

BIFROST: Noise properties of GPS time series

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Abstract. The BIFROST project uses GPS to observe the intra-continental deformation of Fennoscandia caused predominantly by Glacial Isostatic Adjustment (GIA). The noise in GPS position time series has been proven correlated, so we investigate a fractal model in order to obtain a parameter that can gauge our network stations true velocity uncertainties and utilize an empirical orthogonal function (EOF) to remove the inherent common mode. We employ a Kaiser window to reduce the power spectrum variance and retain independent power estimates based on the window's main lobe width ratio compared to that of a boxcar. As power spectra lack Gaussian distribution properties, we devise a transform that normalizes the power spectrum and subsequently iterate a fractional power law noise model.

We find that the spectral indices for the different velocity components in our network are 0.6 for North, 0.5 for East, and 0.7 for the Vertical. As there is no white noise floor in our power spectra to indicate inevitable system noise it is possible that GPS time series should be sampled more frequently than once per day in order to separate between different uncertainty sources.

Keywords. GPS, time series analysis, power-law noise, uncertainty

1 Introduction

The use of Global Navigation Satellite Systems (GNSS) in general and the Global Positioning System (GPS) in particular have been proven important for geophysical studies for more than a decade. Nevertheless, it is fair to say that the uncertainties of GPS observations are less than fully understood, much related to the difficulties in separating the different uncertainty sources. The

uncertainties are attributed to incomplete models that very broadly can be classified as being related to the satellites (e.g. clocks, phase centre variations, orbital errors), the propagation media (e.g. ionosphere, troposphere) or the receiver units (e.g. clocks, multipath, elevation cut-off, phase centre variations, monumentation). In addition to the electromagnetic signal perturbations, geophysical phenomena such as tidal forces, loading related to ocean, ground water and atmospheric pressure as well as surface humidity, climate related variations and local geological settings impose further uncertainties on the observed variables. The purpose of this paper is twofold: firstly, to apply an empirical orthogonal function (EOF) to remove a common mode noise from a regional network; secondly to assess an unbiased parameter that can be used to scale the formal uncertainties of GPS derived velocities.

In the Baseline Inferences for Fennoscandian

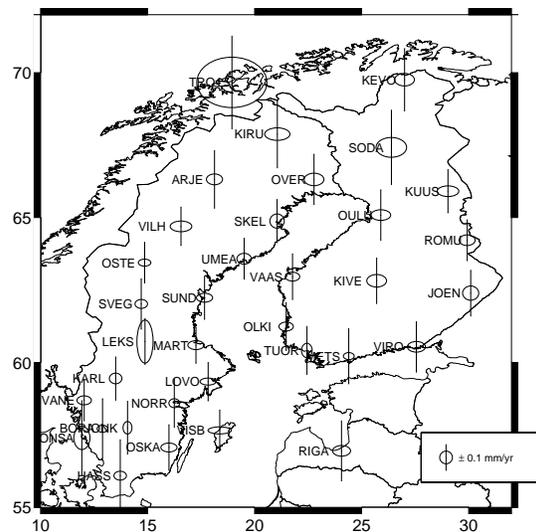


Figure 1. Formal velocity uncertainties from Lidberg et al. (2005). Bars are vertical 1σ , ellipses are horizontal components 1.5σ (scaling based on Bergstrand et al., 2005)

Rebound, Sea level and Tectonics (BIFROST) project, we use GPS to assess the current glacial isostatic adjustment (GIA) that pertains to the unloading of the regional Pleistocene ice sheet in Finland and Sweden. The aseismic GIA process yield station velocities superimposed on the Eurasian tectonic plate motion of order 1 mm/yr horizontally and 10 mm/yr vertically, and we aim at a station velocity determination of ± 0.1 mm/yr, see Fig. 1.

