Optimal Modulation for Cognitive Transmission over AWGN and Fading Channels

Mohammad Reza Khanzadi, Kasra Haghighi and Thomas Eriksson
Department of Signals and Systems, Communication Systems Group
Chalmers University of Technology, Gothenburg, Sweden
{*khanzadi, kasra.haghighi, thomase*}*@chalmers.se*

***Abstract*—This paper proposes a new modulation method for an uncoded cognitive transmission (secondary user transmission) in presence of a Primary User (PU) for AWGN and time-varying flat-fading channels. Interference symbol of the PU is assumed to be known at the transmitter of the Cognitive User (CU) beforehand. Based on this knowledge and using a symbol by symbol approach, we design a CU modulation which can fulfill the coexistence conditions of the CU and the PU. The proposed method is a low-complexity modulation approach in a single (complex-valued) dimension rather than a high dimensional coding scheme, but still it achieves good performance. The robustness of the method is also investigated in case of having an imperfect knowledge about the PU transmitted symbols. An implementation algorithm for our modulation method is presented and its performance is evaluated by experiments.**

***Index Terms*—Cognitive Radio, Fading Channel, Interference Channel, Modulation, Uncoded Communication, Interference Avoidance, Imperfect Side Information.**

I. INTRODUCTION

Cognitive radio [1] is recommended as an option for dynamic and secondary spectrum licensing to overcome the problem of overcrowded and insufficient licensed spectrum [2], [3].

In previous studies, different general techniques for cognitive transmission in presence of the primary (licensed) users have been introduced (e.g., the interweave and overlay techniques [4]). In the interweave technique, the cognitive user (CU) takes advantage of the vacant frequency holes in the spectrum of the primary user (PU). The CU exploits different spectrum sensing methods to find these unoccupied segments of the licensed spectrum of the PU and adapt its transmission to these free frequency bands [5]. On the contrary, in the overlay technique, the CU transmits its information in the same time and frequency as the PU. Having a pre-knowledge about the PU transmitted signal, the CU adapts its transmission to mitigate the interference introduced by the PU transmission while it does not degrade the performance of the PU communication link which is the owner of the licensed frequency band. In this paper our focus is on the overlay technique.

In several information-theoretical studies on the cognitive transmission using the overlay technique (e.g., [6] and [7]), a proper combination of the selfish [8] (dirty paper coding [9]) and selfless [8] scenarios (relay) is suggested in order to fulfill the coexistence conditions [7] of the CU and PU. The coexistence conditions of the cognitive transmission are as follow: Firstly, the PU is not aware of the presence of the CU. It has a fixed transmitter and receiver and is not capable of adapting to the CU's transmission. Secondly, the CU should not degrade the performance of the PU's link by introducing any harmful interference. Although these information-theoretical schemes introduce acceptable rates for coded cognitive radio channels, the infinite length of the codewords (infinite time intervals) and high dimensional coding make them complex for a practical implementation.

To reduce the complexity, a practical method of cognitive transmission in one dimension (a complex-valued dimension) for additive white gaussian noise (AWGN) channel is introduced in [10]. In this work, instead of using the whole sequence of known PU codeword (PU interference), a single transmitted symbol of the PU in each channel use is exploited to produce the transmitted signal of the CU. It is shown that this low complexity method for the uncoded cognitive transmission has a remarkable performance. In [10], the average symbol error probability is used as a measure for evaluating the performance of the CU link. Thus, to design the optimal modulator and demodulator pair of the CU, the demodulator must be redesigned in each round of the modulator optimization. In contrast, in this paper the mutual information [11] between the CU transmitted and received signals is used as the optimization criterion. As this mutual information is not dependent on the demodulation procedure, the demodulator is designed once, after completing the design of the modulator, which makes the optimization less complex.

In addition, we propose a general framework to design the modulator and demodulator of the CU in time-varying flat-fading channels. The method of uncoded cognitive transmission in AWGN case is modified for the fading environment by means of a channel gain distribution quantization technique. The effect of using different power allocation policies on the performance of our method is also investigated in the fading case. Finally, the assumption of having the perfect knowledge about the PU transmitted symbols at the CU transmitter is relaxed and the performance of the method is restudied for this case.

