

# A comparison of schedules resulting from priority rules and mathematical optimization for a real production cell

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## **ABSTRACT**

*In this article we present a complete algorithm using mathematical optimization tools for solving a flexible job shop scheduling problem at Volvo Aero, a company producing high-valued low volume components for the aircraft industry. The goal of the scheduling is to facilitate a higher utilization of the cell while minimizing the total tardiness and the cell throughput time. The production cell consists of ten resources (machines and workstations) whereof five are multipurpose machines that can carry out a variety of operations; the so-called multitask machines work in parallel instead of one dedicated machine for each product. The production cell is studied from a mathematical as well as a logistical point of view. The quality of the schedules is measured by means of the total tardiness and computation time. The resulting schedules from a mathematical optimization model are compared with schedules generated by priority rules, which are similar to today's manual scheduling of the multitask cell. The tests were carried out for different realistic scenarios with regard to work load and product mix. The resulting schedules, which as expected outperformed the two commonly used priority rules, are obtained within minutes for the upcoming shift.*

# 1. INTRODUCTION

*“Hundreds of robots and millions of dollars’ worth of computer controlled equipment are worthless if they are under-utilized or spent their time working in the wrong part because of poor scheduling”* Classen and Malmstrom (1982)

As the above citation indicates production scheduling is of crucial importance to the performance of manufacturing organizations. Still, creating a feasible schedule is many times difficult, and it does not require many machines, products, and constraints until the situation becomes more or less impossible to handle manually. In the literature of manufacturing planning and control it is usually argued that sophisticated algorithms are especially needed in the job shop process, Vollmann et al. (2005). In the job shop every job has its unique arrangement of operations and uses a subset of unique or similar machines at the work centers. Consequently, products move in different directions and priority decisions appear each time a new job arrives at a work center, Jonsson and Mattsson (2009).

In recent years, widespread usage of numerically controlled multi-machines in modern job shops has altered the definition of the classical job shop problem, Baykasoglu and Özbakir (2010) where  $n$  jobs are processed on  $m$  machines and each job has distinct routes. The multi-purpose machines usually perform a number of operations which means that each operation has more than one machine alternative. This means that the classical job shop scheduling problem is complicated by the need to determine a routing policy and consists of two sub problems; assignment to operations to machines, and the sequencing of the operations on these machines, Baykasoglu and Özbakir (2010). This type of extended job shop scheduling problem is called flexible job shop scheduling problem (FJSP).

The FJSP belongs to a class of problems called combinatorial optimization (CO) problems which are very difficult to solve, Pezella et al. (2008). In broad terms the algorithms to tackle CO problems can be classified as complete or approximate algorithms, Blum and Roli (2003). Complete algorithms are guaranteed to find an optimal solution whereas approximate algorithms are not able to guarantee that the solution that has been found is of sufficient quality (ibid). Although complete algorithms at a first glance might seem as the obvious choice the characteristic of the job shop many times lead to computation times too high for practical purposes, Marvin et al. (2006). The difficulties of creating an optimal schedule within reasonable computing time are probably the main reasons why approximate methods have been used in favor of complete algorithms to solve the FJSP.

Several approximate algorithms such as dispatching rules, local search and meta-heuristics such as tabu search, simulated annealing and genetic algorithms have been developed to solve the FJSP. In fact, the production cell studied in this article has been a test case in such an approach, when an evolutionary algorithm has been used, Syberfeldt (2009). Complete algorithms are many times argued not effective for solving FJSP, Pezzella et al (2008). However, in the past decades the development of theory and practice of optimization modeling and methods, together with the development of computer hardware, have decreased computation times significantly, Gayialis and Tatsiopoulos (2004).

In this article we show that a complete algorithm that solves the mathematical optimization model is able to create production schedules for a real flexible job shop scheduling problem with small enough computation times. The algorithm is used for a production cell consisting of five multipurpose machines that can perform three types of operations.

The paper is organized as follows. In Section 2 we introduce the company where the study was conducted and the multitask cell. Thereafter in Section 3 we describe how the detail planning of the cell is conducted today. In Section 4 the optimization models for scheduling the cell are presented and Section 5 presents the computational results. In Section 6 there is a short discussion followed by conclusions in Section 7.

