

# OPTIMIZATION OF SCHEDULES FOR A MULTITASK PRODUCTION CELL

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## ABSTRACT

### **Purpose of this paper**

To optimize production schedules of a real production cell containing multi-purpose machines working as a job-shop in a complex logistic environment producing aircraft engine components. The planning and control of the production in the so called multitask cell results in a complex combinatorial problem.

### **Design/methodology/approach**

A mathematical optimization model of the multitask cell has been developed and tested on real data instances. The current production planning prerequisites have been studied in order to choose an appropriate objective function.

### **Findings**

Production plans resulting from the optimization model, are compared with schedules formed by the First in First Out (FIFO) and Earliest Due Date (EDD) priority rules, which are similar to the current manual planning of the multitask cell.

### **Practical implications**

The proposed scheduling principle will shorten lead times, and provide a more efficient use of the resources of the multitask cell. The effects are most significant at times with high utilization of the cell.

### **What is original/value of paper**

The development of theory and practice of optimization, together with the development of computer hardware during the past decades, enable the utilization of optimization as a tool for computing efficient production schedules in a complex logistic environment.

*Keywords: Production planning, Job-shop schedule, Optimization, Priority functions*

## INTRODUCTION

Volvo Aero is currently investing to build up production capability and capacity to avoid outsourcing and to retain production within Sweden. One of these investments is the so called "multitask cell" which is a new production cell containing ten resources. The production cell is supposed to carry out a large variety of jobs since five of the cell's resources are multi-purpose machines that are able to process three different types of operations (milling, turning and drilling). As this production cell constitutes a significant investment it needs to be utilized as efficient as possible. The ability to plan and control the use of the production cell, and other similar ones, is therefore of major importance to maintain Volvo Aero's competitiveness.

The scheduling and control of all ten resources in the multitask cell obviously results in a complex combinatorial problem. The production cell was therefore delivered with a scheduling algorithm based on a simple priority function. This algorithm is, however, judged not to be efficient enough for job scheduling. A master's thesis project was therefore carried out during 2006, in co-operation with Mathematical Sciences at Chalmers University of Technology (Jansson, 2006). The study indicated that there exists a significant potential for increasing the efficiency of the cell; but this requires that a better suited scheduling algorithm is implemented. The scheduling algorithm required needs to be very advanced in functionality as well as in execution speed. Literature studies showed that there was a lack of theory on such mathematical modeling techniques for real life logistic problems. Therefore, Volvo Aero and Chalmers Mathematical Sciences applied for funding of a PhD-project by The Swedish Research Council.

### 1.1. Background

Volvo Aero is active in the aerospace industry and develops and produces aircraft and rocket engines in cooperation with world-leading companies in the aircraft industry. The focus is on complex and advanced structures and rotors for medium and large aero-engines, such as the components shown in Figure 1.1.



*Figure 1.1 Some static and rotating aero-engine parts manufactured by Volvo Aero.*

The aerospace industry is special in the sense that the requirements on quality and on tolerances in manufacturing are extremely high due to flight safety regulations. Several types

of complex products and low production volumes together with expensive machines that are extremely difficult to move constitute the main reasons for the jumbled flow (Hopp & Spearman, 2008), which is the reality of the current production process.

The multitask cell was built with the aim of achieving a higher degree of machine utilization, reducing product lead times and being flexible both with regard to product mix and to processing type. At the time this article is written, the multitask cell is executing about 30 different operations on eight different products. The process structure of the production is a job shop environment, i.e. each product follows a specific predetermined routing (Brucker, 2007). Each product typically visits the multitask cell multiple times on its way to completion.

## 2. THE MULTITASK CELL

The multitask cell consists of ten resources (machines and workstations), one input/output conveyor and two stocker cranes, one for transporting the products and one for transporting the tools to the resources. There is also a stock for processing tools with a tool supplying robot, which collects the tools for the upcoming processing operations.