II. MATHEMATICAL FORMULATION OF THE MODEL

Information messages of the PU are represented as a discrete random variable Ω_1, uniformly distributed over the set $\{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$. During each channel use, one of the realizations of the Ω_1 is transmitted. This message is modulated

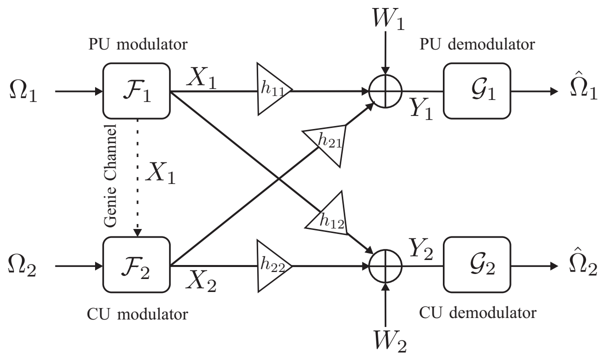

Fig. 1: System Model.

by the modulator function $\mathcal{F}_1 : \{\omega_{1,1}, \ldots, \omega_{1,M_1}\} \to X_1 \in \mathbb{C}$ of the PU. The output of $\mathcal{F}_1$ is the complex-valued transmitted signal of X_1. At the receiver, a complex Gaussian noise W_1, zero mean with variance equal to σ_1^2 is added to the X_1. The received signal $Y_1 = h_{11}\mathcal{F}_1(\Omega_1) + W_1 = h_{11}X_1 + W_1$ is demodulated by the demodulation function $\mathcal{G}_1 : Y_1 \in \mathbb{C} \to \{\omega_{1,1}, \ldots, \omega_{1,M_1}\}$.

In our model, in which the PU has a fixed and non-adapting design, $\mathcal{F}_1$ and $\mathcal{G}_1$ are two fixed functions and cannot be adapted in presence of the CU. For the given demodulator of the PU, decision regions $\mathcal{B}_{\omega_{1,i}}$ are also fixed and can be defined as

$$\mathcal{B}_{\omega_{1,i}} = \{y_1 | \mathcal{G}_1(y_1) = \omega_{1,i}\}, \quad i = \{1, \ldots, M_1\} \tag{1}$$

which is the set of received signals y_1 that results in the output $\omega_{1,i}$ of the demodulator function.

In presence of the CU, the received signal of the PU is $Y_1 = h_{11}\mathcal{F}_1(\Omega_1) + W_1 + h_{21}X_2 = h_{11}X_1 + W_1 + h_{21}X_2$ where X_2 is the complex-valued transmitted signal of the CU that will be introduced in more detail later. Assuming complex Gaussian noise and additive channel, the conditional probability density function (pdf) of the received signal Y_1 given Ω_1 and X_2 can be written as

$$\begin{aligned} & f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i}, x_{2,ij}) \\ & \quad = \frac{1}{2\pi\sigma_1^2} \exp\Big(-\frac{1}{2\sigma_1^2} |y_1 - h_{11}x_{1,i} - h_{21}x_{2,ij}|^2 \Big). \end{aligned} \tag{2}$$

In the single PU case where the CU is not present, the average symbol error probability of the PU using the demodulation function $\mathcal{G}_1(Y_1) = \hat{\Omega}_1$ is equal to

$$P_e(\text{Single PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is not transmitted}). \tag{3}$$

In the presence of the cognitive user, the average symbol error probability is

$$P_e(\text{PU}) = \Pr(\hat{\Omega}_1 \neq \Omega_1 | X_2 \text{ is transmitted}). \tag{4}$$

We assume that the transmitter of the CU is aware of the transmitted symbol of the PU in each channel use by means of a genie aided channel [8]. The receiver of the CU, however, is not aware of this message but only a posterior probability mass function (pmf) of the PU's modulation. The discrete random variable Ω_2 represents information messages of the CU and is defined uniformly over the set $\{\omega_{2,1}, \ldots, \omega_{2,M_2}\}$. The modulator of the CU $\mathcal{F}_2 : \{\omega_{2,1}, \ldots, \omega_{2,M_2}\} \times \mathbb{C} \to X_2 \in \mathbb{C}$ maps Ω_2 and the known transmitted signal from the PU (X_1) to the proper complex-valued signal X_2 which will be transmitted later. At the receiver of the CU, a complex Gaussian noise W_2 with mean zero and variance σ_2^2 is added to this signal. The received signal Y_2 is demodulated by demodulator function $\mathcal{G}_2 : Y_2 \in \mathbb{C} \to \{\omega_{2,1}, \ldots, \omega_{2,M_2}\}$.

