THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Optimization of truck tyres selection

ZUZANA NEDĚLKOVÁ

Division of Applied Mathematics and Statistics
Department of Mathematical Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
AND UNIVERSITY OF GOTHENBURG

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Department of Mathematical Sciences
Chalmers University of Technology
and University of Gothenburg
412 96 Gothenburg, Sweden
Telephone: +46 (0)31 772 5352
Author email: `zuzana@chalmers.se`

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Zuzana Nedělková

Department of Mathematical Sciences,
Chalmers University of Technology
and University of Gothenburg

Abstract

This thesis, which consists of an introduction and five appended papers, concerns the optimal selection of tyres for a variety of vehicle configurations as well as operating environments. The selection problem stems from a project cooperation between Chalmers University of Technology and Volvo Group Trucks Technology. We analyze the selection problem from a mathematical optimization point of view. The overall purpose is to reduce the tractive energy required to run the vehicle. We develop a computationally efficient vehicle dynamics model of the vehicle, the tyres, and the operating environment. The tyres are represented by a surrogate model of the rolling resistance coefficient, which measures the energy losses caused by the tyres. The properties of the surrogate model called for a methodology for connecting expert knowledge about a general simulation-based function with its radial basis function interpolation.

An algorithm for the solution of a large set of instances of a simulation-based optimization problem with continuous variables has been developed and tested on a set of problem instances. This algorithm enables an efficient computation of approximately optimal tyre designs (represented by continuous variables) for each vehicle configuration and operating environment specification. A splitting algorithm for simulation-based optimization problems with categorical variables has been developed and evaluated on a set of test problems. This algorithm outperforms all algorithms applicable to this class of optimization problems, and finds an approximately optimal tyres configuration. Since each execution of this algorithm requires many computationally expensive evaluations of the simulation-based objective function, it cannot be used to solve the full tyres selection problem. The two latter algorithms are then combined to enable the efficient solution of many instances of a simulation-based optimization problem with categorical variables. The resulting algorithm is applied to a couple of instances of the tyres selection problem.

Our experiments show that the optimization methodology developed enables a computationally efficient solution of the truck tyres selection problem, in the combinatorial domain of possible vehicle configurations and operating environment specifications. Putting our methodology into practice will involve many challenges besides the problems studied in this thesis; however we have shown that our methodology can be utilized in the sales tool at Volvo.

Keywords: simulation-based optimization, categorical variables, truck tyres, rolling resistance coefficient, vehicle dynamics, surrogate model, radial basis function, efficient solution, approximately optimal solution

Appended papers

Paper I: Nedělková, Z., Lindroth, P., Strömberg, A.-B., and Patriksson, M., *Integration of expert knowledge into radial basis function surrogate models*, Optimization and Engineering, 17(3), pp. 577–603, 2016.

Paper II: Nedělková, Z., Lindroth, P., and Jacobson, B., *Modelling of optimal tyres selection for a certain truck and transport application*, accepted for publication in International Journal of Vehicle Systems Modelling and Testing.

Paper III: Nedělková, Z., Lindroth, P., Patriksson, M., and Strömberg, A.-B., *Efficient solution of many instances of a simulation-based optimization problem utilizing a partition of the decision space*, accepted for publication in Annals of Operations Research.

Paper IV: Nedělková, Z., Cromvik, C., Lindroth, P., Patriksson, M., and Strömberg, A.-B., *A splitting algorithm for simulation-based optimization problems with categorical variables*, under review for publication in Engineering Optimization.

Paper V: Nedělková, Z., *Efficient solution of many instances of a simulation-based optimization problem with categorical variables utilizing a partition of the decision space*, manuscript.

Publications not included in this thesis

1. Šabartová[1], Z., Lindroth, P., Strömberg, A.-B., and Patriksson, M., *An optimization model for truck tyres selection*, Proceedings of the 4th International Conference on Engineering Optimization, edited by A. Araujo, Instituto Superior Técnico, Universidade de Lisboa, Lisbon, Portugal, pp. 561–566, 2014.

2. Šabartová, Z., Lindroth, P., Patriksson, M., and Strömberg, A.-B., *Optimizing truck tyres—How to improve the realism of simulation-based optimization through physical constraints*, ORbit, medlemsblad for Dansk Selskab for Operationsanalyse og Svenska OperationsAnalysFöreningen, 23, pp. 12–14, 2014.

3. Šabartová, Z., Jacobson, B., and Lindroth, P., *A joint model of vehicle, tyres, and operation for the optimization of truck tyres*, Proceedings of the 4th International Tyre Colloquium: Tyre Models for Vehicle Dynamics Analysis edited by P. Gruber and R. S. Sharp, University of Surrey, Guildford, UK, pp. 177–186, 2015.

4. Odrigo, A., El-Gindy, M., Pettersson, P., Nedělková, Z., Lindroth, P., and Öijer, F., *Design and development of a road profile generator*, International Journal of Vehicle Systems Modelling and Testing, 11(3), pp. 217–233, 2016.

5. Andréasson, N., Evgrafov, A., Patriksson, M., with Gustavsson, E., Nedělková, Z., Sou, K. C., and Önnheim, M., *An Introduction to Continous Optimization*, 3rd edition, Studentlitteratur, Lund, 2016.

[1]The author's surname changed from Šabartová to Nedělková in 2015.

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Zuzana Nedělková
Gothenburg, January 2018

List of abbreviations

BARON	Branch-And-Reduce Optimization Navigator
CVM	Complete Vehicle Model
DAE	Differential Algebraic Equation
DIRECT	DIviding RECTangles
FCC	Fraunhofer–Chalmers Research Centre for Industrial Mathematics
FEA	Finite Element Analysis
GTA	Global Transport Application
GSP	Global Simulation Platform
RBF	Radial Basis Function
RRC	Rolling Resistance Coefficient
NOMAD	Nonlinear Optimization by Mesh Adaptive Direct Search
ODE	Ordinary Differential Equation
PERF	PERFormance Vehicle Analysis Tool
quadDS	Discrete Search with Quadratic Underestimation
SVS	Strategic Vehicle Specification
UOIT	University of Ontario Institute of Technology
Volvo GTT	Volvo Group Trucks Technology
VTM	Vehicle Transport Models

Contents

1 Introduction

This thesis constitutes the result of the project *TyreOpt—Fuel consumption reduction by tyre drag optimization*, performed in a cooperation between Volvo Group Trucks Technology (GTT), the Department of Mathematical Sciences at Chalmers University of Technology and University of Gothenburg, and the Department of Applied Mechanics at Chalmers University of Technology. The project was financed by the Swedish Energy Agency and Volvo GTT.

The purpose of the thesis is to describe and model the practical truck tyres selection problem, introduce the scientific areas utilized in the appended papers, and clearly describe the connections between the real-world problem and the mathematical problems studied. Some relevant sub-areas of mathematical optimization as well as of the general problem of selecting components of a system are more extensively studied in the appended papers.

Some of the results presented in this PhD thesis are previously presented in the licentiate thesis (Šabartová, 2015).

1.1 Background

In order to improve the truck energy efficiency, various energy losses must be minimized. For a long-haul highway driving of a loaded truck the energy losses consist of the engine losses 60–65%, the transmission losses 5%, the aerodynamics losses 10–15%, the tyres losses 10–15%, the braking losses 10%, and the other losses 3% (e.g., losses in the vertical damping, losses while stationary); see Martini (2016, Ch. 1) and National Research Council (US), Committee to Assess Fuel Economy Technologies for Medium- and Heavy-Duty Vehicles (2010). It was identified in (Hoever, 2014) that the truck tyre losses can be even higher, representing up to 40% of the fuel consumption. Accordingly, by decreasing the tyre losses, the vehicle's fuel consumption can be substantially reduced. In this thesis we develop a mathematical optimization model with the objective to minimize the energy losses caused by the truck tyres.

The energy losses caused by the tyres can be represented by a complex function, which is influenced by many parameters, such as inflation pressure, vehicle speed, axle load, tyre type and material, tyre radius and other tyre dimensions, tyre temperature, and tread pattern (see Nedělková (2016)). In this thesis, we identify the most influential parameters and construct a composite function, describing how each of the selected parameters influences the energy losses caused by the tyres.

Volvo trucks are used in a variety of markets differing in characteristics concerning operating environments and legislations; this has led to a high degree of specialization and truck customization (see Lindroth (2011, Ch. 1)). Therefore, a great variety of truck configurations as well as tyres must be offered. Since there is an enormously large set of combinations of vehicles and tyres offered it is in practice impossible to solve the computationally demanding tyres selection optimization problem for each individual vehicle. To overcome these difficulties a specialized optimization strategy has been developed and is presented in this thesis.

1.2 Purpose and aim

The overall goal of the PhD project is to improve the transportation energy efficiency by minimizing the energy losses caused by the tyres. To achieve this goal we develop a procedure—and subsequently a practical tool—based on mathematical optimization, for identifying an optimal configuration of tyres from an existing tyre database, given the configuration and the operating environment of a vehicle; see Figure 1. Optimal configurations for a large number of customers need to be found in a computationally efficient way, such that it can be incorporated into a sales tool. Satisfying the main goal will lead to a better energy efficiency of road transport.

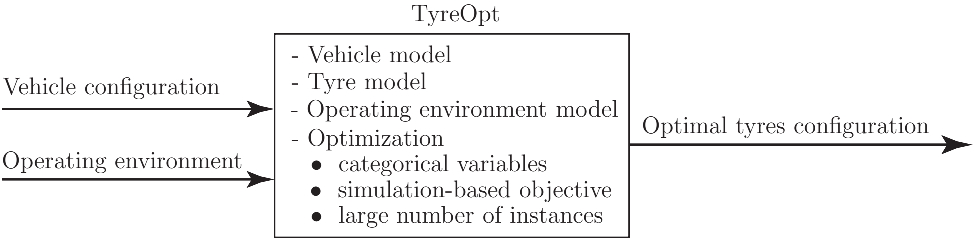

Figure 1: The aim of the TyreOpt project is to find the optimal configuration of tyres for each customer's specification of the vehicle and its operating environment. First, models of the vehicle, the tyres, and the operating environment are established. Then the simultaneous optimization of a large number of instances of a simulation-based optimization problem with categorical variables is employed to find the optimal tyres configuration for each specific vehicle.

