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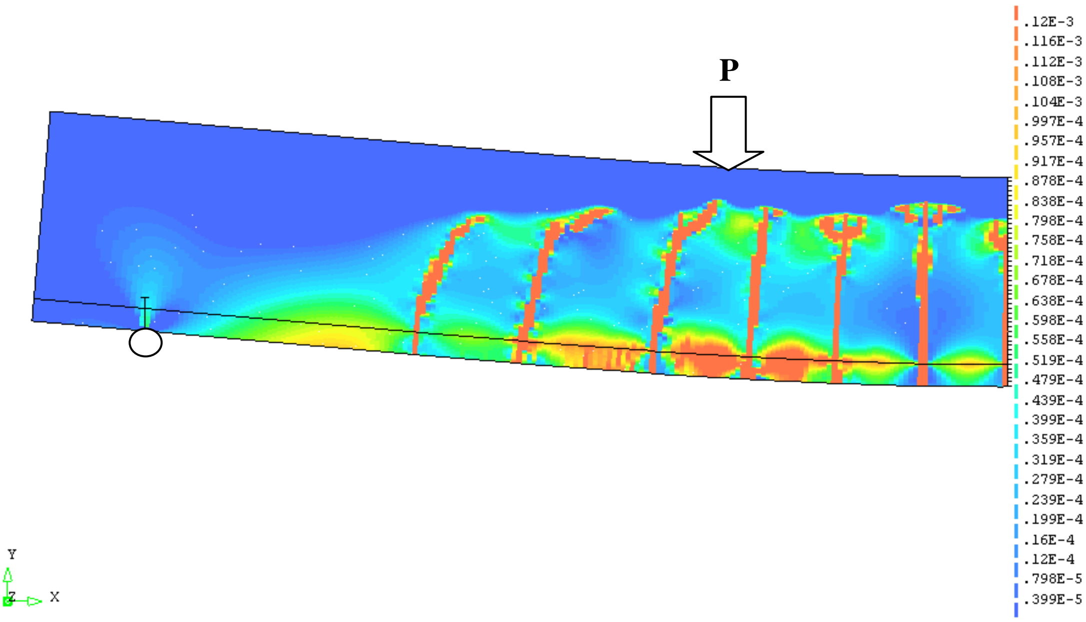

Fibres in reinforced concrete structures - analysis, experiments and design

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Department of Civil and Environmental Engineering
Division of Structural Engineering
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2008

THESIS FOR THE DEGREE OF LICENTIATE

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ISSN 1652-9146
Lic 2008:3
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Cover:
Crack pattern from finite element analysis of beam with fibre content V_f =0.25% and three reinforcement bars of diameter 6 mm.

Chalmers Reproservice
Göteborg, Sweden 2008

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ABSTRACT

Potential benefits from fibres in concrete are improved crack control and the possibility of more slender structures. The extent of the crack control depends, among other factors, on the amount of fibres added, and plays a great role for durability. As of today there exist no generally accepted design and analysis procedures, and if the technique with fibres is to move forward, there is a need for development of such methods. As a part of the present work, an investigation of currently available design methods (proposed) was made. In addition a selection of analysis methods for fibre-reinforced concrete specimens in bending was studied. The main characteristics and comparisons of the investigated design and analysis methods are presented in a report Jansson (2007), and also in an article based on the report, Jansson *et al.* (2008b).

Although several technical committees have proposed design methods, these methods are mainly intended for design in the ultimate limit state (ULS). Therefore, in order to control and understand crack growth in fibre-reinforced concrete, methods aimed at serviceability limit state design are needed.

The present work has been carried out with this in mind and the aim, in the long run, is to develop a method which can be used to predict crack widths, i.e. small crack widths relevant for the serviceability limit state (SLS). The work includes experimental evaluation in the form of four-point beam-bending tests to investigate flexural behaviour and wedge-splitting tests to obtain material properties in the form of a stress-crack opening (σ-w) relationship. Finite element analyses (FEA) of the tested beams were performed. This is the tool which, in combination with fracture mechanics, is believed to have the potential to provide the desired results regarding crack-width prediction.

From the work presented here, the FEA results indicate that a rather simplified (σ-w) relationship is sufficient for calculations in the ULS, while for the SLS a more refined σ-w) relationship may be required. The multi-linear (σ-w) relationship, which was investigated and used in the present work, appears to yield more accurate results during the early stage of the analysis, i.e. the cracking stage.

Keywords: Fibre-reinforced concrete, crack width, post cracking, design methods, analysis method, stress crack-width relationship

Fibrer i armerade betongkonstruktioner - analys, experiment och dimensionering

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SAMMANFATTNING

Potentiell vinst med fibrer i armerade betongkonstruktioner är förbättrad sprickontroll samt möjlighet till slankare konstruktioner. Vidden av sprickontroll beror bland annat på mängden tillsatta fibrer, och har stor inverkan på beständigheten. I dagsläget finns inga allmänt vedertagna dimensionerings- och analysmetoder, och om tekniken att armera med fibrer ska kunna utvecklas, finns ett behov för utveckling av sådana metoder. Som en del av detta licentiatarbete gjordes en inventering av tillgängliga dimensioneringsmetoder (föreslagna). Utöver det studerades ett urval av analysmetoder för fiberarmerade betongkroppar. Utmärkande egenskaper, samt jämförelser mellan de undersökta dimensionerings- och analysmetoderna återfinns i en rapport, Jansson (2007).

Trots att flera tekniska kommittèer har tagit fream förslag på dimensioneringsmetoder, så är dessa metoder främst avsedda för dimensionering i brottstadiet. Därför, för att kunna kontrollera och förstå spricktillväxt i fiberarmerad betong, behövs metoder som är avsedda för dimensionering i bruksstadiet.

Det häri presenterade arbetet har utförts med detta i åtanke och målet, i ett längre perspektiv, är att finna/utveckla en metod som kan användas till att förutsäga sprickbredder, dvs. de små sprickbredder som förekommer i bruksstadiet. Arbetet innefattar experimentell utvärdering i form av fyrpunkts balkböjning för att undersöka böjbeteende och kilspräckprovning för att ta fram materialegenskaper i form av spänning-spricköppning (σ-w) samband. Finita elementanalyser utfördes för de testade balkarna. Det är detta verktyg, i kombination med brottmekanik, som i detta arbete anses ha potential att åstadkomma de önskade resultaten med avseende på sprickbreddsbedömning.

Resultaten från det häri presenterade arbetet indikerar att ett förenklat σ-w samband är tillräckligt noggrant för beräkningar i brottstadiet, medan det för bruksstadiet krävs ett mer detaljerat samband. Det multilinjära σ-w sambandet, vilket undersöktes och användes i detta arbete, förefaller ge något bättre resultat för den tidiga delen av analysen, dvs. sprickstadiet.

Nyckelord: Fiberarmerad betong, sprickbredd, residual, dimensioneringsmetod, analysmetod, spänning-spricköppningsamband

LIST OF PUBLICATIONS

The following papers are included in this thesis:

Paper I

"Design methods of fibre reinforced concrete: a-state-of-the-art review". Jansson A. Löfgren I. and Gylltoft K. Submitted to *Nordic Concrete Research* in February 2008.

Paper II

"Applying a fracture mechanics approach on FRC beams, material testing and structural analysis". Jansson A. Löfgren I. and Gylltoft K. Submitted to *Journal of Advanced Concrete Technology* in February 2008.

The following publications have been written as a part of the presented work, but are not included in this licentiate thesis:

Publication I

"Analysis and design methods for fibre reinforced concrete: a state-of-the-art report". Jansson A. Chalmers report no 2007:16, 196 pages.

Publication II

"Applying a fracture mechanics approach to material testing and structural analysis of FRC beams". Jansson A. Löfgren I. and Gylltoft K., conference paper presented at FRAMCOS-6 Catania, Italy. June 2007.

Contents

Preface

The work presented in this licentiate thesis was suggested by AB Färdig Betong / Thomas Concrete Group together with Chalmers University of Technology. It is a continuation of the work on fibre-reinforced concrete structures conducted at Chalmers by Ingemar Löfgren. The present work was carried out between February 2006 and March 2008 at Chalmers University of Technology, at the Department of Civil and Environmental Engineering, Division of Structural Engineering, Concrete Structures.

First of all, I would like to thank my supervisor and examiner, Prof. Kent Gylltoft, for having given me the opportunity to work on this research project, for creating an inspiring environment, and for the valuable discussions we have had throughout the work, as well as my assistant supervisor, Ph.D. Ingemar Löfgren, for sharing his thorough knowledge and giving valuable advice. I would also like to extend my appreciation to Prof. Ralejs Tepfers who has enthusiastically shared his broad and deep knowledge, and to my colleague Helen Broo, who cannot be thanked enough for her patience with answering my constant flow of questions.

Penultimate, but not last, are thanks to all of my colleagues at the Department, who have all, in one way or another, assisted with the many theoretical and practical problems encountered, as well as for their good humor making the work more enjoyable.

Finally, but not least, I would like to express my sincere gratitude to the companies that made this project possible through a donation to Chalmers: Thomas Concrete Group and AB Färdig Betong. For his involvement in the project, I would also like to thank Prof. Tomas Kutti. In addition, Bekaert Sweden is appreciated for having supplied fibres to the experiments. Furthermore, I thank my family for their unlimited support and for reminding me what is important besides work.

It is my hope that this licentiate thesis will be read and reviewed critically, and that any viewpoints, comments and suggestions regarding its content will be directed to me.

Göteborg, February 2007

Anette Jansson

Notations

Upper case letters

E	Modulus of elasticity of matrix
F_{sp}	Splitting load in the wedge-splitting test
F_{v}	Vertical load in the wedge-splitting test
G_{F}	Specific fracture energy
G_{f}	Specific energy dissipated during fracture
F_{sp}	Splitting load in the wedge-splitting test
F_{v}	Vertical load in the wedge-splitting test
l_{f}	Fibre length
M	Bending moment
N	Normal force
N_{b}	Number of bridging fibres
$N_{\mathrm{f.WST}}$	Number of fibres per unit area in a fractured specimen
V_{f}	Volume fraction of fibres

Lower case letters

a_1	Initial slope of the bi-linear σ-w relationship
a_2	Second slope of the bi-linear σ-w relationship
b_2	Intersection of the bi-linear σ-w relationship with the y-axis
d_{f}	Diameter of fibre
f_{c}	Compressive strength
f_{t}	Tensile strength
f_y	Yield stress of reinforcing steel
f_u	Ultimate tensile capacity of reinforcing steel
h	Height of beam section
l_{ch}	Characteristic length
s	Length of non-linear hinge region
s_{rm}	Average crack spacing
w	Crack opening

Greek letters

α	Wedge angle in the wedge-splitting test
δ	Deflection
ε	Strain
v	Poisson's ratio
ρ	Reinforcement ratio
μ	Coefficient of friction
θ	Crack opening angle
η_{b}	Fibre efficiency factor
σ	Stress
$\sigma(w)$	Stress as a function of crack opening
τ_{b}	Bond strength

Abbreviations

AR-GFRC	Alkali Resistant Glass Fibre Reinforced Concrete

CMOD	Crack Mouth Opening Displacement
CoV	Coefficient of Variance
EC 2	Eurocode 2
FEA	Finite Element Analysis
FEM	Finite Element Method
FRC	Fibre-Reinforced Concrete
GFRC	Glass Fibre Reinforced Concrete
HSC	High-Strength Concrete
HPFRCC	High-Performance Fibre-Reinforced Cementitious Composite
PVA	Polyvinyl acetate
RC-65/35	Specification of Dramix® fibre (65/35 = aspect ratio / length)
SFRC	Steel Fibre-Reinforced Concrete
SCC	Self-Compacting Concrete
SIFCON	Slurry Infiltrated Fibre Concrete
SIMCON	Slurry Infiltrated Mat Concrete
WST	Wedge-Splitting Test

1 Introduction

During the past decades the concrete construction field has experienced a growing interest in the advantages fibre reinforcement has to offer. Between the different fibres available, e.g. steel, synthetic, glass, and natural fibres, the steel fibre is probably the most investigated and most commonly used. Fibre reinforcement today is mainly used in applications such as industrial floors, overlays, and sprayed concrete, although other application areas exist. Some of the potential benefits of fibres in concrete are improved crack control and the possibility of designing more slender structures. However, the extent of the crack control depends to a large extent on the type and amount of fibres added.

From a durability point of view it is essential to control the cracking process and, moreover, being able to predict crack widths and crack pattern as well as to design a structure that exhibits the desired behaviour. This behaviour, of course, depends on a number of different factors such as structural type and size, type of concrete and amount and type of reinforcement, and, not at least, the casting procedure. In general, to achieve crack control, large amounts of conventional reinforcement are needed, especially in structures where only very small crack widths ($w \leq 0.1$ mm) are allowed. Negative effects from large amounts of reinforcement are that: structural dimensions often need to be larger than what is needed for load bearing capacity in order to make space for all the steel; the heavy labour placing it; and also difficulties with pouring the concrete past the tightly placed reinforcement bars of the steel cage. By using fibres in combination with or instead of the conventional reinforcement, these drawbacks may be reduced or even completely avoided.

Fibre reinforced concrete (FRC) is a cement based composite material reinforced with discrete, usually randomly distributed, fibres. The objective with adding fibres to a concrete mix is to bridge discrete cracks and thereby providing for increased control of the fracture process and also to increase the fracture energy (i.e. yields a more ductile behaviour).

Combining concrete with dispersed “fibres” consisting of grains of steel left-overs was an idea patented already in 1874 by the American A. Berard, thus creating a new more ductile material. Today steel and synthetic fibres are used for both non-structural and structural purposes, where the latter (e.g. polypropylene and nylon) has mainly been used to control the early cracking (plastic-shrinkage cracks) in slabs, Löfgren (2005). Although it has been found that adding fibres to concrete mainly enhances the post-cracking properties in terms of a more ductile behaviour and reduced crack widths, it still remains to show that these enhanced mechanical properties can be predicted with reasonable accuracy and that they can be incorporated into design methods.

1.1 Literature survey

A literature survey revealing the current state of research for FRC has been carried out as a part of this licentiate work. It was found that the available literature is extensive. The current understanding of the behaviour of fibre-matrix interfacial mechanics is based on a number of pullout studies using single or multiple fibres, and development of theoretical models. Some of the major studies in the field are e.g. Bentur *et al.*

(1985), Gopalaratnam and Shah (1987), Namur and Naaman (1989), Bentur and Mindess (1990), Stang and Shah (1990), Wang *et al.* (1990a), Wang *et al.* (1990b), Li *et al.* (1993), Leung and Li (1991), Chanvillard and Aïtcin (1996), Kullaa (1994), Li and Stang (1997), Grünewald (2004). Regarding flexural behaviour of FRC several theoretical approaches have been proposed, see e.g. Lok and Pei (1998), Lok and Xiao (1999) and Ezeldin and Shiah (1995) for purely analytical models, and Zhang and Stang (1998) for a semi-analytical one. An analysis model developed for finite element calculations is described by Barros and Figueiras (2001). Flexural behaviour of FRC in terms of cracking may be found in Rossi (1999) and Stang *et al.* (1995). For a majority of the currently available design methods though, the material properties/structural behaviour is proposed to be determined from structural tests such as beam bending or uniaxial tension tests, e.g. RILEM TC 162-TDF (2003), CNR-DT 204/2006 (Draft 2006), DAfStb UA SFB N 0146 (2005 (In German).), and Swedish Concrete Society (1997). The number of FRC structural applications is increasing and some worth mentioning are tunnel linings, see e.g. Nanakorn and Horii (1996), and Kooiman (2000) and suspended flat slabs without any conventional reinforcement, e.g. Gossla (2006). It should also be mentioned that new concepts and new techniques are continuously being developed, see e.g. Shah and Kuder (2004) and Li (2002), for a comprehensive overview, see Bentur and Mindess (2006). In the here presented work, fracture mechanics was implemented in finite-element modelling, simulating four-point bending of steel-fibre-reinforced full-scale beams, with focus on the small crack widths that occur in the serviceability limit state. The combination fracture-mechanics / finite-element modelling is a concept gaining interest among researchers in the field, see e.g. Kanstad and Dössland (2004), Plizzari and Tiberti (2007), and Tlemat *et al.* (2006).

1.2 Aim

The aim with the present work was to investigate available design methods, evaluate, and present an overeview, presented in a state-of-the-art report. In addition, by means of experiments and non-linear fracture mechanics the flexural behaviour of reinforced FRC members was to be investigated. However, the overall goal for this project is to develop a method which can be used to predict crack widths especially in the serviceability limit state.

The reason for the work being focused on ability to predict crack widths is mainly due to the lack of common design methods regarding FRC structures. Since it has been concluded that one of the main benefits from fibres is the improved crack control, e.g. decreased crack widths, (e.g. Stang and Aarre (1992) **a**nd Schumacher (2006)) and as existing methods are primarily focused on design in the ultimate limit state, it seems natural to focus the work on methods for determination of the flexural behaviour of FRC, including sufficient crack-width predictions.