## **2. THE CASE STUDY**

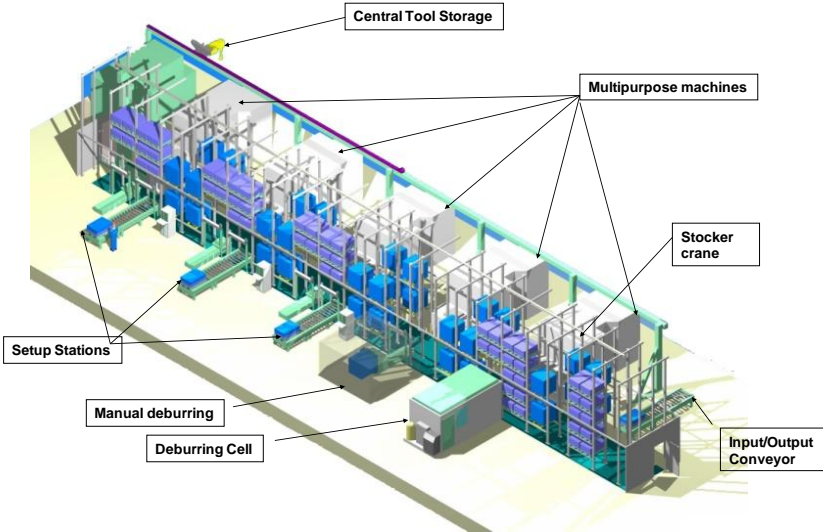
The study is conducted at Volvo Aero, an aerospace industry developing and producing aircraft and rocket engines in cooperation with world-leading companies in the aircraft industry. The focus is on complex and advanced structures and rotors for medium and large aero-engines.

The logistic conditions at Volvo Aero production are: (1) Expensive machines which are extremely difficult to move. (2) Several types of complex high-valued products with low production volumes. (3) The requirements on quality and on tolerances in manufacturing are extremely high due to flight safety regulations. (4) Expensive production fixtures are needed for processing. (5) Difficult to get rid of excess stock.

The list above constitutes the main reasons for the complex jumbled flow, which is the reality of the current production process at Volvo Aero.

One of Volvo Aero's recent investments is the so called "multitask cell" which is a production cell containing ten resources (see Figure 1). The production cell is supposed to carry out a large variety of jobs since five of the cell's resources are multi-purpose machines that are able to process three different types of

operations (milling, turning and drilling). The multitask cell was built with the aim of achieving a higher degree of machine utilization, reducing product lead times and being flexible both with regard to product mix and to processing type. Presently, the multitask cell is executing about 30 different operations on eight different products. Each product typically visits the multitask cell multiple times on its way to completion.



**Figure 1:** Overview of the multitask cell.

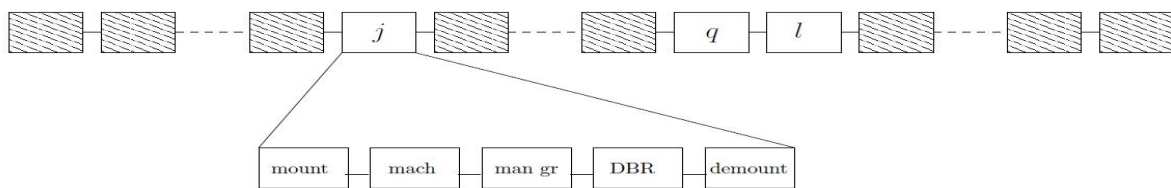
The parts that are ready to be processed are the parts checked-in at the input conveyor but not yet put into a fixture at a set-up station. After check-in, the parts are transported by a stocker crane to special storage locations inside the multitask cell. There are also storage areas in the cell for parts already mounted in fixtures. The ten resources on which the jobs are to be scheduled are listed in Table 1. Each part to be processed in the multitask cell follows a specific routing through the listed resources, which consists of three to five operations, starting and ending by the mounting and removing of fixtures at a setup station. The second operation in this routing is always processing in one of the multitask machines. Some parts need manual and/or robot deburring. The routing inside the cell is illustrated for a job in Figure 2.

**Table 1:** The resources of the multitask cell

Resource	Description
MC1-5	Five main processing multipurpose machines, which can perform milling, turning and drilling
ManGr	One manual deburring station
DBR	One deburring robot cell
MDM1-3	Three set-up stations in which the parts are mounted in and removed from fixtures

## 2.1. The queue of jobs

The job performed in the multitask cell, is only a part of the complete routing for a product, see Figure 2. A typical routing contains about twenty operations, whereof about five are processed in the multitask cell. Hence, the objective for the scheduling is to enhance the detail planning for the jobs within the cell and to enable an efficient utilization of the cell.



