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A Distributed Positioning Algorithm for Cooperative Active and Passive Sensors

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***Abstract*—The problem of positioning of an unknown target in cooperative active and passive wireless sensor network is addressed. Two-way time of arrival and time difference of arrival measurements in active and passive nodes are used to estimate the position of the target. A maximum likelihood estimator (MLE) can be employed to solve the problem. Due to nonlinear nature of the cost function in the MLE, the iterative search might converge to local minima resulting in large error of estimation. To avoid the MLE drawback, we formulate the problem of positioning as the intersection of some convex sets. To find the position estimate, we apply projection onto convex sets approach which is robust and can be implemented in a distributed manner. Simulations are performed to compare the performance of the MLE and new method.**

I. INTRODUCTION

Nowadays wireless sensor network (WSN) has been vastly considered for both civil and military applications. Accurate positioning of node is one of the important task which has great effect on the performance of every WSN [1]. Most literatures assumed that there are some reference nodes, also called anchor nodes, which can be used to estimate the position of an unknown target [2], [3]. In general, there are various positioning algorithms based on time of arrival (TOA), time difference of arrival (TDOA), received signal strength (RSS), and angle of arrival that can be used in different applications [4].

Two-way TOA (TW-TOA) has been considered as an effective approach in literature [5] due to advantageous such as no need to reference clock. In this approach, the reference nodes send a signal to target, and wait for response from it. The round trip delay between reference and target gives an estimation of distance. It is evident that the more reference nodes are used for TW-TOA ranging, the more accurately the target position can be estimated. Since, in practice, there are some limitations in increasing the number of reference due to power constraints, the idea of cooperation between reference nodes was proposed [5] to decrease the number of transmission. In this method, some reference nodes, which we call them primary reference nodes (PRNs), initiate range estimation by sending a signal. The target replies to the received signals by sending an acknowledgement. Suppose that there are some other reference nodes which can listen to both signals, henceforth we call them secondary reference nodes (SRNs).

It has been shown that the SRNs can help the PRN to estimate the target position more accurately. In fact, it is possible to get the same performance with less the PRNs when measurements from the SRNs are involved in positioning process. In this model, the PRNs are active nodes and the SRNs play the role of passive nodes. The model considered here is based on cooperation between active and passive reference nodes which is different from targets cooperation in cooperative network.
In this paper, we assume that the SRNs are able to receive signal from both target and the PRNs. Therefore the SNRs are able to measure the TDOA between target signal and PRN's signal. In this case, a maximum likelihood estimator (MLE) derived in [5] can be employed to improve positioning accuracy compared to non-cooperative approach. Due to nonlinear nature of cost function in the MLE, the iterative search may converge to local minima and then resulting in high error of estimation. Using geometric interpretation, we formulate the positioning problem as finding the intersection of some convex sets resulted from the TW-TOA and TDOA measurements in SRNs and PRNs. Successive orthogonal projection onto discs and ellipsoid can be employed to solve the optimization problem of cooperative positioning. The proposed algorithm is robust and converges after a few iterations. The distributed nature of algorithm allows it to be implemented in an iterative approach.

This paper is organized as follows. Sec. II explains the signal model considered in this paper. Positioning algorithms are explained in Sec. III and simulation results are discussed in Sec. IV. Finally Sec. V concludes the paper.

II. SIGNAL MODEL

We consider a 2D network (the generalization to 3D is straightforward, but is not explored here). Let the sensor network consists of $N+M$ reference nodes located at known position, $\mathbf{z}_i = [x_i, y_i]^T \in \mathbb{R}^2, i = 1, ..., N+M$. Suppose N PRNs are used to measure the TW-TOA between the PRNs and unknown target, and suppose that M SRNs are able to listen and measure signals transmitted by the PRNs and target. For simplicity, we assume that the first Nth sensors are primary nodes and the other M sensors are secondary nodes.
Let us define $\mathcal{C} = \{(i,j) : i = 1, ..., N, j = N+1, ..., N+M\}$ as the set of all pairs of active and passive sensors which are connected. The TW-TOA measurement between reference node i and the target, located at coordinates $\boldsymbol{\theta} = [x, y]^T \in \mathbb{R}^2$

can be written as [5]