1.3 Limitations

Although this thesis includes an investigation of different fibre materials available today, the conducted experiments and analyses are based on one single type of fibre, namely a hooked- end steel fibre: Dramix^{TM} RC-65/35-BN. The work has been focused on fibre-reinforced self-compacting concrete with softening post-cracking behaviour.

1.4 Scientific approach

Experiments as well as computer analyses have been carried out. The experiments were of two kinds; wedge splitting of cubic specimens to determine material properties, and four-point beam bending to investigate structural behaviour. In order to gain further knowledge of the material/structural behaviour, finite element (FE) analyses were performed using the computer software Diana, see TNO (2005). Also here two kinds of analyses were performed, inverse analyses to obtain the material properties in tension and simulation of the beam-bending tests for better understanding of the flexural behaviour.

1.5 Outline

This licentiate thesis consists of two articles and a summary of the work done. The purpose of the summary is to connect the different parts of the performed work, put them in their context and also to further explain certain areas.

In Chapter 2 the experimental work is explained. The four-point beam-bending test (4PBT) is addressed only briefly, since it is a well known test method. The wedge splitting test, on the other hand, is described more in detail. Chapter 3 treats the finite element analyses both for the 4PBT and the inverse analysis (which is based on the WST). The modelling aspects are explained and difficulties are pointed out and discussed. Throughout the text, the aim is to put each part of the work in its context enabling the reader to follow the intended path of the project from start to finish and to see the connection to the demands from the concrete society. Finally in Chapter 4 new and old fibre technology is addressed followed by a discussion in Chapter 5 of everything that has been treated in the previous Chapters.

The first article in this licentiate thesis is a summary of the report "*Analysis and design methods for fibre reinforced concrete - a state-of-the-art report*", Jansson (2007), which is a review of ten of the most currently proposed design methods for FRC. The report also includes an investigation of six analysis methods for FRC proposed by different researchers in the field. Article number 2, "*Applying a fracture mechanics approach on FRC beams, material testing and structural analysis*", is a continuation of the conference paper, "*Applying a fracture mechanics approach to material testing and structural analysis of FRC beams*", Jansson *et al.* (2007). In the conference paper fracture mechanics and finite element modelling based on a bi-linear σ-w relationship was treated and the continuation consists of the same analyses using a multi-linear σ-w relationship (as presented in Chapter 4 in the present work).

2 Experiments

2.1 Test programme

Four-point beam bending tests (4PBT) were carried out on five series with a total of 15 beams. The test programme is outlined in Table 1. In each series of three beams a different amount of fibres was used: 0, 0.25, 0.5 and 0.75% by volume (or 0, 19.6, 39.3, and 58.9 kg/m^3). In addition two sizes of longitudinal reinforcement were used, where each beam was reinforced with 3 reinforcement bars of diameter either ϕ6 or ϕ8 mm. The fibres used were Dramix^{TM} RC-65/35-BN (fibre-aspect ratio = 65 and length = 35 mm). For details see Jansson *et al.* (2007) and Gustafsson and Karlsson (2006).

The concrete used was self-compacting (a slump-flow of 550 to 650 mm) and had a *w/b*-ratio of 0.55. The mix compositions, as well as compressive strengths for each mix can be found in Gustafsson and Karlsson (2006). In order to determine the compressive strength of the used FRC and obtain an estimation of the tensile strength, for each mix, compression testing of cubic specimens was carried out at the laboratory of Thomas Concrete Group AB in Göteborg. The WST, the inverse analyses for the bi-linear σ-w relationship as well as the four-point beam-bending tests were performed at Chalmers University of Technology in Göteborg by Master students Sanna Karlsson and Martin Gustafsson, supervised by Ingemar Löfgren and Tomas Kutti at Thomas Concrete Group. The FE-analyses of the beams and the inverse analyses for multi-linear σ-w relationship were carried out by the author.

Table 1. Outline of the test programme.

Series	Fibre content % / [kg/m^3]	Reinforcement number and diameter [mm]	Number of beams	Number of WST cubes	Number of Compression cubes
1	-	3 ϕ8	3	9	6
2	0.5 / 39.3	3 ϕ8	3	9	6
3	0.5 / 39.3	3 ϕ6	3	9	6
4	0.25 / 19.6	3 ϕ6	3	9	6
5	0.75 / 58.9	3 ϕ6	3	9	6

2.2 Material testing

2.2.1 Compressive strength

To determine the compressive strength of each mix, compressive tests on cubic specimens were conducted. For the fibre reinforced mixes the obtained compressive strengths varied between 37 MPa for the Series with fibre volume V_f =0.75% and 39 MPa for the Series with V_f =0.25%. For the mix with no fibres added the compressive strength was 47 MPa.

2.2.2 Properties of the conventional reinforcement

The reinforcement bars were produced from a TEMPCORE steel with yield-stress capacity f_{sy} = 660 MPa and f_{su} = 784 MPa for the re-bars with 6 mm diameter, and for the re-bars with 8 mm diameter f_{sy} = 590 MPa and f_{su} = 746 MPa.

2.2.3 Tensile fracture behaviour - Wedge Splitting Test

The tensile fracture behaviour of the fibre-reinforced concretes was determined by conducting wedge-splitting tests (WST), see Figure 1. Nine cubic specimens were tested for each series, adding up to a total of 45 specimens. The tensile stress-crack opening (σ-w) relationships were obtained for each mix by conducting inverse analysis following a procedure presented by Löfgren et al. (2005), see Section 3.2 for details on inverse analysis. This approach, in previous studies by the author and other researchers, has been shown to yield reliable results; see e.g. Meda *et al.* (2001), Löfgren (2005), Löfgren et al. (2005), and Löfgren *et al.* (2008). Since the fibre bridging stress is influenced by the number of fibres crossing the fracture plane and, when the stress-crack opening relationship is determined from a material test specimen of different size and shape compared to the full-scale specimen, it may be necessary to consider any difference in fibre orientation between the specimens. A smaller specimen, as e.g. the cubic WST specimen, is likely to experience a fibre orientation closer to 2D, while a larger specimen most likely approaches a 3D fibre distribution. Therefore the number of fibres was counted in all the WST specimens and an average experimental fibre efficiency factor was determined for each mix. The experimental fibre efficiency factor, $\eta_{b.WST}$, was calculated as:

$$\eta_{b.WST} = \frac{N_{f.WST}}{V_f / A_f} \qquad (1)$$

where $N_{f.WST}$ is the number of fibres per unit area, V_f is the fibre volume fraction, and A_f is the cross-sectional area of a fibre. The experimental fibre efficiency factor, $\eta_{b.WST}$, should be compared to the fibre efficiency factor, $\eta_{b.beam}$, for the tested beams, which depends on whether the fibres have a free (random) or biased orientation. For the tested beams, the fibre efficiency factor was determined theoretically using an approach suggested by Dupont and Vandewalle (2005). For a 2D fibre distribution the theoretical fibre efficiency factor equals $2/\pi\approx0.64$, while for a 3d distribution it equals 0.5, Krenchel (1975).

To account for the differences in fibre efficiency factor between the WST specimens and the tested beams, the stress-crack opening relationship obtained from the inverse analyses was reduced with the ratio between the two fibre efficiency factors, according to:

$$\sigma_{b.beam}(w) = \sigma_{b.WST}(w) \cdot \frac{\eta_{b.beam}}{\eta_{b.WST}} \qquad (2)$$

The test setup is shown in Figure 1. A cast groove (depth 22mm) is provided for the splitting load to be applied and a starter notch (53mm for small cube and 78mm for larger cube) is cut in the groove in order to ensure crack propagation. To prevent horizontal cracking, a vertical notch with depth 25mm is cut on the front and on the back face of the cube (Figure 1). Two steel platens with roller bearings are placed partly on top of the specimen, partly into the groove, and through a wedging device the splitting force, F_{sp}, is applied. The applied vertical load, F_v, is monitored during the test, as is the horizontal deformation (CMOD) at the level of the roller bearings. If the friction of the roller bearings is ignored, the splitting load, F_{sp} (horizontal), is calculated by the wedge angle α as in Equation 3 (in this case α = 15°).

$$F_{sp} \approx \frac{F_v}{2\tan(\alpha)} \qquad (3)$$

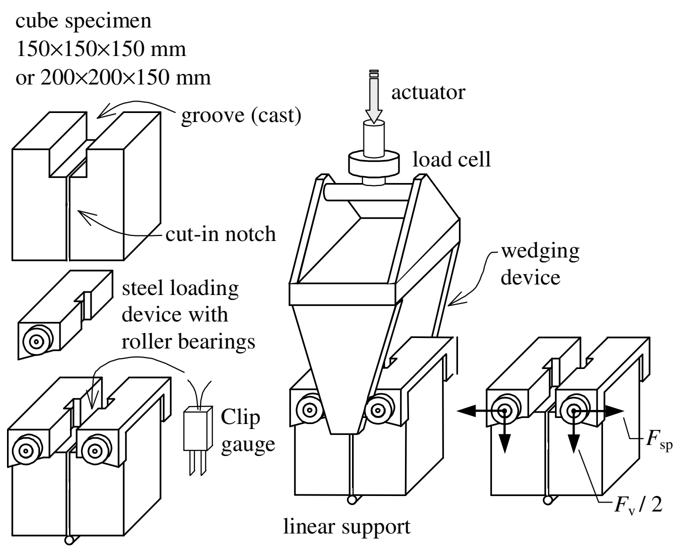


Figure 1-Principle of the wedge-splitting test method, from Löfgren (2005).

Results from WST

To enable easier comparison the nine results for each series have been combined into one average cuve. The average measured splitting load versus CMOD curve for each series is shown in Figure 2.

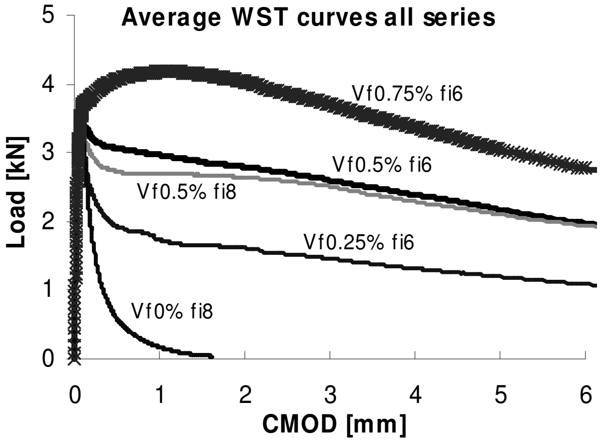


Figure 2 - Average curves from WST results

2.3 Beam bending

The beams were simply supported (rollers at both ends) with a span of 1800 mm and subjected to a four-point load, according to Figure 3, with a distance between the loads of 600 mm. The tests were conducted with deflection control, and during the tests the following parameters were measured: load; deflections and support settlements; and at two points the width of a crack were measured. The deflection was measured at mid-span and at the loads; at all measuring points two displacement transducers were used. For details see Jansson *et al.* (2007) and Gustafsson and Karlsson (2006).

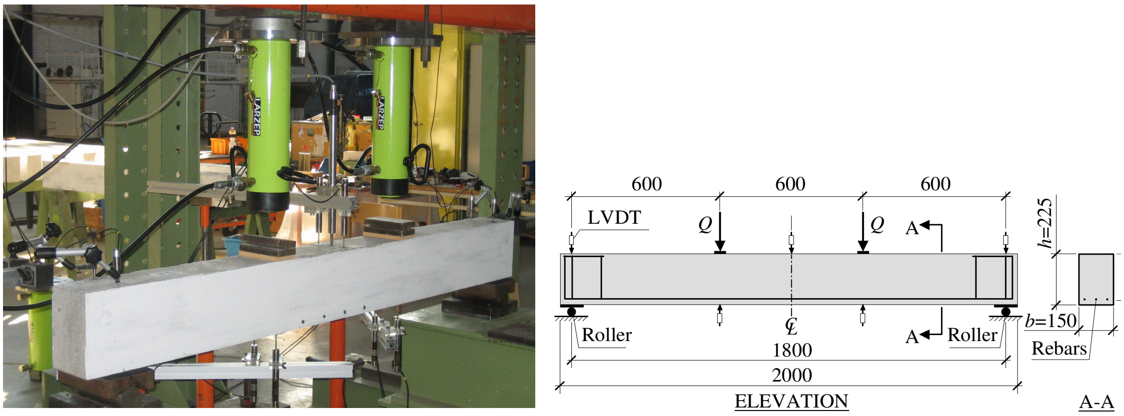


Figure 3 - Setup for the beam-bending tests, a) photo from laboratory and b) dimensions and loading conditions, from Gustavsson and Karlsson (2006).

For easier comparison of experiments and FE results, in each test series, the resulting curves from the three beams have been combined into one average curve, (in total five average curves). In Figure 4 the results from the 4PBT and the average results are shown.

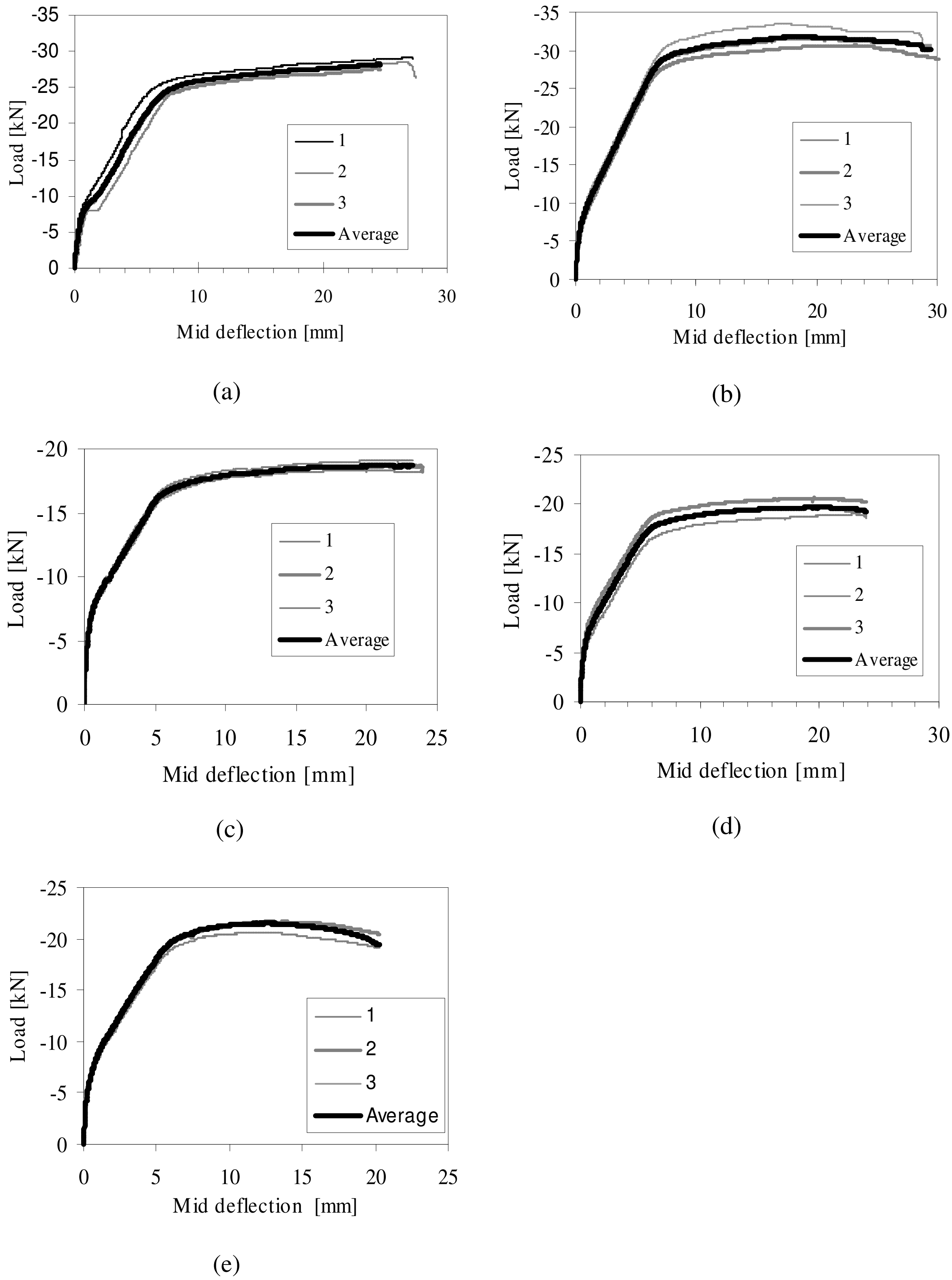


Figure 4 - 4PBT results: a) V_f=0% rebar ϕ8; b) V_f=0.5% rebar ϕ8; c) V_f=0.25% rebar ϕ6; d) V_f=0.5% rebar ϕ6 and e) V_f=0.75% rebar ϕ6.