The received signal of the CU is $Y_2 = h_{22}\mathcal{F}_2(\Omega_2, X_1) + W_2 + h_{12}X_1 = h_{22}X_2 + W_2 + h_{12}X_1$. Based on the demodulation function $\mathcal{G}_2(Y_2) = \hat{\Omega}_2$, the average symbol error probability for the CU is $P_e(\text{CU}) = \Pr(\hat{\Omega}_2 \neq \Omega_2)$. The conditional pdf of the received signal Y_2 given Ω_2 and X_1 is written as

$$\begin{aligned} & f_{Y_2|\Omega_2,X_1}(y_2|\omega_{2,j}, x_{1,i}) \\ & \quad = \frac{1}{2\pi\sigma_2^2} \exp\Big(-\frac{1}{2\sigma_2^2} |y_2 - h_{12}x_{1,i} - h_{22}x_{2,ij}|^2 \Big). \end{aligned} \tag{5}$$

For the given demodulator of the CU, decision regions $\mathcal{B}_{\omega_{2,j}}$ can be defined as

$$\mathcal{B}_{\omega_{2,j}} = \{y_2 | \mathcal{G}_2(y_2) = \omega_{2,j}\}, \quad j = \{1, \ldots, M_2\}. \tag{6}$$

$\mathcal{B}_{\omega_{2,j}}$ is a set of received signals y_2, where $\omega_{2,j}$ is the result of the CU's demodulator.

Along with the definition of the cognitive radio as a wireless device which can sense and adapt its transmission to the environment [1], $\mathcal{F}_2$ and $\mathcal{G}_2$ (and decision regions $\mathcal{B}_{\omega_{2,j}}$) are not fixed and can be designed adaptively according to the requirements of the different scenarios.

III. CONSIDERATE METHOD

We want to design the optimal modulator and demodulator of the CU for the uncoded cognitive transmission to fulfill the coexistence conditions. The problem is formulated as an optimization in which the focus is on maximization of the performance of the CU link as well as avoiding the possible detrimental effects on the PU performance.

To design the modulator, the mutual information between the transmitted information message Ω_2 and the received signal Y_2 is used as a criterion for the CU link performance in the optimization. This mutual information $I(Y_2; \Omega_2)$ is used for a special case of symbol by symbol cancellation of the known interference in [12]. It is easy to show this mutual information is equal to the communication rate of the CU in this case.

As the CU is limited by its transmission power, we have a constraint on the power of its transmitted signal X_2.

$$\begin{aligned} E|X_2|^2 = & \frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} |\mathcal{F}_2(\omega_{2,j}, x_{1,i})|^2 \\ = & \frac{1}{M_1 M_2} \sum_{i=1,j=1}^{M_1,M_2} |x_{2,ij}|^2 \leq P_{\text{CU}} \end{aligned} \tag{7}$$

where P_{CU} is the maximum allowed CU transmission power.

In addition to the power constraint, another constraint must be added to the optimization in order to guarantee the performance of the PU link. This new constraint can be formed by comparing the performance of the PU in two cases of the absence and presence of the CU. The performance measure which we suggest for the PU communication link is its average symbol error probability.

Based on these definitions, the optimization for design of the modulator $\mathcal{F}_2$ can be written as

$$\begin{aligned} &\underset{x_{2,ij}\in\mathbb{C}}{\text{maximize}}\ I(Y_2;\Omega_2) \\ &\text{subject to}\ \begin{cases} P_e(\text{PU}) \le P_e(\text{Single PU}) \\ E|X_2|^2 \le P_{\text{CU}} \end{cases} \end{aligned} \tag{8}$$

The $I(Y_2;\Omega_2)$ is calculated in (9) at the top of the next page and the PU average symbol error probability in presence of the CU is computed as

$$\begin{aligned} P_e(\text{PU}) &= 1 - \frac{1}{M_1}\sum_{i=1}^{M_1}\int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1}(y_1|\omega_{1,i})\mathrm{d}y_1 \qquad (10) \\ &= 1 - \frac{1}{M_1 M_2}\sum_{i=1,j=1}^{M_1,M_2}\int_{\mathcal{B}_{\omega_{1,i}}} f_{Y_1|\Omega_1,X_2}(y_1|\omega_{1,i},x_{2,ij})\mathrm{d}y_1. \end{aligned}$$

