VERTICAL GROUND HEAT EXCHANGERS: A REVIEW OF HEAT FLOW MODELS

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ABSTRACT

The ground may be used as a heat source, a heat sink or as a heat storage medium by means of vertical ground heat exchangers. Over the years various analytical and numerical models of varying complexity have been developed and used as design and research tools to predict, among others, the heat transfer mechanism inside a borehole, the conductive heat transfer from a borehole and the thermal interferences between boreholes. This paper is based on reviews of scientific work and provides a state-of-the-art review of analytical and hybrid models for the vertical ground heat exchangers. It details and compares various models for short and long term analysis of heat transfer in a borehole. The paper also highlights the strengths and limitations of these models from design and research points of view.

1. INTRODUCTION

Ground source heat pump (GSHP) systems are rapidly becoming state-of-the-art in the field of heating, ventilation and air conditioning (HVAC). A recent study (Lund et al. 2005) has reported that the annual energy use of the ground source heat pumps grew at a rate of 30.3 % and that the installed capacities of the GSHP systems increased by 23.8 % between the years 2000 and 2005. The tremendous growth of the GSHP systems is attributed to their high energy efficiency potential, which results in both environmental and economic advantages. These advantages can be further enhanced by optimizing the systems. One of the key challenges in this optimization is modelling of the ground heat exchanger (GHE).

Application of GHE Models

The modelling of a GHE is an intricate procedure and so far determination of the long-term steady-state temperature response has been the predominant modelling application. Even this basic task usually involves many simplifying assumptions. In more common real-world situation, however, GHEs exhibit transient responses that will last for long as well as short-term intervals. Long or short is decided by the frequency content of the load variations in relation to the thermal properties of the GHE.

The temperature response of a GHE depends on the heat transfer inside the borehole and the heat conduction across the boundary of the borehole. Heat transfer inside the borehole is characterized by its thermal mass and its heat transfer resistance. This resistance may be purely conductive, if the borehole is filled with grout or a viscous liquid. It may contain also a convective term if there is groundwater flow (advection) or thermally induced convection in a water-filled hole. The heat flow from the borehole also depends on various other factors such as the location of the considered borehole in the borehole field and its thermal interaction with the adjacent boreholes.

Single borehole systems, which are mostly used in residential applications, can be designed by considering only the long-term response of their GHEs. The two most critical design criteria for these systems, the appropriate design length of the GHE and the need for balancing of the ground loads, can both be determined using long-term response of the GHE.

Multiple borehole systems, on the other hand, are generally used for energy storage and are more common for commercial applications. In this case the short-term response of these GHE systems has significant impact on the efficiency of the whole GSHP system. Hence, for these systems short-term response of the GHE is equally important as the long-term response.

Model Developments

Over the years various analytical and numerical models of varying complexity have been developed and used as design and research tools. Among other things, they can be used to predict the heat transfer mechanism inside a borehole, the conductive heat transfer from a borehole and the thermal interferences between boreholes. Some of the most noteworthy numerical models include the work of Eskilson and Claesson (1988), Muraya (1994), Zeng et al. (2003) and Al-Khoury et al. (2005; 2006). Numerical models are attractive when the aim is to obtain very accurate solutions or in parametric analysis. However, most numerical models of GHEs have limited flexibility and extended computational time requirements. Therefore they cannot be directly incorporated into building energy simulation software and hence they have limited practical application.

Hybrid models, however, provide a feasible alternative. Such models have been presented e.g. by Eskilson (1987) and Yavuzturk (1991) and they are used to calculate special temperature response functions numerically. These response functions can then be incorporated into the building simulation software as databases and hence can be used without the inherent disadvantages of numerical models. Analytical models, despite being less precise than numerical models, are preferred in most practical applications because of their superior computational time efficiencies and better flexibility for parameterized design. The imprecision in the results of the analytical models correspond to the underlying modelling assumptions made when deriving analytical solutions for GHE. It must, however, be kept in mind that uncertainties regarding the quality of input data may be more significant than uncertainties due to model approximations.