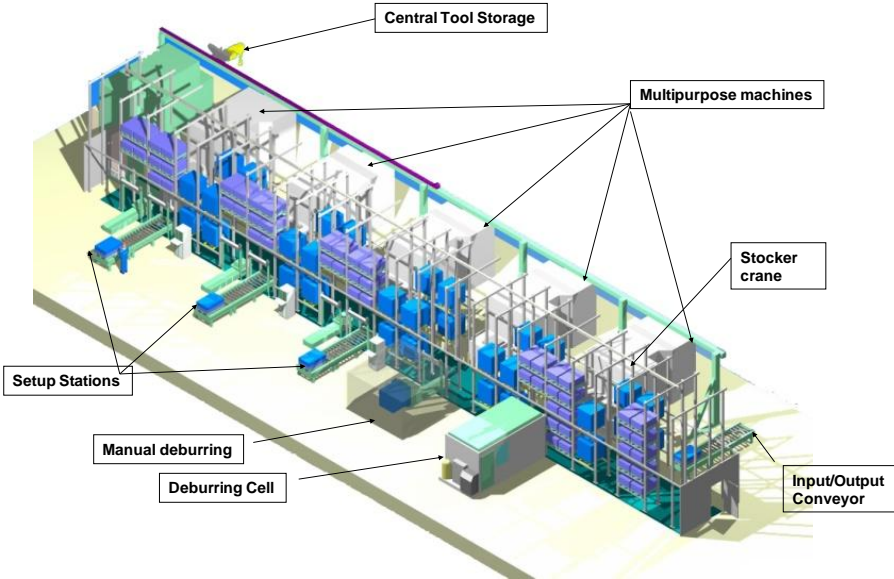


Figure 2.1 Overview of the multi-task cell

The queue of parts that are ready to be processed are the parts checked-in at the input conveyor but not yet put into a fixture at a set-up station. After check-in, the parts are transported by a stocker crane to special storage locations inside the multitask cell. There are also storage areas in the cell for details already mounted in fixtures and hence in between route operations. For an overview of the multitask cell, see Figure 2.1.

Table 2.1 The resources of the multitask cell.

Resource	Description
MC1-5	Five main processing multipurpose machines, which can perform milling, turning and drilling

ManGr	One manual deburring station
DBR	One deburring robot cell
MDM1-3	Three set-up stations in which the parts are mounted in and removed from fixtures

The ten resources on which the jobs are to be scheduled are listed in Table 2.1. Each part to be processed in the multitask cell follows a specific route through the listed resources. Each route consists of three to five route operations, starting and ending by the mounting in and removing of fixtures at a setup station. The second operation in the route is always processing in one of the multipurpose machines. Some parts need manual and/or robot deburring. A route for a typical job in the multitask cell is illustrated in Figure 2.2.

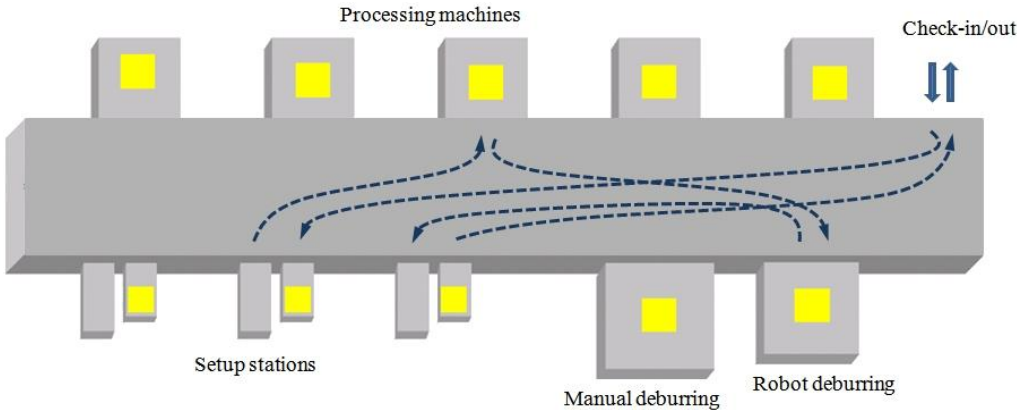


Figure 2.2 A schematic view of the multitask production cell. A possible route through the cell for a job with the four operations mount – machining – robot deburring – demount (remove fixture) is marked out with dotted lines and arrows.

