

Stability design of steel structures

Comparison of approaches presented in *Eurocode 3* on how to consider imperfections and second-order effects

Master's thesis in the Master's program Structural Engineering and Building Technology

VIKTORIA BODÉN
STINA LUNDIN

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CHALMERS
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Department of Architecture and Civil Engineering
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Steel Structures

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Supervisor: Svante Severinsson, ELU Konsult AB

Technical support: Christoffer Svedholm, ELU Konsult AB

Examiner: Mohammad Al-Emrani, Structural Engineering, Chalmers

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Department of Architecture and Civil Engineering

Division of Structural Engineering

Steel Structures

Chalmers University of Technology

SE-412 96 Gothenburg

Telephone +46 31 772 1000

Cover: An illustration of steel frames made in Abaqus showing global buckling modes.

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Abstract

The design procedure to consider imperfections and second-order effects is more complex for frame structures than individual members. The design procedures for individual members are relatively well described in EN1993-1-1 (2005). However, questions arise when studying a frame where the design should consider both system and member effects.

There are currently three design approaches in EN1993-1-1 (2005) to verify the instability due to buckling of frames and the difference between the approaches still remains unknown. This leaves many questions regarding interpretation for the designer and creates need for increased understanding. Therefore, the purpose of this thesis project was to compare and establish guidelines for the different approaches presented in EN1993-1-1 (2005), regarding consideration of imperfections and second-order effects for steel structures.

To achieve this, a literature study was performed concerning structural stability and frame design procedures. In addition, two studies were conducted. The first one considered different ways to implement imperfections for a pinned column in the FE-software Abaqus. The same column was also investigated for different ways to consider both imperfections and second-order effects analytically. The second study compares the three approaches to consider imperfections and second-order effects for a sway frame. It was conducted as a parametric study where the slenderness of the frame was varied. Only in-plane global buckling in the strong-direction of the cross-section was studied. The considered profile for all members were the standard hot-rolled profile IPE300 and the analyses were performed by using elastic cross-sectional response.

For the pinned column the study showed that all methods produces similar utilisation for stocky columns, while the difference increases with increasing slenderness. The same tendencies was also showed for the study regarding the frame design procedures. In general, further studies needs to be conducted in order to fully understand the links between the parts within the design code concerning stability analysis.

Keywords: Eurocode 3, instability, buckling, imperfections, second-order effects, frame design.

Dimensionering av stålkonstruktioner med hänsyn till stabilitet
Jämförelse av metoder presenterade i *Eurokod 3* gällande beaktning av
imperfectioner och andra-ordningens effekter

Examensarbete inom masterprogrammet Structural Engineering and Building Technology

VIKTORIA BODÉN
STINA LUNDIN

Institutionen för Arkitektur och Samhällsbyggnadsteknik
Avdelningen för Konstruktionsteknik
Stålkonstruktioner
Chalmers Tekniska Högskola

Sammanfattning

Inom stabilitetsanalyser ska imperfectioner och andra-ordningens effekter tas hänsyn till, vilket kan anses mer komplext för ramkonstruktioner jämfört med enskilda element. Dimensioneringsprocessen för enskilda element är relativt tydligt beskrivet i EN1993-1-1 (2005). Däremot uppkommer otydligheter vid dimensionering av ramkonstruktioner då effekter på både system- och komponentnivå bör tas hänsyn till.

I EN1993-1-1 (2005) finns det i dagsläget tre tillvägagångssätt för att kontrollera stabiliteten för ramkonstruktioner. Skillnaden mellan dessa tillvägagångssätt är inte fullständigt undersökt, vilket kan leda till misstolkningar och därför finns ett behov för ökad förståelse. Syftet med detta examensarbete var därmed att jämföra och fastställa riktlinjer för de olika metoderna i EN1993-1-1 (2005), gällande imperfectioner och andra-ordningens effekter för stålkonstruktioner.

För att uppnå detta, genomfördes först en litteraturstudie inom strukturell stabilitet och tillvägagångssätten för dimensionering av ramkonstruktioner. Dessutom genomfördes även två studier på olika strukturer. Första studien behandlar olika sätt att implementera imperfectioner för en ledad pelare och genomfördes med FE-analys i programvaran Abaqus. För samma pelare undersöktes också olika metoder analytiskt för att ta hänsyn till både imperfectioner och andra-ordningens effekter. Den andra studien jämför tre tillvägagångssätt för att ta hänsyn till imperfectioner och andra-ordningens effekter på en svajram. Detta genomfördes som en parameterstudie där slankheten av ramen varierades. Studierna är begränsade till att endast studera global buckling i planet, där den starka riktningen av elementens tvärsnitt har studerats. Profilen som används i studierna är en standard varmvalsad IPE300 och analyserna som utförts har endast beaktat elastisk respons.