After solving this optimization, the optimal modulator $\mathcal{F}_2$ is given and the decision regions for the correct demodulation are defined based on the maximum likelihood rule (11).

$$\begin{aligned} \hat{\omega}_{2,j} =& \mathcal{G}_2(y_2) \qquad (11) \\ =& \underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\text{argmax}} f_{Y_2|\Omega_2}(y_2|\omega_{2,j}) \\ =& \underset{\omega_{2,j}\in\{\omega_{2,1},\ldots,\omega_{2,M_2}\}}{\text{argmax}} \sum_{i=1}^{M_1}\Big\{ \\ & \exp\Big(-\frac{1}{2\sigma_2^2}|y_2 - h_{12}x_{1,i} - h_{22}\mathcal{F}_2(\omega_{2,j},x_{1,i})|^2\Big)\Big\}. \end{aligned}$$

Now, the secondary transmission is just a lookup table. Based on the PU transmitted signal (message) and the CU transmitted information message, we look inside the designed modulator table and find the proper CU transmitted signal.

IV. CONSIDERATE METHOD IN THE FADING CASE

In this section, we propose a fairly general framework to design the modulator and demodulator of the CU in the time-varying flat-fading channels. Similar to the AWGN case, we maximize the performance of the CU link while the coexistence conditions are fulfilled. We assume that the distribution of each channel (h_{11}, h_{12}, h_{21} and h_{22}) is known for the CU. In addition, the CU is completely aware of the states of the channels during each channel use. These assumptions are provided by the CU capability to listen and observe the channel states. To design the CU modulator and demodulator, the channel gain distributions of all four independent channels are quantized. For example, the continuous channel gain $\gamma_{11} = |h_{11}|$ is quantized to K_{11} discrete samples γ_{11}^i using the quantization regions $[\gamma_{11}^a, \gamma_{11}^b)_{i=1,\ldots,K_{11}}$. $\gamma_{11,i}^a$ and $\gamma_{11,i}^b$ are the boundaries of each quantization region and the probability of being in each region is defined as

$$\alpha_{11}^i = F(\gamma_{11,i}^b) - F(\gamma_{11,i}^a) \tag{12}$$

where $F(\gamma_{11})$ is the cumulative distribution function (cdf) of the h_{11} channel gain.

All four independent channel gain distributions ($\gamma_{11}, \gamma_{12}, \gamma_{21}$ and γ_{22}) are quantized that results in $K_{11} \times K_{12} \times K_{21} \times K_{22}$ independent combination of the quantized channel gains. The CU modulator and demodulator can be designed for each combination similar to the AWGN case. The constraint for respecting the PU link is its average symbol error probability over all of the quantized channel gain combinations. The CU performance criterion is the average mutual information $I(Y_2;\Omega_2)$ over all different combinations. In order to limit the transmission power of the CU, two different power constraints, namely short-term and long -term average power constraints [13] are used as follow:

A. Short-Term Average Power Constraint

Using the short-term average power constraint [13], there is a constant power limit (P_{CU}) on the transmission power of each combination ($P_{k,l,r,s}$) independently. In order to design the CU modulator in this case, the optimization problem is rewritten as (13). The designed modulator is again a lookup table. The CU can find a suitable transmitted signal from this table in each channel use, knowing the PU transmitted signal and the instantaneous channel gain values.

To design the demodulator, the CU forms the likelihood function (11) for each combination of the channel gain quantized values. The proper likelihood function in each channel use can be found based on the instantaneous channel gain values and the demodulation is done using the received signal.

B. Long-Term Average Power Constraint

Here, a long-term average power constraint strategy [13] is employed. In other words, instead of limiting the average power of each combination independently, the average transmission power over all combination of the quantized channel gain values is limited to P_{CU}. This power constraint can be written as

$$\sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^k\alpha_{12}^l\alpha_{21}^r\alpha_{22}^s P_{k,l,r,s} \le P_{\text{CU}} \tag{14}$$

where $P_{k,l,r,s} \in \mathbb{R}^+$ is the proper transmission power of each combination which also must be found inside the optimization problem.

