

Magnetomotive instability and generation of mechanical vibrations in suspended semiconducting carbon nanotubes

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Abstract. We investigate the electromechanics of a freely suspended semiconducting carbon nanotube subjected to a magnetic field H in the current-biased regime and show that self-excitation of mechanical nanotube vibrations can occur if H exceeds a critical value H_c of the order of 10–100 mT. The effect can be detected by measuring the magnetic field dependence of the time-averaged voltage drop across the nanotube, which has a singularity at $H = H_c$. We discuss the applications of the device as an active, tuneable radiofrequency oscillator.

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Nanoelectromechanical systems (NEMS) containing a suspended carbon nanotube (CNT) vibrating at radiofrequencies (RF) have received increasing attention recently. The advantages of CNT mechanical resonators include their high resonance frequencies (up to the GHz range), their low dissipative losses [1] and the possibility of tuning the resonance frequency by adjusting the tension in the tube [2, 3]. CNT-based NEMS devices have already shown a great potential for a plethora of technological applications, including mass sensing [4, 5] and tunable high-frequency electronics [2, 3, 6, 7]. However, most of the devices that have been realized so far are passive resonators, which perform frequency filtering of the incoming RF-signal [2, 3, 6, 7]. Here, we propose an active oscillator based on a current-biased doubly-clamped CNT for which the conductance depends monotonically on the nanotube deflection. Such a deflection-sensitive resistance has been demonstrated for semiconducting single-walled CNTs suspended over a gate electrode [2]–[4, 8] and has been used for the detection of their vibrations [2]–[4]. The active feedback is provided by the Lorentz force induced by a constant magnetic field directed perpendicular to the current-carrying CNT. We show that, by applying a constant external current in a sufficiently high magnetic field, we obtain mechanical instability leading to self-excitation of mechanical oscillations at a frequency close to the mechanical resonance frequency of the doubly-clamped nanotube. Furthermore, we show that the mechanical instability results in oscillations of the voltage drop across the tube accompanied by a deviation of the time-averaged voltage from the value obtained for a static nanotube.

The proposed oscillator device is shown in figure 1. A semiconducting CNT is suspended over a gate electrode and subjected to a homogeneous magnetic field H perpendicular to the nanotube and parallel to the gate electrode. The separation between the gate and the nanotube depends on the deflection $z(t, x)$ of the tube from its equilibrium straight configuration (x is the coordinate along the nanotube). We will assume that the nanotube mechanics is completely characterized by the amplitude of its fundamental bending mode $u(t)$, and, consequently, let the time dependence of the mechanical deflection have the form $z(t, x) = u(t)\varphi_0(x)$, where $\varphi_0(x)$ is the normalized profile of the fundamental mode [9]. This assumption captures the essence of the physics behind the phenomena to be considered below, for which the dependence of the mutual capacitance $C_G(u)$ between the nanotube and gate on the amplitude of the fundamental mode is central. This is because the concentration of charge carriers in the semiconducting CNT $\rho(C_G(u), V_g)$ is a function of both the gate voltage V_G and $C_G(u)$, which means that ultimately the resistance $R(\rho(C_G(u), V_G))$ of the nanotube will depend on its deflection.

When an external current-source feeds a current I_0 to one of the leads, a Lorentz force proportional to the current through the CNT and the magnetic field causes a deflection of the tube. For displacements on a scale much smaller than the length of the nanotube, its curvature is negligible and consequently one can consider the Lorentz force to be the same at every point. Since the force is almost uniform over the tube, it is mainly the fundamental bending mode that will be affected, which provides another justification for our assumption that all higher modes can be neglected. Assuming the CNT to be an elastic beam whose motion can be described by linear continuum mechanics, we arrive at the following set of equations⁶ for the time evolution of the voltage drop $V(t)$ across it and the amplitude of the fundamental mode $u(t)$:

$$\begin{aligned} m\ddot{u}(t) + \gamma\dot{u}(t) + \kappa u(t) &= LHI, \\ C\dot{V}(t) &= I_0 - I, \quad I = V/R(u(t)). \end{aligned} \tag{1}$$

⁶ Since the voltage bias across the trench needs to be relatively high, typically a few mV, a simple estimation shows that the magnetically induced electromotive force can be ignored.