$$\hat{t}_i = \frac{r_i(\boldsymbol{\theta})}{c} + \frac{n_{T,i}}{2} - \frac{n_{i,T}}{2}, \quad i = 1, ..., N, \tag{1}$$

where $r_i(\boldsymbol{\theta}) = \|\mathbf{z}_i - \boldsymbol{\theta}\|$ is the distance between the ith PRN and the point $\boldsymbol{\theta}$, $n_{i,T}$ is the TOA estimation error at the target node for the signal being transmitted from the ith PRN, and $n_{i,T}$ is the TOA estimation at the ith PRN for the signal being transmitted from the target node. The estimation errors are modeled as zeros mean Gaussian random variables with variances $\sigma^2_{T,i}$ and $\sigma^2_{i,T}$, i.e., $n_{T,i} \sim G(0, \sigma^2_{T,i}), n_{i,T} \sim G(0, \sigma^2_{i,T})$. Lastly, c is the speed of light.

Suppose that the SRNs are able to measure the TOA for received signal from the target and the PRNs. The TOA estimate of ith PRN in jth SRN is

$$\hat{t}_{i,j} = T_{o_i} + \frac{r_{i,j}}{c} + n_{i,j}, \quad (i, j) \in \mathcal{C}, \tag{2}$$

where ith PRN sends its signal at time reference T_{o_i} which is not known to the SRN and $n_{i,j}$ is modeled as Gaussian random variable $n_{i,j} \sim G(0, \sigma^2_{i,j})$. Suppose that the replied signal from the target to this signal is also received in jth SRN. The TOA estimation for this signal is

$$\hat{t}^T_{i,j} = T_{o_i} + \frac{r_i(\boldsymbol{\theta})}{c} + \frac{r_j(\boldsymbol{\theta})}{c} + n_{i,T} + n_{T,j}, \quad (i, j) \in \mathcal{C}. \tag{3}$$

Having these two measurements in the SRN, i.e., 2 and 3, we are able to measure the TDOA between ith PRN and the target which corresponds to distance from ith PRN to the target plus distance from the target to the jth SRN.

III. POSITIONING ALGORITHMS

In this section, we propose an iterative algorithms to extract position information based on measurements collected in the PRNs and SRNs. To find some insight into the problem, let us consider Fig.1 where one PRN starts the TW-TOA measurement with the target. The PRN sends a signal to the target, and the target replies to this signal. Here, we assume that the turn-around time in the target is extremely small and so we can neglect it. Suppose that two other nodes (SRN1 and SRN2) listen to both signals. Since the distance between reference nodes is known, in the secondary node, it is possible to estimate the time reference from (2), hence the SRNs are able to estimate the overall distance from the PRN to the target and the target to the SRN as follows

$$\hat{r}^T_{i,j} = c(\hat{t}^T_{i,j} - \hat{T}_{o_i}) =$$
$$r_i(\boldsymbol{\theta}) + r_j(\boldsymbol{\theta}) + \frac{c}{2}n_{j,T} + \frac{c}{2}n_{i,T} - cn_{i,j}, \quad (i, j) \in \mathcal{C}, \tag{4}$$

where $\hat{T}_{o_i}$ is an estimation of T_{o_i}.
From (1), the distance estimate to target in the ith PRN is

$$\hat{r}_i = c\hat{t}_i = r_i(\boldsymbol{\theta}) + \frac{c}{2}n_{i,T} - \frac{c}{2}n_{T,i}, i = 1, ..., N. \tag{5}$$

It is obvious that there are correlations between the TW-TOA and the TDOA measurements in the PRNs and SRNs. Considering collected measurements in both PRNs and SRNs, the MLE can be obtained as [5]

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \{ (\frac{2}{c^2\sigma_i^2} - \frac{1}{a_i c^4 \sigma_i^4})(\hat{r}_i - r_i(\boldsymbol{\theta}))^2$$
$$- \frac{1}{a_i} (\sum_{j=N+1}^{M+N} \frac{\hat{r}^T_{i,j} - r_i(\boldsymbol{\theta}) - r_j(\boldsymbol{\theta})}{4c^2\sigma_j^2}) - \frac{\hat{r}_i - r_i(\boldsymbol{\theta})}{a_i c^2 \sigma_i^2} \sum_{j=N+1}^{N+M}$$
$$\frac{(\hat{r}^T_{i,j} - r_i(\boldsymbol{\theta}) - r_j(\boldsymbol{\theta}))}{2c^2\sigma_j^2} + \sum_{j=N+1}^{N+M} \frac{(\hat{r}^T_{i,j} - r_i(\boldsymbol{\theta}) - r_j(\boldsymbol{\theta}))^2}{2c^2\sigma_j^2} \}, \tag{6}$$

where

$$a_i = \frac{1}{2c^2\sigma_T^2} + \frac{1}{2c^2\sigma_i^2} + \sum_{j=N+1}^{M+N} \frac{1}{4c^2\sigma_j^2}. \tag{7}$$