As a result of the project, both academia and Volvo GTT have gained a better understanding of how the tyres selection can be modelled and optimized. The benefit for Volvo GTT is the ability to offer a better fit of the tyres selected for each vehicle configuration and operating environment specification, and to provide tyres recommendations to its customers. Another goal is to spread the knowledge about mathematical optimization and its profitable utilization within the company, to emphasize the requirements for a successful optimization (i.e., provision of numerical measures of qualities), and to give an example of what a production company can gain from the use of mathematical optimization. The benefit for academia is an insight into the truck tyres selection process and knowledge about the application of mathematical optimization to a real industrial problem, and where the biggest differences lie between real industrial problems and academic ones; see Section 4. The expected benefit for the customers is a guide for searching and choosing among the available tyres which, in this case, reduces the monetary costs as well as the environmental footprints of transport operations.

In this thesis we interpret and model the tyres selection problem mathematically and develop a methodology to solve this complex problem. In order to retain the tractability of the mathematical model a lot of information about the tyres, the vehicle configuration, and the operating environment are left out of the model. A discussion on the left-out information and how it can be dealt with is presented in Section 4.

1.3 Limitations

We consider only long-haul straight driving on hard surfaces in this thesis.

The focus of this thesis is on the mathematical optimization framework for the truck tyres selection optimization problem. The different models used to evaluate the tyre related functions are to be considered as inputs to this framework. The methodology developed is modular, such that sub-models can easily be replaced.

The fuel consumption, measured by the energy consumption, is aimed to be minimized by performing a structured tyres selection. New unworn tyres are to be selected from an existing tyre database. We neglect the influence of the temperature on the tyre behaviour.

1.4 Previous research

The research presented in this thesis is related to the previous research project *Product Configuration with respect to Multiple Criteria in a Heterogeneous and Dynamic Environment within an Extended Enterprise*. The project was performed at Volvo GTT in cooperation with the Fraunhofer–Chalmers Research Centre for Industrial Mathematics (FCC) and the Department of Mathematical Sciences at Chalmers University of Technology and the University of Gothenburg, and is presented in the PhD thesis by Lindroth (2011). In the thesis a product development problem was interpreted and formulated in a mathematical optimization framework, and several resulting subproblems were considered in the papers appended to the thesis.

The motivation for the research project presented in Lindroth (2011) comes from the effort to offer the optimal truck configuration to each Volvo GTT customer based on its transport mission and operating environment. For economical reasons, these configurations have to be produced with a limited set of technical solutions. This is done by a common architecture, in which technology is shared such that the same components are used in different combinations, leading to a relatively small number of components but a very large number of possible configurations. In Lindroth (2011) the configuration problem is analyzed from a mathematical optimization perspective. The problem of deciding on a good product to offer to the customer is then formulated within a multi-objective optimization framework.

In the scientific literature, there are previous attempts to optimize certain tyre design parameters; see, e.g., Carlo et al. (2004) in which the cornering stiffness of the tyre is optimized wrt. multiple objectives. The tyres' performance is also tested by tyre suppliers. These tests usually consider only a single vehicle specification and a few operating environments. We aim to solve the tyres selection optimization problem—a special case of the products selection problem, described in, e.g., Ho and Tang (1998)—to a much greater extent and higher complexity, i.e., optimize several tyre design parameters simultaneously for all possible combinations of the vehicle configuration and the operating environment.

1.5 Outline

The remainder of this thesis is organized as follows. In Section 2 the current status of the tyres selection process at Volvo GTT is described. This is our starting point for determining the framework. Section 3 presents the scientific areas utilized in the thesis. The variables and the objective function used to model the truck tyres selection problem, and the tyre and vehicle models used to evaluate the tyre related functions, are discussed in Section 4. In Section 5, we summarize the appended papers, in which the selected parts of the problem studied are presented. Finally, in Section 6 the main contributions of this thesis as well as topics for future research are reviewed.

2 Current status of the tyres selection process at Volvo GTT

At Volvo GTT a gradually developed sales system with an associated organization is used to select tyres. This system enables the customer—via a dialogue with a sales person—to specify the required vehicle configuration (see Section 2.1) and the intended usage of the vehicle (see Section 2.2) in a systematic way. The sales system then provides a large set of feasible tyres for each truck; these tyres are suitable for the potential utilization, and also fit the truck dimensionally. The current practice for selecting the tyres configuration is then usually based on experience and customer input. This practice could, as demonstrated in this thesis, be improved by means of scientific measures and methodologies. An additional factor complicating the selection of tyres is the increasing number of combinations of steered and non-steered axles in future long vehicle combinations (Kati et al., 2014), which may have up to 30 tyres, as illustrated in Figure 2.

Figure 2: A long vehicle combination with eleven axles and 26 tyres.

The tyres selection process is to be improved using the results from the research presented in this thesis. We want to find an optimal combination of tyres, that is, suggest which tyres from the set of feasible tyres that should be used in order to minimize the fuel consumption. Since we are selecting tyres from an existing tyre database we assume that constraints, such as ride comfort and safety requirements, are fulfilled by the feasible tyres when introduced to the market by the tyre suppliers. We aim to make the tyres selection only for the truck, which is manufactured by Volvo (the first three axles in Figure 2), and not for the trailer, which is also excluded from the vehicle model.