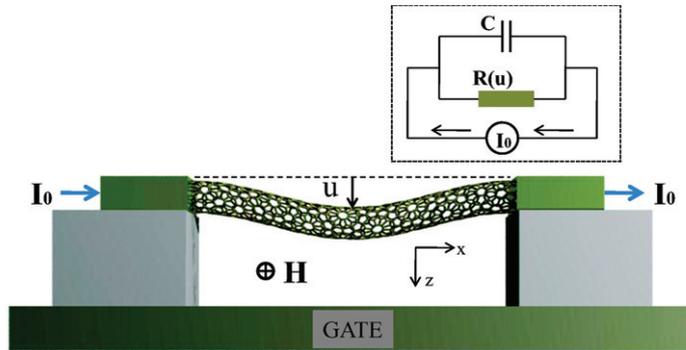


Figure 1. Sketch of the proposed active oscillator device. A semiconducting CNT is suspended over a gate electrode and connected to an external dc current source. An external magnetic field, applied perpendicular to the direction of the current, gives rise to a Lorentz force that deflects the tube towards the gate, which affects the resistance and provides a feedback mechanism that for large enough magnetic fields leads to self-sustained nanotube oscillations (see text). The inset shows an equivalent electric circuit of the device.

Here, m and L are the effective mass and length of the suspended part of the nanotube, κ and γ are the effective spring and damping constants and C is the capacitance of the junction.

The system of equations (1) has a time-independent solution given by

$$u(t) = u_0 = LHI_0/\kappa, \quad (2)$$

$$V(t) = V_0 = R\left(\frac{LHI_0}{\kappa}\right)I_0. \quad (3)$$

A linear stability analysis of this solution yields a secular equation for the Lyapunov exponents λ of the form

$$P(\lambda, \beta) = (\lambda^2 + Q^{-1}\omega_0\lambda + \omega_0^2)(\lambda + \omega_R) - \beta\omega_0^2\lambda = 0, \quad (4)$$

where $\omega_0 = \sqrt{\kappa/m}$ is the eigenfrequency of the fundamental mode, $\omega_R = 1/R(u_0)C$, and $Q = \sqrt{\kappa m}/\gamma$ is the quality factor. The parameter

$$\beta = \frac{LHI_0}{\kappa\ell_R}, \quad \text{where } \ell_R \equiv \left(\frac{-1}{R(u)} \frac{\partial R(u)}{\partial u} \Big|_{u=u_0} \right)^{-1}, \quad (5)$$

is proportional to the Lorentz force and will be referred to as the magnetomechanical coupling parameter of the system.

A rough estimate of the characteristic length ℓ_R can be obtained from the Drude-model conductance of a fourfold degenerate one-dimensional (1D) semiconducting CNT of length L and radius r_0 . Assuming that the CNT is suspended over a large gate electrode that is kept at a fixed voltage V_g , we follow Zhou *et al* [15] and approximate the conductance in the limit of low carrier densities as

$$G \approx G_0 \left(\frac{3\pi r_0}{4e} c_g V_g \right)^2. \quad (6)$$

Here, c_g is the gate capacitance per unit length and $G_0 = (2e^2/\pi\hbar)(l_0/L)$, where l_0 is the electronic scattering length at high energies. Since $c_g = c_g(u)$ depends on the distance $L_g = h - u$ between the nanotube and gate, it is the only parameter in (6) that varies with the deflection u . Because $R(u) = 1/G(u)$ it follows that the characteristic length can be expressed as

$$\ell_R \equiv \left(\frac{2}{c_g(u)} \frac{\partial c_g(u)}{\partial u} \Big|_{u=u_0} \right)^{-1}. \quad (7)$$

In this approximation, the characteristic length is entirely defined by the geometry of the system and does not depend on the gate voltage V_g .

The capacitance of a CNT suspended a distance h above the gate electrode can be approximated by that of a cylinder above an infinite plate. Provided that $r_0 \ll h \ll L$ and assuming that the bending, u , of the CNT is small compared to h one finds that $c_g = 2\pi\epsilon_0/\ln(2h/r_0)$. Using this result in (7), we estimate that $\ell_R \sim (h/2) \ln(2h/r_0)$, which for the reasonable experimental parameters $r_0 = 1$ nm and $h = 0.2$ μm yields $\ell_R = 0.6$ μm .

It turns out that there is a simple relation between the value of the coupling parameter β and the sign of the differential resistance in the static regime. Indeed, from the time-independent solution (3), we obtain the relation

$$\frac{dV_0}{dI_0} = R(u_0)(1 - \beta), \quad (8)$$

from which we see that the differential resistance in the time-independent regime is positive if $\beta < 1$, while for $\beta > 1$ it is negative.

