge to the ML estimate. The EM algorithm has been applied to a MAP-EM receiver framework for OFDM [1], [4]. However, these receivers experience a dramatic complexity increase when applied to QAM, making them impractical. In this paper we avoid this complexity increase through a different association of parameters in the E- and M-steps of the EM algorithm. Rather than considering the unknown data symbols as parameters to be estimated, as in [1], [4], here we consider the channel and CFO as parameters to be estimated, and treat the data symbols as nuisance parameters. We propose the following setup for the EM processor parameters, as follows: Denote by  $\mathcal{D}_{\text{ch}} = \{\mathbf{h}_{d,t}, 1 \leq d \leq D, 1 \leq t \leq N\}$  the (unknown) channel parameters, and by  $\mathcal{D}_o$  the other (unknown) parameters of the problem. For example, in the CFO case,  $\mathcal{D}_o = \{\epsilon\}$ . We treat  $(\mathcal{Y}, \mathbf{c})$  as complete data and  $\mathcal{D}_{\text{ch}}, \mathcal{D}_o$  as M-step parameters. This will allow for efficient calculation of the a posteriori symbol probability at each iteration of the EM algorithm, as shown below.

1) *Formulation of M-step:* With the new set of parameters proposed herein, the M-step in the  $i$ -th EM iteration, is the maximization over  $(\mathcal{D}_{\text{ch}}, \mathcal{D}_o)$  of the following metric

$$\mathbb{E}_{\mathbf{c}|\mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)}} \{\log \Pr(\mathcal{Y}|\mathbf{c}, \mathcal{D}_o, \mathcal{D}_{\text{ch}})\}. \quad (6)$$

We observe that due to the independence of the noise on different antennas and at different transmissions

$$\log \Pr(\mathcal{Y}|\mathbf{c}, \mathcal{D}_o, \mathcal{D}_{\text{ch}}) = \sum_{d,t} \log \Pr(\mathbf{y}_{d,t}|\mathbf{c}, \epsilon, \mathbf{h}_{d,t}), \quad (7)$$

in which, due to (1),

$$\begin{aligned} \log \Pr(\mathbf{y}_{d,t} | \mathbf{c}, \epsilon, \mathbf{h}_{d,t}) &\propto \|\mathbf{y}_{d,t} - \frac{1}{Q} \mathbf{W} \mathbf{\Gamma} \mathbf{W}^H \tilde{\mathbf{C}} \mathbf{W}_L \mathbf{h}_{d,t}\|^2 \\ &\propto Q \mathbf{h}_{d,t}^H \mathbf{W}_L^H \tilde{\mathbf{C}}^H \tilde{\mathbf{C}} \mathbf{W}_L \mathbf{h}_{d,t} \quad (8) \\ &\quad - 2\Re\{\mathbf{y}_{d,t}^H \mathbf{W} \mathbf{\Gamma} \mathbf{W}^H \tilde{\mathbf{C}} \mathbf{W}_L \mathbf{h}_{d,t}\}, \end{aligned}$$

where  $\propto$  indicates proportionality up to irrelevant additive and positive multiplicative constants.

2) *Solving the M-step:* Assuming we can compute

$$\mathbf{S}_2 = \mathbb{E}_{\mathbf{c}|\mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)}} \left\{ \tilde{\mathbf{C}}^H \tilde{\mathbf{C}} \right\} \quad (9)$$

and

$$\mathbf{S}_1 = \mathbb{E}_{\mathbf{c}|\mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)}} \left\{ \tilde{\mathbf{C}} \right\}, \quad (10)$$

then the maximization in (8) over  $\mathbf{h}_{d,t}$  can be performed analytically

$$\hat{\mathbf{h}}_{d,t}^{(i)}(\epsilon) = \frac{1}{Q} (\mathbf{W}_L^H \mathbf{S}_2 \mathbf{W}_L)^{-1} (\mathbf{y}_{d,t}^H \mathbf{W} \mathbf{\Gamma}(\epsilon) \mathbf{W}^H \mathbf{S}_1 \mathbf{W}_L)^H. \quad (11)$$

The maximization over  $\epsilon$  can be carried out numerically, after substituting  $\hat{\mathbf{h}}_{d,t}(\epsilon)$  into (7):

$$\begin{aligned} \hat{\epsilon}^{(i)} = \arg \max_{\epsilon} \sum_{d,t} & (Q \mathbf{h}_{d,t}^H \mathbf{W}_L^H \mathbf{S}_2 \mathbf{W}_L \hat{\mathbf{h}}_{d,t}^{(i)}(\epsilon) \\ & - 2\Re\{\mathbf{y}_{d,t}^H \mathbf{W} \mathbf{\Gamma}(\epsilon) \mathbf{W}^H \mathbf{S}_1 \mathbf{W}_L \hat{\mathbf{h}}_{d,t}^{(i)}(\epsilon)\}). \quad (12) \end{aligned}$$

This optimization can be performed in either serial, or parallel search fashion, after partitioning the range of  $\epsilon$  values into equal intervals, for example. In this paper, this search method is applied. However, any other suitable optimization methods could be also used with success [11]. Alternatively, an estimate of  $\mathbf{h}_{d,t}$  can be found through a conditional maximization (CM) argument, by solving (11), only for  $\epsilon = \hat{\epsilon}^{(i-1)}$ . The estimate of  $\epsilon$  is then determined by solving (12), where we substitute the CM estimate of  $\mathbf{h}_{d,t}$ .