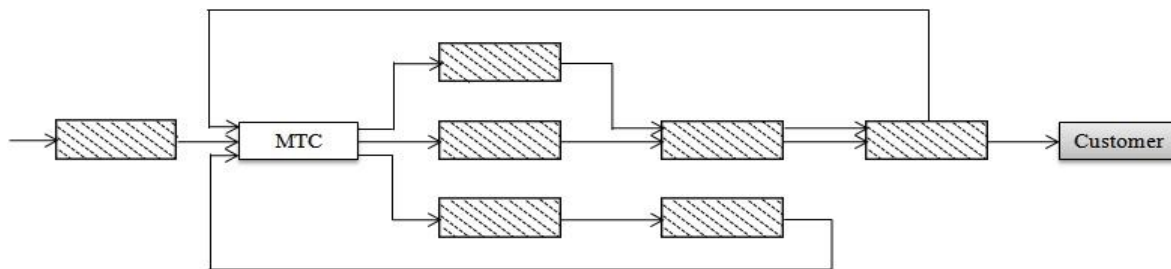
**Figure 2:** Routing of a part with the jobs  $j$ ,  $q$  and  $l$  to be processed in the multitask cell; the dashed jobs are to be performed outside the cell. The routing inside the cell is shown for job  $j$ .

Since each part passes through three different phases before the processing in the multitask cell, the queue of jobs to the multitask cell, can be divided into three categories: planned orders not yet released, i.e. existing only in the planning system; released orders, or so called production orders, i.e. physical parts being processed outside the cell on their way to the multitask cell; jobs checked-in into the multitask cell, i.e. parts inside the multitask cell waiting to be processed. Some of the jobs in the third category are to be processed on parts that will return to the multitask cell for subsequent jobs.

## 3. CURRENT DETAIL PLANNING OF THE MULTI-TASK CELL

In the existing Enterprise Resource Planning (ERP) system there are two lists available, which propose the job priority for incoming jobs. One is based on the Earliest Due Date (EDD) priority rule and the other is based on First In First Out (FIFO) priority rule. There is also an existing built-in scheduling algorithm in the control system of the multitask cell. Studies made in a master thesis, Jansson (2006) indicated that this built-in algorithm was not well suited for the logistical situation of the multitask cell. The logistical conditions of the multitask cell has recently been studied in another master thesis, Pettersson (2010), where the current detail planning has been described. The planning of today is done manually by a detail planner with the help of the mentioned EDD-list and other priorities based on the current logistical situation. The decision on which job to schedule on which machine is made by a group planner together with the detail

planner. As each job is only allowed to be processed in a subset of the multitask machines, this is not a simple task. Even though the processing machines are of the same kind, they are not identical, and certain jobs have requirements on extremely low tolerances due to flight safety issues. This is one of the reasons why some jobs can only be processed in specific machines. As a consequence of the low product volumes and the fact that the machines are difficult to move, most of the parts have different routes through the factory, and the situation with regard to incoming jobs is hard to get hold of for a manual planner, see Figure 3.



**Figure 3:** An illustration of a possible production path for one product through the factory.

The main objective for the detail planners and the manager of the multitask cell, are delivery precision and a high utilization of the cell. Moreover, the group planner considers that short lead times are important, Pettersson (2010).

#### **4. AN OPTIMIZATION MODEL FOR SCHEDULING OF THE CELL**

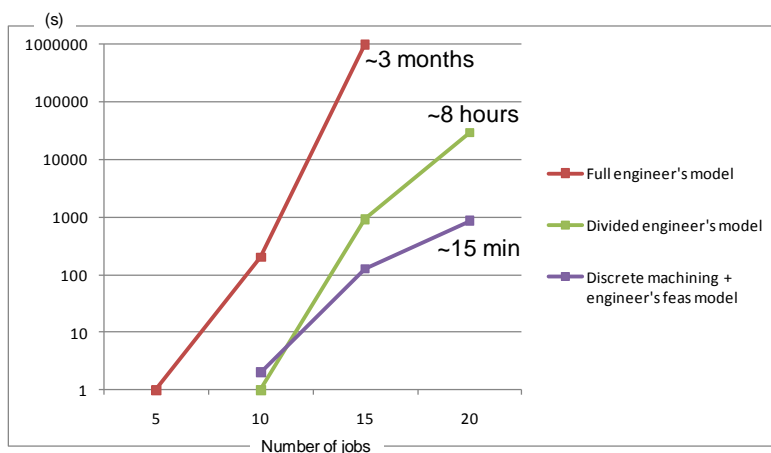
We have modeled the scheduling of jobs in the multitask cell described in the previous section using mixed integer programming techniques, see e.g. Wolsey (1998). As pointed out in the introduction, the use of exact algorithms for job shop problems has encountered problems with high computation time. This was unfortunately also the case for the first model developed for the multitask cell including all ten resources, called the full engineer's model in Figure 4. One instance took about 3 months to reach optimum for a queue of 15 jobs.

