Optimization of 16-point Ring Constellations in the Presence of Nonlinear Phase Noise

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Abstract: The optimum radius distribution in terms of the average symbol error rate (SER) for different 16-point signal sets is derived numerically for a fiber channel limited by nonlinear phase noise. The results show up to 2.4 dB performance improvement.

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1. Introduction

The high demand for increasing the data rate of fiber-optical channels has attracted tremendous efforts to design signal sets with higher spectral efficiencies [1,2]. Among the fiber-induced impairments, nonlinear phase noise (NLPN) is a major limitation particularly in long-haul transmission [3]. NLPN is caused by the interaction between the signal and amplified spontaneous noise (ASE) via the fiber Kerr nonlinearity. Different approaches have been investigated for combating the effect of NLPN, e.g., optical hardware methods, electronic compensation with predistortion, and post compensation. Lau and Kahn [4] proposed a maximum likelihood (ML) detector for phase-shift keying modulated signals and a two-stage detector for the 16-QAM constellation. Although this method performs close to the ML detector in the highly nonlinear regime, there is no consideration about optimizing the 16-point constellation.

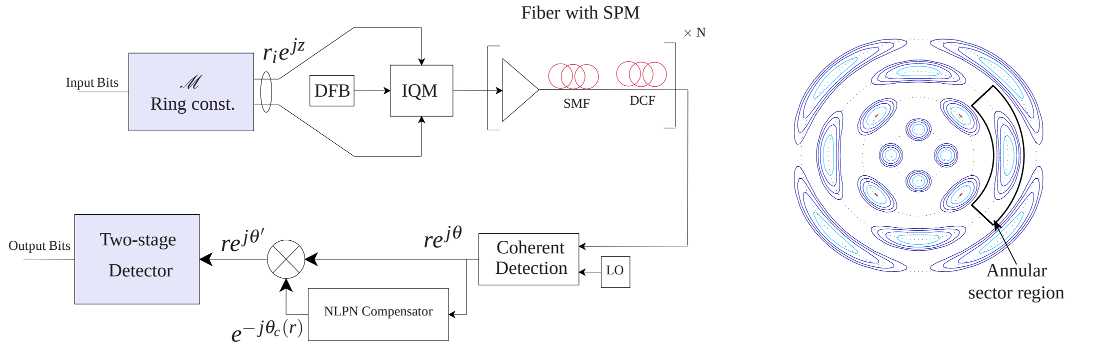
In this paper, we consider the nonlinear postcompensator proposed in [4] for a fiber link with a distributed amplification. A ring constellation with 16 points is optimized for this system to make it robust against the residual phase noise after the nonlinear compensator. For this purpose, different geometrical shapings are examined in terms of the minimization of the average symbol error rate (SER) for a wide range of transmitted powers. The numerical results show that the radii of the rings can be chosen so that 2.4 and 2 dB performance improvements are achieved in the linear and nonlinear regime, respectively. This is an extension of previous results which were limited to the nonlinear regime [5].

2. System Model

We consider a fiber-optical link with SPM produced via the Kerr effect, exploiting a distributed amplification which compensates the fiber loss completely [4]. The system uses a 16-point ring constellation consisting of I rings with K_i, $i=0,\dots,I$, equally spaced phase points with a nonreturn-to-zero (NRZ) pulse shape. The vector $(r_1,\dots,r_I)$ represents the scaled radius distribution of the constellation such that $\sum_{i=1}^{I} K_i r_i^2 = 16P_{\mathrm{t}}$, where P_{t} is the average transmitted power. In this paper, we use $(K_1 - K_2 \dots - K_I)$ to represent a ring constellation with $K_1,\dots,K_I$ points in rings $1,\dots,I$, respectively. According to the proposed block diagram in Fig. 1(a), the mapper $\mathscr{M}$ produces complex I/Q symbol $r_i e^{jz_k}$, where $z_k = \frac{2k\pi}{K_i}$, $0 \le k \le K_i - 1$. The optical I/Q modulator (IQM) transforms the generated complex symbol to an I/Q modulated signal. The ASE noise generated by inline amplifiers is modeled as complex zero-mean circularly symmetric Gaussian random variables with variance σ^2 [4] in two polarizations for a fiber link with the total length L. As in [6] and [4], we also assume that the sole impairment is the noise within the bandwidth of the optical signal, neglecting the effect of chromatic dispersion.