2.1 Vehicle configuration

At Volvo GTT the principles of the platform-based product development (Jiao et al., 2007; Simpson et al., 2001) are exploited. A large number of shared parts is combined to create a great variety of product configurations. A truck is specified by its so-called *variants*, each of which belongs to a certain *variant family*, as illustrated in Figure 3. The variant families represent physical choices, such as engine type or frame width, as well as operation-related features, such as the type of roads that the truck is aimed for. A certain truck configuration is completely defined by its variants. The combinations of variants are linked to actual physical parts of the truck.

```
FM 84F R 24-HDV   UWVTAPPR ULEGC    TYPE-FM  8*4      RIGID
BRAND-V  RC-ROUGH TC-CONST CURV-H   COG-BAS  DIRT-MED T-HILLY
MA-BAS   ULSA     GCW70.0  ENG-VE13 EGR-H    FAL20.0  RAL32
STWPOS-L RAP7940  FFL1275  FAP3040  VW2600   CHH-XHIG VH4.0
UADR     NR-80EC  UADRC    ATL15    ATU-VH   ETOR2400 D13K460
EAP-LHD2 FLYHM-AL OILS-PL  FUEFE2SW XFUEL50  TNK-SING MTNK-R
FUELTS56 ULFUEL   UGAS     DIESEL   UTFUEL   FTCL-BAS RFUEL445
FTANK-AL FUFF-AS  FCAP-L   PRIM-EL  UAUXFUEL UFUEQWH  UFTI
STRAP-P  TURBO-S  TURB-HD  EXD-VERR EASP7    UEXSH    EASO-HV
EXST-ST  HS-BAS   EBR-VEB+ EM-EU6   ADTP-L   EML-BAS  ADB032
PADB80   UADTC    RAG-ADBL SCRS-EHH ACL1ST-S AIRIN-HI CCV-C
EAS-SD   UWILDBAR BUGNET   CPSR-VAR FAN-VIE2 USPEEDDU USPLIMS
SPEED90  URSPED   CRUISEC  OBDEP-B  ENGPROT  OBDEC1   24V
2BATT170 BATTD128 UBAMAFRE BATTIND  BBOX-L   TAS-ANA  24AL110B
UESTAID  HL-ASYMR DRL-LED  HL-BAS2  AHS-LB   UHLPPRO  POSLMP-W
UIDLAMP  HL-ADJ   UCLSTAT  URPLCR   TL-LED   UARLIG   BLIGHT-E
TLB-BAS  UARL     INLI-BAS MARKL-SR URESTS   WLC-PKH  UASL
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Figure 3: An excerpt from a truck specification. Each entry is a code for a variant in some variant family. The variant *8*4* belongs to the variant family *Axle arrangement* and describes that the truck has eight wheels, thereof four driving. The variant *RC-ROUGH* belongs to the variant family *Road condition* and describes that the truck will be operated on badly maintained roads. The variant *T-HILLY* belongs to the variant family *Road topography* and describes that the truck will be operated on hilly roads.

A typical truck configuration consists of about 500 valid variants, each from a variant family containing two or more variants. Thus, the number of possible configurations is huge, even though not all variants can be combined due to different kinds of documented *restrictions*; see Figure 4 for illustrations of some truck configurations. The sales system contains a small subset of the complete set of variant families from which the actual variant is chosen by the customer.

2.2 Discretization of the operating environment

Once a specific truck is defined, it is also necessary to introduce an environment in which it should operate. The operating environment is characterized by many parameters, e.g., topography, road conditions, and curve density. The properties of the

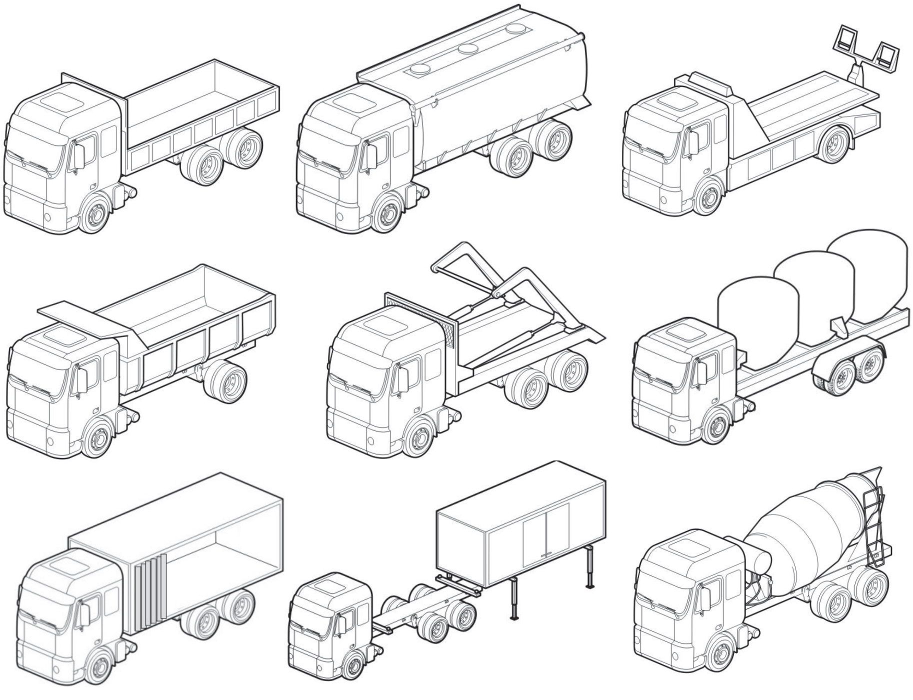

Figure 4: A selection of vehicle configurations, illustrating the variety of vehicles produced by Volvo GTT. The variants specify, e.g., the axle configuration. Hence, e.g., the upper- and right-most configuration does not correspond to the same variant for the axle configuration as the lower- and right-most configuration.

operating environment are changing over the driving cycle and are hard to measure. However, discretizations of these properties were introduced by Edlund and Fryk (2004) in order to design vehicles adapted to the actual customer needs rather than to the worst-case conditions possible for any customer. The Global Transport Application (GTA) defines a number of vehicle-independent parameters that specify differences in driving and transport conditions for vehicle operations worldwide (see Edlund and Fryk (2004)). Among the GTA parameters defining a certain transport application are the operating cycle (divided into the four well-defined classes *Stop&Go*, *Local*, *Regional*, and *Long Distance*), the road condition (divided into *Smooth*, *Rough*, *Very Rough*, and *Cross Country*), and the topography (divided into *Flat*, *Predominantly Flat*, *Hilly*, and *Very Hilly*). Some of the GTA parameters, as, e.g., road conditions, are considered as variant families in the product structure as well as in the sales system to link specific parts of the truck as well as to forbid certain variant combinations. The operating environment is completely defined by fifteen GTA parameters, each containing three or more classes. Thus, the number of classes of operating environments is also very large.

2.3 Tyre specification

An extensive database of tyres, which can be used by the trucks produced, was developed in a cooperation between Volvo GTT and its tyre suppliers; see Figure 5 for an illustration of differences between tyres. The database contains some information about each tyre (e.g., rolling resistance coefficient class, noise class, load capacity, axle load at each given pressure value, width, diameter, sidewall height, weight, brand name, and recommended applications). Requesting additional information from tyre suppliers appeared to be problematic and requiring a fairly long waiting time.

In this thesis, the most important tyre parameters will be used to define the decision variables in the optimization model developed for solving the tyres selection problem; see Section 4, Nedělková (2016), and **Paper II**.

Figure 5: Three tyres available in Volvo GTT's tyre database.

2.4 Complexity of the tyres selection process

A complex system is formed by a set of interconnected components. To understand the behaviour of a complex system one must understand the behaviour of its components and their interactions (see Bar-Yam (1997, Ch. 0)). To solve the tyres selection problem we need to model a complex system consisting of the three interacting systems of the vehicle, the tyres, and the operating environment.

Complexity can, according to (Bar-Yam, 1997, Ch.0), be defined as the amount of information needed to describe the complex system[2]. The complexity of the truck and the operating environment model varies with the scale in which the truck is described. The tyre model has to be complex enough to differentiate between different tyre designs. Therefore, solving the tyres selection problem for each customer is time consuming. However, the main complexity lies in the need to identify the respective optimal tyre configurations, among the tyres in the extensive tyre database,

[2]Note that this definition differs from that of computational complexity, which is essentially a function of the amount of information needed to describe the system.

for the large number ($\sim$ 300 000 trucks sold every year out of $\sim 10^{120}$ possible vehicle configurations (Voronov, 2013)) of vehicle configurations operating in the large number of environments.

In **Paper III** and **Paper V** we develop a methodology allowing for the solution of the computationally expensive truck tyres selection problem only for *strategic vehicle specifications* (SVSs, see Lindroth (2011, Ch. 2)) and then for assembling the optimal tyres configurations for the other vehicle configurations in a computationally efficient way. The concept of SVS was introduced to systematize and simplify the production and development processes for trucks at Volvo GTT (see Lindroth (2011, Ch. 2)).

3 Scientific areas covered

Since this thesis is driven by an industrial application, it utilizes theory from several separate, although interconnected, scientific areas.

The most important scientific areas for our application are *Engineering design* (see Section 3.1), *Vehicle dynamics* (see Section 3.2), and several subdisciplines of optimization, such as *Global optimization* (see Section 3.3), *Simulation-based optimization* (see Section 3.4), and *Optimization with categorical and discrete variables* (see Section 3.5). Summaries of these areas are given below, the respective extent roughly representing their importance for this thesis.

3.1 Engineering design

Computationally expensive design problems are becoming common in manufacturing industries. The design of complex systems usually involves multiple disciplines and computationally intensive processes such as finite element analysis (FEA) for the system simulation. Therefore, the engineering design problems usually cannot be solved by an exact analysis. One needs to start with an approximate assessment and set up idealized simplifications, and then construct models to represent the real physical conditions. These simple models having limited validity range are called *surrogate models*. Surrogate modelling techniques in engineering design, e.g., splines, regression, Kriging, and artificial neural networks, are surveyed in Fu (2002) and Wang and Shan (2007). Having a surrogate model, common optimization methods can be applied to search for an approximation of an optimal design. The process towards a final design usually involves gradual enhancements of the accuracy of the surrogate model utilizing design space exploration (e.g., Jakobsson et al. (2010)) and various model validation techniques, such as cross validation (Meckesheimer et al., 2002), bootstrapping, and split sample (Queipo et al., 2005).

3.2 Vehicle dynamics

To evaluate the tyre related functions when solving the tyres selection problem certain vehicle and tyre models have to be used. Some of these models are discussed next. The most suitable models are then selected based on several criteria (computational efficiency of the models and possibility to easily change the specifications of tyres) and used further; see also **Paper II**, Chen and Prathaban (2013), and Kolář (2015).

Tyre models

When considering the tyre behaviour we need to be aware that the tyre is a very complex structure because of its construction (it is composed of several layers) and of its material. The tyre behaviour is related to its deformation at the ground contact. The basic task of a tyre is to provide a support of the vehicle in the normal direction to the road, to smooth the road irregularities, and to transmit forces in the ground plane that allow motion along the desired trajectory (Wong, 2008). In addition to the basic tasks, there are other expectations from the tyres, related to rolling resistance, noise, ride comfort, handling performance, etc. resulting in a lot of different kinds of tyres available on the market. For trucks, there are in particular two distinct groups of tyres manufactured: steer tyres and drive tyres.

To transmit the forces between the vehicle and the road two main phenomena, the adhesion/sliding and the deformation of the tyre and the road, have to be modelled. Since we focus on hard road surfaces we can neglect the deformation of the road. The adhesion/sliding effects are caused by molecular attraction between the tyre and the road in the contact patch. A certain amount of energy is dissipated in and under the wheels. The adhesion/sliding depends on the temperature, the damping properties, and the deformations of the surfaces in contact (Genta and Morello, 2009, Ch. 2). The tyre deformation is caused by the forces straining the tyre structure. The deformation of the tyre contact patch generates a reaction force to the road allowing the vehicle to be controlled in both the longitudinal and the lateral directions (Genta and Morello, 2009, Ch. 2).

The rolling resistance is caused by the deformation of the tyre and is defined as the loss of longitudinal force on the vehicle body, as compared to the longitudinal force, which would have been transferred with an ideal wheel. The rolling resistance coefficient (RRC) is defined as the rolling resistance force divided by the normal force, F_z, i.e.,

$$RRC := \frac{e}{R} = \frac{\frac{T}{R} - F_x}{F_z}, \tag{1}$$

where F_x denotes the longitudinal force on the wheel, T denotes the applied torque, e is the normal force offset and R denotes the tyre radius; see Figure 6. For a free rolling tyre, where $T = 0$, RRC becomes simply

$$RRC = -\frac{F_x}{F_z}.$$

Additional tyre characteristics might be required to model the interaction between the road and the vehicle, such as, the lateral slip stiffness, the lateral stiffness, the longitudinal stiffness, the longitudinal slip stiffness, the contact patch length, relaxations lengths for longitudinal slip and slip angle, the vertical stiffness, and the wheel moment of inertia.