For a high quality factor, $Q \gg 1$, and weak magnetomechanical coupling, $\beta \ll 1$, the (approximate) solutions of equation (4)—together with a third root that is always real and negative—are

$$\lambda_{1,2} = \frac{\omega_0}{2} \left(\frac{\omega_0\omega_R}{\omega_R^2 + \omega_0^2} \beta - \frac{1}{Q} \right) \pm i\omega_0 \left(1 - \frac{\beta}{2} \frac{\omega_0^2}{\omega_R^2 + \omega_0^2} \right). \quad (9)$$

From equation (9) it follows that the two Lyapunov exponents $\lambda_{1,2}$ have a positive real part when

$$\beta > \beta_c = \frac{1}{Q} \left(\frac{\omega_0}{\omega_R} + \frac{\omega_R}{\omega_0} \right). \quad (10)$$

This means that for large quality factors, $Q \gg \max(\omega_0/\omega_R, \omega_R/\omega_0)$, the regime of time-independent charge transport through a static CNT becomes unstable with respect to self-excitation of mechanical vibrations if the coupling parameter β exceeds the critical value β_c . The natural way of reaching this self-excitation regime is to increase the magnetic field H , since increasing I_0 could lead to overheating. Hence, we choose to characterize a particular setup by the critical magnetic field H_c above which the system will be unstable. From equation (10) it follows that for large quality factors and $\omega_0 = \omega_R$ the critical magnetic field is⁷

$$H_c = \left(\frac{2\ell_R}{L} \right) \frac{\kappa}{QI_0}. \quad (11)$$

⁷ Using the estimate $\omega_0/2\pi \approx 0.1$ –1 GHz the optimal value of the capacitance, C_{opt} , is found to lie approximately in the interval 1–100 fF.

If we take $\ell_R \approx 0.1\text{--}1\ \mu\text{m}$, $L \approx 1\ \mu\text{m}$, $\kappa \approx 10^{-5}\ \text{N m}^{-1}$, $Q \approx 1000$ and $I_0 \approx 0.1\ \mu\text{A}$, we obtain a critical magnetic field of the order of $H_c \sim 10\text{--}100\ \text{mT}$.

So far we have analyzed the situation when the mechanical friction was very small. It turns out that the time-independent regime becomes unstable for large enough magnetomechanical coupling even if the mechanical subsystem is overdamped, that is, if $Q \ll 1$. Indeed, the system becomes unstable when the real part of the Lyapunov exponents changes sign from negative to positive. It means that at a certain critical point $\beta = \beta_c$, the secular equation (4) has two purely imaginary roots $\lambda_{1,2} = \pm i\omega_c$. These are obtained from the two equations

$$\begin{aligned} \text{Re } P(i\omega_c, \beta_c) &= \omega_R \omega_0^2 - \omega_c^2 (\omega_R + Q^{-1} \omega_0) = 0, \\ \text{Im } P(i\omega_c, \beta_c) &= -\omega_c (\omega_c^2 - Q^{-1} \omega_0 \omega_R - \omega_0^2 (1 - \beta_c)) = 0, \end{aligned} \quad (12)$$

whose general solution is

$$\begin{aligned} \beta_c &= \frac{1}{Q} \frac{\omega_0 \omega_R + Q(\omega_0^2 + \omega_R^2)}{\omega_0(\omega_0 + Q\omega_R)}, \\ \omega_c &= \omega_0 \sqrt{\frac{Q\omega_R}{\omega_0 + Q\omega_R}}. \end{aligned} \quad (13)$$

It is important to note that β_c is always positive. In particular, from this it follows that the instability can occur only when the direction of the magnetic field is such that it deflects the nanotube towards decreasing resistance. In our geometry, it means that the Lorentz force must be directed towards the gate electrode.

From equations (13) it follows that for small quality factors, $Q \ll \max(1, \omega_0/\omega_R)$, and when the coupling parameter β has just overcome the critical value $\beta_c \cong \omega_R/(Q\omega_0)$, the static regime becomes unstable with respect to oscillations at angular frequency $\omega_c \approx \sqrt{\omega_0 \omega_R Q}$, which is always smaller than the mechanical resonance frequency. It is worth noting that, for an overdamped mechanical system, the critical value of the coupling parameter is bigger than unity, and as discussed previously this implies that the instability in this regime could be considered to be caused by a negative differential resistance. Estimating the critical magnetic field with the same parameters as before, but now with $Q = 0.1$, we obtain $H_c \approx 100\text{--}1000\ \text{T}$, which is too large for experimental observation. Therefore, for the remaining part we will focus our attention on the low-dissipation regime, which can be achieved at least at low temperatures [1, 10].