3) *Solving the E-step:* The E-step involves the computation of  $\mathbf{S}_1$  and  $\mathbf{S}_2$ . Observe that  $\mathbf{S}_1$  is a diagonal matrix with as  $q$ -th diagonal element

$$\mathbf{S}_{1,q} = \sum_{c \in \mathcal{C}_{\text{QAM}}} c \times \Pr(c_q = c | \mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)}), \quad (13)$$

Similarly,  $\mathbf{S}_2$  is a diagonal matrix with as  $q$ -th diagonal element

$$\mathbf{S}_{2,q} = \sum_{c \in \mathcal{C}_{\text{QAM}}} |c|^2 \times \Pr(c_q = c | \mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)}). \quad (14)$$

To reduce the computational complexity,  $\mathbf{S}_2$  can be approximated by the identity matrix. Note that this approximation is exact for constant modulus constellations. With this approximation, the matrix inversion in (11) can be precomputed.

The a posteriori probabilities  $\Pr(c_q = c | \mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(i-1)}, \hat{\mathcal{D}}_o^{(i-1)})$  are computed efficiently, on a per carrier basis, as follows:

$$\begin{aligned} \Pr(c_q = c | \mathcal{Y}, \mathcal{D}_{\text{ch}}^{(i-1)}, \mathcal{D}_o^{(i-1)}) &\propto \\ \Pr^{(k-1)}(c_q = c) \prod_{d,t} & \Pr(\mathbf{y}_{\text{mod},q,d,t}^{(i-1)} | c_q = c, \hat{\mathbf{h}}_{d,t}^{(i-1)}, \hat{\epsilon}^{(i-1)}). \quad (15) \end{aligned}$$

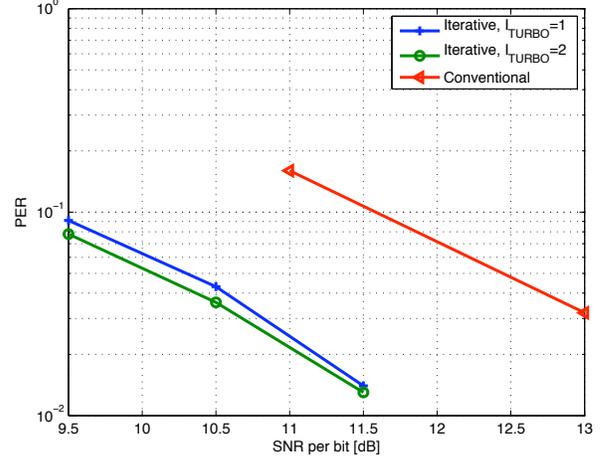


Fig. 2. Results for PER (no ARQ) with optimized Turbo processing and comparison to the conventional receiver.  $Q = 64$ ,  $M = 16$ ,  $I_{\text{EM}} = 2$  and  $I_{\text{TURBO}} = 2$ . Normalized Doppler 0.02.  $L = 8$ ,  $L_P = 16$ ,  $\epsilon = 0.03$ ,  $D = 1$ .

with

$$\mathbf{y}_{\text{mod},d,t}^{(i-1)} = \frac{1}{Q} \mathbf{W} (\hat{\mathbf{\Gamma}}^{(i-1)})^H \mathbf{W}^H \mathbf{y}_{d,t} \quad (16)$$

where  $\hat{\mathbf{\Gamma}}^{(i-1)}$  is the CFO-related matrix,  $\mathbf{\Gamma}$  with  $\epsilon = \hat{\epsilon}^{(i-1)}$ , and the coded symbol probability  $\Pr^{(k-1)}(c_q = c)$  is calculated through the a priori LLR of the  $k-1$  APP decoder output. Finally, due to Gaussian statistics

$$\begin{aligned} \Pr(\mathbf{y}_{\text{mod},q,d,t}^{(i-1)} | c_q = c, \hat{\mathbf{h}}_{d,t}^{(i-1)}, \hat{\epsilon}^{(i-1)}) &\propto \\ \exp\left(-\frac{\|\mathbf{y}_{\text{mod},q,d,t}^{(i-1)} - c[\mathbf{W}_L]_{\text{row } q} \hat{\mathbf{h}}_{d,t}^{(i-1)}\|^2}{\sigma_Z^2}\right) \quad (17) \end{aligned}$$