3 Analyses

3.1 Analytical analysis

In addition to the FE analyses, an analytical approach was used. The analytical model is based on the non-linear hinge model as proposed by Olesen (2001a; Olesen (2001b), and described by RILEM TC 162-TDF (2002). The non-linear hinge model, in its turn, is based on non-linear fracture mechanics and the fictitious crack model, originally proposed by Hillerborg *et al.* (1976) and Hillerborg (1980). To be able to model the behaviour of reinforced FRC members, the non-linear hinge model was further developed by Löfgren (2003) to consider: (1) the non-linear stress-strain behaviour in compression; (2) a multi-linear stress-crack opening relationship; and (3) a multi-linear strain-hardening relationship for the reinforcement. The sectional forces N and M are obtained for each crack opening, *w*, by an incremental deformation of the crack angle θ^*, see Figure 5.

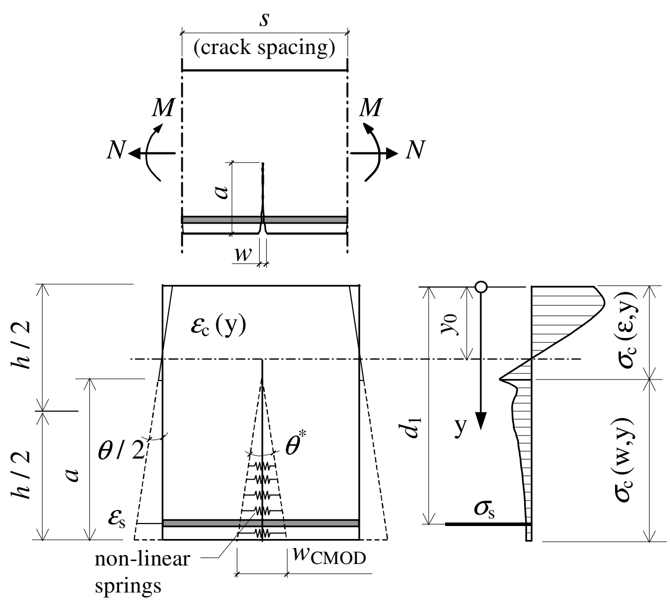


Figure 5 - Non-linear hinge model with conventional reinforcement, from Löfgren (2005).

Fictitious crack model

The fictitious crack model for FRC, as proposed by Hillerborg *et al.* (1976) and Hillerborg (1980), accounts for the stresses that are transferred over the crack by the fibres by the so called σ-w relationship. As long as stress transfer occurs, the crack is referred to as a fictitious crack as opposed to a real crack with zero stress transfer. Outside the fracture area elastic stresses are assumed. Stress distribution for the fictitious crack model is shown in Figure 6, and the main parameters are the modulus of elasticity, the tensile strength, the fracture energy and the shape of the σ-w curve. For plain concrete the shape of the σ-w curve does not vary much and the fracture energy together with the tensile strength are the most important properties. For FRC, on the other hand, the shape of the σ-w curve varies considerably depending on the quality of the concrete and the type and amount of fibres used. Also, the fracture energy is of no interest, since a stress free crack opening in most FRCs does not occur until the crack opening is very large.

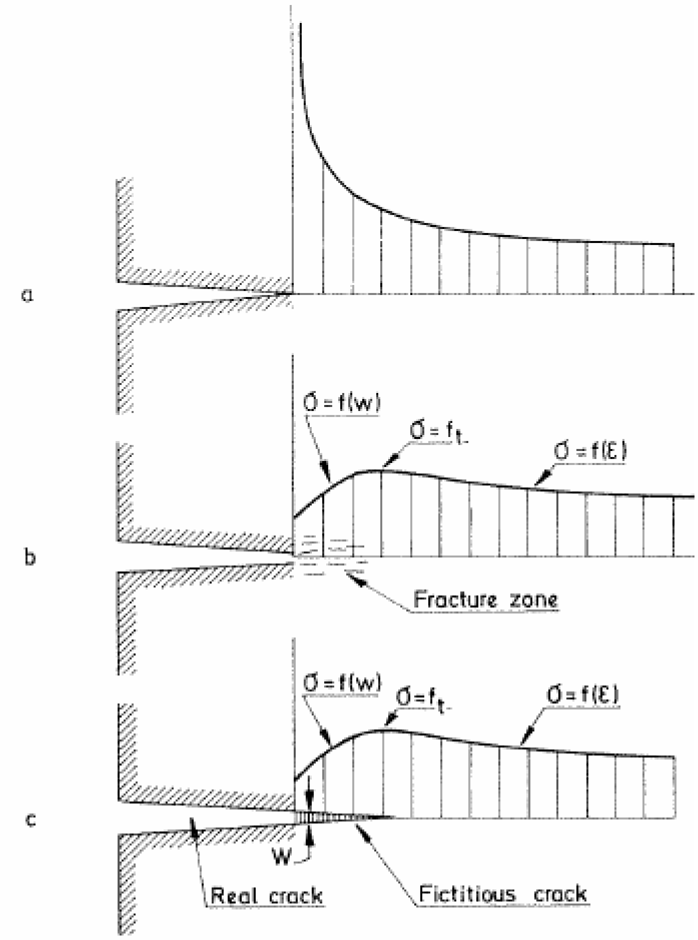


Figure 6 Stresses at crack tip; a) theory of elasticity, b) influence of fracture zone taken into account and c) fracture zone shown as fictitious crack. From Hillerborg (1980).

Results

In Table 2 and Figure 7 the maximum loads Q_{max} from the 4PBT and the analytical analyses are compared. It can be seen that the agreement is acceptable for some Series and quite good for the others, but generally the models tend to overestimate the ultimate load. The analytical model appears to be a useful tool for calculations of load bearing capacity in the ultimate limit state, and in addition it may be used for calculation of crack width with reasonable result, Löfgren (2005).

Table 2. Maximum loads from experiments and analyses.

Q_{max} [kN]					
	$V_f0\ \phi8$	$V_f025\ \phi6$	$V_f05\ \phi8$	$V_f05\ \phi6$	$V_f075\ \phi6$
Exp	*28.4*	*18.7*	*31.8*	*19.8*	*21.5*
Analytic	*29.8*	*21.2*	*33.4*	*23.4*	*26.1*
$Q_{analytic}/Q_{exp}$	*1.05*	*1.13*	*1.05*	*1.18*	*1.21*

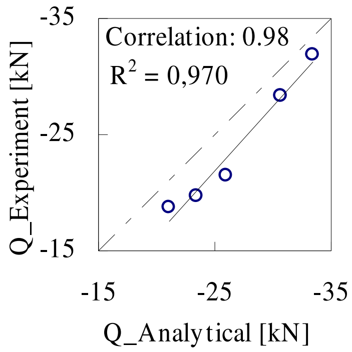


Figure 7- Comparison of maximum load from experiments and from the analytical approach.

3.2 Inverse analysis

By curve fitting, so called inverse analysis, it is possible to obtain a tensile stress versus crack opening (σ-w) relationship which is representative for the material in question. In this case the wedge-splitting tests were simulated with FE analyses and the resulting curves (load-crack opening) were compared with the results from the WST experiments. A σ-w relationship is chosen and used as material input for the FE analysis and the exact shape of it is obtained by a trial and error process. When the FE results agree with the experimental results with enough accuracy, the resulting σ-w relationship is considered representative for the material in the WST specimens. For larger specimens (e.g. the tested beams), the fibre distribution and orientation are likely to differ from the more confined conditions in the WST cubic specimens. Therefore an adjustment of the so called fibre-efficiency factor is done. For the choice of σ-w relationship there are several options, e.g. drop-constant, bi-linear, and multi-linear, to mention a few, see Figure 8.

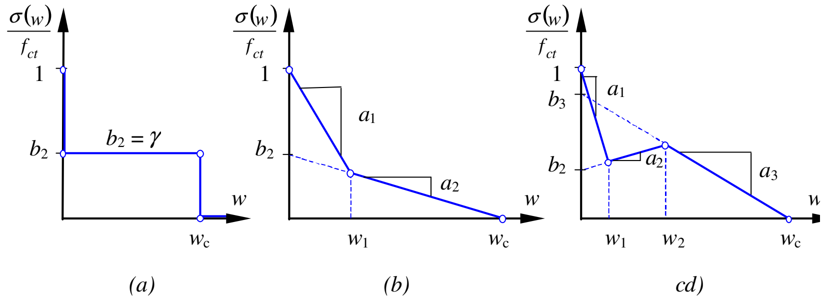
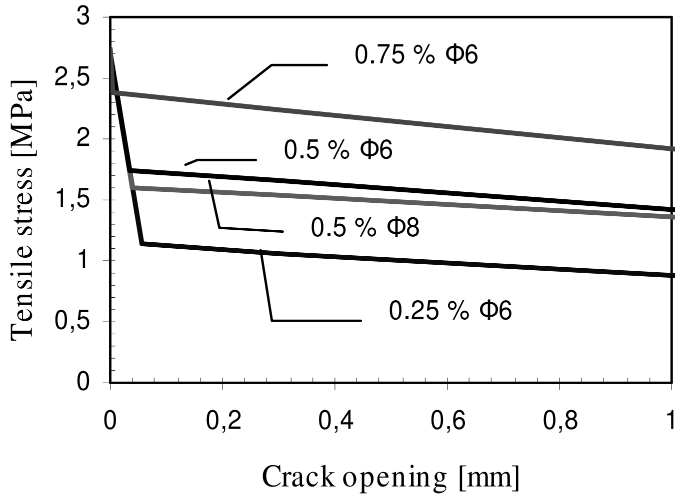


Figure 8 Different σ-w relationships: (a) drop-constant; (b) bi-linear; and (c) polylinear (or multilinear). From Löfgren (2005).

3.2.1 Bilinear σ-w relationship

In a first attempt a bi-linear σ-w relationship was chosen. This is generally considered to describe the material behaviour of FRC with sufficient accuracy if the main goal is to predict an overall behaviour in terms of crack spacing and maximum moment capacity. It is described by five parameters: E_c, f_t, a_1, a_2 and b_2, (Figure 8b), where E_c is Young's modulus, f_t is tensile capacity of the matrix, a_1 is the slope of the first branch, a_2 is the slope of the second branch and b_2 is the point where the second branch crosses the y-axis.

In this work wedge-splitting tests (WST) were performed in order to obtain the material properties of the different concrete mixes (see Section 2.2.3). In the inverse analysis, different combinations of the parameters E_c, f_t, a_1, a_2 and b are put together until the average load-CMOD curve from the WST (for each series) can be recreated with predetermined accuracy. (CMOD=crack mouth opening displacement). Naturally there exists no unique solution, which should be kept in mind when judging the results from the FE-analyses. The bi-linear σ-w relationships were obtained by using an optimisation program in Matlab®© developed at DTU by Østergaard *et al.* (2003) and are shown in Figure 9a. Figure 9b shows a comparison of the results from the inverse analyses and the WST results.



(a)

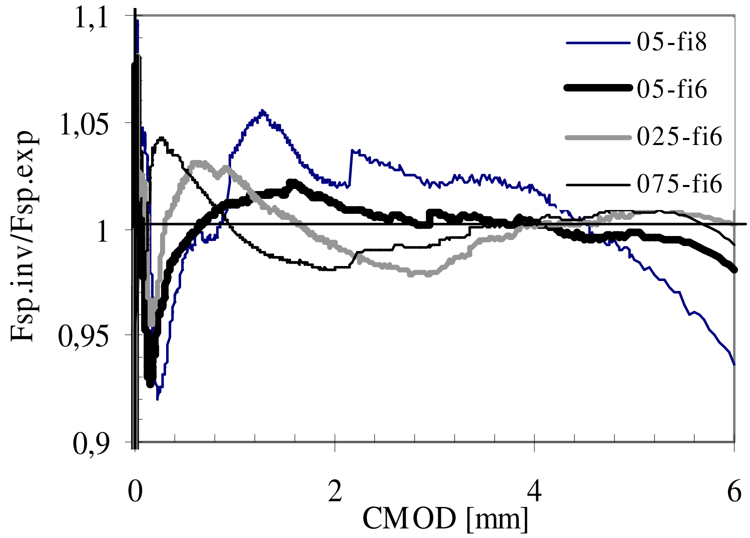


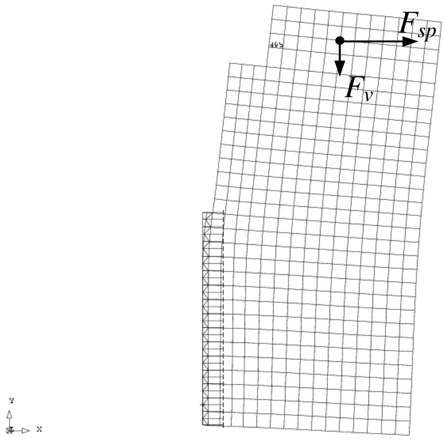
Figure 9 a) Bi-linear σ-w relationship for the full-scale elements adjusted to account for differences in the fibre efficiency factor and b) the ratio $F_{sp,inv}/F_{sp,exp}$ i.e. evaluation of agreement between inverse result and experiments.

3.2.2 Multilinear σ-w relationship

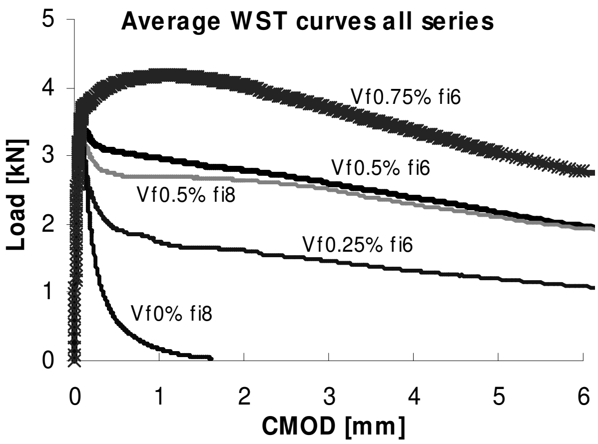
In a second attempt a multi-linear σ-*w* relationship was used. In this case the inverse analyses were carried out by the author as a combination of manually changed values in the σ-*w* relationship and finite element analysis of a WST specimen. The reason for choosing a multi-linear relationship was to investigate if better agreement could be achieved with the laboratory results. Of special interest was to see if crack widths and crack spacings can be simulated/predicted with higher accuracy especially in the area of the crack widths found in the serviceability limit state.

The inverse analyses were carried out for the same WST curves as used for the first work with the bi-linear σ-w relationship. For the FE analyses the WST cube was modelled in half (symmetry) using the discrete crack approach (Figure 10a). With this approach no characteristic length is needed since the crack is predetermined to occur exactly in the elements along the symmetry line. In reality the horizontal splitting load is achieved from the applied vertical load which is transformed into horizontal loading through the roller bearings. In the inverse analysis both the vertical and the horizontal loads must be applied in the loading point (Figure 10a). The horizontal component was obtained by using Equation 3 and the two forces were applied through stepwise incrementation. The average results in the form of load-CMOD curves from the experimental wedge-splitting tests are shown in Figure 10b.

Starting with a relevant choice of tensile strength, ft, and elastic modulus, E, a multi-linear σ-w curve (with an approximate shape in accordance with σ-w curves obtained in uniaxial testing performed by other researchers), is used as input in an FE simulation of a wedge splitting test. The FE results, in the form of load-CMOD curves are then compared with the average curves from the experimental WST results (Figure 11). In Figure 12 the obtained multi-linear σ-w curves are shown. The agreement is checked by taking the ratio of the experimental load for each measured crack opening and the load in the FE results (Figure 11d). If the ratio is close to unity ($0.95<F_{Exp}/F_{FE}<1.05$), the obtained σ-w relationship is assumed acceptable to represent the tensile material characteristics. The fracture energy, which is a major factor in fracture mechanics, is kept the same in the FE analysis as in the WST experiments by the actual agreement of the two resulting load-CMOD curves.



(a)



(b)

Figure 10 – a) FE model of the WST specimen, b) WST results for each series.

The load-CMOD curves from the WST testing were recreated with better agreement than the ones in the first attempt where a bi-linear σ-w relationship was used, compare Figure 9b and Figure 11d. An inverse analysis does not yield one unique answer regarding the values in the σ-w relationship. It was noted though, that if the tensile capacity f_t is chosen within a relevant range (for the actual concrete strength class), the FE results for the beam bending analyses will differ only slightly. (This is valid for the present work where the σ-w relationships were adjusted with respect to fibre efficiency factor.)