Due to nonlinear nature of the cost function (6), when using iterative search, it may converge to local minim resulting in large estimation error. In the sequel, using geometric interpretation, we propose an iterative method to estimate target position. Suppose there is no noise in the TOA estimation,

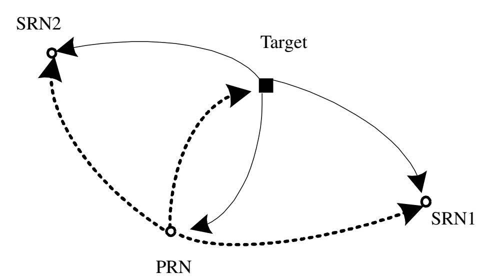

Fig. 1: Primary node initiates transmitting signal to the target and the target replies to the received signal. Both signals are received in the secondary nodes

based on the TW-TOA measurement, it is obvious that the target can be found on a circle with distance r_i centered around $\mathbf{z}_i$. On the other hand, TDOA measurement in the SRN defines an ellipse with foci $\mathbf{z}_i$ and $\mathbf{z}_j$. In absence of noise, target can be found in the intersection of some circles and some ellipses. Let us define disc and elliptic set as follows.
For the ith PRN, consider the disc

$$\mathcal{D}_i = \{\boldsymbol{\theta} \in \mathbb{R}^2 : r_i(\boldsymbol{\theta}) \le \hat{r}_i\}, \tag{8}$$

and for ith SRN and jth PRN, consider elliptic set $\mathcal{E}_k$ as follows

$$\mathcal{E}_k = \{\boldsymbol{\theta} \in \mathbb{R}^2 : r_i(\boldsymbol{\theta}) + r_j(\boldsymbol{\theta}) \le \hat{r}^T_{i,j}\}. \tag{9}$$

Now, the target can be found in the intersection of sets $\mathcal{D}_i, i = 1, ..., N$ and $\mathcal{E}_k, k = 1, ..., NM$,

$$\hat{\boldsymbol{\theta}} \in \mathcal{J} = \bigcap_{i=1}^{N(1+M)} \mathcal{J}_i, \tag{10}$$

where $\mathcal{J}_i = \mathcal{D}_i$ for $i <= N$ and $\mathcal{J}_i = \mathcal{E}_i$ if $i > N$. For the case of empty intersection, which can occur due to measurement noise, the estimator finds a point that minimizes the sum of distance to the sets $\mathcal{J}_i, i = 1, ..., N(1+M)$, i.e.,

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{k=1}^{N(1+M)} \|\boldsymbol{\theta} - \mathcal{P}_{\mathcal{J}_k}(\boldsymbol{\theta})\|^2 \tag{11}$$

where $\mathcal{P}_{\mathcal{J}_k}(\boldsymbol{\theta})$ is the orthogonal projection of $\boldsymbol{\theta}$ onto convex set $\mathcal{J}_k$.

Algorithm 1 ***CE-POCS***

Initialization $\boldsymbol{\theta}^0$ is arbitrary
for $k = 0$ until convergence **do**
  if $\boldsymbol{\theta}^k \in \mathcal{I}_k$ **then**
    $\boldsymbol{\theta}^{k+1} \Longleftarrow \boldsymbol{\theta}^k$
  else

    if k mode $N(1+M) \leq N$ **then**
      $\mathcal{P}_{\mathcal{I}_k}(\boldsymbol{\theta^k}) \Longleftarrow \frac{\boldsymbol{\theta}-\mathbf{z}_i}{\|\boldsymbol{\theta}-\mathbf{z}_k\|}\hat{r}_k$
    else
      $\mathcal{P}_{\mathcal{I}_k}(\boldsymbol{\theta^k}) \Longleftarrow [F_k^{-1}[\frac{[F_k\boldsymbol{\theta}]_{2\times2}}{\|[F_k\boldsymbol{\theta}]_{2\times2}\|}\ 1]^T]_{2\times2}$
    end if
    $\boldsymbol{\theta}^{k+1} \Longleftarrow (1-\lambda_k)\boldsymbol{\theta}^k + \lambda_k \mathcal{P}_{\mathcal{I}_k}(\boldsymbol{\theta}^k)$
  end if
end for