**2.1. The jobs to be scheduled**

The job performed in the multitask cell, is only a part of the complete routing for a product, see Figure 2.3. A typical routing contains about twenty operations, whereof about five are processed in the multitask cell. Hence, the objective for the scheduling is to enhance the detail planning for the jobs within the cell and to enable an efficient utilization of the cell.

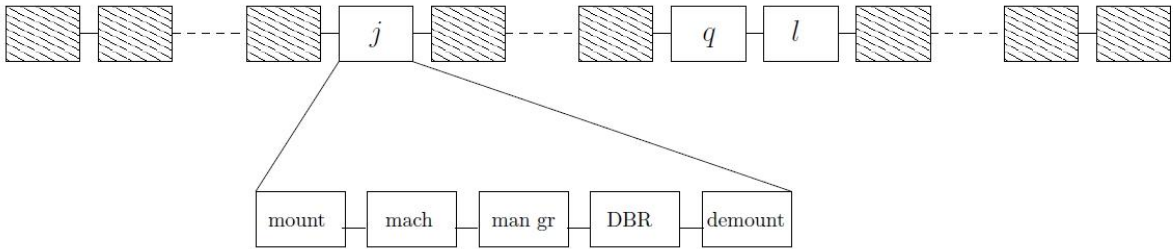


Figure 2.3 Routing of a part with the jobs *j*, *q* and *l* to be processed in the multitask cell; the dashed jobs are to be performed outside the cell. The route operations inside the cell are shown for job *j*.

The queue of jobs to the multitask cell, can be divided into three categories, since each part passes through three different phases before the processing in the multitask cell:

- planned orders not yet released, i.e. existing only in the planning system;
- released orders, or so called production orders, i.e. physical parts being processed outside the cell on their way to the multitask cell;
- jobs checked-in into the multitask cell, i.e. parts inside the multitask cell waiting to be processed.

### **3. CURRENT DETAIL PLANNING OF THE MULTITASK CELL**

In the existing Enterprise Resource Planning (ERP) system there are two reports used at Volvo Aero, which propose the job priority. One is based on the Earliest Due Date (EDD) priority rule and the other is based on First In First Out (FIFO) priority rule. There is also an existing built-in scheduling algorithm in the control system of the multitask cell. Studies made in the master thesis (Jansson, 2006) indicated that this built-in algorithm was not well suited for the logistical situation of the multitask cell.

The prerequisites for the logistics of the multitask cell has recently been studied in another master thesis (Pettersson, 2010), where the current detail planning has been described. The planning of today is done manually by a detail planner with the help of the mentioned EDD-list and other priorities based on the current logistical situation.

The decision on which job to schedule on which machine is made by a group planner together with the detail planner. As each job is only allowed to be processed in a subset of the multitask machines, this is not a simple task. Even though the processing machines are of the same kind, they are not identical, and certain jobs have requirements on extremely low tolerances due to flight safety issues. This is one of the reasons why some jobs can only be processed in some machines.

As a consequence of the low volumes of the products and the expensive machines difficult to move, most of the parts have different routes through the factory, and the situation with regard to incoming jobs is hard to get hold of for a manual planner, see Figure 3.1.