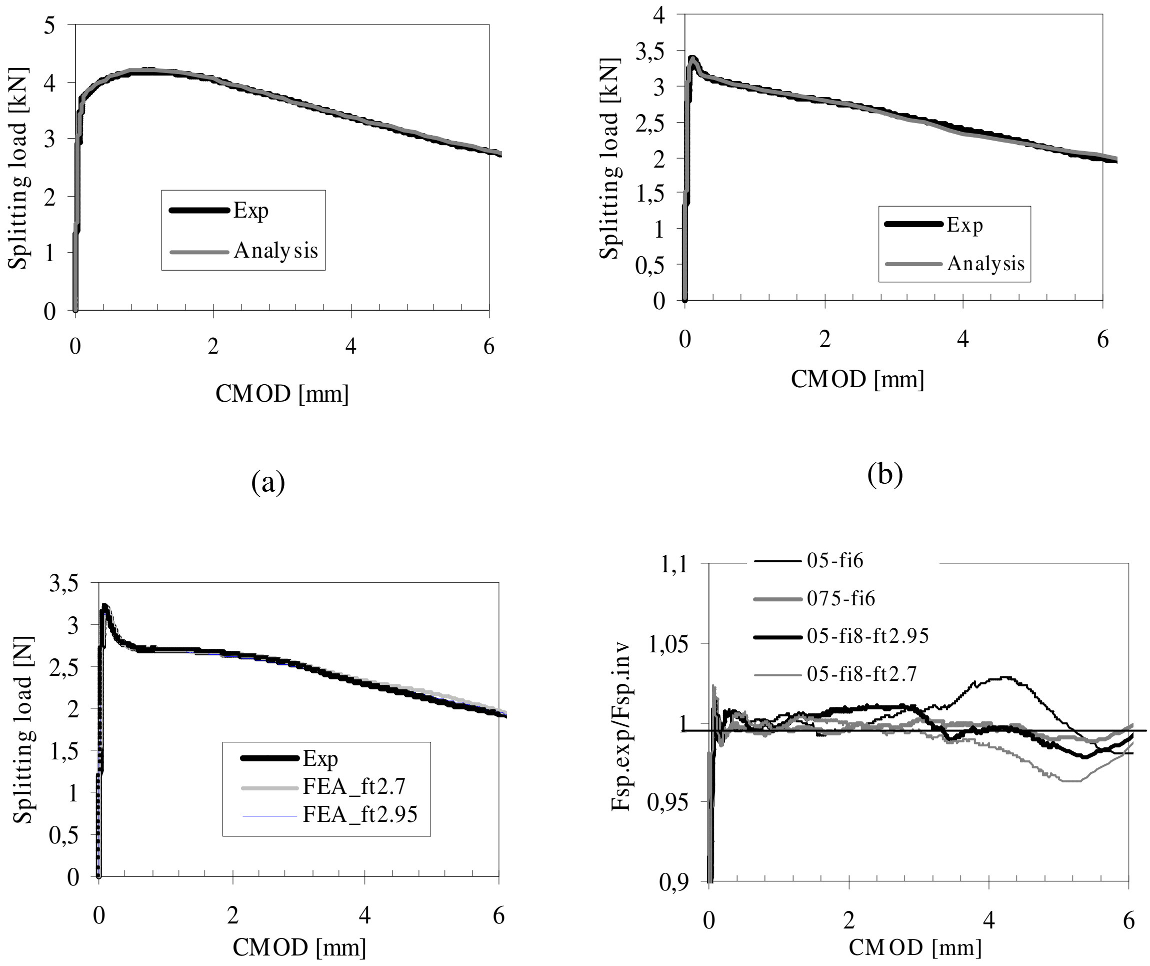


Figure 11- Inverse analysis of multi linear σ-w relationship, a) for V_f =0.75%; b) for V_f =0.5% ϕ6; c) for V_f =0.5% ϕ8 and d) comparison of inverse results with experiments through ratio F_{exp}/F_{FE},inv.

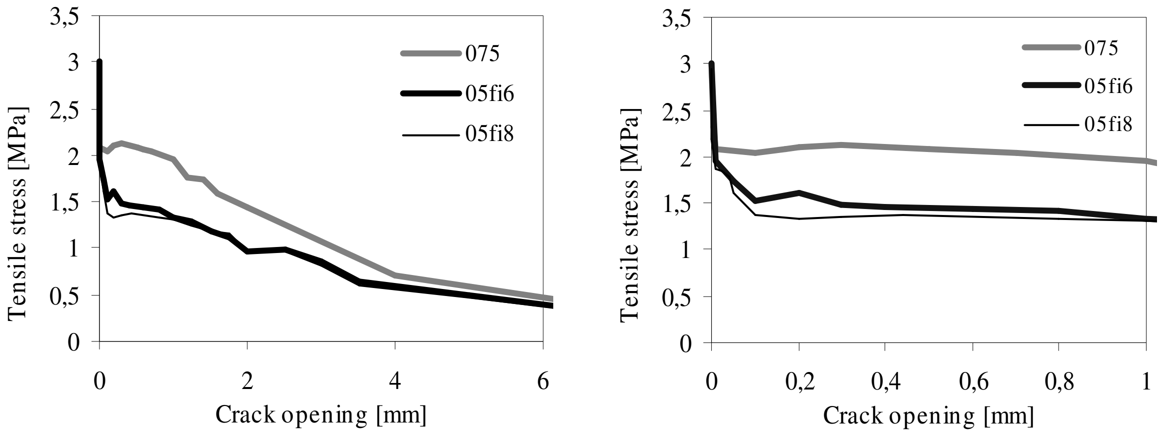


Figure 12 The obtained multi-linear σ-w *relationships where right hand figure shows only the part up to w = 1 mm.*

3.3 FE analysis

All FE analyses in this thesis were carried out using the software Diana, developed at TNO offices, Delft, the Netherlands.

3.3.1 FE model for beam bending

Due to symmetry only half the beam was modelled, meaning that in the model the nodes along the symmetry line (in this case the mid-section of the beam) are restricted to move in the longitudinal direction of the beam. A symmetry approach also means that only half of the crack located in the elements at the symmetry area is modelled, hence the fracture energy for this crack should be halved. The beam-bending specimens were modelled in 2D using four-node quadrilateral isoparametric plane stress elements for the concrete. This element is based on linear interpolation and Gauss integration.

Two approaches were used when modelling the reinforcement bars; the bond-slip approach and the embedded approach. For the former the reinforcement bars were modelled with truss elements and the connection between the truss elements and the concrete elements was modelled with non-linear interface elements given relevant bond-slip characteristics to simulate the connection between the steel and the concrete, see Paper II Jansson *et al.* (2008a). For the latter the reinforcement was modelled using the embedded concept, meaning that the reinforcement has no degrees of freedom of its own. Instead it is a part of the concrete, but given steel properties at the location of the reinforcement bars. The two concepts may be useful in different areas; the bond-slip is necessary if the main interest is to study crack patterns and crack openings, while the embedded concept provides enough information of the load-bearing capacity if the crack pattern is of little interest. The latter requires less input (e.g. no bond-slip properties) and thus it is simpler and less time consuming to perform. It should be noted that the embedded approach was used only in the first part of the work, where bi-linear σ-w relationships were used, see Publication II, Jansson *et al.* (2007).

3.3.2 Loading

A stepwise incremental deformation was applied in a point 600 mm from the support centre so the span of the beam was evenly divided in three parts. In reality the load is distributed through a loading plate of size 100 x 150 mm^2 and the same is true for the supports. To simulate the loading and support plates in the experiments, the nodes next to the actual loading point were connected using the TIE command. The desired behaviour was to relate the downward movement of the nodes on the right side of the loading point to a corresponding upward movement of the left hand nodes. See Figure 13 for model view and mesh and Figure 14 for explanation of the TIE command.

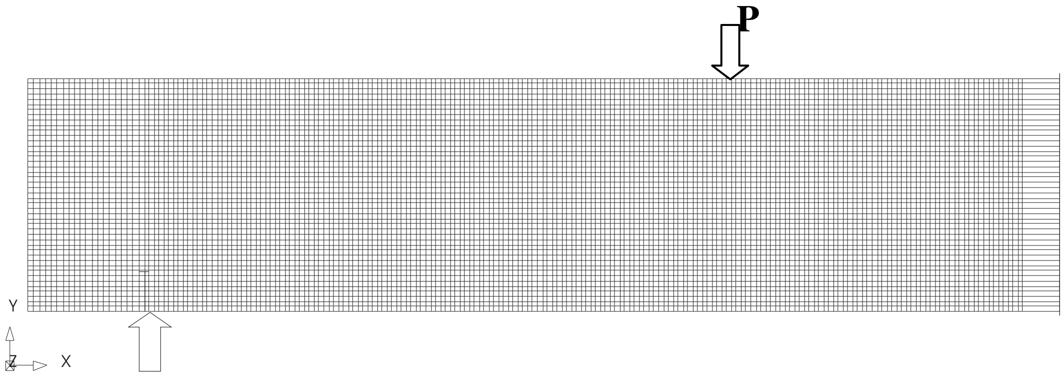


Figure 13 Diana beam model.

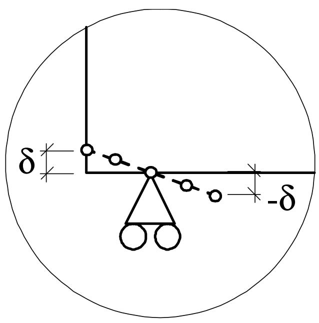


Figure 14 - Modelling of a support plate with the TIE command. I.e. forcing the nodes on each side of (e.g.) the support to have the same vertical displacement but in the opposite direction.

3.3.3 Crack model

The cracking was modelled using a total strain-based concept, where rotation of the cracks is allowed. Total strain means that no distinction is made between different types of strains, thus only orthogonal cracking can be handled. In most engineering problems, though, this is fully sufficient, Rots.J.G). The fact that the crack is allowed to rotate means that the crack faces will be located perpendicular relative to the principal strain direction. In addition a smeared crack approach was used, where the complete cracking zone is smeared out over a certain width, l_{ch} = characteristic length.

3.3.4 Input parameters

Several factors govern the results in the FE simulations, and choosing correct input is not completely straight forward for FRC. Besides the three factors addressed below, it should be mentioned that also the size of the steps in the incremental loading and the mesh element size may affect the result, e.g. a very small step size may cause numerical problems, while a too coarse mesh may not correctly capture the flexural behaviour. In the present work though, the mesh size is rather fine and since the characteristic length for the SFRC beams was user-defined, the influence from the last factor was avoided.

3.3.4.1 σ-w

The stress-crack-width relationship (σ-w) describes the tensile behaviour of the FRC material for each possible crack width. Although tensile properties are preferably determined through direct testing, i.e. with uniaxial tension tests, the difficulties in setting up this test makes the use of indirect test methods, e.g. beam-bending and plate tests necessary, Nour and Massicotte (2007) and Martinola *et al.* (2007). With indirect test methods it is not easy to distinguish between the actual behaviour of the fibre system as such and the properties referring to the type and size of test specimen, thus leading to uncertainties in how to transfer the material characteristics obtained in e.g. bending to direct tensile properties. Irrespective of the choice of indirect test method, the σ-w relationship must be determined through curve fitting, so called inverse analysis, see Section 3.2. Which shape of the σ-w relationship to choose, depends on the purpose of the analysis. Löfgren (2005) found that the shape of the σ-w relationship not only influences the maximum moment, but also the crack propagation stage. A consequence is that, if a highly simplified/idealised σ-w relationship is used (e.g. a drop constant), it will not be possible to accurately predict the service behaviour (crack widths and flexural stiffness) – but fortunately, the predicted load resistance is less sensitive to this choice.

3.3.4.2 Characteristic length

For the smeared crack concept (which was used here) the crack is smeared out over an assumed cracking width (perpendicular to the crack surface) which may be refered to as the characteristic length. It is important to choose this length correctly. For plain concrete (if using bond-slip concept) the characteristic length is about the width of one mesh-element. For FRC, on the other hand, it is more complicated; it is usually more than one element width but never more than the average crack spacing and this depends on the amount of fibres, i.e. the σ-w relationship. Hence, the crack is assumed to be smeared out over a length which is up-to half the crack spacing on each side of the actual crack. If no experimental results are available, the crack spacing must be estimated to a value which is relevant for the actual FRC material. Since the characteristic length is chosen as multiples of the mesh element size, it is important that the mesh is fine enough in the cracking areas, so that a representative choice can be made. To check if the choice was acceptable, once the analysis is run, the crack pattern has to be checked to see that the cracks actually have localised in a distinct number of element rows. If the crack pattern is diffuse (i.e. a large zone of cracked elements), another choice must be done. It should be mentioned that the choice of characteristic length has quite a large impact on the load bearing capacity in the analysis. A too small value for the characteristic length tends to over estimate the load resistance and the deformation capacity.

3.3.4.3 Bond slip

In CEB/FIP Model Code 90 (1990) formulas to derive bond-slip relationships for plain concrete are proposed. In the FE-analyses performed by the author of the FRC beams the same relationship was used as would have been used for plain concrete with comparable compressive strength. This relationship was obtained with the recommendations given in MC90 for plain concrete with compressive strength f_c=38 MPa, assuming good bond conditions.

In theory and also in accordance with observations from experimental testing (e.g. Schumacher (2006)), the bond-slip properties for FRC are considered the same as for plain concrete for situations where spalling does not occur. Should spalling occur, however, it is believed that the fibres provide extra confinement, thus leading to improved bond-slip behaviour than would have been the case with plain concrete. Since, theoretically, the bond-slip relationship ought to be altered due to the confinement provided by the fibres, and since it is well known that fibres may decrease crack spacing and, in large enough amounts, even contribute to a larger number of finer cracks (crack distribution), a modification of the bond-slip relationship for FRC appears appropriate.

3.3.5 **Results-Comparison of beam bending FEA/Exp**

Series 1 - $V_f = 0\%$-$\phi 8$

For plain concrete, as in Series 1, the fracture energy, G_F, together with the tensile strength, f_t, may be used as the only parameters governing the fracture process, Hillerborg (1980). The characteristic length was taken as the default value given by the used FE program (Diana), i.e. l_{ch} equals the length of the diagonal of the mesh element, (here approximately 7 mm). It can be seen in Figure 15 that there is a sudden drop on the load-deflection curve from the FE analysis. This is probably due to that an already located crack suddenly opens up more, leading to numerical difficulties in the FE program. Possibly there was a need for a decrease of the deflection for one or several elements (due to unloading in combination with the sudden opening of the aforementioned crack). Since the test was deformation controlled, this was not possible and an alternative solution (incorrect) was the only option besides abortion of the analysis. For series1 the experimental average crack spacing, s_{rm}, was equal to 78 mm and for the FE analyses, s_{rm} was approximately 80 mm, see Figure 16.

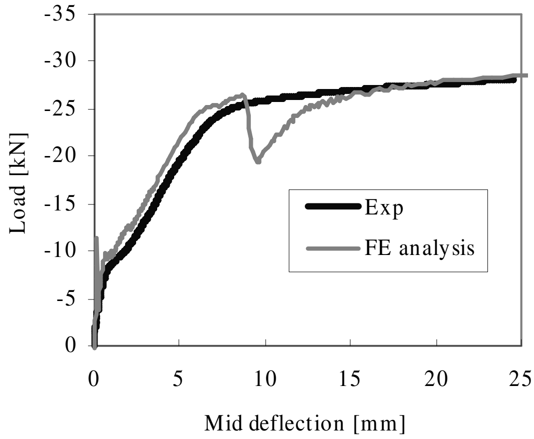


Figure 15 - Load-deflection curves, FEA vs average exp for Series1 (V_f=0%).

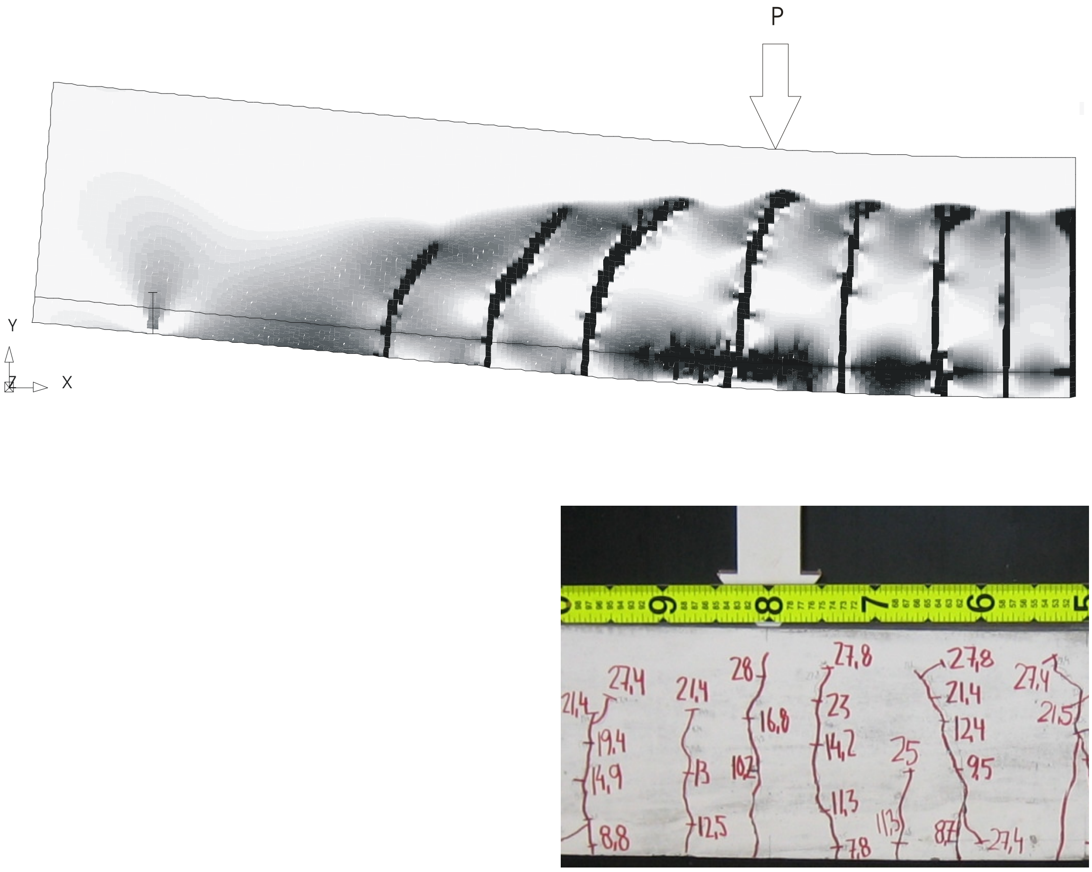


Figure 16 - Crack pattern for Series 1; a) FEA and b) exp.