Hence, the CU can adapt its transmission power based on the channels condition. For instance, assume the h_{22} channel has a small value because of the fading. In this case, the CU transmits with small amount of power. On the other hand, when the value of the interference channel from the CU transmitter to the PU receiver (h_{21}) is small, the CU can transmit with more power without degrading the performance of the PU link. Since the channel values are time-varying,

$$\begin{aligned}I(Y_2;\Omega_2) &= H(\Omega_2) - H(\Omega_2|Y_2)\\ &= \sum_{j=1}^{M_2}\int_{-\infty}^{\infty} f(y_2,\omega_{2,j})\log p(\omega_{2,j}|y_2)\mathrm{d}y_2 - p(\omega_{2,j})\log p(\omega_{2,j}) = \sum_{j=1}^{M_2} p(\omega_{2,j})\int_{-\infty}^{\infty} f(y_2|\omega_{2,j})\log\frac{f(y_2|\omega_{2,j})}{f(y_2)}\mathrm{d}y_2\\ &= \sum_{i=1,j=1}^{M_1,M_2}\Big\{p(\omega_{1,i})p(\omega_{2,j})\int_{-\infty}^{\infty} f(y_2|\omega_{1,i},\omega_{2,j})\log\frac{\sum_{k=1}^{M_1} f(y_2|\omega_{1,k},\omega_{2,j})p(\omega_{1,k})}{f(y_2)}\mathrm{d}y_2\Big\}\\ &= \frac{1}{M_1M_2}\sum_{i=1,j=1}^{M_1,M_2}\Big\{\int_{-\infty}^{\infty} f(y_2|\omega_{2,j},x_{1,i})\log\frac{\frac{1}{M_1}\sum_{k=1}^{M_1} f(y_2|\omega_{2,j},x_{1,k})}{f(y_2)}\mathrm{d}y_2\Big\}.\end{aligned}\tag{9}$$

$$\begin{aligned}&\underset{x_{2,ij}\in\mathbb{C}}{\text{maximize}}\ \sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^k\alpha_{12}^l\alpha_{21}^r\alpha_{22}^s I(Y_2;\Omega_2|\{|h_{11}|=\gamma_{11}^k,|h_{12}|=\gamma_{12}^l,|h_{21}|=\gamma_{21}^r,|h_{22}|=\gamma_{22}^s\})\\ &\text{subject to}\ \begin{cases}\displaystyle\sum_{k=1,l=1,r=1,s=1}^{K_{11},K_{12},K_{21},K_{22}} \alpha_{11}^k\alpha_{12}^l\alpha_{21}^r\alpha_{22}^s P_e(\text{PU})|_{|h_{11}|=\gamma_{11}^k,|h_{12}|=\gamma_{12}^l,|h_{21}|=\gamma_{21}^r,|h_{22}|=\gamma_{22}^s} \le \sum_{k=1}^{K_{11}} P_e(\text{Single PU})|_{|h_{11}|=\gamma_{11}^k}\\ P_{k,l,r,s} = E|X_2|^2 \le P_{\text{CU}}\end{cases}\end{aligned}\tag{13}$$

maximizing the CU performance using the long-term policy is a sort of "water-filling" power adaptation strategy [14]. This strategy results in spending the power in those combinations of the quantized channel gain values for which the CU link has a better performance. The modulator optimization is similar to (13) with a difference in the power constraint as (14). The demodulator is designed in the same way as the short-term average power case.

V. IMPLEMENTATION AND NUMERICAL RESULTS

In this section, the implementation method of our uncoded cognitive transmission for the antipodal binary modulation (BPSK) case is investigated.

In this case, the PU has two information messages $\omega_{1,1} = 0$ and $\omega_{1,2} = 1$ ($M_1 = 2$) with the same probability of transmission. The PU transmission power is P_{PU} and its transmitted signals are $x_{1,1} = -\sqrt{P_{\text{PU}}}$ and $x_{1,2} = \sqrt{P_{\text{PU}}}$.

The CU also has two equal probable information messages $\omega_{2,1} = 0$ and $\omega_{2,2} = 1$ ($M_2 = 2$). The transmitted signals $x_{2,ij}$ must be found for different scenarios.

A. AWGN Case

Our method of implementation is stimulated by the optimal cancellation method of known interference in [12]. Under our assumptions of the BPSK case, there are four ($M_1 \times M_2 = 4$) different choices of $x_{2,ij}$ transmitted signals as below.

	$x_{1,1} = -\sqrt{P_{\text{PU}}}$	$x_{1,2} = \sqrt{P_{\text{PU}}}$
$\omega_{2,1} = 0$	$x_{2,11}$	$x_{2,21}$
$\omega_{2,2} = 1$	$x_{2,12}$	$x_{2,22}$

The probability densities of the $x_{1,i}$, $\omega_{2,j}$ and the white Gaussian noise are symmetric. Therefore, we have $x_{2,ij} \in \{-a,-b,a,b\}$ where a and b are positive real constants. First, a and b must be found. Then $x_{2,ij}$ must be mapped to the set $\{-a,-b,a,b\}$. As a and b are not ordered, there will be $\frac{4!}{2!} = 12$ possibilities for this mapping set.