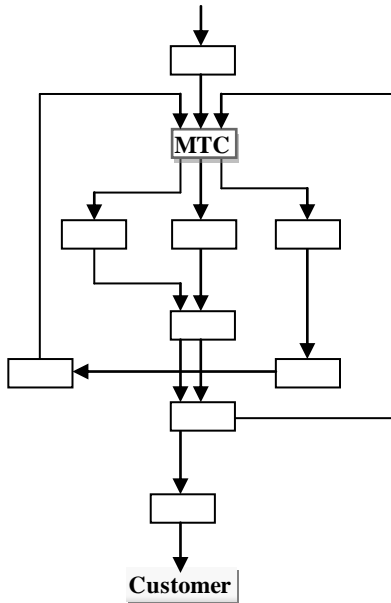


Figure 3.1 An illustration of a possible production path for one product through the factory. (Pettersson, 2010)

The main objective for the detail planners and the manager of the multitask cell, are delivery precision and a high utilization of the cell. Moreover, the group planner of the cell considers that low lead times are important (Pettersson, 2010).

#### 4. OPTIMIZATION MODEL FOR SCHEDULING OF THE CELL

We have modelled the scheduling of jobs in the multitask cell described in the previous section using mixed integer programming techniques, see e.g. (Wolsey, 1998).

During the past decades the development of theory and practice of optimization modelling and methods, together with the development of computer hardware, have decreased computation times by several magnitudes. However computing job shop schedules is still an NP-hard problem (Garey and Johnson, 1979), and the first developed model including all ten resources turned out to have too long computation time for a realistic case.

The model was therefore decomposed into two. The first model finds an optimal sequence of operations for each of the five processing machines; the second model then generates a feasible schedule for all ten resources, with the optimal sequence for the five processing machines as input data. In this article we present the model finding an optimal sequence of operations for the processing machines. The second model is however based on the same logic, apart from some minor details and the fixing of some variables to the result from the first model. These two models work well, but the computation times still need to decrease.

Therefore we have developed two additional models, using discrete time variables (van den Akker et al., 2000), in order to solve the optimal sequences of operations for the processing resources, and the results gained from one of these models are very promising. Since the computation times vary a lot with the input data and with the hardware and software used, more tests are needed before stating any general conclusions. However, a schedule can probably be generated within minutes for the coming shift, which is necessary for a successful implementation of the model in the multitask cell control system.

## 4.1. Assumptions

All the processing tools are assumed to be available and transported to the appropriate resource on time for each route operation. The availability of fixtures and personnel for the manual work in the cell is also considered to be sufficient. This is however not always the case and how this best can be included in the model is an area of future studies.

## 4.2. Input data

The following input data are available: the planned release dates and due dates for the multitask cell; which machines are allowed for each route operation and each job; the route operation processing time; the internal transportation time between the resources of the multitask cell; the time when each resource will be available if occupied at time  $t_0$ , which is the starting time of the schedule to be made; the planned lead time between two jobs in the multitask cell, which are to be processed for the same part; the planned lead time from the actual position of a part to its arrival at the multitask cell.

## 4.3. Notation

### 4.3.1. Definition of sets

The set of jobs to be scheduled are denoted by  $\mathcal{J}$ . The set of resources of the multitask cell, listed in Table 2.1, is denoted by  $\mathcal{K}$ .

Some jobs are to be processed on the same individual part; all pairs of such jobs adjacent in the routing, form the set  $\mathcal{Q}$ . For the part, which routing is illustrated in Figure 2.3, the set  $\mathcal{Q}$  contains the pairs  $(j, q)$  and  $(q, l)$ .

### 4.3.2. Definition of parameters

Table 4.1 The parameters of the mathematical program.

Parameter	Description
$\lambda_{jk}$	equals 1, if job $j$ can be processed on resource $k$ ; equals 0, otherwise.
$r_j$	the release date of job $j$ , (expressed in hours from time $t_0$ ).
$d_j$	the due date of job $j$ , i.e. the point in time when the last route operation of job $j$ (i.e. the remove of the fixture) is planned to be completed.
$a_k$	the time when resource $k$ will be available the first time.
$v_{jq}^m$	the planned lead time between the completion time of job $j$ and the start of the machining operation of job $q$ , where $(j, q) \in \mathcal{Q}$ .
$p_j^m$	the processing time of the machining operation of job $j$ .