For a disc with radius $\hat{r}_i$, the projection function is simply defined as follows

$$\mathcal{P}_{\mathcal{J}_i}(\boldsymbol{\theta}) = \begin{cases} \boldsymbol{\theta}, & \text{if } r_i(\boldsymbol{\theta}) \leq \hat{r}_i \\ \frac{\boldsymbol{\theta}-\mathbf{z}_i}{\|\boldsymbol{\theta}-\mathbf{z}_i\|}\hat{r}_i, & \text{otherwise.} \end{cases} \tag{12}$$

For elliptic set, we need to project a point onto an ellipse. Here we consider a geometric solution to the elliptic projection problem. A general form of an ellipse can be expressed as

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0. \tag{13}$$

The coefficients of an ellipse can be obtained versus known positions of reference nodes and distance estimate to the target. Suppose there is no noise in the TOA measurements. From (4), for primary node i and secondary node j we have

$$r_i(\boldsymbol{\theta}) + r_j(\boldsymbol{\theta}) = r_{i,j}^T = \|\mathbf{z}_i - \boldsymbol{\theta}\| + \|\mathbf{z}_j - \boldsymbol{\theta}\|. \tag{14}$$

Moving one term of (14) to left and squaring both sides, we have

$$\|\mathbf{z}_i - \boldsymbol{\theta}\|^2 = {r_{i,j}^T}^2 - 2r_{i,j}^T\|\mathbf{z}_j - \boldsymbol{\theta}\| + \|\mathbf{z}_j - \boldsymbol{\theta}\|^2. \tag{15}$$

With some similar manipulations, we have

$$\begin{aligned} &4{r_{i,j}^T}^2((x-x_i)^2 + (y-y_i)^2) = \\ &a^2x^2 + b^2y^2 + c^2 + 2acx + 2bcy + 2abxy, \end{aligned} \tag{16}$$

where $a = 2(x_i - x_j)$, $b = 2(y_i - y_j)$, and $c = x_j^2 - x_i^2 + y_j^2 - y_i^2$. Finally the general form of ellipse is

$$\begin{aligned} &(a^2 - 4{r_{i,j}^T}^2)x + 2abxy + (b^2 - 4{r_{i,j}^T}^2)y^2 + 2(ac + 4{r_{i,j}^T}^2 x_i)x \\ &+ 2(bc + 4{r_{i,j}^T}^2 y_i)y + c^2 - 4{r_{i,j}^T}^2(x_i^2 + y_i^2) = 0. \end{aligned} \tag{17}$$

Therefor the coefficients, A, B, C, D, E, and F can be computed from (17). Equation (13) can be written in a matrix form as

$$\mathbf{z}^T M \mathbf{z} = 0, \tag{18}$$

where $\mathbf{z} = [x\ y\ 1]^T$ and symmetric matrix $M \in \mathbb{R}^3$ is defined as follows

$$M = \begin{bmatrix} A & B & D \\ B & C & E \\ D & E & F \end{bmatrix}. \tag{19}$$