Milne et al. (2001, 2004) fitted the GPS velocity observations to a three-layered viscoelastic earth model response to an ice model by Lambeck et al. (1998) and found an optimal model with 120 km lithospheric thickness and upper and lower mantle viscosities of 8×10^{20} Pas and 1×10^{22} Pas, respectively. However, in Milne et al. (2001) the station velocities in the north eastern part systematically showed larger misfits compared to the model. These stations are known to be strongly affected by snow accumulation on the radomes, as first observed in the network by Jaldehag (1996). The analyst's approach to mitigate the effect of snow on radomes has varied somewhat arbitrarily in the past depending on study theme, and to gauge whether the observation uncertainties allow the misfit or if the ice model has to be reassessed, we need an unbiased estimate of the true station velocity uncertainties.

Bergstrand et al. (2005) confirmed the inferred rheological variables by Milne (2001) in an independent baseline analysis that required a scaling of formal GPS uncertainties by a factor no more than 1.5 to fit a root-mean square < 0.8 mm/yr for the 630 baselines in the network, including the snow affected stations. In this context, 'baselines' means estimating the velocities from individual site-to-site time series. The baseline method is robust as it eliminates a lot of common mode uncertainties in the horizontal direction related to e.g. satellite orbits, mapping functions and systematic errors at reference stations, but yields results that are hard to compare to other techniques such as tide gauges and precise levelling. Milne et al. (2001) and subsequent articles used a GIPSY network solution thoroughly presented by Johansson et al. (2002), and one conclusion by Bergstrand et al. (2005) was that common mode had not significantly affected the single site inferred viscosity parameters. However, we propose EOF as an alternative to baselines to remove the common

mode, partly because it is more sensitive to vertical variations. The scaling of the formal uncertainties mentioned above largely pertains to the fact that the processes that drive the error sources to some extent are autocorrelated.

Langbein and Johnson (1997) applied a maximum likelihood estimator (MLE) to electronic distance measuring (EDM) observations and estimated the frequency-power relation with

$$P_x(f) = P_0 \left(\frac{f}{f_0} \right)^{-\kappa} \quad (1)$$

where f is the temporal frequency, P_0 and f_0 are normalizing constants, and κ is the power law or spectral index (Mandelbrot and Van Ness, 1968). This application to geodetic time series was subsequently adapted to GPS by a number of authors, e.g. Zhang et al. (1997), Mao et al. (1999), Williams et al. (2004) and Beavan (2005). Classic examples of different noise behaviour assign integer values to κ and include white ($\kappa=0$), flicker ($\kappa=1$), and random walk noise ($\kappa=2$). Integer κ 's are convenient in many ways but are often chosen with little forethought, as there is no physical reason why κ should be restricted to these values. As pointed out by several authors (e.g. Mandelbrot (1983), Turcotte (1997), Williams (2003), and Williams et al. (2004)), a wide range of observable geophysical phenomena are also represented by non-integer, or fractal κ . Geophysical phenomena generally have more power at low frequencies, i.e. there is a correlation in time between different observations. Processes in the ranges $-1 < \kappa < 1$ and $1 < \kappa < 3$ are termed 'fractal white noise' and 'fractal random walk', respectively and processes with κ larger than one are non-stationary. We use the general term 'coloured noise' to encompass the whole range of κ , where redness pertains to $\kappa > 0$, blueness to $\kappa < 0$ and whitening to κ approaching 0. In our case, the validity of the power law model is limited at high frequencies by the sampling theorem

$$f_{\text{sampling}} \geq \frac{f_{\text{signal}}}{2} \quad (2)$$

and at low frequencies by the rebound signal that is assumed to be smooth and deterministic. Inevitably, we have to remove a biased signal from the time series before we can analyze the noise, and the lowest frequencies will be affected.

With a set of 19 months long time series from a regional network in southern California, Zhang et al. (1997) were unable to distinguish between a pure

fractal noise model with index 0.4 and a combination of white and flicker noise. Mao et al. (1999) used three-year time series from a global network and concluded that GPS time series could be estimated with a combination of white and flicker noise but didn't preclude the possibility of fractal white or fractal random walk noise. Williams et al. (2004) investigated a large number of global and regional time series with an average length of 3.4 years and found that the noise in global time series was best characterized with a combination of white and flicker noise, but that regional analysis should solve for spectral index along with the noise amplitudes in the MLE algorithm. The above authors (partially save Zhang et al. (1997)) advocated an a priori assumption that white noise is a significant contributor in GPS daily time series although Beavan (2005, section 5.1) noticed that the white noise component of the PositionNZ network time series often tended towards zero.