Truck tyres have a lower RRC than passenger vehicle tyres (approximately half, see (Wong, 2008)) mainly because of their high inflation pressure (9 bar when compared to 3 bar for passenger vehicle tyres) and their dimension. The truck tyres are developed with the aim to minimize the RRC, because the fuel economy is critical for trucks.

Truck tyre measurements show that the rolling resistance and other tyre related functions are dependent on operation conditions (Xiong and Tuononen, 2015). Therefore, apart from modelling the influence of the tyre specification, the tyre model needs to reflect the operating conditions. It is not easy to create a model that realistically represents the full behaviour of the tyre under all possible driving

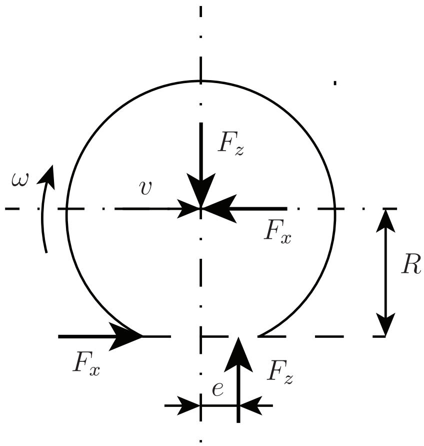

Figure 6: Force distribution on a free rolling tyre rolling forward with a translational speed v and rotational speed ω.

conditions. Trying to do so would result in a very complex and computationally expensive model, which would be practically impossible to handle. Therefore, many simpler tyre models for different use cases have been developed. The tyre models can be classified as follows:

- *Empirical models*
  The empirical models consider the tyre as a black box. The tyre deformation is created and the tyre forces are then measured. Then, a model of the dependence of the tyre forces on different parameters is built. These models are usually provided in a closed form and hence require a low computational effort. However, a lot of expensive tyre measurements have to be done in order to populate the model and it is often difficult to generalize the model for varying tyre designs. Examples of such models are Pacejka's magic formula (Pacejka and Besselink, 2012, Ch. 4) and the combined tyre model (Redrouthu and Das, 2014).

- *Theoretical models*
  For the theoretical models a simple physical representation of the tyre is used. These models are typically developed for specified operating conditions and are computationally efficient. However, a certain amount of experimental data (not as much as for the empirical models) to validate the model needs to be obtained using specialized equipment. Examples of theoretical models are the brush model (Wong, 2008), the extended brush model (Davari et al., 2017), the string model (Potts et al., 1977), the Dahl tyre model (Chou, 2004), and the SWIFT tyre model (Besselink et al., 2004).

- *Physical models*
  The physical models require a large computational effort because they are

based on detailed tyre descriptions. These models are typically created using computer simulations, guaranteeing an accurate and quantitative fit of the tyre characteristics to the real measured tyre behaviour in the considered conditions. No additional experimental data are needed for a validated model. An example of such complex physical models are the FTire tyre model (Gipser, 2005) and the UOIT FEA truck tyre model (Ali et al., 2013).

The UOIT FEA truck tyre model (Ali et al., 2013) is used to build the tyre model used in the TyreOpt research project because it allows for investigations of the influence of several tyre design parameters as well as some parameters describing the vehicle configuration and the operating environment. Further details on the tyre model development can be found in Section 4.3 and **Paper II** .

Vehicle models

The study of vehicle dynamics starts with studying the interfaces between the vehicle and its environment. The vehicle tyres are the primary force interface for the motion in the ground plane. The vehicle is also subjected to aerodynamic forces and, in case of road inclination, gravity forces in the ground plane. Another kind of interaction is the driver. When the interfaces are successfully modelled the vehicle dynamics in the longitudinal, the lateral, and the vertical directions have to be considered. Since we aim to do the tyres selection for long-haul straight driving, mainly the vehicle dynamics in the longitudinal direction is studied. The vehicle dynamics model consists of models of the body, of the transmission, of the engine, and of other systems (e.g., brakes). The tyre models can also be seen as a part of the vehicle dynamics model.

The vehicle dynamics model is a model of a dynamic system. Mathematically, this corresponds to a system of differential algebraic equations (DAEs). The system of DAEs is typically converted into a system of ordinary differential equations (ODEs) of the state variables (typically velocities and positions). The resulting system of ODEs is then solved numerically using well approved numerical methods (Arnold et al., 2011).

There are many differences (Jacobson, 2017) between heavy trucks and passenger cars, resulting in different requirements and expectations of their owners but also in a need for a specialized modelling approach. Trucks are typically owned by companies and each truck is bought for a certain transport task. The life, counted in the covered distance, of a truck is typically ten times longer than that of a passenger car. The life time fuel cost is typically five times the vehicle purchase price, as compared to passenger cars with these costs being equal. The life time cost of tyres is approximately half of the truck purchase price. A truck has up to ten times less power installed per gross vehicle weight than a passenger car. The weight of the load on a truck can be up to four times the weight of the empty vehicle while the payload on passenger cars is lower than the empty car weight. Trucks often have many steered axles, while passenger cars normally have only the front axle steered. The center of gravity of trucks is typically located higher than the center of gravity of passenger cars leading to a higher probability of roll over of a truck.

We have mainly focused on the vehicle models for the longitudinal direction, allowing to investigate the interaction of the vehicle and tyres in a specified operating environment, available at Volvo GTT, but one can find similar models of the same type in other organizations as well. Below is a list of all relevant available vehicle models.

- *PERF (PERFormance)*
  PERF is a longitudinal dynamics fuel consumption model used at Volvo GTT capable of evaluating the vehicle power train performance criteria on simple longitudinal driving cycles. It mainly aims to assess the fuel economy, and suitability of a particular vehicle specification for a desired environment. PERF is written in FORTRAN (SunSoft, 1995); it is therefore complicated to modify. This limits any changes of the tyre definition, which is very simple (the rolling resistance coefficient is constant over the driving cycle).

- *VTM (Vehicle Transport Models)*
  VTM is a forward simulation environment allowing to assess a vehicle dynamics behaviour through evaluating vehicle traction and handling models of a large number of different vehicle configurations manufactured by Volvo GTT. The models are created in MATLAB/Simulink (MATLAB, 2012); therefore changes of the tyre definition (Pacejka's magic formula is used (Pacejka and Besselink, 2012, Ch.4)) are simple. A VTM vehicle model of a typical transport vehicle, a tractor with semitrailer, has ten state variables for the motion in road plane, ten state variables for the motion out-of-road plane, and from ten to twenty additional state variables for the wheels, the actuation systems, and the driver dynamics.

- *GSP (Global Simulation Platform)*
  The vehicle longitudinal dynamics is modelled within GSP, which is used at Volvo GTT, with the aim to compute the fuel consumption. A well documented implementation of GSP in MATLAB/Simulink allows editing components of the power train system, which is used for assessment of the vehicle performance in various drive cycles. The model of the chassis is very simple and neglects suspension dynamics in the vertical and the lateral directions, so the influence of the different vehicle configurations cannot be tracked. The tyre–road contact is modelled without the rolling resistance and the longitudinal slip. Also, the lateral tyre slip is not modelled.

- *CVM (Complete Vehicle Model)*
  CVM is a very detailed structure and vibration vehicle model built in the NASTRAN FEA software (MSC Nastran, 2004) and used at Volvo GTT. The vehicle structure is defined by discrete elements and evaluated with a finite element analysis (FEA) solver. The model is computationally expensive and to work with the model requires a special training. The tyre model is relatively simple and consists of linear springs and dampers taking into consideration the slip

equations. CVM evaluates the static analysis, the handling, the ride comfort, and the durability of the vehicle considered.

- *Computationally efficient model of a heavy duty truck*
  A computationally efficient model of a heavy duty truck was developed by Chen and Prathaban (2013) for the need of the TyreOpt research project. The inverse dynamics principle was applied and a simple model of the power train was built in MATLAB/Simulink (MATLAB, 2012) for fuel consumption evaluation.

  The inverse dynamic principle, sometimes called backward simulation, means that a vehicle motion is prescribed by its velocity profile as a function of time. The force required to attain the velocity profile is calculated backwards through the propulsion system. The backward simulation results in less state variables and more computationally efficient vehicle model than the forward (natural causality) simulation, but it fails when handling some situations, such as, when the propulsion system cannot generate enough power to follow the velocity profile.

  The tyre model is based on an interpolation of sample points simulated by the UOIT FEA tyre model (Ali et al., 2013). A simple tyre wear model based on Grosch and Schallamach (1962) and several constraining events (such as, startability, ride comfort, and handling) were implemented as well. The main advantage of the model is its computational efficiency. However, many trade-offs and simplifications were done, e.g., that dependencies between the vehicle and the power train are missing, the cornering is not modelled, and many tyre dependent parameters are given fixed values leading to problems to use it to select between a large set of different tyre designs.

The best balance between the accuracy of the prediction of influence of the tyre design variation and the computational efficiency is found with VTM combined with a modular surrogate tyre model. VTM is employed to build the vehicle model used in the TyreOpt research project. Further details on the vehicle model development can be found in Section 4.4 and **Paper II**.

To allow for the use of our results in the future, when new and enhanced tyre and vehicle models might be available, the methodology for solving the tyres selection problem has to be modular, such that the sub-models can easily be replaced.

3.3 Global optimization

Consider the optimization problem to

$$\begin{aligned} &\text{minimize} \;\; F(\mathbf{x}), \\ &\text{subject to} \;\; \mathbf{x} \in X, \end{aligned} \tag{2}$$

with the decision variables $\mathbf{x} \in \mathbb{R}^m$. The goal is to find a global optimal solution $\mathbf{x}^* \in X$ that minimizes the *objective function* $F : \mathbb{R}^m \to \mathbb{R}$ over the *feasible set* $X \subset \mathbb{R}^m$.

The feasible set X is typically determined by a number of *equality* and/or *inequality constraints* involving the decision variables.

According to Bazaraa et al. (2006), a point $\mathbf{x}^* \in X$ is a *global minimum* of F over X if it holds that

$$F(\mathbf{x}^*) \leq F(\mathbf{x}), \quad \mathbf{x} \in X. \tag{3}$$

A point $\mathbf{x}^* \in X$ is a *local minimum* of F over X if there exists a neighbourhood $\mathcal{N}(\mathbf{x}^*)$ of $\mathbf{x}^*$ such that

$$F(\mathbf{x}^*) \leq F(\mathbf{x}), \quad \mathbf{x} \in \mathcal{N}(\mathbf{x}^*) \cap X, \tag{4}$$

where the definition of $\mathcal{N}(\mathbf{x}^*)$ varies with the definition of X (cf. Hu et al. (2008)). When $X \subset \mathbb{R}^m$ is considered, $\mathcal{N}(\mathbf{x}^*)$ is an open Euclidean ball centered at $\mathbf{x}^*$ and with a bounded radius.

Assuming that the set X is convex and that the function F is convex on the set X, the fundamental theorem of global optimality (e.g., Bazaraa et al. (2006, Theorem 3.4.2)) implies that any local minimum of F over X is also a global minimum. Therefore, when a problem fulfills these convexity assumptions, it is enough to apply a local optimization algorithm to find a global minimum of the problem (2). Local optimization algorithms typically possess a lower computational complexity than their global analogues (Bazaraa et al., 2006, Ch. 7).