The model was therefore decomposed into two. The first model finds an optimal sequence of operations for each of the five processing machines; the second model then generates a feasible schedule for all ten resources, with the optimal sequence for the five processing machines as input data. In Thörnblad et al. (2010) we have presented the first model, which finds an optimal sequence of operations for the processing machines. The second model is however based on the same logic, apart from some minor details and the fixing of some variables to the result from the first model. These two models work well, but the

computation times still needed to decrease. Test instances with 20 jobs had a mean of 8 h before reaching optimum.

Therefore we have developed two additional models, using discrete time variables, van den Akker et al. (2000), in order to solve the optimal sequences of operations for the processing resources, and the results gained from one of these models are very promising. The computation times vary a lot with the input data and with the hardware and software used, and a few test instances with 20 jobs have required 1 h to find optimum, but a mean of all calculations made so far is 15 minutes. More tests are needed before stating any general conclusions, but all instances of this size can probably be generated within a few minutes for the coming shift, as we can stop the algorithm with an optimality gap we choose, say 0.1%, which is totally acceptable in real production. Such small computation times are necessary for a successful implementation of the model in the multitask cell control system.

In Figure 4 below the mean computation times for the mentioned models are marked out in a graph with a logarithmic scale on the y-axis. The computations have been carried out on a 4 Gb quad-core Intel Xeon 3.2 GHz system using AMPL-CPLEX12 as optimization software.



**Figure 4:** Mean computation times to optimum of three different models.

## 4.1. Assumptions

All the processing tools are assumed to be available and transported to the appropriate resource on time for each route operation. The availability of fixtures and personnel for the manual work in the cell is also considered to be sufficient. This is however not always the case and how this best can be included in the model is an area of future studies.

## 4.2. The objective function

The main objective of the optimization is to minimize the total tardiness, but in order to also differ between jobs that are completed on time or before its due date, i.e. jobs with zero tardiness, the sum of the completion times is added to the objective. This means that the jobs considered in the planning, are scheduled as early as possible. For the computational result presented in this article, the objective function employed in the first model—to find an optimal sequence of operations for the machining resources—is given by

$$\text{Minimize } \sum_{j \in \mathcal{J}} (s_j + h_j) .$$

A full presentation of the mathematical model is made in Thörnblad et al. (2010) and is not presented in this article.

## 5. COMPUTATIONAL RESULTS

### 5.1. Test scenarios

Six different test scenarios have been used for the computations. Each scenario consists of 20 jobs, which are assumed to be checked-in into the multitask cell, i.e. ready to be processed, at time  $t_0=0$ . Three scenarios were created based on real production data from one day in March 2010. One scenario was left as it was, one was altered to include a larger proportion of short jobs, and the third was altered to include a larger proportion of long jobs. In these scenarios, all jobs are late at time  $t_0=0$ .

Three other scenarios were created analogously, however, based on a scenario of a high volume case. This was created by the technician of the multitask cell together with a master planner and is a realistic case of a future product mix. In the three latter scenarios, approximately half of the jobs are assumed to be late at time  $t_0=0$ .

We have been gathering real production data for 2 months at the time writing this article, and the jobs checked-in have varied from 4 to 23. There is a maximum of about 30 jobs checked-in, due to limitation in storage. The assumption of 20 jobs checked-in is realistic, since during the period we have been gathering real production data, the workload of the multitask cell has been low. The computational results for these real scenarios will be presented in August 2010 at the PLAN research conference.