### 4.3.3. Definition of variables

We have defined two sets of binary variables and three groups of continuous time variables, namely

- $z_{jk}$ , which is 1 if job  $j$  is allocated to resource  $k$ , 0, otherwise;
- $y_{jqk}$ , which is 1 if job  $j$  is processed before job  $q$  on resource  $k$ , 0, otherwise;
- $t_j$ , the starting time of the machining operation of job  $j$ ;
- $s_j = t_j + p_j^m + p_j^{pm}$ , the completion time of job  $j$ , where  $p_j^{pm}$  is the sum of the postmachining route operations;
- $h_j = \max(s_j - d_j; 0)$ , i.e. the tardiness of job  $j$ .

## 4.4. The objective function

The main objective of the optimization is to minimize the total tardiness, but in order to also differ between jobs that are completed on time or before its due date, i.e. jobs with zero tardiness, the sum of the completion times is added to the objective. This means that the jobs considered in the planning, are scheduled as early as possible. For the computational result presented in this article, the objective function employed in the first model—to find an optimal sequence of operations for the machining resources—is given by

$$\text{Minimize } \sum_{j \in \mathcal{J}} (s_j + h_j).$$

## 4.5. The planning constraints

Each job has to be scheduled exactly once which is expressed by the constraints

$$\sum_{k \in \mathcal{K}} z_{jk} = 1, \quad j \in \mathcal{J}.$$

Moreover, a job may only be planned on a resource that is allowed for this job, which is given by the constraints

$$z_{jk} \leq \lambda_{jk}, \quad j \in \mathcal{J}, k \in \mathcal{K}.$$

### 4.5.1. Realistic release dates and related constraints

When dealing with real world problems one may come across problems that one might not think of in theory. An example is that the release dates given by the ERP system might be in the past, even for parts that are processed elsewhere and have not yet arrived at the multitask cell. Hence, in order to get hold on a realistic point in time when a job is available for processing in the multitask cell, each job is assigned a new release date which is calculated as the maximum of the release date indicated by the system and the planned lead time from the actual position of the part; a part already checked-in into the multitask cell is given the release date 0.

The following constraints express that the starting time of job  $j$  has to be at or after its release date and that it cannot be scheduled on a resource before this resource is available,

$$t_j \geq r_j, \quad j \in \mathcal{J},$$

$$t_j \geq a_k z_{jk}, \quad j \in \mathcal{J}, k \in \mathcal{K}.$$

#### 4.5.2. Precedence constraints

In order to prevent two jobs belonging to the same production order from being scheduled too close in time, these jobs are separated by the planned lead time between the jobs to be performed in the multitask cell. This is represented by so called precedence constraints, according to

$$t_q \geq s_j + v_{jq}^m, \quad (j, q) \in \mathcal{Q}.$$

For jobs that are to be processed by the same resource, the starting time of an operation must be scheduled after the completion of the previous operation scheduled on the same resource, according to the constraints

$$y_{jqk} + y_{qjk} \leq z_{jk}, \quad j, q \in \mathcal{J}, k \in \mathcal{K}, j \neq q,$$

$$y_{jqk} + y_{qjk} + 1 \geq z_{jk} + z_{qk}, \quad j, q \in \mathcal{J}, k \in \mathcal{K}, j \neq q,$$

and  $t_j + p_j^m - M(1 - y_{jqk}) \leq t_q, \quad j, q \in \mathcal{J}, k \in \mathcal{K}, j \neq q.$

## 5. COMPUTATIONAL RESULTS

### 5.1. Test scenarios

Six different test scenarios have been used for the computations. Three scenarios were created based on real production data from one day in March 2010. One scenario was left as it was, one was altered to include a larger proportion of short jobs, and the third was altered to include a larger proportion of long jobs. In these scenarios, all jobs are late at time  $t_0=0$ .