For implementation of the optimization (8), first the real values between 0 to $\sqrt{P_{\text{CU}}}$ is quantized uniformly and a grid of possible values for the a and b is made. Then the optimization is done as follows:

- **Step 1:** Find all of the combinations of the grid points for a and b that can fulfil the power constraint (7) which can be rewritten as $\frac{a^2+b^2}{2} \le P_{\text{CU}}$.
- **Step 2:** For the set of a and b found in Step 1, form the 12 possibilities of the set $\{-a,-b,a,b\}$.
- **Step 3:** Find all of the combinations from the result of Step 2 which can fulfil the constraint of average symbol error probability of the PU by calculating the $P_e(\text{PU})$ using (10) and comparing the result with the $P_e(\text{Single PU})$.
- **Step 4:** For the result set of the Step 3, the $I(Y_2;\Omega_2)$ is calculated using (9) and the set which can maximize this value is chosen as the proper transmitted signal of the CU ($x_{2,ij}$). The infinite integration inside (9) is computed numerically exploiting the Simpson's rule.

Result of using the CU considerate method is compared with the single user, the optimal interference cancellation [12] and the interference cases in Figure 2. Figure 3 shows the PU link performance for the different scenarios of the Figure 2. The CU cancels out a large portion of the interference by using the optimal cancellation method [12] and its $I(Y_2;\Omega_2)$ is close to the no-interference (single user) case . But as it can be seen in Figure 3, the PU link performance is degraded and its P_e is increased. The CU in considerate scenario performs much better than the interference case (interference without cancellation). On the other hand, its performance is degraded compared to the optimal cancellation case. However, this degradation is the result of the same symbol error probability for the PU link before and after presence of the CU (Figure 3). Figure 2 also depicts the effect of changing the PU link performance in the single user case on the CU performance in the considerate method. Improving the performance of the PU link (decreasing the P_e), the CU must care more about the PU link compared to its own link. As a result, the selfless side of the method is dominant compared to the selfishness and performance of the CU link is decreased.

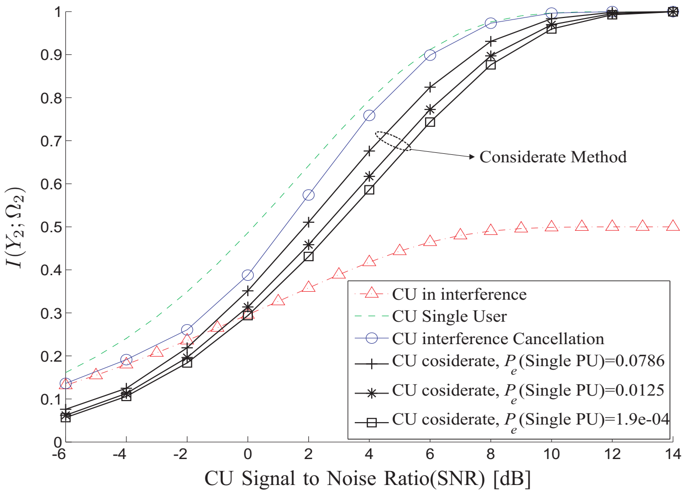

Fig. 2: The CU performance in terms of mutual information vs. the CU SNR in different AWGN scenarios. Transmission power of the CU = 1, Transmission power of the PU = 1.

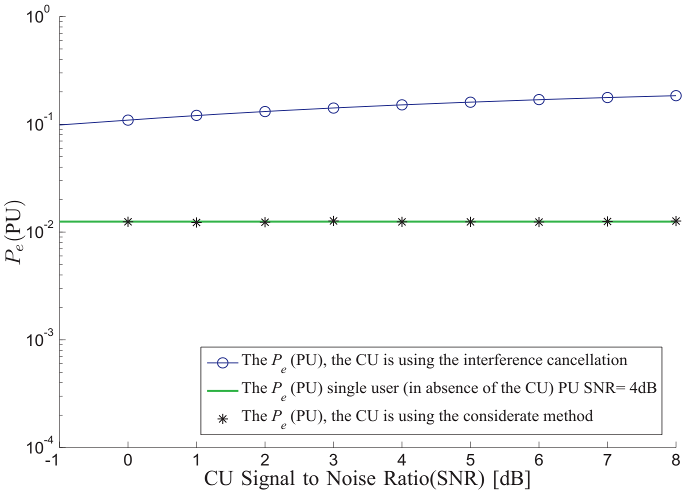

Fig. 3: The PU performance in AWGN scenarios vs. CU SNR. PU SNR = 4 dB.