Williams (2003) and Williams et al. (2004) commented upon the large computational burden involved in the MLE approach, and in the discussion Williams (2003) suggested a simplified route to obtain rate uncertainty estimates. We suggest a new, simple method to estimate the rate uncertainties. In the long run, this will enable us to efficiently gauge the true uncertainties of station velocities and also identify stations with abnormal behaviour. A much simplified route is viable when time series are "long enough" and we investigate whether our time series fulfil this apparently loose

criterion. We investigate the influence of two different GPS processing software and strategies on the estimation of κ in the regional BIFROST network, using a previously published data set (JGR02) and a recently processed solution of data from 1996 to 2004 (Lidberg et al., 2005; henceforth JG05).

2 Data and Processing

The data in this study are acquired with the Swedish and Finnish geodetic reference networks SWEPOS and FinnRef and the adjacent IGS stations in Riga and Tromsø. The sites have generally been chosen with a free sky view at angles $>10^\circ$ and often lower; antenna monuments are, with few exceptions temperature regulated to suppress thermally related vertical variations and mounted on unweathered crystalline bedrock. Data with 30 s sampling interval have been used to obtain daily position estimates and these position time series constitute the data set used for interpretation in this study. The primary reason for reprocessing has been optimal station velocity estimation, and the variables in the different solutions have been chosen independently. Thorough presentations of the networks, processing strategies etc. have been given by Scherneck et al. (2002), Johansson et al. (2002), and Lidberg et al. (2005). Previous publications have applied different editing strategies to the data before interpreting the results; the current analysis is applied to the original data time series. Data availability for the series, significant gaps, etc are shown in Fig. 2.

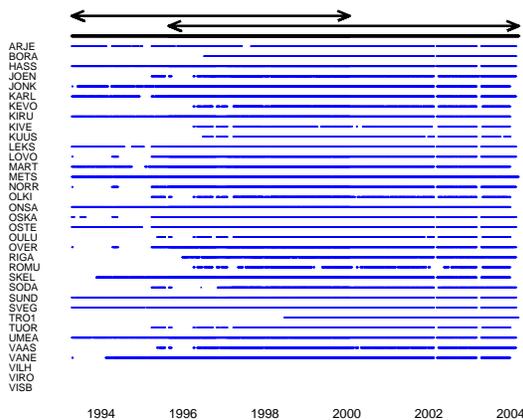


Figure 2. The 35 stations' data availability for the two solutions. The time spans for JGR02 (1993-08 – 2000-05) and JG05 (1996-01 – 2004-05) are indicated with the arrows above the data set.

3 Method

We aim to characterize the noise in the different solutions and ran an editing loop that removed outliers larger than 5σ in the time series. This removal is conservative compared to the 3σ in the edited solution of Johansson et al. (2002), and removed less than 1% from the total data set, compared to e.g. the 1-4% in Nikolaidis (2002) and 4.8% in Beavan (2005). We also note that the stations that are most affected by snow still use close to every observation, in contrast to earlier studies (e.g. Scherneck et al., 2002). When data gaps are present, Langbein and Johnson (1997) suggested adding white noise to their EDM interpolate, and was able to match a value that corresponded to

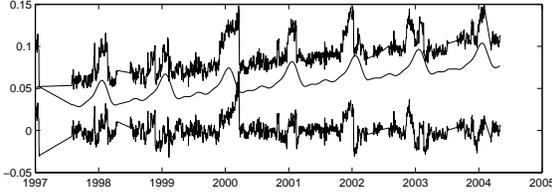


Figure 3. The effect of the rate and periodic fit removal from the original time series for the vertical component for station KUUS. From top to bottom: raw data series, linear trend and seasonal component fit, and residual after quasi-deterministic component removal. Vertical scale in meters.

presumed instrumental noise. Previous authors followed this suggestion; with the higher continuity of GPS measurements compared to EDM, the minor importance of the high frequency end for GIA evaluation and the unknown structure of GPS noise in mind we refrain from mixing observed and synthetic data. We therefore exclude voids in the time series for the linear fit and include them with a zero value in the following unbiased autocovariance estimate in the spectral evaluation.