The way in which the instability evolves is largely dependent on the magnitude of β . For a coupling parameter just above the threshold, $\beta_c \lesssim \beta \ll 1$, we may analyze the development of the instability with the ansatz $u(t) = u_0 + A(t) \sin(\omega_0 t)$, assuming $A(t)$ to be a slowly varying function on the scale $1/\omega_0$. Substituting this ansatz into equation (1) and averaging over the rapid oscillations [11], we obtain an equation for \dot{A} ,

$$\dot{A}(t) = a_1 \omega_0 A(t) \left[\left(\frac{\beta - \beta_c}{\beta} \right) + b_1 \frac{A^2(t)}{(2\ell_R)^2} \right], \quad (14)$$

where

$$\begin{aligned} a_1 &= \frac{\beta}{2} \frac{\omega_0 \omega_R}{\omega_0^2 + \omega_R^2}, \\ b_1 &= \frac{4\omega_0^4 - 5\omega_0^2 \omega_R^2 + 3\omega_R^4}{2(\omega_0^2 + \omega_R^2)(4\omega_0^2 + \omega_R^2)} + \frac{1}{2} \left(\frac{3\omega_R^2 - \omega_0^2}{\omega_0^2 + \omega_R^2} \right) \frac{\partial \ell_R}{\partial u_0} - \frac{1}{2} \ell_R \frac{\partial^2 \ell_R}{\partial u_0^2}. \end{aligned} \quad (15)$$

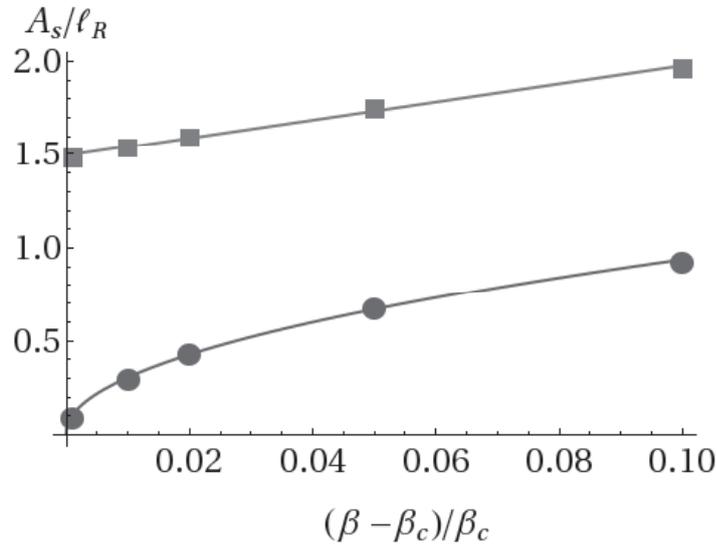


Figure 2. Saturation amplitude A_s for self-excited vibrations of a semiconducting CNT of quality factor $Q = 100$, normalized to the characteristic length ℓ_R , as calculated from equations (1) and (17) using the initial conditions $u(0) = 0$, $\dot{u}(0) = 0$ and $V(0) = 0$. Results in the ‘soft’ instability regime ($\omega_R = \omega_0$) for different values of the magnetomechanical coupling parameter β , which are all larger than but close to the critical onset value β_c , are marked by solid circles, while solid squares are used to mark results in the ‘hard’ instability regime ($\omega_R = 2\omega_0$). The solid lines are a guide to the eye.

From equation (14) it follows that for $\beta > \beta_c$ ($H > H_c$) an initially small amplitude $A(t = 0) \ll 2\ell_R$ will at first increase exponentially in time. There are then two different scenarios for the further development, depending on the sign of the coefficient b_1 . If $b_1 < 0$ and $\beta - \beta_c \ll \beta_c$ the amplitude saturates at the value

$$A_s = 2\ell_R \sqrt{\frac{H - H_c}{|b_1|H_c}}, \quad (16)$$

which vanishes for $\beta = \beta_c$ ($H = H_c$) and corresponds to a ‘soft’ instability. For $b_1 > 0$, the amplitude may saturate at a finite value, which does not vanish as $\beta \rightarrow \beta_c$ (from above). This is called a ‘hard’ instability. To illustrate the different scenarios of soft and hard instabilities we have performed a number of computer simulations of the system of equations (1) using the expression