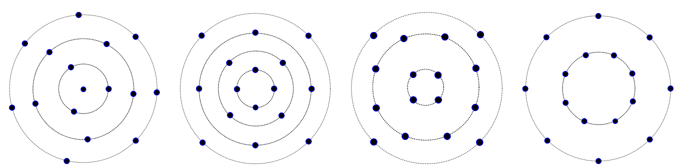
The normalized received amplitude r is defined as the amplitude of the received electric field at the output of the coherent receiver, divided by σ. The amplitude-dependent phase rotation $\theta_c(r)$ (caused by SPM) of the received signal is removed by the nonlinear maximum likelihood NLPN compensator [4] (as shown in Fig. 1(a)). Finally a two-stage detector extracts the transmitted information bits by first detecting the ring and then the phase from the compensated received signal. Due to exploiting this suboptimal detector, any phase offset between the rings doesnot change the performance of the system. Here, using the results in [4], the joint probability density function (pdf) of the normalized amplitude (r) and compensated phase (θ') for a received symbol from an M-PSK constellation with transmitted phase θ_0 and transmitted power P_{t}, is obtained by

$$f_{R,\Theta'}(r,\theta') = \frac{f_R(r,\rho)}{2\pi} + \frac{1}{\pi}\sum_{m=1}^{\infty} |C_m(r)| \cos\left[m\left(\theta' - \theta_0\right)\right],$$



(a) (b)

Fig. 1. a) The system block diagram. b) The pdf of the complex signal $re^{j\theta'}$, with contour values $10^{-3}, 10^{-2.5}, \ldots, 1$ ($L = 5000$ km and $P_\mathrm{t} = 0$ dBm).



(a) (1-3-5-7) (b) (4-4-4-4) (c) (4-8-4) (d) (8-8)

Fig. 2. Ring constellations from [1, 4]

where $f_R(r, \rho)$ is the Ricean pdf of the received amplitude, $\rho = P_\mathrm{t}/\sigma^2$ is the signal-to-noise ratio (SNR), and the Fourier coefficients $C_m(r)$ are given in [4]. This pdf was derived for a large number of fiber spans ($N > 32$).

3. SER of 16-point Ring Constellations

In this section, we analyze the SER of a 16-point ring constellation exploiting the two-stage detector [4]. In this detection method, the annular sector (see Fig. 1(b)) is used instead of the exact Voronoi region as the decision region in the detector. Since the distribution of noise in the radius direction is Ricean, the ML decision boundary circles with radii $\mu_i,\ i = 1, \ldots, I-1$, between rings are obtained by intersecting the two Ricean pdf's. Here, μ_i is the radius of the circle which is the ML decision boundary between rings i and $i+1$ (normalized by σ). We compute the probability of correct detection of a transmitted symbol $s_i = r_i e^{j0}$ selected from ring i and initial phase $\theta_0 = 0$ by $P_{\mathrm{c}_i} = \Pr\{R \in [\mu_{i-1}, \mu_i) \wedge \Theta' \in [-\pi/K_i, \pi/K_i)\}$, where $i = 1, \ldots, I$, $\mu_0 = 0$, and $\mu_I = \infty$. Using the symmetry of the ring constellation, it is readily seen that the total SER of a16-point ring constellation is $\mathrm{SER} = 1 - \sum_{i=1}^{I} K_i P_{\mathrm{c}_i}/16$.

This probability can be computed by taking the integral of the pdf over the annular sector of the symbols in different rings i and eventually we obtain

$$\mathrm{SER} = 1 - \frac{1}{16}\sum_{i=1}^{I}\left(P_{r_i} + K_i \sum_{m=1}^{\infty} \frac{2\eta_{m,i}}{m\pi} \sin\left(\frac{m\pi}{K_i}\right)\right), \tag{1}$$

where

$$P_{r_i} = Q\left(\sqrt{2}\frac{r_i}{\sigma}, \sqrt{2}\mu_{i-1}\right) - Q\left(\sqrt{2}\frac{r_i}{\sigma}, \sqrt{2}\mu_i\right), \quad \eta_{m,i} = \int_{\mu_{i-1}}^{\mu_i} |C_m(r)|\mathrm{d}r,$$

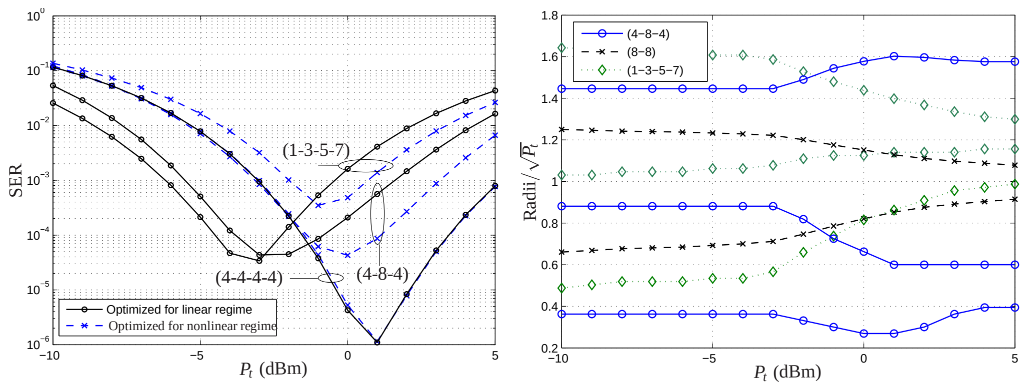
and $Q(x, y)$ is the Marcum Q function [7].