Aim of the Review

This article presents a literature review of the most significant analytical and hybrid solutions used for modelling of the GHE. The purpose of the article is to present the noteworthy GHE models which can be readily used by designers and researchers engaged in the modelling of GSHP systems. The solutions discussed are mainly for the simplest case of a single borehole because the solution becomes complex in case of multiple boreholes due to the thermal interactions between the boreholes. The simplifying assumptions and the resulting limitations of the analytical models when deriving the temperature responses for GHE are also discussed.

The GHE modelling approaches can be divided in two main categories. In the first category are the conventional models which are used to calculate the required borehole depth by predicting its long term performance. These models usually consider the heat transfer from a GHE in a steady-state and model it using long time-steps. Short time-step models, on the other hand, focus more on the transient heat transfer in GHEs. The time step for these models is in the hourly or sub-hourly range. Following the general approach, we have categorized the GHE models under the headings of long-term and short-term response.

2. LONG-TERM RESPONSE

In this overview we will consider three types of models, the infinite and the finite length line sources and the cylindrical source. This includes numerical as well as analytical solutions.

Infinite Length Line Source – Analytical Method

The very first significant contribution to modelling of GHEs came from Ingersoll et al. (1954) who developed the line source (LS) theory of Kelvin (1882) and implemented it to model the radial heat transfer. The GHE is assumed to be a line source of constant heat output and of infinite length surrounded by an infinite homogeneous medium. The classical solution to this problem, as proposed by Ingersoll, is:

$$T - T_0 = \frac{\dot{q}_b}{4\pi\lambda}\int_{1/4Fo}^{\infty}\frac{e^{-u}}{u}\,du = \frac{\dot{q}_b}{4\pi\lambda}\,E_1\left(\frac{1}{4Fo}\right), \qquad Fo = \frac{at}{r_b^2} \tag{1}$$

Eq. (1) is an exact solution to the radial heat transfer in a plane perpendicular to the line source. As the temperature response at the wall of the borehole is sought, the dimensionless time, i.e. Fourier number (Fo), is based on the borehole radius (r_b). Many researchers have approximated the exact integral of eq. (1) using simpler algebraic expressions. Ingersoll et al. (1954), for instance, presented the approximations in tabulated form. Hart and Couvillion (1986), on the other hand, approximated the integral by assuming that only a certain radius of the surrounding ground would absorb the heat rejected by the line source. Various other algebraic approximations of the exact integral of eq. (1) can be found in 'Handbook of mathematical functions' (Abramowitz & Stegun. 1964) and similar mathematical handbooks.

The LS model can be used with reasonable accuracy to predict the response of a GHE for medium to long-term ranges. Ingersoll and Plass (1948) have recommended using LS models only for applications with Fourier numbers >20. The model cannot be used for smaller Fourier numbers as the solution gets distorted for the shorter time scales because of its line source assumption. The classical LS solution also ignores the end effects of the heat source as it assumes the heat source to have infinite length.

Cylindrical Source – Analytical Method

The cylindrical source (CS) method is another established analytical way of modelling heat transfer in GHEs. This method provides a classical solution for the radial transient heat transfer from a cylinder surrounded by an infinite homogeneous medium. The cylinder, which usually represents the borehole outer boundary in this approach, is assumed to have a constant heat flux across its outer surface. The solution has the following general form.

$$T - T_0 = \frac{\dot{q}_b}{\lambda}\underbrace{\frac{1}{\pi^2}\int_0^{\infty}\frac{e^{-u^2 Fo}-1}{(J_1^2(u)+Y_1^2(u))}\left[J_0(r_b^* u)Y_1(u) - J_1(u)Y_0(r_b^* u)\right]\frac{du}{u^2}}_{G-factor} \tag{2}$$

The integral is often referred to as the G-factor in literature. As with the exact integral in the LS method, the G-factor has also been approximated using various tabular and algebraic expressions. Ingersoll et al. (1954), Kavanaugh (1985) and more recently Bernier (2001) have all made important contributions.