$$R(u)/R_0 = (1 + e^{-2(u-u_0)/\ell_R})/2, \quad (17)$$

where $u_0/\ell_R = \beta$, to model the deflection dependence of the nanotube resistance. In the first series of simulations, we chose $\omega_R = \omega_0$, for which formulae (15) predict a soft instability, whereas in the second series of simulation we chose $\omega_R = 2\omega_0$, for which formulae (15) predict a hard instability. The results for the saturation values of the mechanical oscillation amplitude A_s , plotted for different values of β in figure 2, confirm the behavior found in the approximate analytical analysis above.

We note in passing that in the case of a hard instability, self-excitation results in a large amplitude $\sim \ell_R$ of the mechanical oscillations and hence nonlinear effects in the nanotube dynamics become important. Therefore, to analyze this regime in detail one would have to consider nonlinear continuum mechanics. This would, however, take us too far outside the scope of this paper.

The result of the instability discussed above is a stationary regime that is characterized not only by an oscillating mechanical displacement of the nanotube, but also by an oscillating voltage across the trench. In the regime of high quality factors, or more precisely when $\omega_R \gg \omega_0/Q$, the oscillation frequency will be very close to the mechanical resonance frequency $\omega_0/2\pi$, typically of the order of 0.1–1 GHz. Detecting such rapid voltage oscillations may pose serious experimental challenges. The nonlinearity of the system does, however, result in a nonzero deviation of the time-averaged voltage $\bar{V}(H)$ from the static value $V_0(H)$. If $A_s \ll \ell_R$, the time-averaged voltage can be approximated to lowest order with the formula

$$\frac{\bar{V}(H) - V_0(H)}{V_0(H)} = \left(\frac{\partial \ell_R}{\partial u_0} + \frac{\omega_R^2 - \omega_0^2}{\omega_R^2 + \omega_0^2} \right) \left(\frac{A_s}{2\ell_R} \right)^2. \quad (18)$$

Thus, in the case of a soft instability, the square of the saturation amplitude, A_s^2 , can be calculated from formula (16) and is found to be proportional to $(H - H_c)/H_c$. Accordingly, in this case, one finds that

$$\frac{\bar{V} - V_0}{V_0} \propto \frac{H - H_c}{H_c} \theta(H - H_c), \quad (19)$$

where $\theta(x)$ is the Heaviside step function. Thus, at the critical point $H = H_c$, a soft instability manifests itself as a jump in the derivative $\partial \bar{V}/\partial H$, whereas a hard instability would show up as a discontinuous jump in \bar{V} . Hence, the instability could be detected by measuring the time-averaged voltage drop across the suspended nanotube as a function of magnetic field. To illustrate this phenomenon we again performed computer simulations, the results of which are shown in figure 3.

It is important to note that for our geometry the characteristic length is of the order of the distance between the nanotube and the gate. Therefore, one can expect strong nonlinear effects in the nanotube dynamics to start to dominate at an amplitude that is just a small fraction of the characteristic length. Hence, the validity of formula (14) is rather limited. However, we believe that formula (18) could still be valid as an approximate relation between the voltage drop and the saturation amplitude. At the other extreme, one could consider systems with a very short characteristic length, for example with a scanning tunneling microscope (STM)-tip positioned above the CNT (see for example, [12, 17]). In this case, the current is determined by the probability for electrons to tunnel from the STM-tip into the CNT, and one could expect a characteristic length of about 0.1 nm.

To conclude, we expect that mechanical vibrations of a suspended CNT can be self-excited in the dc current-biased regime when a sufficiently large external magnetic field is applied perpendicular to the nanotube axis, provided that the resistance of the tube is displacement dependent. We support this claim with an analysis of a classical model of such a system and show that when the dissipation is sufficiently low, the static profile of the nanotube becomes unstable at reasonably weak (10–100 mT) magnetic fields. This magnetomotive instability develops into a steady state characterized by pronounced nanotube vibrations. We also demonstrate that the CNT vibrations change the magnetic field dependence of the time-averaged voltage