#### D. Calculation of the extrinsic LLR and Symbol Probabilities $\Pr(c_q)$ for the APP Decoder

In the beginning of each receiver cycle,  $k$ , ( $1 \leq k \leq I_{\text{TURBO}}$ ),  $i = (k-1)I_{\text{EM}} + 1$ . The detector uses the previous,  $k-1$  cycle a priori coded symbol LLR  $\lambda_{2,N}^{(k-1)}$  (Fig. 1), for  $i = (k-1)I_{\text{EM}} + 1, \dots, kI_{\text{EM}}$ . In the last EM iteration of each detector cycle, i.e. for  $i = kI_{\text{EM}}$ , the extrinsic LLR vector ( $\lambda_{1,N}^{(k)}$ ) is calculated, by employing standard techniques [12], and using the a posteriori probabilities  $\Pr(c_q = c | \mathcal{Y}, \hat{\mathcal{D}}_{\text{ch}}^{(kI_{\text{EM}})}, \hat{\mathcal{D}}_o^{(kI_{\text{EM}})})$  available from the last iteration of the  $k$  receiver cycle.

## IV. RESULTS

Results are presented for  $Q = 64$ ,  $L = 8$ ,  $L_P = 16$ ,  $\epsilon = 0.03$  systems employing a  $\nu = 7$ , rate  $r = 1/2$  FEC convolutional code with generator polynomial  $\{133, 171\}$  in octal notation. In the presented results, QAM with  $M = 16$  symbols is used, while  $I_{\text{EM}} = I_{\text{TURBO}} = 2$  for the iterative receiver. In Fig. 2, results are presented for the packet error rate (PER) for the proposed iterative system without ARQ and the conventional (i.e.  $I_{\text{EM}} = I_{\text{TURBO}} = 1$ ) system. Clearly,

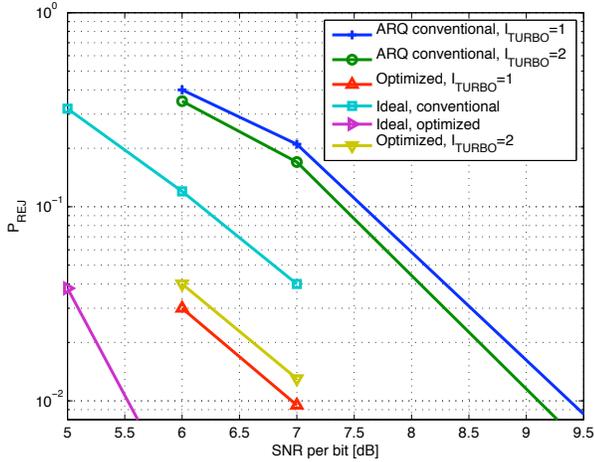


Fig. 3. Results for  $P_{\text{REJ}}$  (ARQ) with optimized Turbo processing and comparison to the ideal and standard Turbo processing cases.  $Q = 64$ ,  $M = 16$ ,  $I_{\text{EM}} = 2$  and  $I_{\text{TURBO}} = 2$ . Normalized Doppler 0.02.  $L = 8$ ,  $L_P = 16$ ,  $D = 1$ ,  $N_{\text{max}} = N_R + 1 = 2$ .

the proposed system with only two EM iterations offers gains in the order of 2.5 dB over its more conventional counterpart, even with the small number of iterations employed.

In Fig. 3, results for an ARQ system with same parameters are presented, with a maximum number of allowable transmissions equal to 2, i.e.  $N_R = 1$ . Here the performance of the proposed receiver for both optimized (i.e. packet combining), and non-optimized (i.e. ordinary ARQ) is compared to the performance of the ideal receiver which has perfect knowledge of the channel and CFO. We see that the optimized system offers about 2.5 dB gain over its non-optimized counterpart, while the optimized performance is within 1 dB of the ideal one in the conventional case, and within 1.5 dB in the optimized case, even with the small number of iterations considered herein.

## V. CONCLUSIONS

In this paper, a novel iterative (turbo) receiver is introduced, suitable for orthogonal frequency division multiplexing (OFDM) employing quadrature amplitude modulation (QAM) and receiver diversity. The system operates over a double-selective channel and includes a carrier frequency offset (CFO). We propose a maximum a posteriori probability expectation-maximization (MAP-EM) receiver with a different EM parameter division than standard methods. This new receiver framework introduces a different parameter division that leads to reduced complexity turbo receivers for QAM signaling, while still achieving close to optimal system performance. The new approach adapts the sum-product algorithm (SPA) parameter framework to the MAP-EM receiver. Thus, in the new receiver framework, the E-step parameters are data symbols, while the M-step parameters are the channel and the CFO. The proposed receiver is shown to achieve excellent performance: it offers high gains compared to a conventional, non-iterative receiver, and it is very close in performance to the

ideal receiver which has perfect knowledge of the channel and CFO, for both optimized and non-optimized ARQ cases, even with the small number of iterations considered here. Thus, the proposed receiver represents a very promising approach for future, QAM-based OFDM systems.

Future work includes harnessing correlation between channel taps across re-transmissions and the extension to time-selective channels with variations within one OFDM symbol.

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