B. Fading Case with Short-Term Average Power Constraint

We assumed the Rayleigh distribution for each independent channel gain. To implement the optimization (13), first the channel gain distributions are quantized using two levels of quantization ($K_{11} = 2, K_{12} = 2, K_{21} = 2, K_{22} = 2$). The four step optimization method of the AWGN case is used independently for each of the sixteen combinations to fulfill the optimization criteria. The final performance measure is the average of $I(Y_2;\Omega_2)$ over all of the combinations. Figure 4 compares the results of considerate method in the Rayleigh fading environment ($\sigma^2 = 0.1$) using the short-term average power constraint and the AWGN case. We also extend the results of the optimal cancellation method [12] to the fading case by means of our channel gain quantization method. The performance of the considerate method with fading is generally less than the AWGN case. But as it can be seen in this figure, the considerate method performance in the fading case is closer to the single user result compared to the AWGN case. As it is discussed before, this improvement is the result of fading in the interference channels (h_{12} and h_{21}) and the PU direct link (h_{11}). Results of the fading case also show that our method performs well even with a low number of quantization levels.

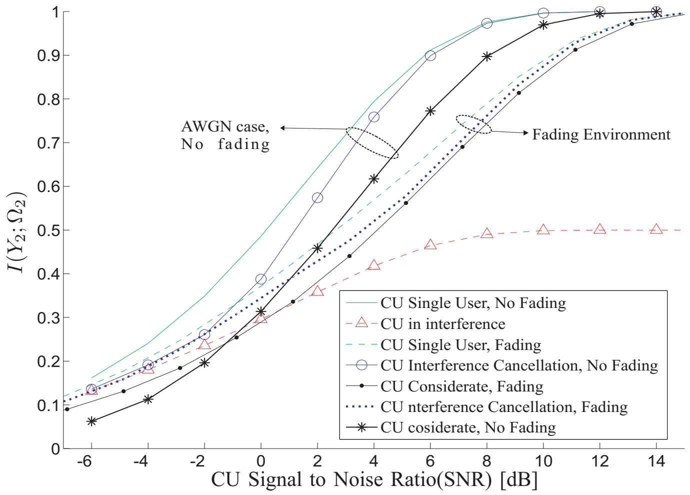

Fig. 4: Comparing the performance of the CU in fading (short-term power policy) and AWGN cases. Transmission power of the CU = 1, transmission power of the PU = 1 and average SNR of the PU = 4 dB.

C. Fading Case with Long-Term Average Power Constraint

The transmission power of each combination is not limited to a constant value in the long-term average power policy and it must be optimized during the optimization. A vector of dynamic power constraints, each elements corresponds to one of the sixteen possible combinations is defined with the initial value of P_{CU}. A numerical gradient decent method with constraint over average power of all combinations is exploited to assign the optimal power to each combination. This power allocation method (water-filling) besides the procedure used in the short-term case implements the optimal CU modulator. Figure 5 compares the results of the considerate method using the short-term and long-term average power policies in the fading environment. By using the long-term method (water-filling), the CU link performance is improved due to the wiser power allocation technique.

VI. IMPERFECT KNOWLEDGE OF THE PU TRANSMITTED SYMBOLS

In the previous sections, the CU was assumed to have a perfect knowledge about the PU transmitted symbols beforehand by means of a genie aided channel. Practically, it means that, we assumed an instantaneous ideal channel between the PU and CU transmitters. Due to the imperfections, a more realistic assumption is that the CU must detect the PU transmitted symbols through an AWGN channel, and transmit the proper signal based on this knowledge and using the designed modulator. Since this AWGN genie channel is noisy, the CU acquires imperfect knowledge of the PU transmitted symbols. Any error in detecting the PU symbols results in a wrong choice of CU transmitted signal. This will cause the performance of the CU to decrease as well as introducing harmful interference into the PU link, which is against the