Three other scenarios were created analogously, however, based on a scenario of a high volume case. This was created by the technician of the multitask cell together with a master planner and is a realistic case of a future product mix. In the three latter scenarios, approximately half of the jobs are already late at time  $t_0=0$ .

Each scenario consists of 20 jobs, which are assumed to be checked-in into the multitask cell, i.e. ready to be processed, at time  $t_0=0$ .

### 5.2. Computational results

Schedules for all the six scenarios have been computed using the optimization model, the EDD priority rule and the FIFO priority rule. The built-in scheduling algorithm and more elaborate dynamic priority functions are not yet implemented in the test bed; this is an area of future research.

In Table 5.1 below, the mean of the results from the three scenarios based on real production data and those based on the high volume case are listed. The computations have been carried out on a 4 Gb quad-core Intel Xeon 3.2 GHz system using AMPL-CPLEX12 as optimization software.

Table 5.1 Computational results. All results are given as a mean per job in the respective test scenario and the percentages are relative to the mean completion time from the optimization.

Case	Scenario	Completion time (h)	Completion time difference (h)	Difference (%)	Tardiness difference (h)	Difference (%)
Real production	Optimization	23.3				
	FIFO	27.5	4.2	18.1	4.2	18.1
	EDD	25.5	2.2	9.5	2.2	9.5
High volume	Optimization	27.4				
	FIFO	33.9	6.5	23.7	4.7	17.1
	EDD	33.5	6.1	22.4	2.6	9.5

The optimization model outperforms the FIFO and EDD scheduling principles, whose tardiness exceeds the optimal value with about 17.5% and 9.5%, respectively. As defined in Section 4.3.3, the completion time is the time from the start of the planning period till the part is removed from the fixture in one of the setup stations. The amount of tardiness at time 0 is just an input data, which no model can alter.

The schedule resulting from the optimization model of the high volume basic case is shown in Figure 5.1. The schedule produced by the EDD priority rule for the five processing machines is shown in Figure 5.2. There is a small gap in the schedule of resource MC5 in the optimal solution. This is due to the fact that jobs 3 and 5 collide in the deburring cell; in order to occupy the fixture as short time as possible, job 3 is scheduled slightly later than necessary.

