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A POSTERIORI ERROR ESTIMATION AND ADAPTIVITY FOR OPTIMAL STEERING OF MECHANICAL SYSTEMS

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Abstract. A particular class of optimal control problems concerns the steering of motion of a mechanical system from an initial state to a target state (target or trajectory control). The motion of the system depends on forces acting as controls via a set of ordinary differential equations. By considering the equations of motion and the relevant kinematic and kinetic limitations, a constrained optimization problem can be formulated where the control forces are sought to minimize a chosen objective function, such as the energy consumption. In this contribution we study the musculoskeletal motion of a human body considered as a discrete mechanical system[3].

For the numerical solution of the optimal control problem, we introduce a discretization of finite element type in time, whereby the optimality conditions are expressed in weak form. Based on the weak form we may employ previous work on a posteriori error estimates based on the pertinent dual problem to estimate discretization errors in both state and control variables. The sources of errors can be traced to specific regions of the state and control time-meshes, which can be used in an adaptive mesh-refinement procedure, since a particular feature of the proposed scheme is that the force (control) and displacement (state) variables are discretized separately.

Earlier work on a posteriori error estimation for optimal control problems have been based on the "optimal control" approach[1,4], whereas the present contribution will use our previous work in error control for parameter identification problems based on a tangent form of the dual problem[2]. Numerical examples indicate that a discretization error in the control variable arises in order to "compensate" for discretization errors in the solution of the equations of motion.

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