Like the LS solution, the CS solution also ignores the end effects of its heat source. It also overlooks the thermal capacities of the fluid and the grout in the GHE. However, the issue of having a constant heat flux across the borehole boundary has been tackled by some researchers by superimposing time-variable loads. The systematic approach of Bernier et al. (2004) deserves a special mention. Based on the CS method, they have modelled the annual hourly variations of a borehole by categorizing the thermal history of the ground into "immediate" and "past" time scales in their so-called Multiple Load Aggregation Algorithm.

Finite Length Line Source – Numerical Method

Eskilson (1987) numerically modelled the thermal response of the GHE using non-dimensional thermal response functions, better known as ***g-functions***. The temperature response to a unit step heat pulse is calculated using the finite difference approach. The model accounts for the influence between boreholes by an intricate superposition of numerical solutions with transient radial-axial heat conduction, one for each borehole. This model is the only one that accounts for the long-term influence between boreholes in a very exact way. The thermal capacities of the GHE elements are however neglected.

The response to any heat input can be calculated by devolving the heat injection into a series of step functions. The temperature response of the boreholes is obtained from a sum of step responses. A representation of g-functions plotted for various borehole configurations is shown in figure 1. The temperature response for any piecewise-constant heat extraction is calculated using eq. (3).

$$T - T_0 = \sum_i \frac{\Delta \dot{q}_i}{2\pi\lambda} \cdot g\left(\frac{t - t_i}{t_s}, r_H^*, \dots\right), \qquad t_s = \frac{H^2}{9a} \qquad (3)$$

Here, the change in heat extraction at time t_i is $\Delta\dot{q}_i$ .The dots in the argument of the g-function refer to dimensionless parameters that specify the position of boreholes relative to each other. The limitation of the numerically calculated g-functions lies in the fact that they are only valid for times greater than ($5r^2_b$ /a), as estimated by Eskilson. This implies times of 3-6 hours for typical boreholes as noted by Yavuzturk (1999). Another practical aspect of the g-functions is that these functions have to be pre-computed for various borehole geometries and configurations and then have to be stored as databases in the building energy analysis software.

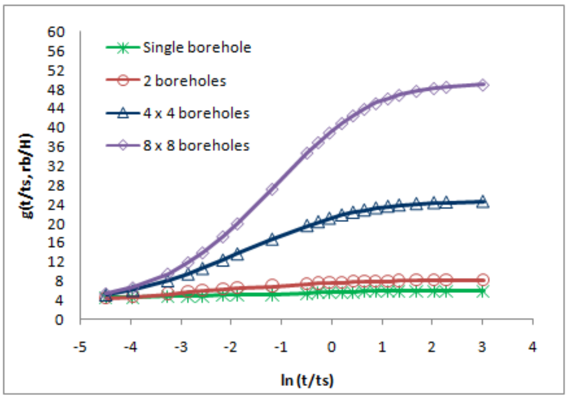

Figure 1: Eskilson's g-functions for various borehole configurations

Finite Length Line Source – Analytical Solution

Many researchers have tried to determine ***analytical g-functions*** to address the flexibility issue of numerically computed g-functions. Eskilson (1987) himself developed an analytical g-function expression, which was later adopted by Zeng et al. (2002). The explicit analytical g-function is determined using a line heat source with finite length. The temperature at the middle of the borehole of the length H is taken as the representative temperature when calculating the heat transfer between the borehole and the fluid. The mathematical expression for this analytical g-function is:

$$g(Fo, r_H^*) = \frac{1}{2}\int_0^1 \left\{ \frac{erfc\left[\frac{\sqrt{r_H^{*2} + [0.5-(z/H)]^2}}{2\sqrt{Fo}}\right]}{\sqrt{r_H^{*2} + [0.5-(z/H)]^2}} - \frac{erfc\left[\frac{\sqrt{r_H^{*2} + [0.5+(z/H)]^2}}{2\sqrt{Fo}}\right]}{\sqrt{r_H^{*2} + [0.5+(z/H)]^2}} \right\} \mathrm{d}\left[\frac{z}{H}\right] \quad (4)$$

Lamarche and Beauchamp (2007) used a similar approach to calculate their analytical g-function. However, they used the integral mean temperature along the borehole depth, z, instead of considering the middle point temperature. Their approach provided a much better match to the numerically calculated g-functions than that proposed by Zeng et al. (2002).