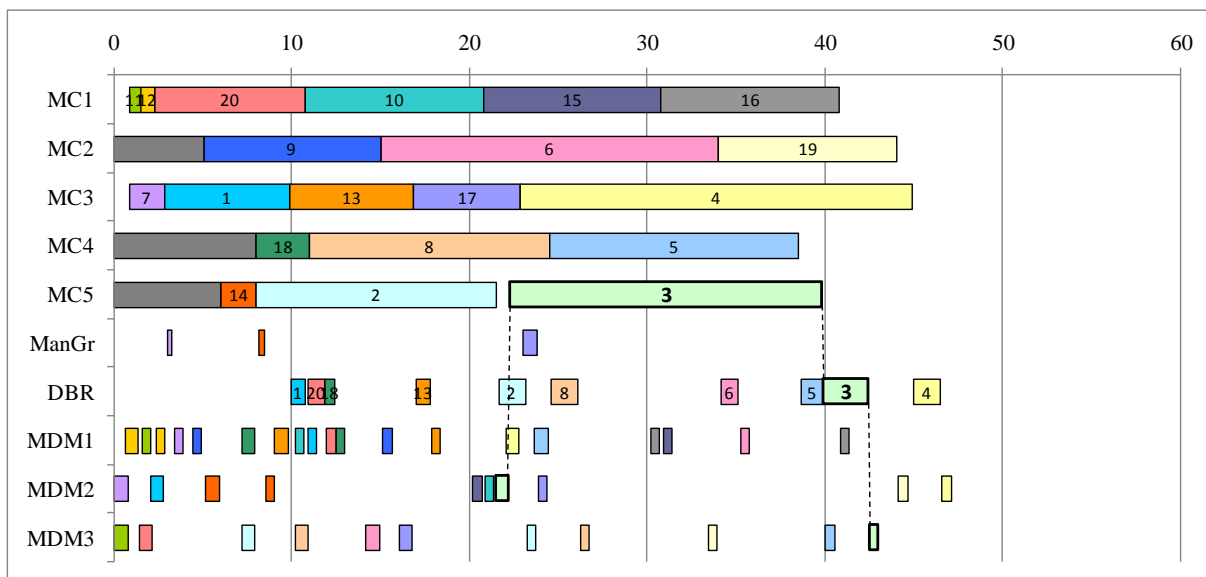


Figure 5.1 An optimal schedule for the high volume basic case. The route of job 3 is indicated by dotted lines.

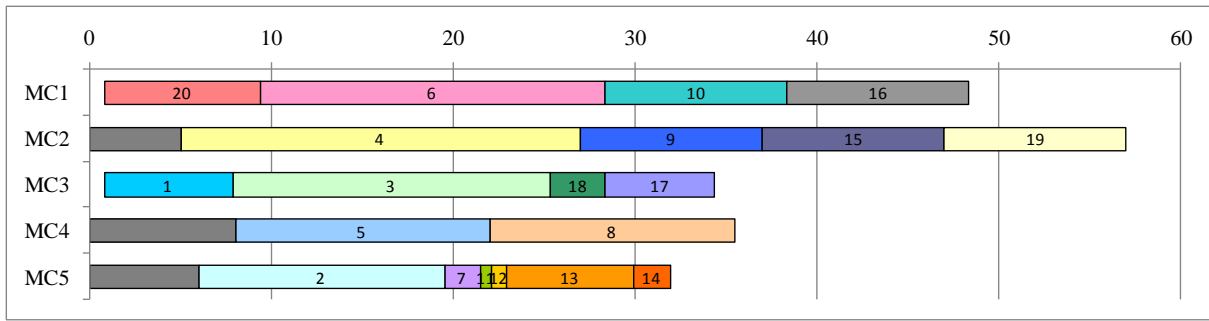


Figure 5.2 A schedule for the machining resources resulting from the EDD priority rule.

## 6. DISCUSSION

Even though the use of simple priority functions may seem uncomplicated, it is really a complicated task in practice since all jobs are not allowed to be processed on all machines. In order to find a feasible schedule, one might be forced to delay the next job in the priority list, since the allowed machines may be busy with other jobs.

Another fact that makes the scheduling complicated is that the surroundings of and the situation in a work-shop is constantly changing (Stoop & Wiers, 1996). The test scenarios made for this study were assumed to be static, and all jobs in the queue were checked-in into the multitask cell at time  $t_0$ . The optimization model is however capable of handling jobs on their way to the multitask cell, i.e. the release dates of those jobs are greater than  $t_0$ . Let us consider an example with two processing machines. Assume the jobs arrive at the cell in the order illustrated in Figure 6.1. The length of each bar indicates the time it takes to process the corresponding job and the time-line indicates when the jobs arrive. Inside each bar the job number and the corresponding allowed resources are indicated.

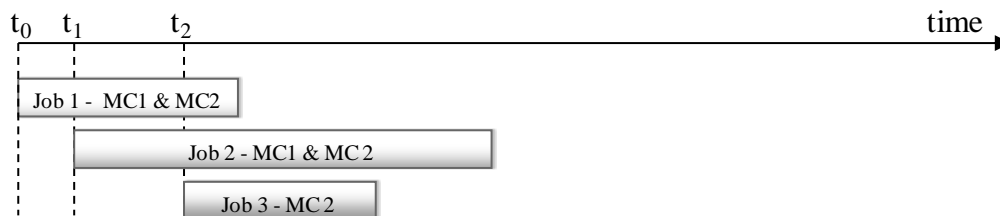


Figure 6.1 Example sequence of jobs arriving to the multitask cell.