4. Optimizing the 16-point Ring Constellation for a Fiber Channel with NLPN

We carry out the optimization of a ring constellation with 16 points in two steps: First, we decide the number of points in each ring. In the second step, exploiting a numerical optimization technique, the radius distribution of the constellation is determined for a given SNR in order to minimize the SER of the system. Since large amplitude values contribute more NLPN, for high transmitted power (i. e., in the nonlinear regime), it is advantageous to have few points in the outer rings. On the other hand, the inner rings are more vulnerable to noise at low transmitted power (in the linear regime). Overall, the 16-point constellations shown in table 1 are selected in the first step based on the structures introduced in [1, 4] (see Fig. 2).

Table 1. The optimized16-point ring constellations for $L = 3500$ km.

Ring constellation $(K_1, \ldots, K_I)$	Linear regime radius distribution $(r_1, \ldots, r_I)/\sqrt{P_t}$, $P_t = -10$ dBm	Nonlinear regime radius distribution $(r_1, \ldots, r_I)/\sqrt{P_t}$, $P_t = 0$ dBm	[ref]
(1-3-5-7)-Fig. 2(a)	(0, 0.49, 1.03, 1.64)	(0, 0.82, 1.13, 1.44)	[1]
(4-4-4-4)-Fig. 2(b)	(0.28, 0.63, 0.98, 1.6)	(0.283, 0.625, 0.981, 1.6)	[1]
(4-8-4)-Fig. 2(c)	(0.36, 0.88, 1.45)	(0.27, 0.66, 1.58)	[4]
(8,8)-Fig. 2(d)	(0.67, 1.25)	(0.82, 1.15)	[1]



(a) (b)

Fig. 3. a) The optimized radii of (4-8-4), (1-3-5-7), and (8-8) versus P_t. b) The SERs of the optimized ring constellations.

Fig. 3(a) shows the SERs of the numerically optimized ring constellations introduced in table 1 based on the optimization in linear ($P_t = -10$, solid lines) and nonlinear ($P_t = 0$, dashed lines) regime for three of the constellations introduced in table 1. As expected, constellations having few symbols in the inner ring is best in the linear limit, whereas few symbols in the outer ring is optimal in the nonlinear regime. Moreover, the radius distribution of the ring constellations (4-8-4) and (4-4-4-4) for different transmitted powers have been shown in Fig. 3(b). In the simulations, we assumeda bandwidth of the optical signal 42.7 GHz, a spontaneous emission factor of 1.41 [4] at a wavelength of 1550 nm, an attenuation coefficient of 0.25 dB/km, and a fiber length of 3500 km. As seen in Fig. 3(a), the performance of the (4,8,4) constellation can be improved by 2.4 and 2 dB by using a suitable radius distribution in the linear and nonlinear regime, respectively at SER = 10^{-4}.

5. Conclusions

Optimization of some 16-point ring constellations have been performed in order to minimize the symbol error rate. This optimization is performed for a two-stage detector, which is suitable for practical implementation. The results illustrate that one may gain 2.4 dB SER performance in the linear regime by exploiting the radius distribution in this regime rather than nonlinear limit. It can also be noticed that in the nonlinear regime 2 dB gain is achieved with this approach in a dispersion-managed fiber channel limited by nonlinear phase noise.

References

1. R. J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, “Capacity limits of optical fiber networks,” *J. Lightw. Technol.*, vol. 28, no. 4, pp. 662–701, Feb. 2010.
2. E. Agrell and M. Karlsson, “Power-efficient modulation formats in coherent transmission systems,” *J. Lightw. Technol.*, vol. 27, no. 22, pp. 5115–5126, Nov. 2009.
3. J. Renaudier and G. Charlet, “Coherent detection for long haul transmission systems,” *IEEE Laser, Electro-Optics Society Annual Meeting*, pp. 214–215, Oct. 2007.
4. A. P. T. Lau and J. M. Kahn, “Signal design and detection in presence of nonlinear phase noise,” *J. Lightw. Technol.*, vol. 25, no. 10, pp. 3008–3016, Oct. 2007.
5. M. C. Niaz and L. Beygi, “Optimization of 16-QAM constellation in the presence of nonlinear phase noise,” *Proceedings of the 6th International Conference on Frontiers of Information Technology*, Dec. 2009.
6. K. P. Ho and J. M. Kahn, “Electronic compensation technique to mitigate nonlinear phase noise,” *J. Lightw. Technol.*, vol. 22, no. 3, pp. 779–783, Mar. 2004.
7. G. J. Proakis and M. Salehi, *Digital Communications*, 5th ed. McGraw-Hill, 2008.