Algorithm 2 ***CE-POCS***

Initialization $\boldsymbol{\theta}_p^0$ and $\boldsymbol{\theta}_s^0$ are arbitrary
for For k = 0 until convergence **do**
  for For $l = 0$ until a predefined number L **do**
    if $\boldsymbol{\theta}_p^l \in \mathcal{D}_l$ **then**
      $\boldsymbol{\theta}_p^{l+1} \Longleftarrow {\boldsymbol{\theta}_p}^l$
    else
      $\mathcal{P}_{\mathcal{D}_l}(\boldsymbol{\theta}_p^l) \Longleftarrow \frac{\boldsymbol{\theta}_p - \mathbf{z}_i}{\|\boldsymbol{\theta}_p^l - \mathbf{z}_k\|}\hat{r}_l$
      ${\boldsymbol{\theta}_p}^{l+1} \Longleftarrow (1-\lambda_l){\boldsymbol{\theta}_p}^l + \lambda_l \mathcal{P}_{\mathcal{D}_l}({\boldsymbol{\theta}_p}^l)$
    end if
  end for
  for For $j = 0$ until a predefined number J **do**
    if $\boldsymbol{\theta}_s^j \in \mathcal{E}_j$ **then**
      $\boldsymbol{\theta}_s^{j+1} \Longleftarrow {\boldsymbol{\theta}_s}^j$
    else
      $\mathcal{P}_{\mathcal{E}_j}(\boldsymbol{\theta}_s^j) \Longleftarrow [F_j^{-1}[\frac{[F_j\boldsymbol{\theta}_s^j]_{2\times2}}{\|[F_j\boldsymbol{\theta}_s^j]_{2\times2}\|}\ 1]^T]_{2\times2}$
      ${\boldsymbol{\theta}_s}^{j+1} \Longleftarrow (1-\lambda_j){\boldsymbol{\theta}_s}^j + \lambda_j \mathcal{P}_{\mathcal{E}_j}({\boldsymbol{\theta}_s}^j)$
    end if
  end for
  $\boldsymbol{\theta}_k = \left(\boldsymbol{\theta}_c^{L+1} + \boldsymbol{\theta}_s^{J+1}\right)/2$
  $\boldsymbol{\theta}_p^0 = \boldsymbol{\theta}_k$ and $\boldsymbol{\theta}_s^0 = \boldsymbol{\theta}_k$
end for

To project a point to an ellipse, we first find a transform that transforms the ellipse to a unit circle. We subject the point to this transform, project onto the unit circle, and subject the projected point to the inverse transform. A unit circle can be expressed as

$$\tilde{\mathbf{z}}^T I_{-1} \tilde{\mathbf{z}} = 0, \tag{20}$$

where diagonal matrix I_{-1} is

$$I_{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{21}$$

Now, we try to find a function which transforms the ellipse (18) to a unit circle (20). This function first scales the ellipse to a circle, then rotates it, and finally transforms it to origin, i.e., $F = TRS$ [6]. Matrixes T, R, S are defined as follows

$$\mathcal{S} = \begin{bmatrix} s_{1,1} & 0 & 0 \\ 0 & s_{2,2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_{2\times2} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}, \tag{22}$$

where $\mathbf{0} = [0, 0]^T$. The matrix R is given

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{2\times 2} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}, \qquad (23)$$

and

$$T = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} I_{2\times 2} & \mathbf{z}_c^T \\ \mathbf{0} & 1 \end{bmatrix}, \qquad (24)$$

where $\mathbf{z}_c = [x_c, y_c]^T$ is the center of the ellipse. Now the relation between ellipse and unite circle can be expressed as $\mathbf{Z} = F\tilde{\mathbf{Z}}$. To find the matrixes T, R and S, we replace the inverse transform of F in (20) and compare with (18) to yield

$$M = (TRS)^{-T} I_{-1} (TRS)^{-1}, \qquad (25)$$

After some manipulations we get [6]

$$M = \begin{bmatrix} R_{2\times 2} S_{2\times 2}^{-2} R_{2\times 2}^T & -R_{2\times 2} S_{2\times 2}^{-2} R_{2\times 2}^T \mathbf{z}_c \\ -\mathbf{z}_c R_{2\times 2} S_{2\times 2}^{-2} R_{2\times 2}^T & \mathbf{z}_c^T R_{2\times 2} S_{2\times 2}^{-2} R_{2\times 2}^T \mathbf{z}_c \end{bmatrix}. \qquad (26)$$

To find transformed matrix, it is enough to find sub-matrixes $R_{2\times 2}$ and $S_{2\times 2}^{-2}$. Since matrix M is symmetric, we can use the singular value decomposition technique for upper left 2×2 matrix of M, i.e., $M_{2\times 2}$,

$$M_{2\times 2} = U^T \boldsymbol{\lambda} U = R_{2\times 2} S_{2\times 2}^{-2} R_{2\times 2}^T. \qquad (27)$$