At time  $t_0$ , the planner, who is not able to look into the future, only considers job 1, which can be processed in both machines. If he/she chooses to schedule this to machine MC1, job 2 will be scheduled to MC2, which is available when job 2 arrives. At time  $t_2$ , there is nothing the planner can do about the situation but to put job 3 on the queue to MC2, since this is the only allowed resource for this job. The resulting schedule is illustrated in Figure 6.2 while an optimal schedule is shown in Figure 6.3.

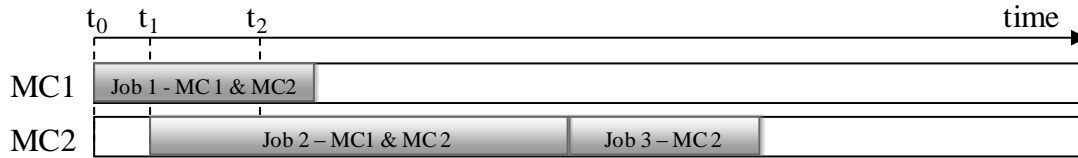


Figure 6.2 The resulting schedule when job 1 is scheduled to MC1.

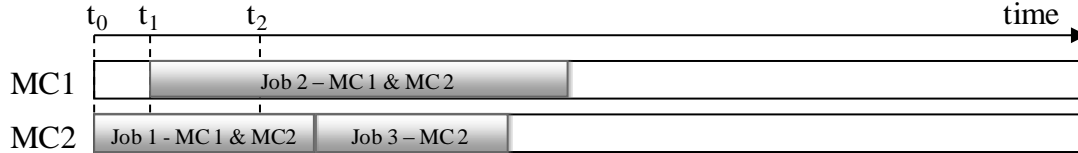


Figure 6.3 An optimal schedule.

The risks of the schedule in Figure 6.2 are that job 3 might be delayed and that MC1 may be idle after processing job 1, with loss of capacity and low utilization as consequences.

Then, consider the schedules in Figure 5.1 and Figure 5.2. Here the jobs 15, 16 and 19 are allowed to be processed only on MC1 and MC2. The result from the EDD rule is that these three jobs are forced to be scheduled with some delay since they are at the end of the priority list.

However, even without those jobs that are easy to point out as badly scheduled, the schedule resulting from the EDD priority rule has a greater total tardiness than the optimal schedule. This fact is not possible to determine just by looking at the schedule, since both schedules are well-filled as a consequence of all jobs being checked-in at time  $t_0$ .

If we had chosen to consider jobs from the other queue categories described in Section 2.1, it might have been easier to determine visually which schedule is better than the other, since there would most probably be significantly more time slots with idle time due to situations like the one described above. However, with an optimal schedule at hand, we are sure whether a sequence is good with regard to tardiness.

## 7. CONCLUSIONS

We have modeled and solved a complex production planning problem with real instance data using mixed integer linear programming techniques. The results, which as expected outperform two commonly used priority rules, are attained within minutes for the coming shift. However, our results come from a small set of data, and more tests are needed in order to be able to draw general conclusions.

Detail planning and control of a flexible system with parallel machines such as the multitask cell is a very complicated task. In practice a decrease in processing time by a few percentages is regarded as a great success. Therefore, shortening the lead times through a production cell like the multitask cell in the range of 10 – 15% ought to create a significant interest in the manufacturing industry.

Another advantage of the proposed methodology is that the schedules produced are *guaranteed* to be of high quality. The proposed scheduling principle will shorten lead times, minimize tardiness and provide a more efficient use of the resources available.

Furthermore, the situation for the planner is constantly changing, making the scheduling even more complex. The optimization model presented in this article is capable of calculating the expected arrival times for incoming jobs and including them in the planning. If the situation changes drastically, as in case of a machine breakdown, a rescheduling of the production plan can easily be made. Moreover, the proposed methodology can be used to evaluate different scenarios of production volume, number of processing machines in the cell, distributions of job characteristics etc. Hence, it can also be utilized as a tool for decision analysis.

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