3. SHORT-TERM RESPONE

Until very recently, most of the solutions for the GHE analysis overlook the short-term response of GHEs. The solutions either completely ignored it or they used oversimplified assumptions. In reality, however, the short-term variations have significant effects on the performance of the heat pump and the overall system. Short-term response of the ground is also critical during heat flux build up stages and for cases with both heating and cooling demands. Studies regarding hourly or sub-hourly thermal energy use and the electrical demands of the ground coupled heat pump system also require the short-term response of the ground to be considered.

In this overview we will describe analytical models of Young (2004), Lamarche and Beauchamp (2007) and Bandyopadhyay et al. (2008) and the implicit numerical model of Yavuzturk (1999). The solutions of the analytical models are presented in their general form. For the exact solutions, the readers are referred to the original literature cited in this overview.

Implicit Numerical Method

The first major contribution to analyze the short-term response of a GHE came from Yavuzturk (1999). He extended Eskilson's concept of non-dimensional temperature response functions to include the short-term analysis using a two dimensional implicit finite volume numerical approach. His model approximated the cross section of the two legs of the U-tube as pie-sectors with constant flux entering the numerical domain for each time step. The model accounted for pipe, grout and flow-related convective resistances.

Yavuzturk noted that the short-term g-functions are typically applicable for times in-between 2.5 min and 200 hours while the long term g-functions are applicable for times longer than 200 hours. As with Eskilson's g-functions the short time-step g-functions of Yavuzturk lack in flexibility and inherit the disadvantages associated with most of the numerically obtained solutions. Due to these reasons, the analytical solutions to predict the short-term response of the boreholes have generated a lot of interest from the researchers.

Analytical Buried Electrical Cable Analogy

Young (2004) modified the classical buried electrical cable (BEC) method. This was developed by Carslaw and Jaeger (1959) to study the heating of the core of an electrical cable by steady current. Young, however, used the analogy between a buried electric cable and a vertical borehole by considering the core, the insulation and the sheath of the cable to represent respectively the equivalent diameter fluid pipe, the resistance and the grout of the GHE. A grout allocation factor (*f*) allocating a portion of the thermal capacity of the grout to the core, was also introduced to provide a better fit for ground heat exchanger modelling. The classical solution to the BEC problem has the following general form as proposed by Carslaw and Jaeger.

$$T - T_0 = \frac{\dot{q}_b}{\lambda} \frac{2\alpha_1^2 \alpha_2^2}{\pi^3} \int_0^\infty \frac{1 - e^{-u^2 Fo}}{u^3\, \Delta(u)} du \tag{5}$$

Analytical Solutions for Composite Media

Lamarche and Beauchamp (2007) developed analytical solutions for short-term analysis of vertical boreholes by considering a hollow cylinder of radius r_e inside the grout which is surrounded by infinite homogeneous ground. The cylinder, the grout and the surrounding ground all represent different media and have different thermal properties. Assuming that the cylinder reaches a steady flux condition much earlier than the adjacent grout, analytical solutions for short-term response of the GHE were developed for two cases of constant flux and of convective heat transfer with known mean fluid temperature. For the first case, with a known heat per unit length ($\dot{q}_b$) the proposed solution is:

$$T - T_0 = \frac{\dot{q}_b}{\lambda} \frac{8\,(\lambda/\lambda_{grout})}{\pi^5 {r_e^*}^2} \int_0^\infty \frac{(1 - e^{-u^2 Fo})}{u^5 ({\phi_c}^2 + {\psi_c}^2)} du, \qquad Fo = \frac{at}{r_e^2} \tag{6}$$

For the second case with convective heat transfer and with a known constant fluid temperature, T_f, the solution takes the following form:

$$T = T_f + (T_0 - T_f)\, \frac{16\,Bi\,(\lambda/\lambda_{grout})}{\pi^4 {r_e^*}^2} \int_0^\infty \frac{e^{-u^2 Fo}}{u^3 ({\phi_c}^2 + {\psi_c}^2)} du \tag{7}$$