In nonconvex optimization problems we have to expect multiple local minima that differ from the set of global minima. The discipline concerning the search for a global minimum is called *global optimization* (see Horst et al. (2000) for an introduction).

Some nonconvex optimization problems possess mathematical properties which make the optimization computationally less complex, e.g., integer programs with the integrality property (see Wolsey (1998, Ch. 3)), which can be reformulated as convex programs, or optimization of Lipschitz functions with a known Lipschitz constant; see Horst et al. (2000, Chs. 2 and 5).

For a general global optimization problem, where the evaluation of the objective function is computationally sufficiently cheap, it is possible to use a method that switches between *local* and *global* phases. During the global phase the whole feasible region is explored, while the local phase is restricted to exploring a local portion of the feasible region. The aim of the local phase is to refine the current solution. The local exploration is performed by sampling more observations in a neighbourhood of the current point, with the aim to find an improved solution in terms of a lower objective function value. Examples of local phases are standard local searches, e.g., the *pattern search* algorithm, originating from Box (1957) and described in detail by Audet and Dennis, Jr. (2004). In contrast, the aim of the global phase is to explore the search domain. The global phase usually generates points without restrictions to neighbourhoods of past solutions. Examples of global phases are the random generation of feasible points or the choice of points, for which a measure of their distance from all previous sample points is maximized (e.g., Jakobsson et al. (2010)). See Locatelli and Schoen (2013) for an overview of methods for global optimization.

One of the most popular methods for solving general nonconvex optimization problems with a computationally sufficiently cheap objective function evaluations is the DIRECT (Dividing RECTangles) algorithm (Jones et al., 1993). The method does not provide any convergence guarantee or any error measure unless some strong assumptions are imposed on the problem properties. The convergence proof for this algorithm is based on the splitting strategy used in the global phase to split the feasible set into hyper-rectangles, in the centres of which the objective function is evaluated, and which will eventually generate a dense set of observations in the feasible set. This type of convergence property is common for all global optimization methods which do not use any prior information about the structure of the problem (Locatelli and Schoen, 2013). The TOMLAB (Holmström, 1999) solver *glbFast* is based on the DIRECT algorithm. Alternatively, nonconvex optimization problems can be solved to global optimality using the algorithm BARON, which is described in Sahinidis (2013) and which is based on the branch-and-reduce approach introduced in Ryoo and Sahinidis (1996). For a review of advances in global optimization see Horst and Pardalos (2013) and Floudas and Gounaris (2009).

The discipline concerning the search for a global minimum when dealing with computationally expensive objective (or constraint) functions is called *simulation-based optimization* and is discussed next.

3.4 Simulation-based optimization

The main assumption in simulation-based optimization is that the objective function f in (2) is not directly available, but must be estimated through a simulation, which implies the absence of analytic derivatives; see Conn et al. (2009) for an overview of algorithms for derivative-free optimization. Computer simulations are extensively used as models of real systems in order to evaluate output responses. Applications of simulation-based optimization are found in engineering design (He and Wang, 2007), manufacturing analysis (Truong and Azadivar, 2003), portfolio selection (Rockafellar and Uryasev, 2000), biomedicine (Iskander and Tumeh, 1989), etc. Simulation-based optimization integrates optimization techniques with simulation analysis.

Optimization problems including simulation-based functions cannot in practice be solved by algorithms requiring many function evaluations, such as, e.g., *direct search methods* (Lewis et al., 2000) or algorithms inspired by physics and/or natural selection (e.g., *genetic algorithms* (Michalewicz, 1996)). Instead, we need to consider global optimization algorithms, in which a surrogate model is constructed, and that mimics the behaviour of the expensive function as closely as possible while being computationally cheap to evaluate; this surrogate model is then optimized. These algorithms are denoted *response surface methods* and are reviewed in Jones (2001), individual algorithms can be found in Jakobsson et al. (2010); Regis and Shoemaker (2005); Jones et al. (1998). The response surface methods iteratively construct the surrogate model of the simulation-based function; see Algorithm 1.

The initial set of sample points at step 0 is created by some design of experiments technique, such as the latin hypercube, introduced in McKay et al. (1979). The strategies to select a new point to evaluate in step 2 differ between specific algorithms. The strategy must balance local and global searches so that the information in the surro-

Algorithm 1 General response surface optimization method

0: Create an initial set of sample points and evaluate the simulation-based objective function on this set.
1: Construct a surrogate model of the simulation-based objective function using the evaluated points.
2: Select and evaluate a new sample point, balancing local and global searches, to refine the surrogate model.
3: Go to step 1 unless a stopping criterion is met.
4: Optimize the surrogate model constructed in place of the simulation-based objective function.

gate model is utilized, but also so that no part of the feasible set is left unexplored. The stopping criterion in step 3 varies between specific optimization problems (e.g., that a maximum allowed number of function evaluations have been made, or that a certain quality measure of the model has been attained wrt. some model validation technique, such as the cross-validation studied in Meckesheimer et al. (2002)). In step 4 the resulting surrogate model is optimized by a global optimization solver and the optimal solution found approximates the optimal solution of the underlying expensive function.

In this thesis we deal with surrogate models constructed as radial basis function (RBF) interpolations (see Wendland (2005, Chs. 1, 6, and 11) and Björkman and Holmström (2000)) of sample points, which often yield good global representations of the expensive function (Buhmann, 2003) and has a closed form expression. The surrogate model can also be obtained as a linear or quadratic approximation (see Bandeira et al. (2012)), Kriging approximation (see Simpson et al. (2001)), a general regression function (see Billups et al. (2013)), or some other kind of interpolation (see Ni and Hu (2004)). The response surface methods utilizing RBF interpolation are described in Gutmann (2001) and in Jakobsson et al. (2010).

Each evaluation of the true simulation-based function is time-consuming, and often there exists some expert knowledge about the true function. In order to improve the accuracy of the surrogate mode, hence, in **Paper I** we have developed a methodology to incorporate existing expert knowledge into the RBF interpolation based on sample points.

Since the computational complexity of solving one simulation-based optimization problem is high and our application requires the solution of a lot of similar such problem instances, in **Paper III** we have developed a computationally efficient optimization algorithm for a combinatorial set of simulation-based optimization problem instances with continuous variables which reduces the variable space and also the number of simulation-based optimization problem instances that have to be solved to optimality.

Our work considers an optimization problem (2) with a computationally expensive objective function F subject to box constraints determining the feasible set X and with categorical variables. The discipline concerning the search for a global minimum when dealing with categorical variables is discussed in the next section.

Since the optimization problem considered is very complex we have decided to stick to one objective function F describing the fuel consumption of a vehicle. Considering additional objectives, e.g., a function measuring tyre wear, ride comfort, or handling quality of the vehicle, would lead to a multiobjective optimization problem requiring specialized optimization algorithms (see, e.g., Miettinen and Mäkelä (2002); Branke et al. (2008) for multiobjective optimization algorithms and Andersson and Krus (2001) for a multiobjective genetic algorithm for multiobjective optimization problems with both continuous and categorical variables).

3.5 Optimization with categorical and discrete variables

In design optimization (see, e.g., Alexandrov and Hussaini (1997); Neumaier et al. (2007); Fuchs et al. (2008)) a design choice is typically modelled by a categorical variable. The categorical variables can be given numerical values, but the values themselves do not have any physical meaning. In real design optimization problems the choices can be made from a specified list of alternatives. A certain alternative corresponds to values of a number of discrete variables, i.e., a point in a multi-dimensional discrete search space. The study of alternatives can be viewed as a branch of combinatorics (Parker and Rardin, 2014, Ch.1).

Some optimization problems with categorical variables can be expressed in a form of a mixed integer optimization problem (Abhishek et al., 2010). Depending on the mathematical properties of the objective function and the constraints the resulting problem falls into one of several classes of mixed integer optimization problems. For each such class algorithms to explore the search space exist. For studies of efficient algorithms for mixed integer optimization problems; see Floudas (1995, Ch. 5) and Nemhauser and Wolsey (1999, Ch. II.4) for mixed integer linear optimization, and Leyffer (1993) and Tawarmalani and Sahinidis (2004) for mixed integer nonlinear optimization. For a survey on discrete optimization see Parker and Rardin (2014, Chs. 1 and 5). A survey on approaches for handling categorical variables in optimization problems is found in Lindroth (2011, Paper IV), which also introduces an algorithm for pure categorical optimization problems.

When it comes to simulation-based optimization problems with categorical variables a limited number of algorithms exists. Algorithms of Abramson et al. (2008) and Audet and Dennis, Jr. (2001) for solving constrained optimization problems in which the variables may be continuous or categorical were recently implemented into NOMAD. Hence, NOMAD (Audet et al., 2009) can be used for optimization problems with categorical variables when a definition of the neighbourhood of the categorical variables is supplied. Algorithms dealing with categorical variables based on pattern search can be found in Sriver et al. (2009). We have developed a splitting algorithm for simulation-based optimization problems with categorical variables in **Paper IV**. In each step of the iterative algorithm a convex approximating subproblem is generated and solved. These subproblems are generated along a search tree created (cf. Svanberg (2002), where the generation of the subproblems of a general nonlinear optimization problem is controlled by so-called moving asymptotes).

A methodology for solving a large set of simulation-based optimization problems with categorical variables is developed in **Paper V**.

4 Optimization of truck tyres selection

Building an optimization model, which typically starts with a quantification of the real-world description of an optimization problem, is the first and yet the most critical step for solving any optimization problem. The resulting optimization model also affects the choice of the optimization algorithm that will be used to solve the problem, the feasibility of the problem, and the computational efficiency of the optimization algorithm chosen.

Optimization in industry is an iterative process, where, at each iteration, a model of the true problem is formulated and an optimal solution is found. Then the outcome is evaluated by experts, with a typical conclusion that some parts of the model are missing or need to be adjusted. Then the model is adjusted and a new iteration takes place. The usage of optimization in industry helps to illuminate and understand the problem, and to create basic data for decision-making. The important ingredients of an optimization problem are