Series 2 - $V_f = 0.5\%$-$\phi 8$

For the same amount of conventional reinforcement, $\phi 8$, as the previous series, but with additional fibre reinforcement, $V_f = 0.5\%$, the average crack spacing measured in the experiements was 59 mm and also for this series a characteristic length of 60 mm was chosen. For the analysis with bi-linear σ-w relationship the average crack spacing, s_{rm}, was 64 mm and for the analysis using a multi-linear relationship s_{rm} =51 mm. In Figure 17 the load-deflection curves from the analyses are compared with the average experimental curve and in Figure 18 the crack patterns from the analyses are shown.

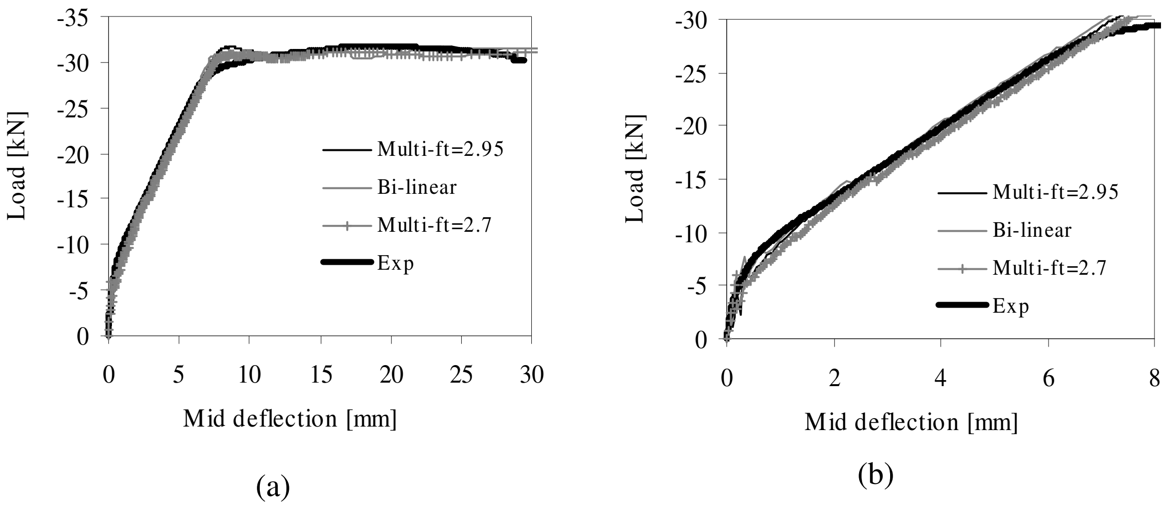
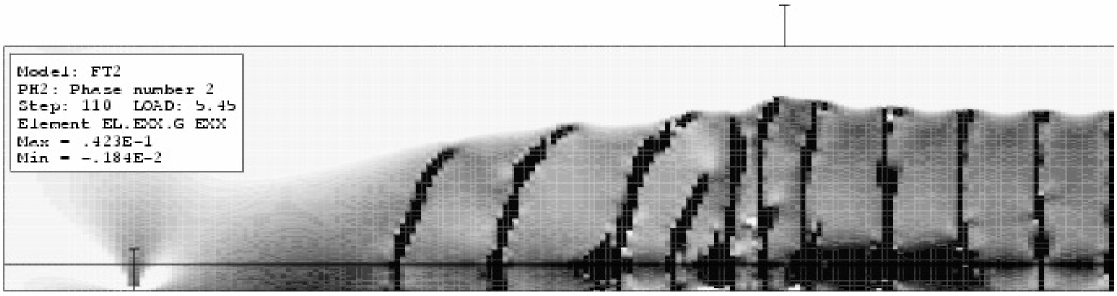
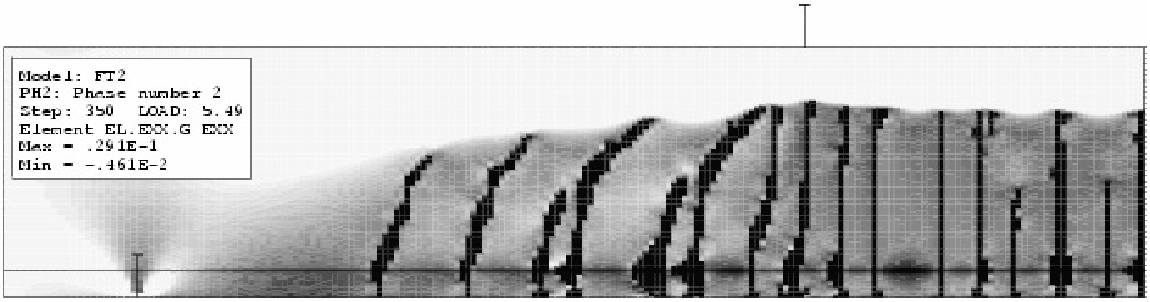


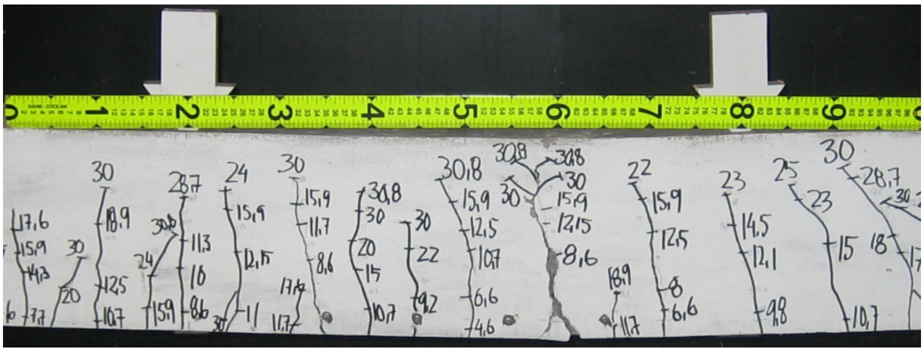
Figure 17 Compare load deflection curves, a) complete curve b) close-up of ascending branch



(a)



(b)



(c)

Figure 18 Crack patterns for V_f =0.5% ϕ8 using a) bi-linear σ-w relation b) multi-linear and c) experiment.

05f8 with varying tensile strength

For this series the multi-linear inverse analyses were carried out with two different tensile strengths, f_t = 2.7 MPa and ft = 2.95 MPa, while the bi-linear analyses were performed with f_t = 2.7 MPa only. The differences between the result curves based on f_t = 2.7 and f_t = 2.95 are small (Figure 19a) and can be considered to be within the range of the experimental results (Figure 19b).

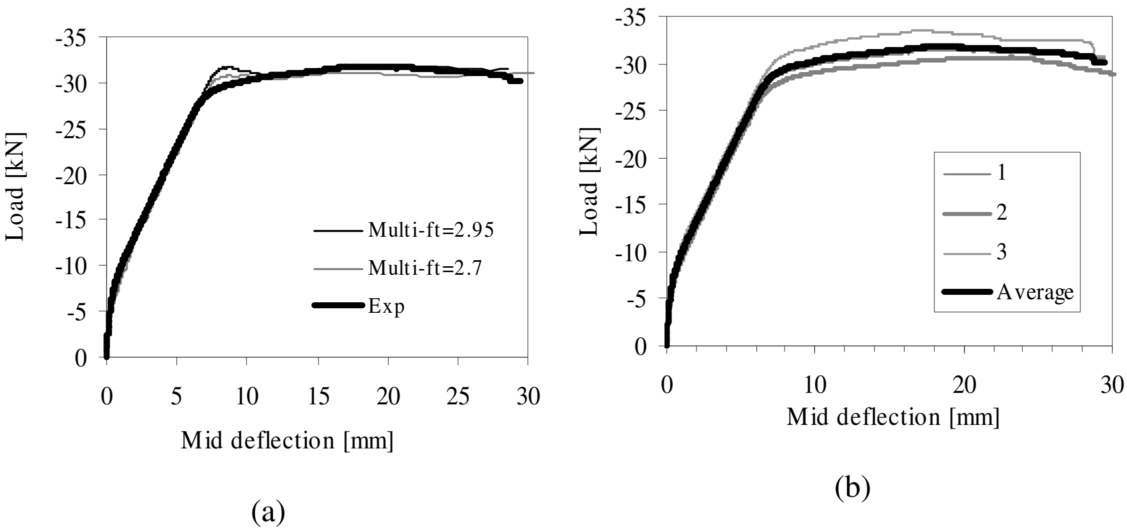


Figure 19 a) Effect of tensile strength on FE beam-bending results. Bond-slip relationship from MC90 for plain concrete and b) average exp curve from the three beam tests of V_f05f8

Series 3 - V_f = 0.5%-ϕ6

For this series an investigation of the influence of varying characteristic length was conducted, and it can be seen in Figure 20a that the choice of characteristic length, l_{ch} (in this study $l_{ch} = h$), has a major impact on the results. If a too small value is chosen, the load resistance (or peak load) will be over estimated, e.g. as for h=30mm in Figure 20a. This means that a too large area was assumed beneath the σ-ε curve, thereby over estimating the fracture energy. In Figure 20b the FEA results are compared with the average experimental curve. Figure 21 shows a close up of the first part of the load deflection curve to enable easier comparison of the curves in the actual cracking zone.

The average crack spacing measured in the experiments was 65 mm, and the characteristic length was chosen as 60 mm. The crack spacing was measured only in the analyses with h = 60 mm and in the analysis with the bi-linear σ-w relationship a crack spacing s_{rm} = 87 mm was obtained, while for the multi-linear one s_{rm} = 57 mm. See Figure 22 for crack patterns from the analyses.

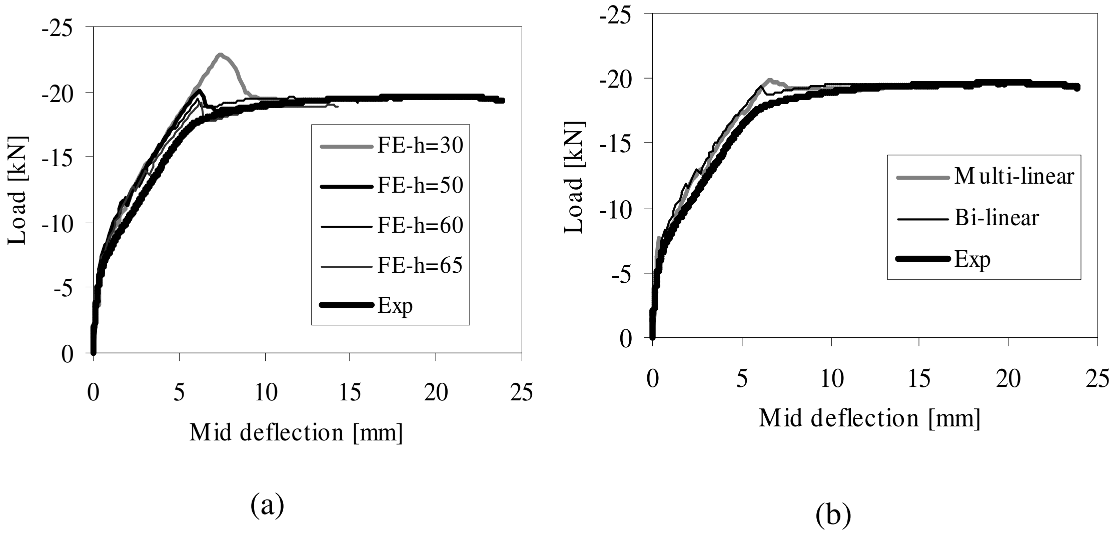


Figure 20 Comparison analysis vs average exp a) Influence of characteristic length, b) bi vs multi linear

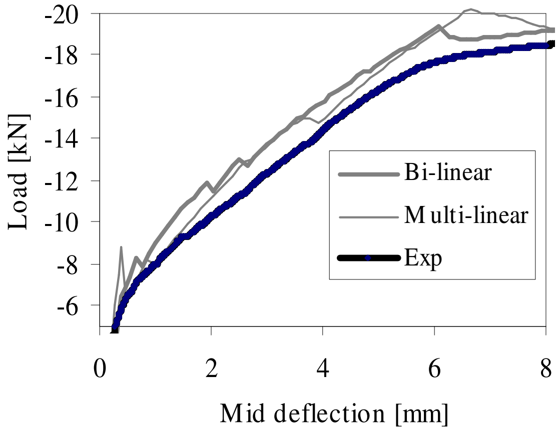
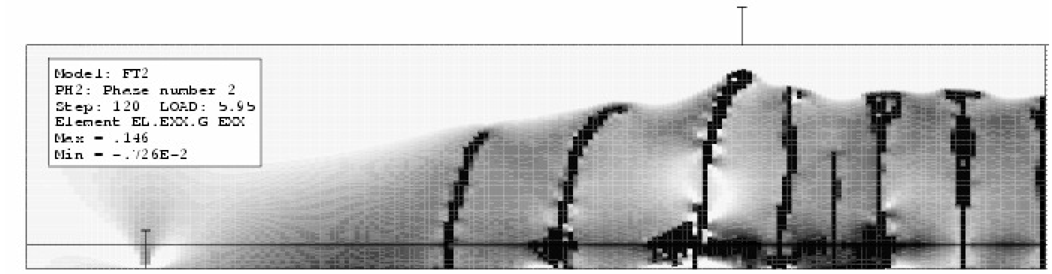
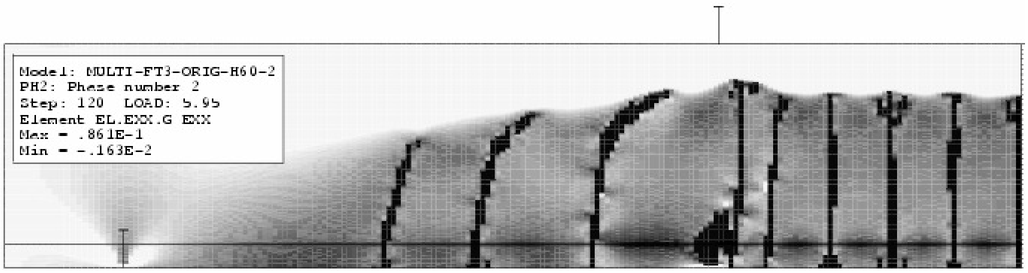


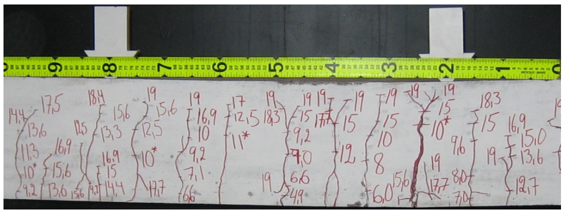
Figure 21 Close-up of first branch of bilinear and multilinear load-deflection curve.



(a)



(b)

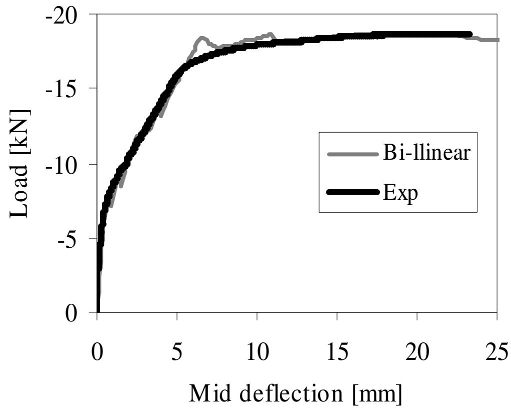


(c)

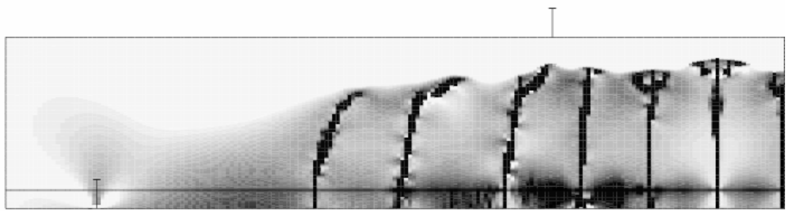
Figure 22 - Crack patterns for a) bi-linear analysis b) multi-linear and c) experiment.