Analytical Virtual Solid Model

More recently Bandyopadhyay et al. (2008) have modelled the short-term response of a GHE in a non-steady-state situation. The model takes the thermal capacity of the circulating fluid into account by the S/S_{core} ratio which is the ratio of the thermal capacity of an equivalent volume to the thermal capacity of the core and also considers the flow related convective heat transfer using Biot number. The circulating fluid in the GHE is modelled as a 'virtual solid' surrounded by infinite homogeneous medium. The heat transferred to the 'virtual solid' is assumed to be generated uniformly over its length. The following classical solution proposed by Blackwell is applicable under these conditions.

$$T - T_0 = \frac{\dot{q}_b}{\lambda_{grout}} \frac{8}{\pi^3} \left(\frac{S}{S_{\mathrm{core}}}\right)^2 \int_0^\infty \frac{1 - e^{-u^2 Fo}}{u^3 (O^2 + P^2)} du \tag{8}$$

DISCUSSION

The short and long term responses of GHE are determined using different approaches. When determining the long-term response of the GHE the geometry of the borehole is often neglected and the borehole is modelled either as a line or as a cylindrical source with finite or infinite lengths. Due to these unrealistic assumptions regarding the geometry of the borehole, the thermal capacities of the borehole elements and the flow-related convective heat transfer inside the borehole are also ignored when analyzing the long term response of the GHE. Bernier (2004) and Nagano (2006) have developed calculation tools using classical CS and LS methods. However, Eskilson's g-function approach, based on the finite LS assumption, is considered as the state-of-the-art and it has been implemented in many building energy simulation software including EED, TRANSYS, Energy Plus and GLEHEPRO.

Short-term response of a GHE, on the other hand, requires more stringent assumptions and the GHE cannot be simply modelled as a line or a cylindrical source. The actual geometry of the borehole is therefore usually retained when determining its short-term response. An equivalent diameter is used for simplifications instead of considering a U-tube with two legs. The equivalent diameter assumption allows taking the thermal mass of the borehole elements and the flow-related convective resistances into account. The short-term g-functions developed by Yavuzturk (1999) are regarded as the-state-of-the-art in determining the short-term response of GHE. Like the g-function approach of Eskilson, the short-term g-function approach has also been implemented in various building simulation and ground loop design software including TRANSYS, Energy Plus and GLEHEPRO.

It is appropriate to highlight that the temperature response of a GHE as predicted by almost all the models is of the following general form.

$$T - T_0 = \frac{\dot{q}_b}{\lambda}\, f(Fo) \tag{9}$$

This indicates that the load on the GHE and its thermal conductivity are the two significant factors in addition to the response of the GHE, i.e. f(Fo). As seen in figure 1, the short-term response of a borehole is independent of size and configuration of its borehole field. The long-term response, however, is strongly influenced by these factors.

CONCLUSION

The solutions predicted by the analytical models are all functions of the $(\dot{q}_b/\lambda)$ ratio multiplied to a function of Fourier number. The simple analytical models described in this paper can be used with reasonable accuracy to predict the response of GHEs with a single borehole. However, there is a shortage of such models when it comes to multiple borehole models. There is a genuine need of an analytical model capable of simulating both the short and the long-term response of the GHE ideally considering all of the significant heat transfer processes related to the GHE and without distorting the actual geometry of the borehole.

NOMENCLATURE

a	ground thermal diffusivity ($m^2\ s^{-1}$)
Bi	Biot number $= \mathrm{hr}/\lambda$
Fo	Fourier number $= \mathrm{at}/\mathrm{r}^2$
H	active borehole depth (m)
J_x	xth − order Basel function of first kind
$\dot{q}_b$	heat flow per unit length of GHE (W m^{-1})
r_x^*	non-dimensional radius
S	unit length thermal capacity (J K^{-1} m^{-1})
t	time (s)
T	temperature (K)
u	integral parameter
Y_x	xth − order Basel function of second kind
z	axial coordinate (m)

Greek Symbols

λ	ground thermal conductivity (W $m^{-1}K^{-1}$)

subscripts

b	considering borehole radius r_b
e	considering radius r_e
f	fluid
H	considering borehole depth
s	steady − state
0	undisturbed

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