3.1 Empirical orthogonal function (EOF)

We assume that the noise in general is stationary but with a considerable superimposed seasonal variation. Before the EOF evaluation, we remove a rate and seasonal component with 4 sinusoidal harmonics Φ from the data through least squares, weighted with the formal uncertainty of each observation. The fit is made to the model:

$$r(t) = \varepsilon(t) + a + bt + \sum_j s_j \sin(\Phi_j t) + \sum_j c_j \cos(\Phi_j t) \quad (3)$$

where $r(t)$ is the original observation (including tectonic plate motion and GIA), $\varepsilon(t)$ is the residual kept for the evaluation and a, b, c and s are coefficients which are disregarded for the rest of this study. The effect of the removal of these quasi-deterministic components is shown in Fig. 3 for one of the most affected stations.

An EOF of the network can be obtained by a numerical search of the eigenvectors that account for the largest variances of the observations. The task is to solve the eigenvalue problem:

$$\mathbf{G}^T \mathbf{G} \mathbf{u} = \lambda^2 \mathbf{u} \quad (4)$$

where \mathbf{G} is a matrix consisting of the observed residual time series $\varepsilon_i(t)$ in each column and $\mathbf{u}_{1 \dots n}$ are the eigenvectors that represent the common mode. The temporal eigenvector

$$\mathbf{v}_1 = \frac{1}{\lambda_1} \mathbf{G} \mathbf{u}_1 \quad (5)$$

is used to remove the common mode from the time series so that

$$\mathbf{G}' = \mathbf{G} - \mathbf{v}_1 \mathbf{v}_1^T \mathbf{G} \quad (6)$$

The common mode reduced \mathbf{G}' is then used for the rest of the evaluation.

3.2 Spectral fit

Mao et al. (1999) used a boxcar window on the time series autocovariance function, and judged from shown power spectra, so did Zhang et al. (1997), Calais (1999) and Williams et al. (2004). On average our time series are longer, and we apply a 4-year Kaiser taper window with shape parameter 3π to the time series' autocovariance functions before evaluating the power spectrum with an ordinary fast Fourier transform (FFT). The Kaiser window yields a power spectrum with less variance compared to that of the boxcar at the expense of spectral resolution (Harris, 1978). In order to get uncorrelated estimates of the frequency powers, we pick independent frequencies ω with an interval based on the main lobe width ratio ρ of the Kaiser and implicit full length boxcar windows in the Fourier domain.

We then notice from Fig. 4 that our power spectra exhibit little or generally no sign of a white noise floor at high frequencies before we make a κ fit to the spectrum. The starting solution is taken from a

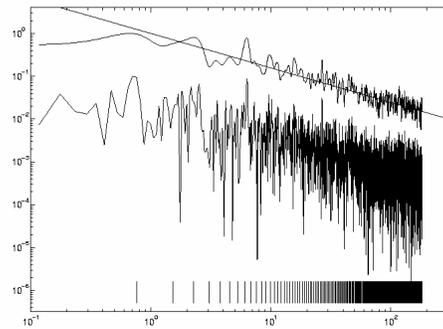


Figure 4. The effect of windowing on power spectrum evaluations. The upper power spectrum is the FFT of the windowed autocovariance function of the SUND station time series Up component. The lower power spectrum is obtained with the implicit boxcar window. The lines at the bottom indicate the independent frequencies based on the lobe width ratio that are used to obtain the spectral fit of the windowed function. The straight line fitted to the upper power spectrum is the initial crude fit.