- the objective, which measures the quality of a solution and is to be minimized or maximized,
- the decision variables, the values of which are to be selected within some given sets, and
- the constraints, which correspond to the design or feature requirements on a solution to be feasible.

The aim of this section is to describe these three components for the tyres selection optimization problem and to introduce the vehicle dynamics models used to evaluate the objective and constraints functions.

We want to find an optimal configuration of tyres, i.e., suggest, for each individual axle, which tyres from the feasible set (here defined by Volvo GTT's tyre database) that should preferably be used in order to minimize the energy losses caused by the tyres. The tyres selection optimization problem cannot be solved through a complete enumeration of all combinations of tyres, since there is a large number of feasible tyres, since the vehicle has several axles which lead to many combinations of tyres, and since the vehicle dynamics simulations are computationally expensive.

4.1 Decision variables

In this thesis we focus on the selection of tyres that are available in the tyre database, i.e., tyres will be represented by categorical variables. Categorical variables, i.e., discrete decision variables that cannot be naturally ordered, are commonplace in engineering design optimization problems (see Alexandrov and Hussaini (1997, Sections 1 and 6) and Fuchs et al. (2008)).

A certain tyre in the database corresponds to specific values of a number of variables (e.g., the tyre diameter) describing the tyre. These variables, which enable the formulation and solution of the tyres selection problem, need to be identified. We typically wish to keep the number of variables at a minimum in order to obtain a practically solvable optimization problem (see also Section 2.4), but sufficiently many variables have to be considered in order to be able to solve the intended real problem.

The tyre specification employed in the TyreOpt project is based on the literature survey in Nedělková (2016) of available tyre models. This enables an identification of the parameters that are most often used to describe the tyres. The selection of decision variables describing the truck tyres must also take into account the available information in Volvo GTT's tyre database about each tyre, as well as the information that can possibly be received from the tyre suppliers. The selected decision variables are

- the inflation pressure,
- the tyre radius, and
- the tyre width.

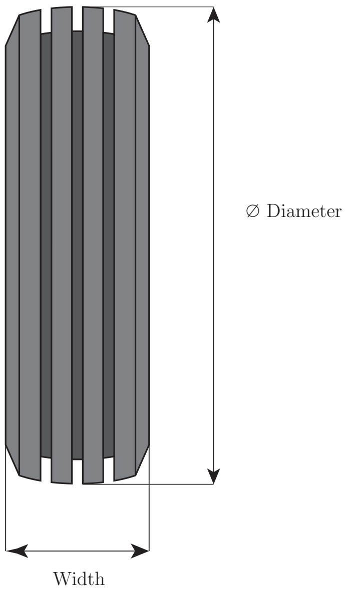

Figure 7: The considered tyre dimensions: tyre diameter and tyre width.

See also Figure 7 for an illustration of the tyre dimensions considered. The tread rubber specification and the groove pattern specification should be considered as decision variables of the tyres selection problem when more information about each tyre is available in the database. The FEA tyre model (Ali et al., 2013) used in the

TyreOpt project allows for investigations of influence of the groove depth and the number of grooves on the RRC. The information about the groove depth should soon be available for each tyre in the tyre database and can be utilized. Further analysis on the importance of the number of grooves for the tyre behaviour should be analyzed. One specific material model was used in the FEA tyre model but the material model as well as its parameters can be varied. The information about the tyre material is considered confidential by tyre suppliers.

There is a variety of other parameters that significantly influence the identification of an optimal tyre configuration and which are not considered so far. Some of them can be considered as fixed (or co-varying with other parameters) and some of them should possibly be included in our optimization models to enable the computation of more accurate and realistic results.

4.2 Objective function and constraints

Engineers often use a combination of experience, computer simulations, and testing to decide on which technical solutions are good and which are not. Within mathematical optimization, we assume real-valued functions (possibly outputs from black-box simulations) to assign a quality measure to each single technical design. It is often a hard and complex task to define an objective to be minimized, measuring the qualities of the technical solutions as functions of the decision variables. Further, it is difficult to keep the total amount of computations at a reasonable level. To receive a good solution in a reasonable computing time, the accuracy of each model included must be compared with the computation time required.

As the full name of the project *TyreOpt—fuel consumption reduction by tyre drag optimization* suggests that we should aim at minimizing the fuel consumption of a vehicle operating in an environment. This can be expressed as to

$$\underset{\mathbf{x}\in X(\mathbf{p})}{\text{minimize}} \quad F(\mathbf{x},\mathbf{p}), \tag{5}$$

where the vector $\mathbf{x}$ denotes the variables describing the tyres, the vector $\mathbf{p}$ denotes the vehicle and operating environment parameters, and the set $X(\mathbf{p})$ contains the feasible tyre designs for a given vector $\mathbf{p}$. Let us assume that the parameters and variables are here differentiated wrt. optimization, but not wrt. the simulation model. The values of the tyre variables are to be found through the use of optimization algorithms.

The fuel consumption, F, of a vehicle operating in an environment (that is, the setting of the vector $\mathbf{p}$) with specific tyres mounted (that is, the setting of the vector $\mathbf{x}$) is to be evaluated using the joint vehicle, tyres, an operating environment model, which is described in **Paper II** and developed in Kolář (2015). The fuel consumption is in our case measured by the tractive energy (measured in MJ) required to run the vehicle for one kilometer. Tyre wear is an important factor as well when solving the tyres selection problem. A preliminary attempt to model tyre wear for the needs of the research project presented in this PhD thesis was performed by Chen and Prathaban (2013); since there is only a limited amount of information available about each tyre, the model is not detailed enough and has not been used so far. A more detailed and accurate model of the tyre wear should be developed. Further, we assume that the tyre wear levels as well as other costs, such as the influence of tyres on fatigue of the vehicle suspension, are similar for all tyres considered. An evaluation of the objective function is required to be computationally efficient in order to keep the tyres selection problem practically solvable.

Since the tyres to be selected are chosen from an existing tyre database we assume that constraints on, e.g., ride comfort and safety requirements, are already fulfilled by the tyre manufacturers when introducing the tyres to the market. We assume that any combination of the tyres fulfilling the constraints mentioned above is a tyre configuration fulfilling these constraints as well. Hence, these constraints are neglected when modelling the tyres selection optimization model.

To model the selected objective function a vehicle dynamics model needs to be developed. This so-called joint model consists of three main components, namely

- vehicle (including a driver model),
- tyres, and
- operating environment.

The model is established with the aim to be computationally efficient, but complete enough to reflect the influence of the selection of tyres on the fuel consumption.

4.3 Tyre model

The fuel consumption of a vehicle is improved while minimizing the energy losses caused by the tyres, represented by the rolling resistance when neglecting the slip.

The function describing the rolling resistance can be represented by a complex function of a large number of variables affecting each other and about which only limited knowledge exists. The rolling resistance is quantified by the rolling resistance coefficient (RRC; see Pacejka and Besselink (2012, Ch. 1)). The instantaneous values of the longitudinal force F_x on the wheel, and the normal force F_z used to determine the instantaneous value of RRC (see the definition of the RRC in (1)) were computed by the FEA truck tyre model developed in Ali et al. (2013). The FEA model is typically run for one instantaneous operating condition, where the tyre deformation and the sliding pattern reach the steady state. The FEA tyre model was chosen because it provides an accurate fit of the tyre characteristics to the real tyre behaviour and no experimental data are required to set up the model. To overcome the difficulty with computationally expensiveness of the FEA tyre model a surrogate model of the RRC function, based on sample points simulated by the FEA tyre model, was constructed. For this purpose, we have chosen an RBF-based interpolation and connected it with the existing expert knowledge about the RRC, i.e., we require the surrogate function to be smooth and possess non-negative values; details are described in **Paper I**.

Linear approximations of additional tyre characteristics required by the vehicle models were created. Such additional tyre characteristics modelled include the lateral slip stiffness, the lateral stiffness, the longitudinal stiffness, the longitudinal slip stiffness, the contact patch length, the relaxations lengths for longitudinal slip and slip angle, the vertical stiffness, and the moment of inertia. All these relations are obtained utilizing experimental data for truck tyres from the tyre database and the available simulations of the FEA tyre model; see Kolář (2015) for details. The tyre model with its inputs and outputs is illustrated in Figure 8.

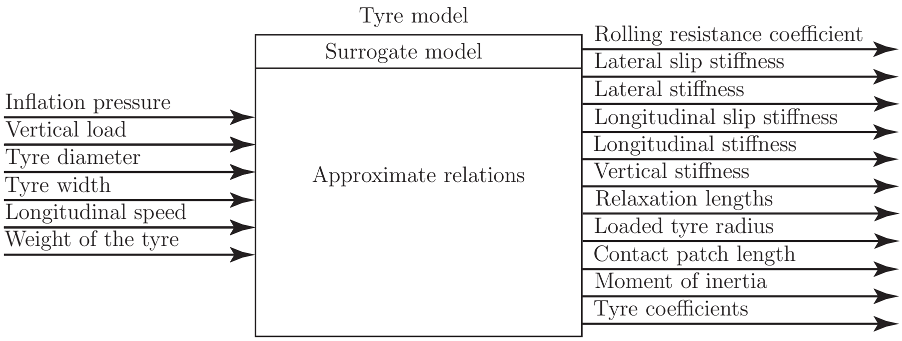

Figure 8: The tyre model, consisting of the surrogate model of the rolling resistance coefficient and approximate relations, is illustrated with its inputs and outputs.

The tyre model should be replaced by a more detailed one allowing for investigations of the tyre material, the groove pattern, and the wheel torque (propulsion and braking situations) on the RRC when additional tyre data are available.

4.4 Vehicle model

A vehicle simulation model reflecting realistically the influence of the tyre design on the tractive energy required to run the vehicle on a specified operating environment is needed for the analysis made in this work. An influence of the tyre design on other costs and properties, such as, the tyre wear and the ride comfort, should be investigated and optimized as a part of the future research. The Volvo in-house developed Matlab/Simulink model VTM was selected because it provides an extensive library of full dynamics simulation models allowing to interchange the vehicle configurations, the loads on the wheels can easily be changed, inclusion of the energy losses due to the tyre slip is possible. The potential but accepted drawbacks of using VTM is less accuracy of the propulsion system dynamics (e.g., gear shifts, driveline oscillations) and higher computational effort required. More information about the vehicle model can be found in **Paper II** and Kolář (2015).

4.5 Operating environment specification

A suitable specification of the operating environment which reflects the intended usage of the vehicle was chosen to complement the vehicle and tyre models as described in **Paper II**. A method of generating a random road profile in three-dimensional space was developed in Odrigo et al. (2016) as part of the TyreOpt project. The profile generated is random, but its most important characteristics (topography, curve density, and surface roughness) are controlled using the GTA parameters (Edlund and Fryk, 2004). The generated roads are independent of the vehicle itself and can be used to compare different vehicles and tyres for the same transport task. With the developed method, many different roads with the same GTA parameters can be generated allowing for robustness analyses. An alternative specification of the operating environment, such as the one being developed by Pettersson (2017), can be used when available.

The vehicle configuration and the operating environment characterization are considered as inputs to the tyres selection problem; see also Figure 1.

The optimization model of the truck tyres selection problem was introduced and all the three parts of the joint vehicle, tyres, an operating environment model were briefly described this section.