It is clear that $R_{2\times 2} = U^T$. To find scaling matrix, (25) can be written based on scaling matrix as follows

$$S^{-T} I_{-1} S^{-1} = \begin{bmatrix} \frac{1}{s_{1,1}^2} & 0 & 0 \\ 0 & \frac{1}{s_{1,1}^2} & 0 \\ 0 & 0 & -1 \end{bmatrix} = (TR)^T M (TR). \qquad (28)$$

Finally the projection of a point $\boldsymbol{\theta}$ outside of an ellipse onto the ellipse can be done as follows

1) Compute transform function $F = (TRS)^{-1}$,
2) Transform point to the new coordinate where the ellipse is transformed to the unite circle, i.e. $\boldsymbol{\theta}_T = F\boldsymbol{\theta}$,
3) Find the projection of $\boldsymbol{\theta}_T$ onto the unite circle, $\mathcal{P}(\boldsymbol{\theta}_T) = \frac{\boldsymbol{\theta}_T}{\|\boldsymbol{\theta}_T\|}$,
4) In final step using inverse transform $F^{-1} = TRS$, the projected point on the unit circle is transformed to a point on the ellipse, i.e, $\mathcal{P}(\boldsymbol{\theta}) = F^{-1}\mathcal{P}(\boldsymbol{\theta}_T)$.

Fig.2 shows how a point outside of an ellipse is projected onto the ellipse. To find the position of the target, we employ orthogonal projection onto circular and elliptical convex sets (CE-POCS) sequentially. Algorithm 1 shows implementation of the CE-POCS. We also suggest a different type of implementation of CE-POCS.

Since there are two different convex sets derived from different measurements, circular and elliptical sets, one method of implementing CE-POCS is applying POCS to circular and elliptical sets individually, i.e., orthogonal projection onto circular convex set (C-POCS) and orthogonal projection onto elliptical set (E-POCS). The accuracy of estimation can be improved by combining the two estimates, namely by computing the average and using it as a new initial value. This procedure is continued for a certain number of iterations. Algorithm 2 shows the modified version of CE-POCS.

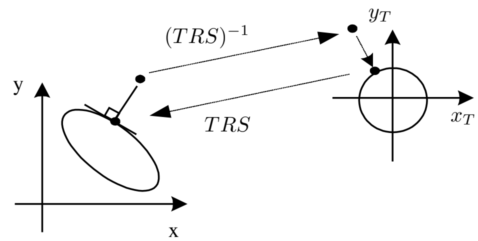

Fig. 2: Projection of a point onto an ellipse

In both algorithms $\{\lambda_k\}_{k=0}^{\infty}$ are relaxation parameters. In the simulation, the relaxation parameters are first set to one, and after a given number k_0 of iteration, decreases as

$$\lambda_k = \left\lceil \frac{k - k_0 + 1}{N} \right\rceil^{-1} \qquad (29)$$

where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

IV. SIMULATION RESULTS

We consider a 100×100 square area. To set reasonable values for variances, we consider Cramér-Rao lower bounds (CRLB). For variance of the TOA estimation, suppose that we are using a signal with 2MHz band width without pulse shaping. The CRLB for the TOA estimation is given by [7]

$$\sqrt{\text{var}(\hat{r}_i)} \geq \frac{c}{2\sqrt{2}\pi\sqrt{SNR}\xi}, \qquad (30)$$

where SNR is the signal to noise ratio and ξ is effective bandwidth which is defined as follows [8],

$$\xi = \left[\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df} \right]^2. \qquad (31)$$

For thermal noise, we assume $N_0 = 1^{-12} W/Hz$.

To compute the SNR, we consider the ensemble mean power at the ith node, measured in dB, which can be modeled as [9]

$$P_i = P_0 - 10\beta \log_{10}(\frac{r_i(\boldsymbol{\theta})}{d_0}) + w_{P_i}, \qquad (32)$$

where β is a path-loss factor and P_0 is the received power in dB at calibration distance d_0. w_{P_i} is a log-normal shadowing term, i.e, $w_{P_j} \sim \mathcal{N}(0, \sigma_{Pi}^2)$. In simulation, we set following values for different parameters in (32)

$$\beta = 2.5, P_0 = -70\text{dBm}, d0 = 1m, \sigma_{Pi}^2 = \sigma_P^2 = 4\text{dB}^2.$$