simple linear fit of P versus ω in the log-log diagram

$$\chi^2 = \sum_{j=p,2p,\dots}^N \left[\log P_{obs j} - (\log P_{0j} - \kappa \log \omega_j / \Omega) \right]^2 \quad (7)$$

to get a model $P^{(0)} = P_0 \cdot (\omega / \Omega)^{-\kappa}$ that is conformant with equation (1). After windowing, the power spectrum estimate is χ_ν^2 -distributed around the expected value

$$\Pr \left\{ \frac{\nu P(\omega)}{\Pi^2} < p \right\} = \chi_\nu^2(p) \quad (8)$$

where ν is the degree of freedom of the estimate (Jenkins and Watts, 1968, ch.6.4.2). In order to fit a model to the observations we assume that the model is identical to the expected spectrum, or

$$\Pi^2 = \bar{P}_0 \omega^b = \Pi^2(\bar{P}_0, b) \quad (9)$$

However, for a least-squares fit the power spectrum random values must be transformed in order to have the property of Gaussian deviates $N(\mu, \sigma^2, p)$, where

$$\mu = \sqrt{\chi_\nu^2\left(\frac{1}{2}\right)} \quad (10)$$

The sought-for transform is

$$F(x) = \frac{\Pi}{\mu} \cdot N\left(\mu, 1; \chi_\nu^{2-1}\left(\frac{\nu x}{\Pi^2}\right)\right) \quad (11)$$

We take the square root of the model power spectrum and search

$$\begin{aligned} \min_{\bar{P}_0^{(n+1)}, b^{(n+1)}} \left\{ \chi^2 = \right. \\ \left. = \sum_j \left[\frac{F(P(\omega_j)) - \Pi(\bar{P}_0^{(n+1)}, b^{(n+1)})}{\Pi^2(\bar{P}_0^{(n)}, b^{(n)})} \right]^2 \right\} \quad (12) \end{aligned}$$

iteratively (we actually use a relaxed version of the iteration, where the next starting value is between the n 'th and $n+1$ 'st solution). In each iteration the transformation F employs the most recent update of the model parameters.

4 Results

The results of the EOF removal and spectral index estimation are shown in Fig. 5. Except the east components of JGR02 and the vertical of JG05, the EOF doesn't significantly change the spectral index estimate but certainly aids in the accuracy of the determination for the horizontal components' spectral indices. North components are adequately described with $\kappa=0.6$, and East components with

$\kappa=0.5$ (though JGR02 east appear slightly whiter before the EOF removal). This difference could possibly be attributed to the geometry of the global net.

The determination of the spectral index for the Up component isn't affected much by the EOF for JGR02. Strong candidates to the explanation are the large data gaps in the Finnish time series, but further investigation is needed to outrule other possible causes. For JG05 whose time series are more complete than JGR02, the spectral index of the vertical component was reduced from 0.8 to 0.7. However, for the extreme cases of high spectral index the EOF failed to whiten the noise. This indicates that this noise is site specific and not a fundamental part of the system.

The obtained degrees of freedom ν for the spectral fit of the two solutions were larger than 6 for JGR02 and larger than 8 for JG05. We feel that this is representative of an adequate trade-off between spectral resolution and variance in our power spectra.

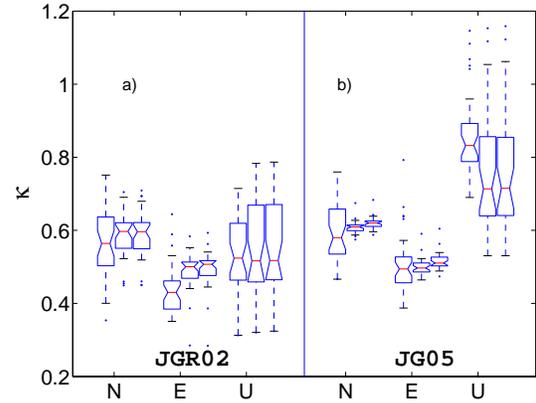


Figure 5. Boxplot results of the North, East and Up components from the analysis of the two solutions. From left to right for each component: the crude results before the EOF removal, the crude results after the EOF removal, and the results after the transformed fit. The horizontal lines of each box show the median and 1 IQR, the whiskers extend to a maximum of 2 IQR from the median and spectral indices outside this range are shown as dots. Box notches graph a robust estimate of the uncertainty about the mean.