Series 4 -V_f = 0.25%-ϕ6

For this Series, which contained the smallest amount of fibres, the FE analyses were carried out using a bi-linear σ-w relationship only. The load-deflection curves are shown in Figure 23a and the crack pattern in Figure 23b. Average crack spacing measured in the experiments was 71 mm, while in the FE analysis it was measured to 88 mm.



(a)

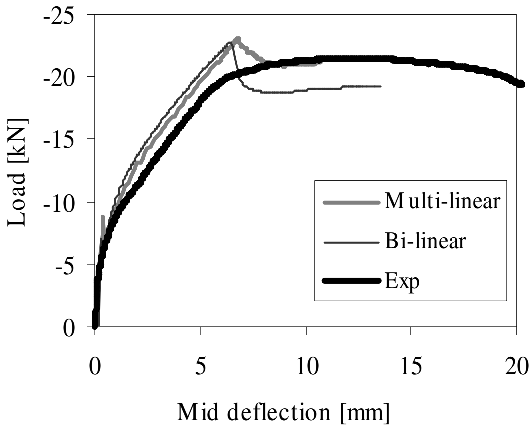


(b)

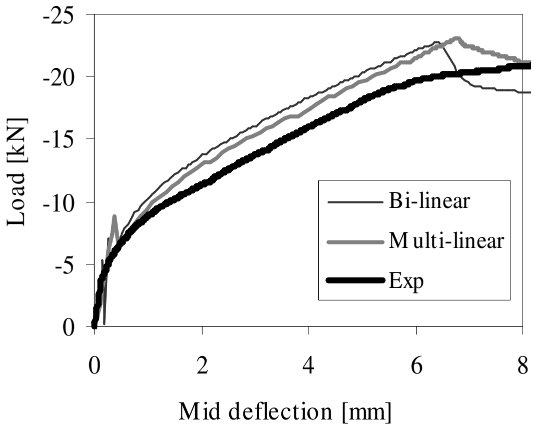
Figure 23 - a) Comparison of load-deflection curves and b) crack pattern for mid deflection=7 mm.

Series 5 -V_f = 0.75%-ϕ6

For the Series with the largest fibre content, series 5, with V_f = 0.75% and three ϕ6 reinforcement bars, the only variation between the analyses was in the form of the two different σ-w relationships, bi-linear and multi-linear. In this case the average crack-spacing from the experiments was 55 mm while from the analysis using the bi-linear σ-w relationship s_{rm} = 48 mm, and from using a multi-linear relationship s_{rm} was measured to approx. 40 mm. A comparison of the load-deflection curves is shown in Figure 24 and the crack patterns in Figure 25.

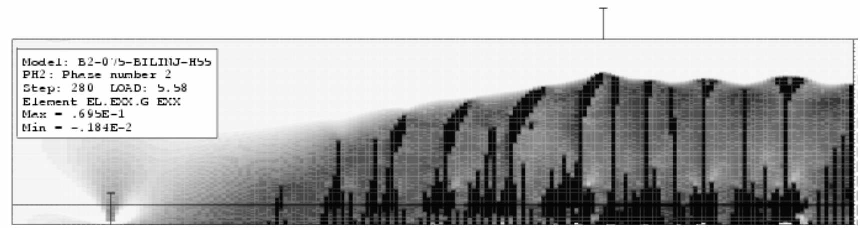


(a)



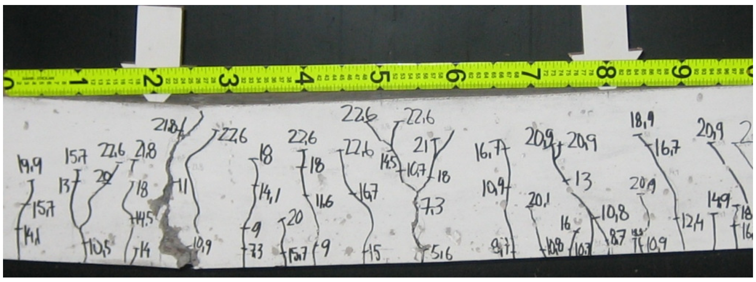
(b)

Figure 24 - Comparisonof load-deflection curves analysis vs average exp, a) complete curve and b) first part of curve.



(a)

(b)



(c)

Figure 25 - Crack patterns for V_f =0.75% ϕ6 using a) bi-linear σ-w, b) multi-linear and c) experiment.

3.3.6 Discussion of the results

The load-deflection curves from the analyses using a bi-linear σ-w relationship show the same agreement with the experimental curves as the results from the analyses using a multi-linear relationship. Even when studying the part of the load-deflection curve which refers to the cracking process, the differences between the bi-linear and the multi-linear analyses are barely noticeable, see Figures 17, 21, 24. From the obtained crack patterns though, the indication is that using a multi-linear σ-w relationship yields a somewhat better agreement with the crack-patterns from the experiments. The largest effect from using a multi-linear σ-w relationship was obtained for Series 5 with $V_f = 0.75\%$ and $\phi 6$ rebars. Here the load-deflection curves differ more, although it is difficult to distinguish which is preferable when studying the curves. Also when looking at the crack patterns in this Series it is difficult to draw conclusions, although it might be concluded that the multi-linear one, in the vicinity of the reinforcement bars, does show slightly more distinct crack localisation. It should be remembered that measuring the average crack spacing in the experiments is a relatively subjective matter, (as it is in the analyes when diffuse cracking is obtained). For instance, how large should a crack be to be considered a crack and at what crack width is a crack visible? It should also be kept in mind that the crack patterns shown in the analysis results are just an indication of the locations of the cracks. The crack location is of course not affected of the choice of plot range, i.e. between which values should the strains be plotted, but the graphical view is.

3.4 Tension rod analysis

In order to investigate how addition of fibres influences the cracking process, a simulation of a tension-rod test was carried out. This was a limited study and only two types of material was investigated; $V_f = 0\%$ in combination with a $\phi 8$ reinforcement bar, and $V_f = 0.5\%$ in combination with a $\phi 8$ reinforcement bar.

3.4.1 Tension rod model

The tension specimen was modelled in 2D, with a length of 500 mm (corresponding to a 1 m long specimen due to symmetry), and width/height equal to 50 mm, see Figure 26**.** The same crack model as for the beam-bending analyses was used, namely a smeared crack-model based on total strain with rotating cracks. The concrete elements were modelled with four-node quadratic isoparametric elements, while the single reinforcement bar, with diameter $\phi = 8$mm, was modelled with truss elements and placed centrically. Element size of the mesh was also in correspondence with the beam analyses; hence an element size of 5 x 5 mm^2 was adopted. A stepwise incremental deformation was applied to one of the end nodes of the reinforcement bar, while the opposite end node was restricted to move in the direction of the load. The end nodes of the reinforcement were locked in the transversal direction as well. Along the left edge of the model, the concrete elements were restricted to move in the loading direction.

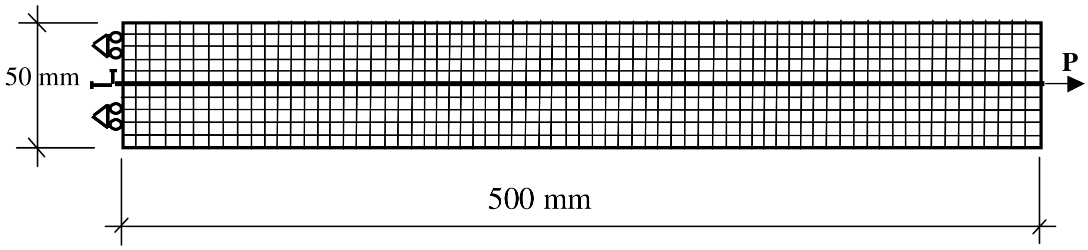
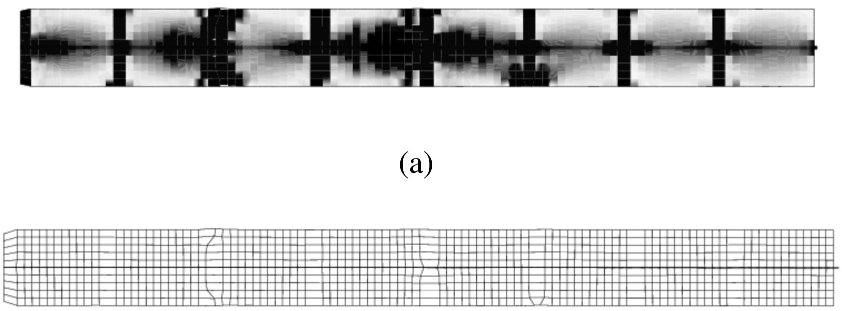


Figure 26 - Model of the tensile specimen.

3.4.2 Tension rod results

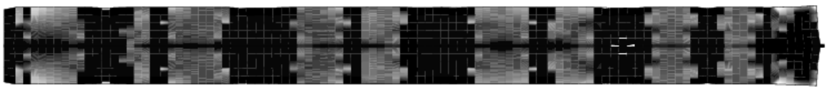
When comparing Figure 27 and Figure 28 it can be seen that, for the two prisms reinforced with the same amount of conventional reinforcement, (ϕ8), the addition of fibres influences the crack-distributive ability. It can be seen, when comparing the two analyses, that the crack openings in the fibre-reinforced prism are visibly smaller than the crack openings in the prism without fibres, compare Figure 27b and Figure 28b, where the deformations in both Figures are magnified by a factor 30. This may also be concluded from Figure 29, where it is seen that the strain in the reinforcement bar is remarkably lower in the prism with fibres. Furthermore, it is observed that the fibre-reinforced prism manages to provide additionally 2 cracks compared with the prism without fibres.



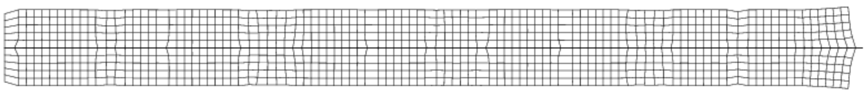
(a)

(b)

Figure 27 - V_f=0% and ϕ8, crack pattern for load/deformation = 1.85 mm a) in terms of contour; b) as deformed mesh with magnification factor = 30



(a)



(b)

Figure 28 - V_f=0.5% and ϕ8, crack pattern for load/deformation = 1.85 mm; a) in terms of contour; b) as deformed mesh with magnification factor = 30

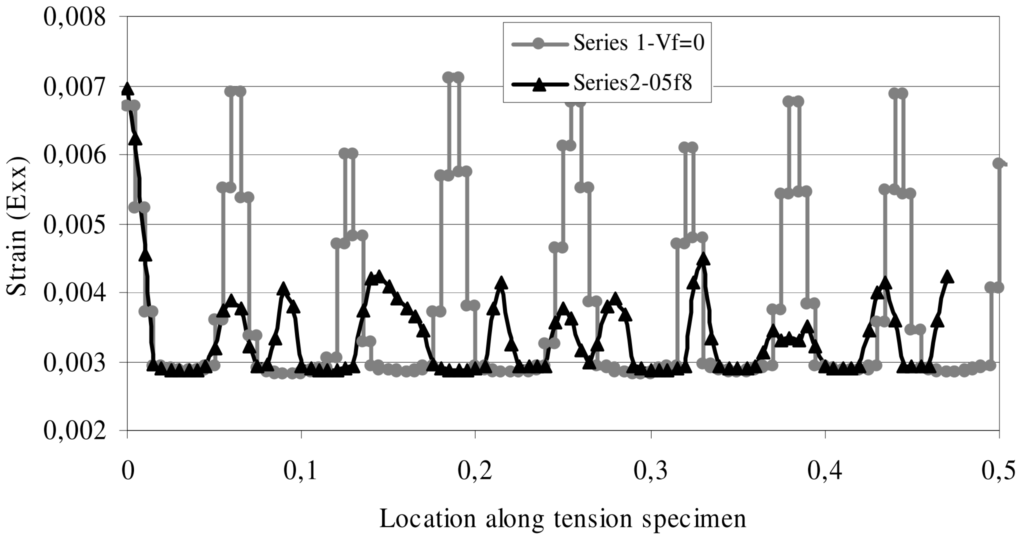


Figure 29 - Comparison of strain distribution along the reinforcement for load/deformation = 1.85 mm.

4 Fibre technology

4.1 General

When adding fibres to concrete, in order to choose the most suitable fibre, it is important to identify the type of effect the fibres are expected to provide. I.e. is the desired benefit of non-structural purpose, such as prevention of early plastic cracking, or is it of structural purpose e.g. in the form of controlled crack widths Löfgren (2005). The effect of fibres can be distinguished at two levels, namely: the micro-level and the macro-level. The micro-level covers a short stage after the linear elastic stage is surpassed, where small cracks arise from initial flaws within the matrix. At increasing load the length of the micro-cracks increases and they coalesce and finally localise into macro-cracks. At a given fibre content, micro-fibres, due to their high number, are more likely to cross these micro-cracks, Rossi *et al.* (1987). For micro-fibres to be effective, they should have a relatively high aspect ratio and stiffness, so that they can restrain the micro-cracks as these propagate into the mortar, Löfgren (2005). If instead an improvement of structural performance is desired, e.g. in bending, then fibres must be chosen that are large enough to bridge macro-cracks, but they also need certain mechanical properties.

According to Naaman (2003), analyses and experimental test results have shown that in order to be effective in concrete matrices, fibres must have the following properties: 1) a tensile strength of approximately two to three orders of magnitude higher than that of concrete; 2) a bond strength with the concrete matrix preferably of the same order as or higher than the tensile strength of the matrix; and 3) unless self-stressing is used through fibre reinforcement, an elastic modulus in tension significantly higher than that of the concrete matrix. In order to avoid detrimental debonding, the Poisson's ratio and the coefficient of thermal expansion should preferably be of the same order for both the fibre and the matrix. However, this could be overcome by methods such as inducing surface deformation to create mechanical anchorage, Naaman (2003).

4.2 Classification of fibres

Short fibres used in concrete can be characterized in different ways Naaman (2003); firstly according to the fibre material: natural organic (cellulose, sisal, jute, bamboo, etc.); natural mineral (such as asbestos and rock-wool); or man-made (e.g. steel, titanium, glass, carbon, polymers or synthetic); secondly, according to their physical/chemical properties such as density, surface roughness, chemical stability, non-reactivity with the cement matrix, fire resistance or flammability; and thirdly according to their mechanical properties: e.g. tensile strength, elastic modulus, stiffness, ductility, elongation to failure, and surface adhesion property.

Moreover, an infinite combination of geometric properties related to the cross sectional shape, length, diameter (or equivalent diameter) and surface deformation can be selected. The cross section of the fibre can be circular, rectangular, diamond, square, triangular, flat, polygonal, or any substantially polygonal shape.

In some fibres the surface is etched or plasma treated to improve bond at the microscopic level, Naaman (2003)**.** In Figure 30 a comparison between end-hooked steel fibres and straight steel fibres.shows the improved pull-out behaviour that can be achieved from deforming the fibres. Typical examples of fibre cross-sectional geometry, and steel fibres are shown in Figure 31 – 32.

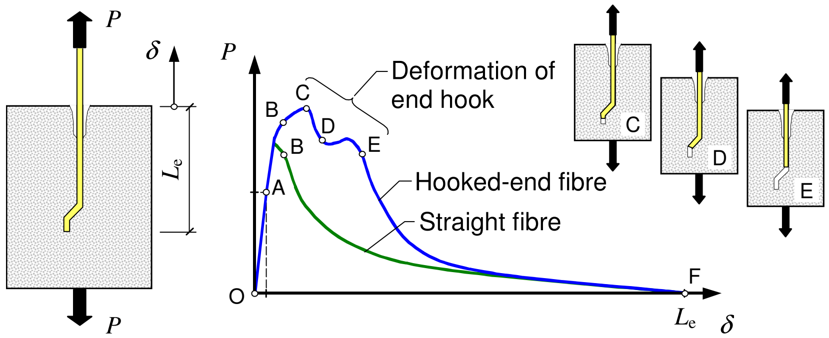


Figure 30 - Example of effect from modified fibre geometry-Typical fibre pull-out relationship between end-slip and load for straight and end-hooked fibre, from Löfgren (2005).