The cumulative density function (CDF) is considered to compare the performance of different methods. The performance of projection onto convex sets is compared with the MLE estimator derived in [5]. We use the same network deployment of [5] where four reference nodes are located at the corners of

the area. To see the performance of cooperative performance, we consider two cases like [5]. In one case, three reference nodes are considered as PRNs and the last one is considered as SRN. In the second case, two non-consecutive nodes on vertexes (on a diameter) of square area are considered as PRNs and the two other nodes play the role of the SRNs. The target is randomly placed inside of that area over a grid of 19×19. For algorithm 2, the procedure is repeated for five iterations. To implement the MLE, Matlab's gradient-based `lsqnonlin` routine [10] is used.
Fig.3 shows the CDF for the MLE and POCS methods. It is obvious that the MLE in cooperation mode shows good performance compared to others. It converges a small percentage of cases to local minima resulting in large error which is not much clear for this deployment, probably due to regular network deployment, but in general it has problem of converging to local minima. It is also seen that cooperation can improve the performance of the CE-POCS for the large error. For this network, we can see that the second type of algorithm with five iterations improves CE-POCS such that it can be compared to the MLE. One important observation is that the gain obtained in the POCS approach due to cooperation is much bigger than that one for the MLE. Fig.4 shows the

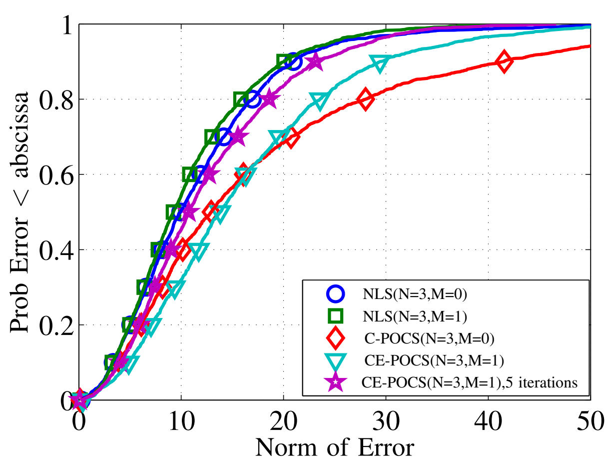

Fig. 3: CDF of cooperative and non-cooperative MLE and POCS for TW-TOA/TDOA measurements (a) N=3, M=0 , and (b) N=3,M=1.

CDF for cooperative and noncooperative algorithms for second case where network contains two PRNs and two SRNs. Again it shows that cooperation improves the performance of both MLE and POCS. For POCS the improved gain for large error is grater than that for MLE. It also shows that algorithms 2 and 1 approximately give the same performance. In other simulations, we observed that algorithm 1 never outperforms algorithm 2.

V. Conclusion

In this paper, we considered the positioning problem in cooperative active and passive sensor network. A maximum likelihood estimator (MLE) can be derived based on measurements in different nodes and the iterative search can be

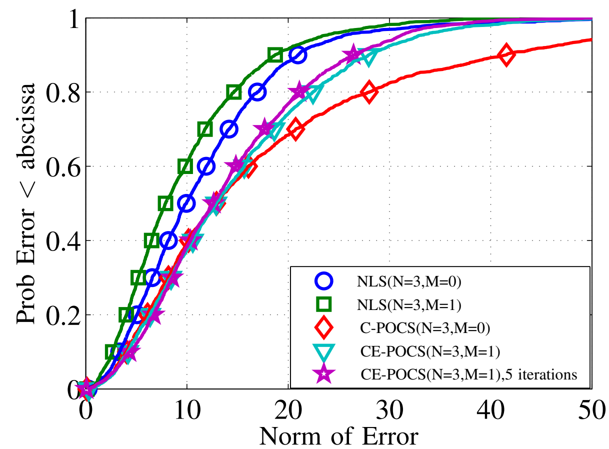

Fig. 4: CDF of cooperative and non-cooperative MLE and POCS for TW-TOA/TDOA measurements (a) N=3, M=0 , and (b) N=2,M=2.

employed to solve it. Due to non linear cost function of the MLE, it needs a good initialization which increases the complexity. Using geometric interpretation, we formulated the positioning problem as the intersection of some convex sets. In fact the positioning problem renders to a feasibility problem which projection onto convex sets can be employed to solve it. The proposed method is a robust technique and can be implemented in a fully distributed manner. Simulation results show a good performance for new method.

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