5 Discussion

JGR02 and JG05 have an overlapping period from 1996 to 2000, and we don't consider this time span long enough to exclude data outside the period. A direct comparison between the different time series' results in Fig. 5 may therefore result in conclusions of little significance, especially as the majority of Finnish stations lack data in the early years.

It is possible that higher order harmonics would improve the fit to the time series and whiten the time series in Fig. 3. However, for stations less affected than KUUS, the risk of fitting non-parsimonious parameters is just as evident. With the high conformity of the network as well as constraints provided by independent observations and model fit to our station velocities, we feel that the chosen parameterization of the linear fit is representative of the factors that are considered important for the network as a whole.

Compared to other methods, e.g. baselines (cf. Bergstrand, 2005) the EOF method is a tool to reduce the common mode noise from a set of GPS time series that requires a small amount of computational code. Another asset is that we find more realistic observation uncertainties on individual stations to constrain the GIA earth and ice models. Results are also more easily compared to those of other independent techniques.

Langbein and Johnson (1997) used an 1100 sample Hann window on their EDM series and noted that the removal of a secular trend from data removes significant amounts of energy from the low frequency part of the spectra and that κ estimates therefore are biased low. This could be complemented with a reasoning of leakage between spectral bins; through our subsampling strategy based on the implicit main lobe width relation to that of the full length boxcar, we only use the straight part of the spectrum and are not affected by the trend removal. This approach reduces the redundancy of the spectral fit slightly, but enhances the structure of the spectral power relation and still leaves enough degrees of freedom to fit fairly complicated models if desired. We also sacrifice the very low frequencies that could be resolved with a rectangular window as we consider these biased after the secular motion reduction in equation (3), anyway.

It appears from Fig. 5 that the devised power fit has little influence on the inferred parameters. However, we investigated the crude first order linear regression power fit for the two solutions (Fig. 6) and found offsets to the ideal that are equivalent to the observed shift in Fig. 5. We also notice that for $\kappa > 1.0$ in JGR02 and $\kappa > 1.2$ in JG05, the crude fit started to diverge but that preliminary tests on the transformed fit for 2500 sample long synthetic data series up to $\kappa = 2.0$ show excellent results. We are thus confident that the obtained results are representative of the time series actual noise content for the chosen model.

An advantage of the scheme is that it is reasonably fast. The first crude fits to the power spectra for the whole network are obtained within 5 minutes and with the relaxed iteration the final results are obtained in less than 30 minutes on an ordinary PC without optimizing the underlying MATLAB script. We are thus provided with a tool that makes a comparison between e.g. different outlier removal strategies and their impact on velocity uncertainties feasible.

Davis et al. (2003) asked whether the estimate of velocity from GPS is fundamentally limited by one or more error sources and if so, what these error

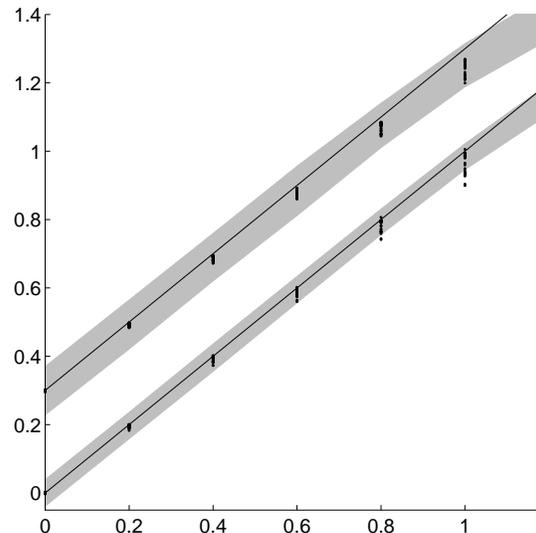


Figure 6. Simulation of the crude power fit for synthetic time series. (JGR02 top, JG05, bottom). Straight lines represent the ideal fit, dots the mean of 1000 formal κ series with gaps equivalent to the original data sets. The gray uncertainty band is the ensemble median \pm the maximum standard deviation. The larger bandwidth for JGR02 is solely attributed to the ROMU station. The offset from the ideal is comparable to that of the crude to normalized fit observed in Fig. 5.