5 A summary of the appended papers

This section summarizes the results gained in the appended papers.

5.1 Paper I: Integration of expert knowledge into radial basis function surrogate models

(coauthored with Peter Lindroth, Ann-Brith Strömberg, and Michael Patriksson)

The main motivation for the research presented in this article was provided by the TyreOpt project, in which we need to explicitly express and optimize a complex simulation-based function describing the rolling resistance coefficient (RRC) of a truck tyre. Many optimization algorithms for solving simulation-based optimization problems are based on a computationally efficient surrogate model of the computationally expensive objective function. It is seldom the case that all important characteristics—referred to as expert knowledge in this paper—such as non-negative values, of the complex function, are automatically inherited by the surrogate model.

We consider several types of expert knowledge about the reality behind a simulation-based function. We show that the utilization of expert knowledge when developing a radial basis function (RBF) interpolation of an unknown function is possible and is often also computationally cheaper than performing additional costly simulations of the unknown function. We have demonstrated that the RBF interpolation can be reformulated as a tractable optimization problem, allowing for the utilization of constraints stemming from expert knowledge. We have also developed a methodology for accomplishing this. The methodology is illustrated on simple example functions and then applied to a function describing the RRC of truck tyres. Numerical results show that the utilization of the expert knowledge typically leads to an increase of the goodness of fit in comparison with an interpolation of the sample points.

My contribution to the paper: I performed the major part of the theoretical development, the writing, and the numerical tests.

The paper is published in Optimization and Engineering, **17**(3), pp. 577–603, 2016. The results from the paper have been presented at the SOAK/NOS6 conference on mathematical optimization and operations research, Gothenburg, Sweden, 2013, at the SIAM Conference on Optimization, San Diego, CA, USA, 2014, and at the 4th International Conference on Engineering Optimization, Lisbon, Portugal, 2014.

5.2 Paper II: Modelling of optimal tyres selection for a certain truck and transport application

(coauthored with Peter Lindroth and Bengt Jacobson)

The main aim of the research leading to this paper is to select—for a truck and its transport application—a configuration of the tyres such that the energy losses caused by these are minimized. We show that neither the rolling resistance coefficient (RRC) classes provided by tyre suppliers nor any other nominal values of RRC evaluated for specific operating conditions are sufficient to do the tyres selection. A surrogate model of the RRC is developed. A tyre model based on the RRC model and approximate relations for other tyre characteristics, such as the longitudinal stiffness, the contact patch length and the vertical stiffness, is introduced. The modularity of the tyre model is demonstrated by coupling it with two different vehicle models (an inverse dynamics model and a full dynamic simulation model) and an operating environment model. The joint vehicle, tyres, and operating environment model created is used to evaluate the energy losses of a vehicle when operating in a given environment with specific tyres mounted. The usage of the joint model is demonstrated by solving a few illustrative instances of the tyres selection problem for different transport applications. The potential savings wrt. energy losses when the selected tyre configurations are used are presented.

My contribution to the paper: I performed the study to show that the RRC classes are not sufficient to do the tyres selection. I developed the tyre model. I co-supervised the Masters students P. Kolář (see Kolář (2015)) and Z. Chen and S. Prathaban (see Chen and Prathaban (2013)), who performed the work of coupling the tyre model with the vehicle models. I collaborated with University of Ontario when developing the road generator used to model the intended operating environment (see Odrigo et al. (2016)). I performed the computational experiments to demonstrate the usage of the joint model and did most of the writing of the publication.

The paper is accepted for publication in International Journal of Vehicle Systems Modelling and Testing. Some ideas from the paper have been presented at the 4th International Tyre Colloquium: Tyre Models for Vehicle Dynamics Analysis, Guildford, UK, 2015.

5.3 Paper III: Efficient solution of many instances of a simulation-based optimization problem utilizing a partition of the decision space

(coauthored with Peter Lindroth, Michael Patriksson, and Ann-Brith Strömberg)

This paper analyzes a special case of a mathematical optimization problem where it is a priori known that its simulation-based objective function is influenced more by its so-called variables than by its parameters. We introduce an algorithm for solving a large set of similar instances (defined by parameter settings) of the simulation-based optimization problem. The algorithm initially finds optimal solutions for a selection of instances using surrogate models of the objective function over the variable space. Subsequently, an approximate optimal solution for any other instance is found by weighting the surrogate models assembled.

The main motivation for the research presented stems from the TyreOpt project, where we aim to find an optimal tyre configuration for each vehicle and environment combination in order to minimize the energy losses caused by the tyre. The need to solve this simulation-based optimization problem efficiently in the combinatorial setting of vehicles and environments led to the development of the algorithm presented. The numerical tests of the algorithm's performance on a set of global optimization problems, differing in both dimension and difficulty, show that the algorithm outperforms a naive approach, which is based on a surrogate model of the objective function over the complete space of variables and parameters, and a standard algorithm for solving simulation-based optimization problems applied to each instance separately.

The methodology developed can be used to efficiently solve a large number of instances of a simulation-based optimization problem with continuous variables, such as the design of a freight aircraft to be utilized for several types of transport missions, or the optimization of charge for melting wrt. the quality of various products and which is desirable to be optimized in real time. For a direct application to the tyres selection problem the approach has to be extended in order to handle categorical variables.

My contribution to the paper: I performed the major part of the theoretical development, the writing, and the numerical tests.

The paper is accepted for publication in Annals of Operations Research. The main ideas presented in the paper have been presented at the 4th International Conference on Engineering Optimization (EngOpt2014), Lisbon, Portugal, 2014.

5.4 Paper IV: A splitting algorithm for simulation-based optimization problems with categorical variables

(coauthored with Christoffer Cromvik, Peter Lindroth, Michael Patriksson, and Ann-Brith Strömberg)

In this paper we present a splitting algorithm, called quadDS (discrete search with quadratic underestimation), that explores the multidimensional discrete search space and is suitable for optimization problems with simulation-based objective functions. The splitting rule is based on the representation of a convex relaxation of the search space in terms of a minimum spanning tree and adopts ideas from multilevel coordinate search. The objective function is underestimated on its domain by a convex quadratic function. Numerical tests on a set of optimization problems are presented to compare the performance of the algorithm developed with that of other existing algorithms.

The main aim of the research leading to this paper is to select—for a vehicle and environment specification—a configuration of the tyres such that the energy losses caused by these are minimized. The tyres selection results in an optimization problem with simulation-based objective function and categorical variables, i.e., integer variables arising from reformulations of discrete multidimensional sets of design points.

The algorithm enables the efficient solution of the tyres selection problem for a limited number of customers corresponding to a specific vehicle configuration and operating environment.

My contribution to the paper: I co-developed the algorithm, conducted the numerical tests, and did most of the writing of the publication.

The paper is under review for publication in Engineering Optimization. The results from the paper have been presented at the International Conference on Operations Research (OR2017), Berlin, Germany, 2017, and at the Swedish Operations Research Conference (SOAK 2017), Linköping, Sweden, 2017.

5.5 Paper V: Efficient solution of many instances of a simulation-based optimization problem with categorical variables utilizing a partition of the decision space

The main motivation behind this paper is the aim to find—for each combination of vehicle and environment specification—a configuration of the truck tyres such that the energy losses caused by them are minimized.

This paper describes how a splitting algorithm for simulation-based optimization problems with categorical variables, developed previously, can be extended to solve a large number of instances of such a mathematical optimization problem in a computationally efficient way. The instances are defined by fixed values of a set of parameters and solving each of the instances provides values of the variables involved. We assume that the objective function value is influenced more by the variables than by the parameters. The splitting algorithm computes an approximately optimal solution over the domain of the variables for each of a selection of parameter settings. Then, approximately optimal solutions for other parameter settings are computed through the construction and optimization of surrogate models, without requiring additional expensive function evaluations.

The methodology developed enables the computationally efficient solution of the truck tyres selection problem, when described in the combinatorial domain of possible vehicle configurations and operating environment specifications. The numerical tests of the methodology lead to the discovery of several assumptions that have to be fulfilled for a successful implementation of the methodology at Volvo GTT.

My contribution to the paper: I performed the theoretical development, the writing, and the numerical tests.

The main idea of the paper has been presented at the International Conference on Operations Research (OR2017), Berlin, Germany, 2017.

6 Conclusions

We now summarize the results of the research behind this thesis, draw conclusions, and suggest future work.

6.1 Main contributions

The activities performed within the TyreOpt research project enable solving approximately the tyres selection problem (having categorical variables) and the tyre design problem (having continuous variables) for many vehicle configurations and operating environment specifications in real-time (up to a minute) during the sales process. The content in the reports and articles authored within the project and their interconnections are illustrated in Figure 9.

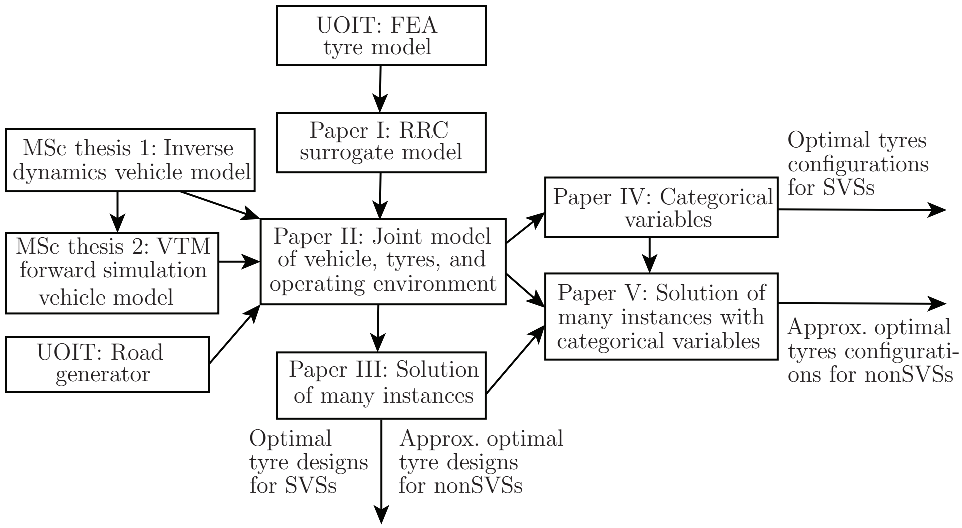

Figure 9: A flowchart illustrating relations between the different activities performed within the research project presented in this thesis, and how they lead to the final goal to find the optimal and approximately optimal tyres configurations for the SVSs and the nonSVSs, respectively.

A survey on how the rolling resistance coefficient as well as other tyre related functions depend on different tyre, vehicle, and operating environment parameters was conducted. Then, some of the important tyre parameters, about which information was available in the tyre database, were selected as variables for the tyres selection optimization problem. A computationally efficient tyre model using radial basis function interpolation has been created, and a methodology for implementing the expert knowledge into has been developed and tested in **Paper I**.