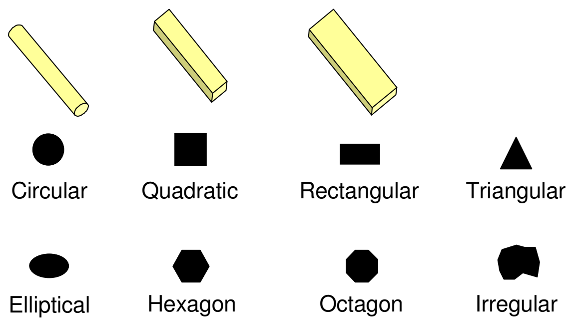


Figure 31 - Examples of cross-sectional geometries of fibres, from Löfgren (2005).

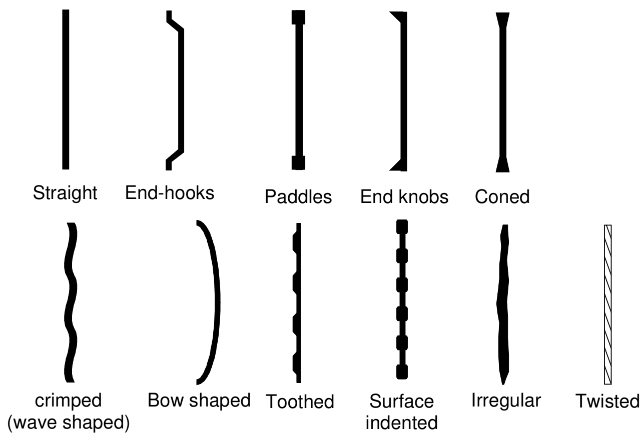


Figure 32 - Examples of typical fibre geometries, from Löfgren (2005).

As mentioned earlier, in order for fibres to increase the strength of composite materials, fibres must have a modulus of elasticity greater than that of the matrix. For cementitious materials this condition is difficult to meet with most synthetic fibres. However, both theoretical and applied research have indicated that, even with low modulus fibres, considerable improvements can be achieved with respect to strain capacity, toughness, impact resistance and crack control of the fibre reinforced concrete composites. In many applications these properties are of much greater significance than a modest increase in tensile (flexural) strength, Zheng and Feldman (1995) and Bentur and Mindess (2006). In addition, according to Bentur and Mindess, synthetic fibres with a high modulus of elasticity in combination with high strength steel fibres have been developed for concrete reinforcement. The basic fibre categories are steel, glass, synthetic and natural fibre materials, and in Table typical physical properties of a few fibres are listed.

Table 3. Physical properties of selected fibres. from Löfgren (2005)

Type of Fibre	*Diameter [μm]*	*Specific gravity [g/cm³]*	*Tensile strength [MPa]*	*Elastic modulus [GPa]*	*Ultimate elongation [%]*
Metallic					
Steel	*5-1 000*	*7.85*	*200-2 600*	*195-210*	*0.5-5*
Glass					
E glass	*8-15*	*2.54*	*2 000-4 000*	*72*	*3.0-4.8*
AR glass	*8-20*	*2.70*	*1 500-3 700*	*80*	*2.5-3.6*
Synthetic					
Acrylic (PAN)	*5-17*	*1.18*	*200-1 000*	*14.6-19.6*	*7.5-50.0*
Aramid (e.g. Kevlar)	*10-12*	*1.4-1.5*	*2 000-3 500*	*62-130*	*2.0-4.6*
Carbon (low modulus)	*7-18*	*1.6-1.7*	*800-1 100*	*38-43*	*2.1-2-5*
Carbon (high modulus)	*7-18*	*1.7-1.9*	*1 500-4 000*	*200-800*	*1.3-1.8*
Nylon (polyamide)	*20-25*	*1.16*	*965*	*5.17*	*20.0*
Polyester (e.g. PET)	*10-8*	*1.34-1.39*	*280-1 200*	*10-18*	*10-50*
Polyethylene (PE)	*25-1 000*	*0.96*	*80-600*	*5.0*	*12-100*
Polyethylene (HPPE)	*-*	*0.97*	*1000-4 000*	*80-150*	*2.9-4.1*
Polypropylene (PP)	*10-200*	*0.90-0.91*	*310-760*	*3.5-4.9*	*6-15.0*
Polyvinyl acetate (PVA)	*3-8*	*1.2-2.5*	*800-3 600*	*20-80*	*4-12*
Natural - organic					
Cellulose (wood)	*15-125*	*1.50*	*300-2 000*	*10-50*	*20*
Coconut	*100-400*	*1.12-1.15*	*120-200*	*19-25*	*10-25*
Bamboo	*50-400*	*1.50*	*200-440*	*33-40*	*-*
Jute	*100-200*	*1.02-1.04*	*250-350*	*25-32*	*1.5-1.9*
Natural - inorganic					
Asbestos	*0.02-25*	*2.55*	*200-1 800*	*164*	*2-3*
Wollastonite	*25-40*	*2.87-3.09*	*2 700-4 100*	*303-530*	*-*

4.3 Fibre types

4.3.1 Steel fibres

Steel fibres are generally made of carbon steel or stainless steel, where the latter is used for structures that require corrosion-resistant fibres. Tensile strengths may be in the range 200-2600 MPa and ultimate elongations between 0.5 and 5%. Although a tensile strength significantly higher than that of the matrix is needed, too strong fibres may have an adverse effect on the reinforcing efficiency ; pullout experiments have shown that in low-strength matrices high tensile strength steel fibres cause more severe matrix spalling around the fibre exit point. Of major importance in order to benefit the most from steel fibres, is that the yield capacity of the fibre is sufficient so that fibre rupture is avoided. Elastic modulus is around 200 GPa, thus greatly exceeding the elastic modulus of the matrix. The originally used straight, smooth steel fibre is rarely seen nowadays in normal-strength concrete due to its insufficient bond with the matrix. However, for high-strength concrete straight brass-coated fibres are quite common; see e.g. Lutfi (2004) and Marcovic (2006). To improve the bond a high aspect ratio (i.e. the ratio fibre length/diameter) is desired. Unfortunately though, there is a limit, and very slender fibres with aspect ratio, l_f / d_f, > 100 tend to cling together in balls, thus reducing workability and possibly also reducing the mechanical properties of the hardened steel-fibre-reinforced concrete (SFRC), the latter due to an uneven dispersion of the fibres. Nowadays, to improve the bond, steel fibres are manufactured in a number of different shapes and types, for examples see Section 4.2 on Classification. Regarding durability of SFRC, it may be concluded from Bentur and Mindess (2006) that durability of SFRC is, if not improved, at least comparable to that of plain concrete.

4.3.2 Glass fibres

Glass-fibre-reinforced cementitious composites (GFRC) have been developed primarily for the production of thin sheet components, (mainly as exterior building-facade panels ACI 544 1R (2002), with a paste or mortar matrix and approx. 5% fibre content, see Bentur and Mindess (2006). Conventional glass fibres have been found to lose strength very quickly due to the high alkalinity of the cement based matrix. From a structural duration point of view, the most interesting type of glass fibre is the one produced from alkali resistant glass, AR-GFRC. The chemical composition of this type of glass differs from conventional borosilicate glass fibres (E-glass) and soda-lime-silica glass fibres (A-glass), mainly by the inclusion of the highly chemically resistive, ceramic material zirconium.

Most commercially manufactured GFRC composites will experience reduction in tensile and flexural strengths and ductility if exposed to an outdoor environment. The strength of fully-aged GFRC composites will decrease to about 40 % of the initial strength and the strain capacity will decrease to about 20 % of initial capacity prior to aging. Furthermore, exposure to natural moisture and temperature cycles will result in cyclical volumetric dimensional changes. The two major ageing mechanisms are: 1) chemical attack and 2) growth of hydration products between the glass filaments, (Bentur and Mindess 2006). Even though commercial AR-GFRC does have higher resistance than E-glass to a chemical attack in the form of an alkaline environment, it is not completely immune and strength reductions have been observed, see Bentur and Mindess. Regarding the second mechanism the hydration product primarily responsible for composite embrittlement, i.e. loss of toughness, is calcium hydroxide, which fills the voids between the glass filaments. Addition of polymer solids, at a minimum of 5 % by volume of total mix, may partly inhibit the embrittlement of the glass fibres, ACI 544 1R (2002).

4.3.3 Synthetic Fibres

Synthetic fibres (SNF) are man-made fibres resulting from research and development in the petrochemical and textile industries. They are increasingly being used for the reinforcement of cementitious materials and many available fibres have been formulated and produced specifically for reinforcement of mortars and concrete, Bentur and Mindess (2006). SNFRC utilizes fibres derived from organic polymers and types that have been used in Portland-cement-concrete based matrices include: acrylic, aramid, carbon, nylon, polyester, polyethylene and polypropylene. Some of the listed fibres are found in commercial applications and have been subject of extensive research, e.g. polypropylene which is being used extensively, while for others there is little research reported. The properties of synthetic fibres vary widely with respect to tensile strength and modulus of elasticity, where especially the latter is most often of a lower magnitude than the one of the matrix.

Polypropylene fibres

Polypropylene fibres can be produced in the form of monofilaments or fibrillated fibres which in turn may be applied either as dispersed fibres or in the form of continuous mats. An advantage of polypropylene fibres is their alkali resistance. Disadvantages are e.g. poor bond with the matrix, sensitivity to sun light and oxygen, and low modulus of elasticity, although, in Denmark, a polypropylene fibre of relatively high tenacity (high modulus of elasticity) has been developed for concrete reinforcement under the trade name Krenit. Its modulus of elasticity ranges from 7-18 GPa and the tensile strength from 500-1200 MPa, Bentur and Mindess (2006). The sensitivity to sun-light leads to extra cautiousness when handling, e.g. store in black plastic bags until needed, Zheng and Feldman (1995) referring to Thomas (1972). Zheng and Feldman also mention that Mai *et al* (1980) reported that for polypropylene cement composites cured for 24h in a 140˚C autoclave, although only a slight loss in tensile strength was observed, the ductility was reduced by approx. 75%. The ductility loss was measured as the reduction of fibre cross-sectional area.

According to Bentur and Mindess (2006) polypropylene fibres may be used in different ways to reinforce cementitious composites. If used in thin sheet components it may provide the primary reinforcement and the fibre volume content must be relatively high, exceeding 5%. Since this is above the critical level in terms of critical fibre volume, the material is referred to as high performance FRC, and producing such a component cannot be done by simply mixing the fibres and the matrix. Instead e.g. hand lay-up of layers of continuous fibre mats, or industrial mechanized processes using special techniques must be used. A second application is as secondary reinforcement where low modulus polypropylene fibres at a content of approx. 0.1% by volume are added to reduce plastic shrinkage, but are not effective for crack control of hardened concrete. Low modulus polypropylene fibres may also be used as fire protection. In case of a fire the fibres melt, leaving empty channels that provide an escape route for the steam produced during the fire, thus preventing spalling of the reinforcement cover, Bentur and Mindess (2006).

Polyethylene fibres

Using conventional batching and mixing techniques, polyethylene fibres can be readily mixed into the concrete at volumes up to 4%. They have been evaluated using either short dispersed fibres mixed with concrete at volumes up to 4%, or a continuous network of fibrillated fibres to produce a composite with about 10% by volume of fibres. At a volume content of 2% the fibres led to a marked post-cracking load-bearing capacity, while at 4% the maximum load in the post-cracking range exceeded the first crack stress. Thus, these fibres seem to be quite effective for crack control Bentur and Mindess (2006). It should be mentioned though, that this type of fibre is prone to creep, see e.g. Peijs *et al.* (2000).

Acrylic fibres

Recently acrylic fibres with high tensile strength (up to1000 MPa) and modulus of elasticity ranging from 14 to 25 GPa have been developed for use in FRC. The bond between fibre and matrix is good and they have been reported to be relatively stable in an alkaline environment, although long term sensitivity to such conditions cannot be ruled out, Bentur and Mindess (2006).

Polyvinyl alcohol fibres (PVA)

High strength PVA fibres have been developed mainly for asbestos replacement. PVA fibres can have a modulus of elasticity on the same order of magnitude as that of a cementitious matrix. Due to the strong chemical bonding though, they tend to rupture instead of being pulled out from the concrete matrix, Bentur and Mindess (2006). Zheng and Feldman mention that autoclaving has a considerably negative effect on both tensile strength and elastic modulus of these fibres.

Polyester

Although polyester fibres are equal in strength compared with e.g. polypropylene fibres, with tensile strength of 280-1200 MPa and elastic modulus of 10-18 GPa, they are not suitable for use in FRC. This is due to their strength and ductility loss over time in the alkaline environment, Bentur and Mindess (2006).

Aramid

Aramid fibres may have a modulus of elasticity as high as 130 GPa if the chemical chains are aligned parallel with the fibre length. The shear modulus in the same direction (longitudinal) as well as the transversal tensile properties are poor. At temperatures higher than 300˚C the fibre may loose all its strength and therefore special evaluation regarding fire safety is needed. The mechanical properties have been studied of cementitious composites reinforced with short discontinuous fibres, produced either by spray technique or mixing, with fibre contents ranging from 1 to 5% by volume. The composites show an ultimate strength and strain higher than those found at first cracking, leading to a tough composite. Durability tests also show good results, indicating that these composites may not be susceptible to immediate durability problems. It should not be ruled out though, that long term exposure to alkaline environment may lead to negative effects, even if a sufficiently long life expectancy probably can be assumed Bentur and Mindess (2006).

Carbon

Carbon fibres are inert to most chemicals and therefore well suited to perform in the alkaline environment of a cementitious composite. Based on different starting materials there are two main processes for manufacturing of carbon fibres; PAN carbon fibres are made from polyacrylonitrile whereas pitch carbon fibres are made from petroleum and coal tar pitch. Their properties may vary over a wide range depending on the production process, i.e. how well the layered graphite planes, constituting the fibre, are aligned parallel with the fibre axis. Although pitch carbon fibres have a modulus of elasticity considerably lower than the PAN fibres it is still in the same range as the elastic moduli of the concrete matrices. Adding of carbon fibres at amounts of about 2-6% by volume have been found to effectively decrease swelling and shrinkage strains. As a result, this composite is dimensionally stable and less sensitive to cracking. It should be mentioned though, that adding of more than approximately 1% by volume will significantly decrease the workability of the mixture and conventional mixing methods cannot be used, Bentur and Mindess (2006).

4.3.4 Natural fibres

Much of the research on the use of natural fibres in cementitious composites has been motivated by the need of a low cost fibre and the availability of such fibres which are high in strength. Wood fibres derived from bamboo or sugar cane have been used for the production of low cost cementitious composites. Other natural fibres are e.g. jute, sisal and coconut fibres.

Natural fibres are sensitive to changes in moisture content and may suffer deterioration or petrification due to the alkaline environment and the continous cement hydration. In e.g. cellulose pulp fibres a moisture increase is associated with reductions in Young's modulus and strength, but an increase in toughness. This effect would most likely be true for other natural fibres as well, Bentur and Mindess (2006).

4.4 Post-cracking behaviour

In contrast to strain softening, which is the behaviour of a majority of the steel-fibre-reinforced applications today, a strain-hardening FRC exhibits an increased load-bearing capacity after cracking, And, in addition, multiple cracking is achieved. Examples of both behaviours are shown in Figure 33. A schematic example of the difference in post-cracking behaviour between unreinforced and two types of fibre-reinforced concrete is shown in Figure 34.

For structural purposes, if the conventional flexural reinforcement is to be completely replaced by fibres, a material exhibiting quasi-strain hardening or pseudo-strain hardening behaviour (i.e. a post-cracking strength larger than the cracking strength, or elastic-plastic response) is a nessecity. For some types of structures, e.g. thin elements subjected only to flexural loading, a material with deflection hardening behaviour would be sufficient. Exceptions are e.g. slabs on grade, walls, tunnel linings, and similar structures, where extreme loading would not lead to catastrophic consequences or where compressive normal forces are present, and thus a strain/deflection-softening material is sufficient, Kanstad and Dössland (2004).

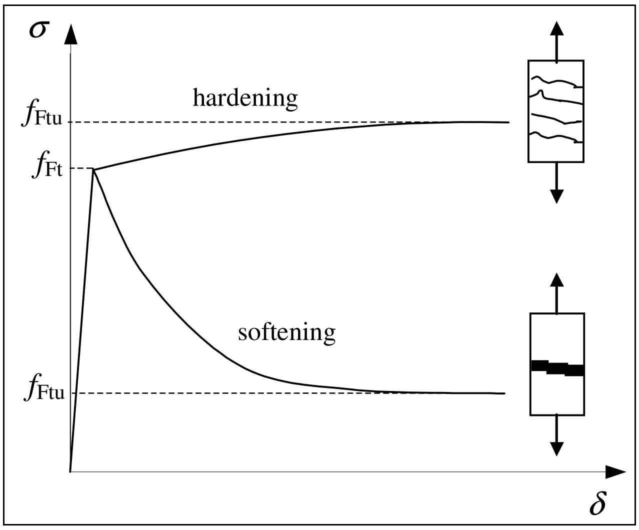


Figure 33 - Tensile behaviour of FRC material, from CNR-DT 204/2006 (Draft 2006).