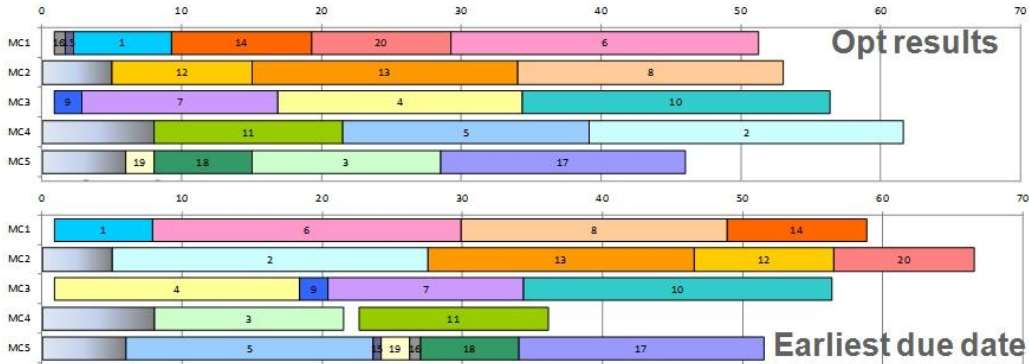
## 5.2. Computational results

Schedules for all the six scenarios have been computed using the optimization model, the EDD priority rule and the FIFO priority rule. In Table 2 below, the mean of the results from the three scenarios based on real production data and those based on the high volume case are listed. The optimization model outperforms the FIFO and EDD scheduling principles, whose tardiness exceeds the optimal value with about 20% and 11%, respectively. The completion time is the time from the start of the planning period till the part is removed from the fixture in one of the setup stations.

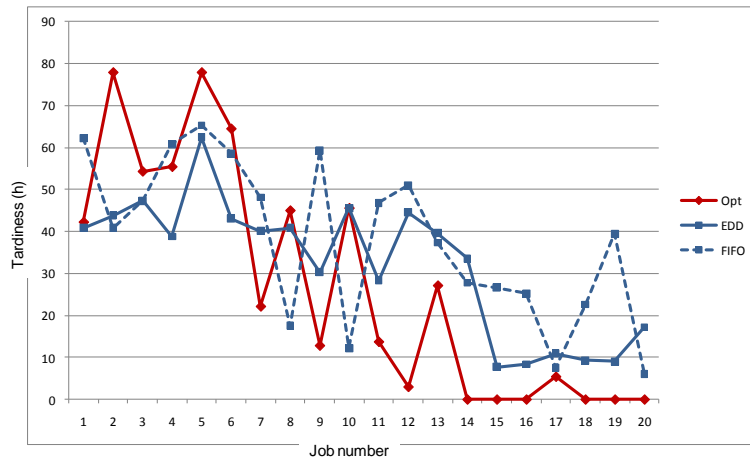
**Table 2:** All results are given as a mean per job and scenario variant and the percentages are relative to the mean completion time from the optimization.

Scenario	Scheduling algorithm	Completion time (h)	Diff from optimal solution	Completion time diff	Tardiness diff (h)	Tardiness diff (%)
Real prod case	OPT	22.9	0	0%	0	0%
	FIFO	26.9	4.0	18.0%	4.0	18.0%
	EDD	25.3	2.4	10.4%	2.4	10.4%
High volume case	OPT	25.4	0	0%	0	0%
	FIFO	33.9	8.5	33.9%	5.7	22.4%
	EDD	32.5	7.1	28.9%	2.9	11.7%

The schedule of the machining resources resulting from the optimization model of the high volume long jobs scenario is shown in Figure 5 together with the schedule produced by the EDD priority rule. The tardiness results for this case are shown in Figure 6. Note that the optimal schedule contains 6 tardy jobs that are later than the EDD schedule, but on the other hand, this schedule also contains 6 jobs with zero tardiness.



**Figure 5:** An optimal schedule for the high volume long jobs scenario compared with a schedule for the machining resources constructed with the EDD priority rule.



**Figure 6:** Tardiness results for all scheduling algorithms for the high volume long jobs scenario.

## 6. DISCUSSION

Even though the use of simple priority functions may seem uncomplicated, it is really a complicated task in practice since the multitask cell is a flexible job shop where all jobs are not allowed to be processed in all machines. In order to find a feasible schedule, one might be forced to delay the next job in the priority list, since the allowed machines may be busy with other jobs. This is exactly what happens in the schedule made with the EDD priority rule shown in Figure 5. In this case, the jobs 12, 14 and 20 are allowed to be processed only on MC1 and MC2 and cannot be scheduled on MC4 which is idle.

Another fact that makes the scheduling complicated is that the surroundings of, and the situation in, a work-shop are constantly changing, Stoop and Wiers (1996). The test scenarios made for this study were assumed to be static, and all jobs in the queue were checked-in, into the multitask cell, at time  $t_0$ . The optimization model is however also capable of simultaneously handling all the jobs that are on their way to the multitask cell.

## 7. CONCLUSIONS

We have modeled and solved a flexible job shop problem with real instance data using mixed integer linear programming techniques. The results, which as expected outperform two commonly used priority rules, are obtained within minutes for the upcoming shift. However, our results are based on a small set of data, and more tests are needed in order to be able to draw general conclusions. A great advantage of the proposed methodology is that the schedules produced are guaranteed to be of high quality. The proposed scheduling principle will shorten lead times, minimize tardiness and provide a more efficient use of the resources available.

## 8. ACKNOWLEDGEMENTS

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