A mathematical model of the truck tyres selection optimization problem has been created. A survey on vehicle simulation models was performed. Then, a computationally efficient joint vehicle, tyres, and operating environment model, for evaluating the objective function of the truck tyres selection problem, was created in **Paper II**. The final joint vehicle, tyres, and operating environment model was developed for the needs of the TyreOpt research project in the master thesis Kolář (2015) with a preliminary study presented in the master thesis Chen and Prathaban (2013). The work on both master theses was co-supervised by the author of this PhD thesis.

A computationally efficient algorithm for solving many instances of a simulation-based optimization problem with continuous variables utilizing a partition of the decision space has been developed and evaluated for a set of test problems in **Paper III**. This algorithm enables to find—computationally efficiently—optimal tyre designs (tyres described by continuous variables) in the combinatorial domain of all possible vehicle configurations and operating environment specifications.

A splitting algorithm, called quadDS, for simulation-based optimization problems with categorical variables, was developed and evaluated for a set of test problems in **Paper IV**. According to our tests, quadDS outperforms all known algorithms (i.e., genetic algorithms (Coleman and Zhang, 2015) and NOMAD (Audet et al., 2009)) that can be used to solve simulation-based optimization problems with categorical variables. Our splitting algorithm can be used to solve the truck tyres selection optimization problem, but cannot be executed in real-time since it requires many evaluations of the simulation-based objective function. A methodology to solve many instances of a simulation-based optimization problem with categorical variables was therefore developed in **Paper V**. In its pre-processing phase quadDS is used to solve a small subset of the instances. Then, an algorithm similar to the one developed in **Paper III** for solving many instances of simulation-based optimization problem with continuous variables is used to solve approximately any other instance without requiring more evaluations of the simulation model. The resulting methodology described in **Paper V** enables a computationally efficient solution of the truck tyres selection problem when described in the combinatorial domain of possible vehicle configurations and operating environment specifications.

We have considered only long-haul straight driving so far, but it would be beneficial to include other transport applications, road curves, and road microstructure. This addition would require changing the tyre model as well as the vehicle model. The road generator (Odrigo et al., 2016) is already prepared to generate more complex operating environments. Thus, the optimization methodology would stay unchanged.

In the next section we present the possible savings of the tractive energy required to run several randomly selected vehicles for one kilometer when the tyres selected by the optimization methodology developed are used in place of the tyres with which the vehicles were sold.

6.2 Results

The quality of the optimization methodology developed in this thesis and the appended papers is demonstrated through its practical use for solving several illustrative instances of the truck tyres selection problem.

Each instance of the truck tyres selection problem is defined by a sold truck specification. The sold trucks considered in the quality assessment are randomly selected from the vehicles sold by Volvo in 2017.

Specifications of the feasible tyres, sorted according to increasing nominal values of the rolling resistance coefficient (RRC), are found in Table 1. Some of the tyres in the tyre database, containing more than 300 different tyres, cannot be distinguished with our limited tyre definition, which is composed by the diameter and width of the tyre and the inflation pressure. Therefore, the tyre database collapses to the 35 (groups of) tyres listed in Table 1.

The optimal tyres configurations for the SVSs obtained by quadDS (**Paper IV**), the approximately optimal tyres configurations for the non-SVSs computed in a computationally efficient way as described in **Paper V**, and the sold tyres configurations are listed in Table 2 with the corresponding values of the true objective function F. The objective function F is evaluated by the joint vehicle, tyres, and operating environment model developed in Kolář (2015) and measures (in MJ) the tractive energy required to run the vehicle for 1 km in the specified operating environment and with the given tyres.

Tyre no	Diameter [mm]	Width [mm]	Pressure [bar]	RRC [N/kN]	Tyre no.	Diameter [mm]	Width [mm]	Pressure [bar]	RRC [N/kN]
1	1038	259	9.00	5.93	19	1076	312	8.50	6.48
2	1020	254	8.50	5.98	20	1094	298	7.50	6.48
3	970	230	6.75	6.07	21	938	306	9.00	6.48
4	962	230	6.75	6.07	22	1080	305	8.00	6.49
5	930	255	8.00	6.15	23	1044	312	8.50	6.51
6	958	276	9.00	6.17	24	1094	312	8.00	6.54
7	1050	279	8.50	6.19	25	950	313	9.00	6.53
8	1018	258	7.25	6.21	26	923	305	8.50	6.57
9	1052	275	8.00	6.22	27	1088	352	8.50	6.85
10	1012	276	8.25	6.24	28	1084	362	9.00	6.88
11	1073	278	7.90	6.25	29	928	361	9.00	7.03
12	1082	286	8.25	6.26	30	948	374	9.00	7.13
13	986	292	9.00	6.30	31	1072	389	9.00	7.15
14	1084	300	8.50	6.36	32	996	386	9.00	7.20
15	926	292	9.00	6.36	33	1073	444	8.50	7.76
16	1044	300	8.50	6.40	34	1072	452	8.50	7.84
17	1000	305	9.00	6.41	35	962	453	9.00	7.88
18	1014	312	9.00	6.47					

Table 1: Specifications of the 35 feasible tyres available for the instances used for assessment of the tyres selection problem considered. The tyres are specified by triplets: tyre diameter, tyre width, and inflation pressure. The nominal RRC values are computed by the tyre model described in Kolář (2015) and **Paper II** at the fixed operating conditions given by a standardized test procedure (ETRMA, 2010).

Under the assumption that the vehicle and tyre models represent the vehicle and the tyre behaviour well enough the optimization methodology developed is able to identify tyre configurations providing 1.51–3.94% lower tractive energy than that

Vehicle no	Topo.	SVS	$\mathbf{x}^v_{\text{opt}}$	$F(\mathbf{x}^v_{\text{opt}}, \mathbf{p}^v)$	$\mathbf{x}^v_{\text{sold}}$	$F(\mathbf{x}^v_{\text{sold}}, \mathbf{p}^v)$	$100\left(F(\mathbf{x}^v_{\text{sold}}, \mathbf{p}^v) - F(\mathbf{x}^v_{\text{opt}}, \mathbf{p}^v)\right) / F(\mathbf{x}^v_{\text{opt}}, \mathbf{p}^v)$
v (w^v*d^v)		nonSVS	$\mathbf{x}^v_{\text{app}}$	$F(\mathbf{x}^v_{\text{app}}, \mathbf{p}^v)$	$\mathbf{x}^v_{\text{sold}}$	$F(\mathbf{x}^v_{\text{sold}}, \mathbf{p}^v)$	$100\left(F(\mathbf{x}^v_{\text{sold}}, \mathbf{p}^v) - F(\mathbf{x}^v_{\text{app}}, \mathbf{p}^v)\right) / F(\mathbf{x}^v_{\text{app}}, \mathbf{p}^v)$
			[tyre no:s]	[MJ/km]	[tyre no:s]	[MJ/km]	[%]
_1 (10*4)_	_2_	_nonSVS_	_1,2,**1**,**1**,1_	_7.455_	_31,31,**19**,**19**,31_	_7.643_	_2.45_
_2 (10*4)_	_2_	_nonSVS_	_2,1,**1**,**1**,1_	_7.297_	_31,31,**19**,**19**,31_	_7.489_	_2.57_
3 (10*4)	2	SVS	1,2,**1**,**1**,1	7.297	14,14,**14**,**14**,31	7.449	2.05
_4 (10*4)_	_2_	_nonSVS_	_2,1,**1**,**1**,1_	_7.297_	_31,31,**19**,**19**,31_	_7.489_	_2.57_
5 (10*4)	2	SVS	1,2,**1**,**1**,1	7.297	32,32,**18**,**18**,32	7.522	3.00
6 (8*4)	2	SVS	1,1,**1**,**1**	7.300	31,31,**19**,**19**	7.471	2.29
7 (8*4)	2	SVS	1,1,**1**,**1**	6.954	31,31,**19**,**19**	7.127	2.43
8 (8*4)	2	SVS	1,1,**1**,**1**	7.300	19,19,**16**,**19**	7.433	1.79
9 (8*4)	2	SVS	15,**1**,**1**,1	6.661	31,**19**,**19**,31	6.849	2.74
_10 (8*4)_	_2_	_nonSVS_	_1,1,**1**,**1**_	_7.093_	_19,19,**19**,**19**_	_7.222_	_1.79_
11 (6*4)	3	SVS	1,**1**,**1**	6.325	19,**19**,**19**	6.355	0.46
12 (6*4)	2	SVS	1,**1**,**1**	6.318	31,**19**,**19**	6.493	2.70
13 (6*4)	3	SVS	1,**1**,**1**	6.325	14,**14**,**14**	6.422	1.51
_14 (6*2)_	_1_	_nonSVS_	_1,**1**,1_	_6.231_	_31,**18**,32_	_6.460_	_3.56_
15 (6*2)	1	SVS	1,**1**,1	6.355	31,**19**,31	6.575	3.35
_16 (6*2)_	_2_	_nonSVS_	_1,1,**1**_	_6.125_	_18,18,**18**_	_6.252_	_2.03_
17 (6*2)	3	SVS	1,**1**,1	6.339	14,**14**,14	6.438	1.54
_18 (6*2)_	_2_	_nonSVS_	_1,**1**_	_3.884_	_32,**18**_	_3.884_	_3.90_
19 (4*2)	2	SVS	1,**1**	3.872	18,**18**	3.872	3.94
20 (4*2)	2	SVS	1,**1**	3.605	25,**25**	3.605	1.34

Table 2: Each vehicle v is characterized by its number of wheels w^v and its number of driven wheels d^v. The positions of the driven wheels are indicated by the bold face. The topographies (1 - flat, 2 - pflat, 3 - hilly, and 4 -vhilly) characterising the operating environments are listed. The nonSVS rows are indicated by italics. The optimal tyres configurations $\mathbf{x}^v_{\text{opt}}$ for SVSs found by quadDS or the approximately optimal tyres configurations $\mathbf{x}^v_{\text{app}}$ for nonSVSs found by the computationally efficient algorithm developed in **Paper V** are reported with the corresponding values of the tractive energy F. The sold tyres configurations $\mathbf{x}^v_{\text{sold}}$ with the corresponding values of the tractive energy F are listed for comparison. The potential savings of the tractive energy required to run the vehicle for one kilometer when the optimal or the approximately optimal tyres are selected in place of the sold tyres are reported as well.

corresponding to the sold tyre configurations. The tractive energy is assumed to be proportional to the fuel consumption of vehicles (Kolář, 2015).

Most of the optimal tyre configurations presented in Table 2 contain the tyre number 1, which has the minimal nominal RRC out of the 35 tyres. The reason is that most of the vehicles considered are operated in environments with flat or predominantly flat topography, where the rolling resistance forms the major part of the resistive forces from tyres.

The results from our test of the methodology developed on Volvo GTTs existing tyre database show that the solution of the truck tyres selection problem in a computationally efficient way, when described in the combinatorial domain of possible vehicle configurations and operating environment specifications, is enabled. It was predicted in the TyreOpt research project plan that a structured tyres selection should lead to 2–4% lower vehicle fuel consumption. This has been confirmed by our numerical results even with a fairly simple tyre description and tyre model. Making the tyre model more accurate—allowing for a more detailed tyre description (i.e., with more than three parameters)—will lead to more feasible tyres available but the savings computed are expected to decrease if we assume that the tyres sold nowadays are selected in a good way within the sales process.

In the next section we propose some areas of future research.

6.3 Future research

It has been identified that the tyre model used for the evaluation of the objective function of the tyres selection problem is not enough detailed. The data about the tyres that is missing in the tyre database should be collected and the tyre model should be improved (e.g., by incorporating numerical representations of the influence of the tyre material, the tread pattern on the RRC, and the wheel torque allowing for simulations of braking and steering situations). The tyre model can easily be replaced within the methodology developed, leading to an improved accuracy of the results since our methodology has been developed with the aim to be modular with respect to all the models used. An internal Volvo GTT project has been initiated to collect additional tyre data allowing for a more complex tyre description which would be of use also for other applications.

The objective function of the tyres selection problem is based solely on the tractive energy required to run the vehicle. A measure of the tyre wear should therefore be included in the objective function. The assumption that constraints, such as ride comfort and safety requirements, are fulfilled by the feasible tyres when introduced to market by the tyre suppliers and their dependence on the vehicle and its usage should be analyzed. Additional constraints on the choice of tyres could be formulated and added to the mathematical model of the tyres selection problem. This would require adjustments of the optimization algorithms as well.

A main goal of this project is to be able to implement the methodology developed into the sales system at Volvo GTT. Putting such a methodology into practice will involve many challenges, such as propagating the data about each tyre from the tyre database and developing an automatic way of generating vehicle models based on their specifications in the sales system. However, we have shown that our methodology can be used as a part of the sales tool at Volvo GTT.

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