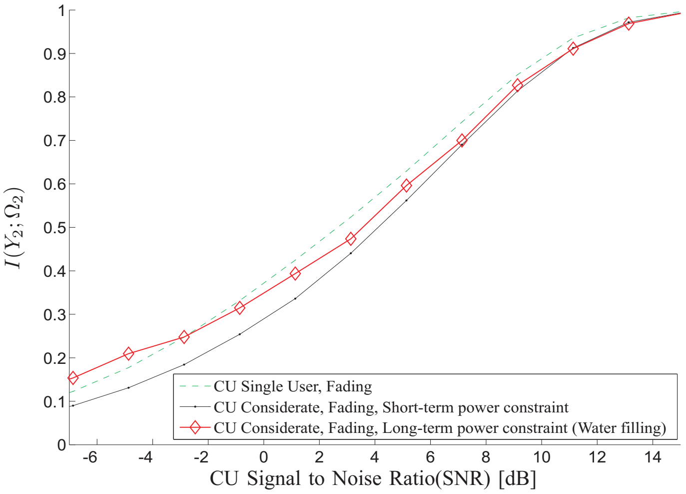

Fig. 5: Comparing the performance of CU using short-term and long-term power policies in a fading environment. Transmission power of the CU = 1, Transmission power of the PU = 1.

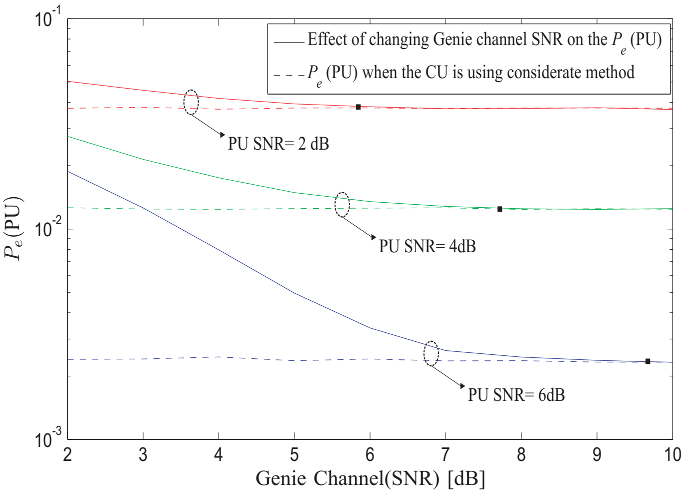

Fig. 6: Effect of changing the genie channel quality (SNR) on the PU performance in AWGN case.

co-existence conditions. The PU link performance in the considerate method for the AWGN channel is evaluated vs. the quality of the genie channel in Figure 6. There is a SNR value of the genie channel in which the quality of the PU link decreases from the perfect knowledge case. This genie channel SNR value is not the same for the PU links with different qualities. To be more specific, the quality of the PU link is a function of the difference between the quality of the PU link and the genie channel. In general, the simulations for the BPSK case show that if we have an imperfect genie channel with the SNR about 4dB higher than the direct PU link, the performance of the method is close to the case in which the CU has the perfect knowledge of the PU transmitted symbols. Broadly speaking, if we assume the path loss [14] as the only factor that decreases the received power, it can be concluded that the distance between the PU and CU transmitters must be less than 0.6 of the distance between the PU transmitter and receiver using the free-space path loss model [14].

VII. CONCLUSION AND FUTURE WORK

In this paper, methods of designing the optimal modulator and demodulator are proposed for the uncoded cognitive transmission in the AWGN and fading channels. Our numerical results show that the CU in this method achieves a notable performance increase without introducing any detrimental effect on the performance of the licensed user. Hence, it can communicate in the same frequency band as the primary (licensed) user. In the fading environment, the channel gain quantization is used to design the optimal CU modulator and demodulator. The long-term average transmitted power policy (water-filling) yields a better performance compared to the short-term strategy. The effect of having imperfect knowledge about the PU transmitted symbols on the performance of the method is also investigated.

In the case of the imperfect knowledge of the PU symbols, the important fact of delay in detecting the PU symbols in the transmitter of the CU should also be considered. The methods to compensate the effects of such a delay can be investigated in the future studies. In the fading case, the performance can be improved by generalizing the method, for example, by taking the quantization regions as unknown parameters into account to be found inside the optimization. This method also has the potential to be extended to the higher number of symbols (higher dimensions) instead of the symbol by symbol strategy which can improve the performance of the CU link.

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