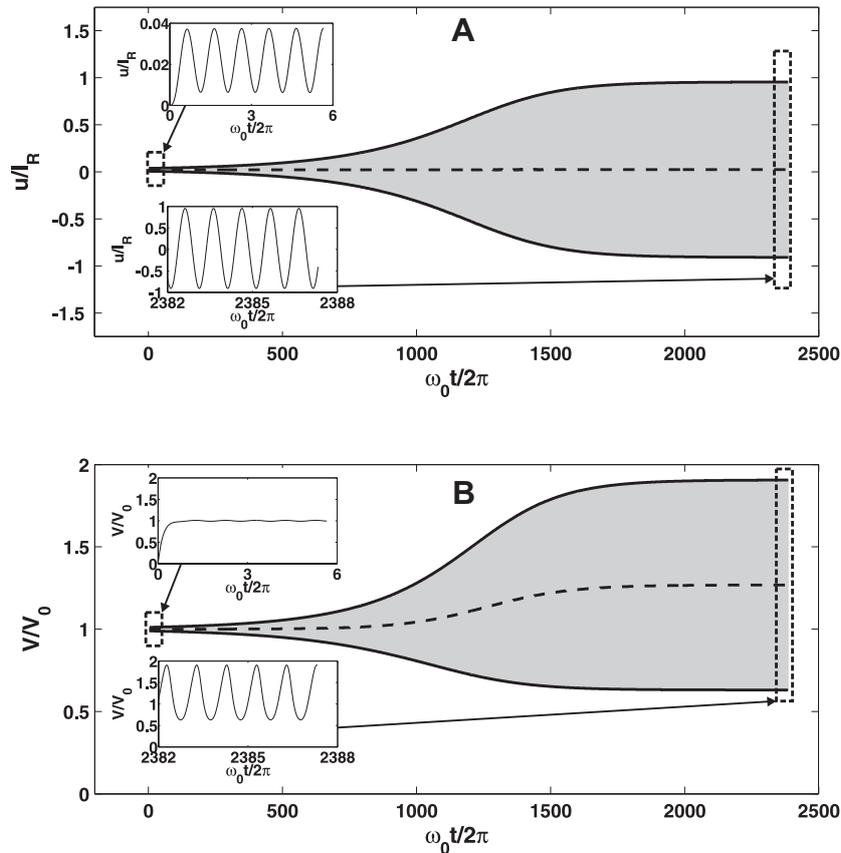


Figure 3. Time evolution of (A) the mechanical deflection of a suspended CNT and (B) the voltage drop over a vibrating nanotube of quality factor $Q = 100$, as calculated from equations (1) and (17) for the RC frequency $\omega_R = \omega_0$, the coupling parameter $\beta = 1.1\beta_c = 0.0219$, and the initial conditions $u(0) = 0$, $\dot{u}(0) = 0$ and $V(0) = 0$. The gray areas span the envelopes of the unresolved oscillations, while the dashed lines mark their time-averaged values. As can be seen, the time-averaged voltage drop provides a measurable signature of the oscillations since it deviates more and more from the static value V_0 as the amplitude of the mechanical oscillation increases. The upper and lower insets display zoom-ins on the oscillations at their onset when the magnetic field has been turned on and when the stationary regime has been reached, respectively.

drop across the nanotube. This phenomenon may be used for the experimental detection of the magnetomotive instability.

Other types of active oscillator based on the sustained self-oscillations of a suspended CNT have been proposed theoretically [17] and have recently been realized experimentally [18]. The approach in [17] relies on the distance-dependent electron injection from an STM-tip into a doubly clamped CNT, whereas in [18] distance-dependent field emission of electrons from a singly clamped CNT to the electrode provides a feedback mechanism that leads to controllable mechanical self-oscillations from a single dc voltage supply. The drawbacks of both these devices are that they require very precise geometry control to obtain self-sustained

oscillations and that the short distance between the CNT and the controlling electrode limits the amplitude of vibration. In contrast, the device that we consider is a suspended-channel CNT field effect transistor—a device that has been successfully fabricated and shown to work in several experimental laboratories [2]–[4]—which does not require short separation between the CNT and the gate electrode (the typical gap is several hundreds of nanometers) and which allows for direct tuning of the mechanical resonance frequency by changing the dc voltage on the gate electrode.

Finally, we note that magnetomotive instability can occur in other suspended semi-conducting nanostructures such as nanowires and graphene; moreover, the conductance of a single-walled CNT can be modulated by its displacement through mechanical stretching [19], even in the absence of a gate electrode. The analysis presented in this paper can be readily applied to all such structures, and the critical magnetic field H_c calculated for any particular device.

Acknowledgments

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