sources are. Zhang et al. (1997) could not distinguish between an a priori chosen model with a white noise floor and one where κ was estimated along with the noise amplitudes. Mao et al. (1999) found a better fit with an a priori chosen model than with estimated κ , but at the same time failed to fit three out of ten synthetic runs with a white and flicker noise combination (Mao et al., 1999, Table 4). Williams et al. (2004) used a larger data set and also advocated a white noise floor in the power spectra and searched for a dominant error source in the time series.

One parameter of the user segment that has received considerable attention in geodetic time series evaluation is the monument stability (Langbein and Johnson (1997), Johnson and Agnew (2000), Williams et al. (2004), Beavan (2005)). Combrinck (2000) gave an introduction to space geodetic monumentation and we adhere to his holistic approach, i.e. that monument stability is more than a strictly geodetic issue. Our stations are essentially invariant to the physical parameters listed by Moore (2004, section 2.3, online) and there is no conspicuous reason to assume dominant monument noise in our network.

We are primarily interested in the true station velocity uncertainties and content with parameters that allow us to scale the formal uncertainties of the observations (e.g. as suggested by Williams, 2003). In BIFROST, the (easily identified!) winter time snow coverage of the northern stations' antennas reddens the noise. Should we be able to completely remove the snow e.g. with hot air fans, the noise for these stations would probably whiten and our velocity estimate uncertainties hence be reduced.

Although of minor importance for the GIA evaluation, we notice that we are generally unable to observe any white noise floor in the higher end of our power spectra. Beavan (2005, section 5.1) used two different MLE packages and with the Williams et al. (2004) package also noticed a recurring absence of white noise in the time series from the PositioNZ network. This observation could possibly be used for evaluation in areas where shorter term movements are targeted and monument stability in not-well-consolidated sediment might be an impediment to the observations, e.g. the Southern California Integrated Geodetic Network (SCIGN), cf. Williams et al. (2004) and Beavan (2005). Given the complexity of the error sources, we question

whether an a priori assumption of white noise properly addresses the noise process in GPS time series before the contributions from individual sources have been duly separated. As we don't observe any white floor in our power spectra, there is evidence that sampling should be more frequent than once per day in order to get smaller error in parameter uncertainties. Given the complexity of the overall uncertainty, we feel that this is a valid result. The next higher order candidate to address could be a semidiurnal tide related signal resolved with a bi-hourly time series. However, with daily sampling interval strategies, the questions by Davis et al. (2003) will probably remain unanswered.

6 Conclusion

We investigated two different GPS time series ensembles from the BIFROST network and used an EOF approach to remove the common mode noise. Our time series have reached a length where it is useful to apply a tapering window to the autocovariance functions before we estimate the power spectra using FFT. Power law models appear to accurately represent the noise process in the BIFROST time series. The spectral indices are in the range 0.5—0.7 for the North, East and Up components and determined with excellent precision in the horizontal.

We are able to determine the spectral power relationship with a simple least squares analysis in the log-log diagram by using uncorrelated spectral bins, but prefer the devised normalizing transformation since it performs better for a wider range of power spectral indices and yields more accurate results. Winter-time snow coverage of antennas still affects some of our position time series, but we now have an unbiased parameter that can be used to scale the individual stations formal velocity uncertainties and still use more than 99% of the observed data.

There is little evidence for a white noise floor in our power spectrum evaluations, and this indicates that a higher sampling frequency than once per day will get smaller errors in parameter uncertainties. This also leads us to the conclusion that a priori assumptions of noise structure may be misleading in a search for dominant error sources.

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