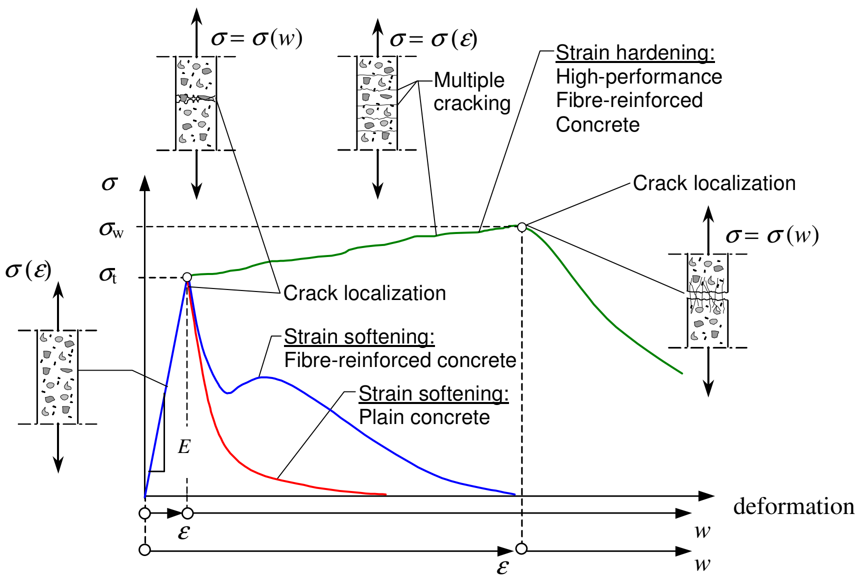


Figure 34 - Difference in tensile behaviour for cement-based materials, from Löfgren (2005)

4.5 High performance fibre reinforced cementitious composites

According to Li (2004), the meaning of high performance can be quite different in different countries. In e.g. Japan it refers to self consolidation (or self compaction), in Europe it often means high durability and in the US it means high strength. Li means that a high performance material must have attributes in both fresh and hardened states, which can not be achieved in normal concrete. Fibres are often many times more expensive than the cost of cement, water and sand. For this reason it is necessary to minimize the amount of fibres and not equate high performance to high fibre content. Instead the high performance should be optimized so that synergistic interactions occur at the fibre, matrix and interface level. Hybrid FRC is a type of high performance material that has not been achieved by simply increasing the fibre amount.

4.5.1 High fibre volume

Strain hardening may be obtained by increasing the fibre amount, although this is not quite as straightforward as it may sound. For the slender fibres that are preferred for improved toughness, the reduced workability at increasing fibre amounts limits the maximum amount of fibres that can be incorporated in the FRC mix. Although this can be overcome by different techniques, such as e.g. SIFCON (slurry infiltrated fibre concrete) and SIMCON (slurry infiltrated mat concrete), these techniques are quite costly. For SIFCON the steel fibres are placed in a mould, with fibre volumes of typically 4 to 12%, and then the concrete paste is infiltrated. While the SIFCON material is labour intensive in terms of placing the fibres, the SIMCON material uses the same type of fibres as used in SIFCON but placed a pre-made mat, thus allowing for higher slenderness of the fibres, Bentur and Mindess (2006).

4.5.2 Densified matrix

Several types of HPFRC systems having extremely dense matrix have been developed. A common feature of these composites is the establishment of an extremely dense matrix with w/c ratio smaller than 0.20 with 2-6% of steel fibres with diameter 0.1-0.2 mm and length of 5-15 mm. Examples are CRC (compact reinforced composite) and DUCTAL®, Bentur and Mindess.

4.5.3 ECC-Engineered Cementitious Composites

By optimising the different components of the FRC, strain-hardening may be achieved without just a mere increase in fibre volume. Concepts for obtaining strain-hardening composites with normal strength matrix and moderate fibre content of about 2 % by volume, was developed by Li and co-workers. This type of composite was named Engineered Cementitious Composites (ECC) and two guiding concepts were identified:

1.) Strength criterion, requiring that the first crack strength does not exceed the maximum bridging stress of the fibres. This is quantified by equations developed by Li and co-workers. For details see Bentur and Mindess (2006).

2.) Energy criterion, assuring that steady-state multiple cracking will occur. Details for quantification of this criterion is found in Bentur and Mindess (2006).

To meet these criteria the bond strength must be controlled. Weak bond will result in low bridging strength compromising the first criterion, while the opposite will lead to a maximum bridging stress at small crack opening, thus compromising the energy criterion. Significant for maximising the ductility, for a given fibre content, is the distribution of the fibres. Uneven distribution reduces the ultimate strain for the composite.

4.5.4 Hybrid FRC

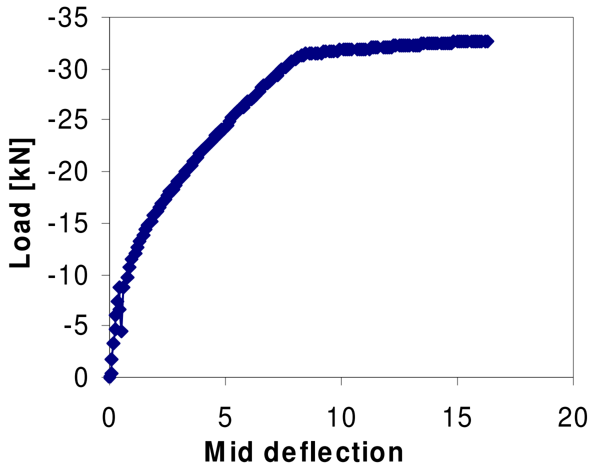
Based on Rossi *et al.* (1987), who distinguish between the different stages of crack formation, the concept of hybrid reinforcement, was proposed in terms of a large volume of short steel fibres to control microcracking and long fibres to bridge macrocracks, Bentur and Mindess (2006). Bentur and Mindess refer to Banthia and Gupta, who classified hybrid synergies into three groups which all include one fibre type that provides toughness in combination with either a stronger stiffer fibre that provides strength, micro fibres that provide crack control, or a type of fibre that changes the properties of the fresh mix.

4.5.5 Engineered fibres

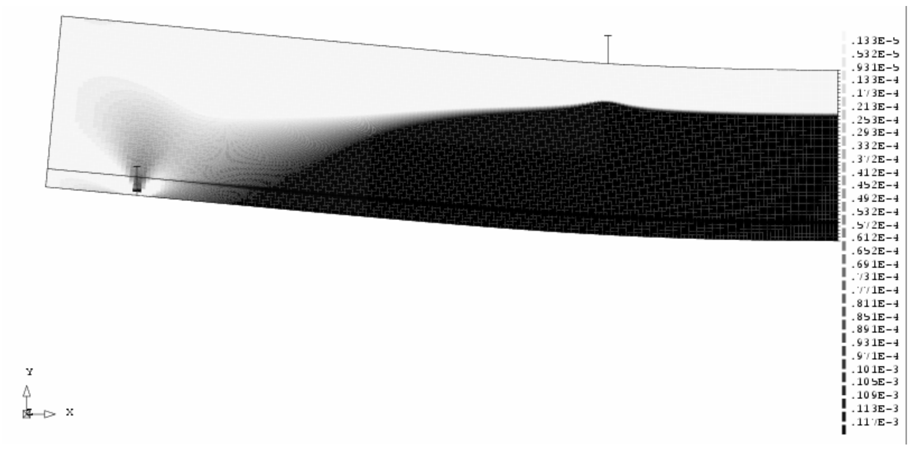
In the late 1990's, research at the University of Michigan led to the development of a new steel fibre of optimised geometry, identified as Torex fibres. The Torex fibre is made of very high strength steel wire, polygonal cross section (primarily triangular or square), possibly with indented sides and twisted along its length. The key feature of this fibre is that when pulled-out from a cement matrix, its resistance increases with increase in slip (the more it pulls out, the harder it resists) and it can be tailored to achieve a level of desirable performance, depending on the type of matrix, Naaman (2003)

4.5.6 Application with deflection-hardening material

The present work has been focused on SFRC with softening post-cracking behaviour. If adjusting the σ-w relationship though, the methodology used may be applied on other types of concrete, e.g. strain/deflection hardening SFRC. As an example a finite element analysis was conducted using a fictive σ-w relationship where the fibre-bridging tensile stress (post-cracking) was higher than the cracking stress. The remainig material properties were the same as used in the earlier described analyses for Series 5 (V_f 0.75% and ϕ6 rebars). The results are shown in Figure 35, where it can be seen that with this type of fibre performance the load capacity is increased significantly compared to the capacity of Series 5, see Figure 24. Moreover, as can be seen in Figure 35, no distinct cracks will form and basically all elements within the tensile zone will crack. This indicates that, as occurs for example in ECC-elements (see Li and co-workers-Bentur and Mindess (2006)), it is possible to achieve multiple cracking with closely spaced cracks (< 10 mm) which are barely opened.



(a)



(b)

Figure 35 - Results from analysis of material with deflection-hardening behaviour, a) load deflection curve and b) crack pattern.

5 Discussion

Based on the need for design methods for fibre reinforced concrete, the work presented in this licenciate thesis has been focused on the part of the structural design which regards small crack widths occurring mainly in the serviceability limit state. An investigation of several design methods for FRC, proposed by different technical committees, indicates that the Italian proposal, CNR-DT 204/2006 (Draft 2006), is heading in the right direction in several matters. This method though, as well as the other investigated methods, is based on rather rough simplifications and is probably better suited for calculations in the ultimate limit state, Jansson (2007).

To investigate the flexural behaviour of SFRC beam specimens, four-point bending tests and FE analyses were carried out. The theory was that instead of directly deriving material properties for the SFRC through, the rather complicated, uniaxial tension test, this could be achieved with a simpler test procedure; in this case the wedge splitting test (WST). It was assumed that the material properties in terms of a stress-crack width (σ-w) relationship would be possible to obtain through inverse analysis based on the WST results. The inverse analyses were carried out with acceptable agreement using both bi-linear and multi-linear shape of the σ-w curve, although the multi-linear σ-w curve did yield noticeably better agreement. For the overall response in the analyses of the beam tests, when comparing with the experimental results, the bi-linear σ-w relationship proved to yield the same degree of accuracy as the multi-linear one. For observations in the serviceability limit state though, the multi-linear σ-w relationship tends to yield a more distinct crack pattern, thus indicating better possibilities for estimation of crack width and crack spacing.

This far in the present work, no attempt has been made to predict/estimate crack widths. First the different mechanisms that govern the flexural response need to be further investigated. One tendency that could be observed in the performed analyses, was that the choice of tensile strength, f_t, for the σ-w relationship, when chosen within reasonable range, does not have a large effect on the load-deformation response. It must be stressed that this applies for the analyses conducted in the present work, where the σ-w relationship was adjusted for fibre efficiency, and may not be generally concluded. It was seen that the choice of characteristic length does have a major influence on the results. This is a drawback when using the smeared crack concept, and research regarding how to correctly determine the characteristic length is necessary. Furthermore, it is believed that the choice of bond-slip relationship will affect the results to quite a large extent. All these factors will be further investigated in future work.

The present work has been limited to steel fibres of one type, and to materials with softening post-cracking behaviour, where the main benefit is the crack-width reducing ability. For fibre-reinforced concrete to compete with ordinary reinforced concrete (RC) on all levels though, materials with hardening behaviour must be made commercially available. High-performance applications are available today, e.g. SIMCON, but these are cost-wise not comparable to ordinary RC. Other attempts to achieve high-performane materials are the ECC, where control of the bond strength and the fibre distribution enables hardening behaviour with only moderate amounts of fibres (≈2%). This is a promising concept, which needs further development to ensure that the needed control can be achieved also in commercial construction and not only in the controlled environment of the laboratory.

Combination of micro and macro fibres, so called hybrid fibre reinforcement, may not lead to a hardening behaviour, although, from a durability point of view it may be an important concept. The micro fibres effectively counteract the fine cracks from shrinkage and drying and also prevents micro cracks from widening. Macro fibres, on the other hand, arrest the crack growth at localised cracking, thus producing a material with less moisture permeability.

Although steel fibres, up to now, have been subjected to more research than any other type of fibre and have been shown to provide cementitious composites with enhanced tensile behaviour, they do have some drawbacks. If for instance aesthetics is important, a negative attribute is that with time rust coloured patches may develop. Perhaps more serious is that due to daily use and environmental exposure the fibres may protrude from the surface as this wears down. One interesting variant of the steel fibre is the so called "engineered fibre". This twisted fibre with square or triangular cross section has the characteristic of exhibiting more resistance the more it is pulled out, Naaman (2003).

Development of fibres made of other materials with properties comparable to those of steel fibres is a continuous process. Synthetic materials are lighter than steel and in general they have tensile strengths equal to or higher than that of the steel fibre, e.g. polypropylene with tensile strengths ranging from 500-1200 MPa. One drawback for most synthetic fibres, is the relatively low elastic modulus (except perhaps for PAN carbon fibres), which may lead to larger elongations over time than has been observed in steel fibres. Thus, crack width reductions in the range of those achievable with SFRC (cracks due to loading) may be difficult with some of the currently available synthetc fibres**,** although PVA fibres used in ECC is an exception**.** In addition, even for high modulus synthetic fibres, there is a tendency for creep, see Peijs *et al.* (2000). Another drawback is the relatively short time that research has been focused on e.g. synthetic fibres for structural applications. This means that it is not fully known how some of these fibres will behave over time, especially in the alkaline environment of concrete. It should also be mentioned that fibres made of synthetic materials most probably will exhibit poorer fire resistance than steel fibres. This must be considered during the design process to ensure proper load bearing capacity corresponding to given requirements. In combination with more heat resistive load bearing materials though, synthetic fibres (e.g. polypropylene) will melt and are believed to provide an escape route for the steam in the concrete, thus preventing fire spalling.

6 Conclusions

In this licentiate thesis a review of different design proposals have been made and experiments and finite element analyses have been conducted on beams loaded in 4-point bending. In addition, a small study on prisms subjected to tensile loading, as well as an investigation of available fibre types and different fibre systems were carried out. The material properties used in the FE analyses of the beams, in terms of σ-w relationships, were obtained from wedge-splitting tests through so called inverse analysis. In a first attempt the FE analyses were based on a bi-linear σ-w relationship. With the aim to better describe the cracking process, in particular the small cracks occurring in the serviceability limit state, in a second attempt the FE analyses were based on a multi-linear σ-w relationship obtained from the same wedge-splitting tests. Based on the results from the above, the following conclusions may be drawn:

- Although the extensive research over the past decades regarding fibre-reinforced concrete has led to a considerable increase in knowledge of the mechanical behaviour and thereby the development of several high-performance concepts, the focus of the work has primarily been on load-bearing capacity (i.e. on the ultimate limit state). Hence design methods intended for the serviceability limit state are needed.

- The investigation of the design methods revealed that the Italian proposal is a comprehensive method. It takes one step forward especially regarding, 1) the introduction of a characteristic length for determination of the strain values from the measured crack openings, and 2) consideration of the difference between test specimen and actual structural size and type when determining the characteristic value. It should be mentioned that the proposed stress-strain relationships are rather simplified and thus this method might be better suited for calculations in the ultimate limit state. In addition, the formula for crack spacing/width needs to be modified to also consider the residual strength.

- From the work presented in this thesis it may be concluded that indirect test methods for material properties are sufficient when focusing on the load-bearing capacity. Furthermore, for these matters, a bi-linear σ-w relationship is fully sufficient.

- The results from the present work indicate that an indirect test method for material properties is sufficient also when focusing on crack widths in the serviceability limit state. In this case though, there are indications that a multi-linear σ-w relationship better describes the cracking procedure. To be able to conclude this, further research is needed.

- From the study of the prism subjected to tensile loading, it was clearly seen that adding fibres to a concrete matrix significantly reduces the crack width.

- To better understand the cracking behaviour and be able to predict crack widths, it is necessary to increase the knowledge of bond-slip behaviour of ribbed bars embedded in fibre-reinforced concrete and also to gain better understanding of how to accurately estimate the characteristic length.

7 Future research

In the concept fracture mechanics combined with finite element analysis, the multi-linear σ-w relationship appears to yield better description of the cracking behaviour of steel-fibre-reinforced concrete than does the bi-linear one. Several factors affect the FEA results though, and these need to be closer studied for this to be concluded. It seems quite possible that the bond-slip properties may change when adding fibres to a concrete mix. In addition it is possible that a different approach to the modelling is necessary. It seems as if the behaviour in the vicinity of the conventional reinforcement is of major interest, then this area of the beam should be modelled in detail and experiment should be conducted on e.g. tensionrods. The continued work in this project will be focused on how to predict the width of the small cracks occuring in the serviceability limit state. In addition, it could be interesting to investigate how different types of fibres influence the cracking process and if a combination of various fibres is to prefer.

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