Resultat för den ledade pelaren visade att skillnad i utnyttjandegrad för metoderna ökar med ökad slankhet. Samma trend följde även studien som är utförd på ramkonstruktioner. Sammantaget, bör fler studier utföras inom området för att helt kunna förstå kopplingarna mellan koncepten beskrivna i Eurokod gällande stabilitetsanalys.

Nyckelord: Eurokod 3, instabilitet, buckling, imperfectioner, andra-ordningens effekter, ramkonstruktioner.

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Viktoria Bodén and Stina Lundin, Gothenburg, June 2019

List of Symbols

Greek lower case letter

- α Imperfection factor [-]
- α_{cr} Minimum force amplifier for the design load to cause elastic instability [-]
- α_h Reduction factor for height h applicable to columns [-]
- α_m Reduction factor for the number m of columns in a row [-]
- $\alpha_{ult,k}$ Minimum force amplifier to reach the characteristic resistance without taking buckling into account [-]
- $\delta_{H,Ed}$ Horizontal displacement at the top of the storey, relative to the bottom of the storey [m]
- η Generalised initial imperfection [-]
- η_{cr} Shape of the elastic critical buckling mode [-]
- η_{init} Amplitude of the imperfection shape [-]
- γ_{M1} Partial factor for resistance of members to instability assessed by member checks [-]
- λ Slenderness [-]
- λ_1 Reference slenderness [-]
- $\bar{\lambda}$ Non-dimensional slenderness [-]
- ϕ Global initial sway imperfection [-]
- ϕ_0 Basic value for global initial sway imperfection [-]
- π Mathematical constant [-]
- σ_{cr} Critical buckling stress [Pa]

Greek upper case letter

- β Critical buckling length factor [-]
- χ Reduction factor due to buckling [-]
- χ_{LT} Reduction factor due to lateral torsional buckling [-]
- Φ Factor to determine the reduction factor χ [-]

$\Delta M_{i,Ed}$ Moments around y- or z-axis due to the shift of the centroidal axis for class 4 [Nm]

$EI\eta_{cr,max}$ Bending moment due to η_{cr} at the critical cross section [-]

Roman lower case letter

e_0 Initial bow imperfection [m]

$e_{0,d}$ Design value of initial bow imperfection [m]

f_y Yield stress [Pa]

h Height of structure [m]

i Radius of gyration [m]

k_{ii} Interaction factor [-]

m Number of columns in a row [no]

n Reflector of the buckling mode [-]

v Lateral deflection [m]

v_0 Initial geometrical imperfection [m]

v_{max} Maximum second-order deflection [m]

Roman upper case letter

A Cross sectional area [m^2]

C_m Equivalent uniform moment factor [-]

E Young's modulus [Pa]

F_{cr} Elastic critical buckling load for global instability mode [N]

F_{Ed} Design load on the structure [N]

H_{Ed} Design value of the horizontal reaction at the bottom of the storey due to the horizontal loads and fictitious horizontal loads [N]

I Second moment of area [m^4]

K Amplification factor [-]

L Member length [m]

L_{cr} Critical buckling length [m]

M_{cr} Elastic critical moment for lateral-torsional buckling [Nm]

M_{Ed} Design bending moment [Nm]

$M_{Ed,II}^{II,max}$ Maximum second-order bending moment induced by first-order moment [Nm]

M_{el} Elastic bending moment [Nm]

M_{max} Maximum bending moment [Nm]

M_{pl}	Plastic bending moment [Nm]
M_{Rd}	Design moment resistance [Nm]
M_{Rk}	Characteristic moment resistance [Nm]
$M_{\eta^{II}initi,m,max}^{II}$	Maximum second-order bending moment for the equivalent member [Nm]
$N_{b,Rd}$	Design buckling resistance [N]
N_{cr}	Elastic critical buckling force [N]
N_{Ed}	Design axial force on the structure [N]
$N_{Ed,max}^{II}$	Maximum second-order axial force induced by first-order axial force [N]
N_{Rd}	Design normal force resistance [N]
N_{Rk}	Characteristic normal force resistance [N]
U	Utilisation ratio [-]
V_{Ed}	Total design vertical load on the structure at the bottom of the storey [N]
W	Sectional modulus [m^3]

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1

Introduction

In design of steel structures, the structural engineer needs to consider imperfections and at times second-order effects with respect to instability. Any real structural member has imperfections and these may either be due to geometrical defects, residual stresses or due to eccentricity of applied load. Second-order effects take into account the effect of the deformation of the structure under loading. These effects may strongly influence the behaviour and the ultimate load of the structure and are therefore of importance to consider when required.

The design procedure to consider imperfections and second-order effects is more complex for a frame structure than individual elements. The design procedures for individual members are relatively well described in Section 6.3 in EN1993-1-1 (2005). However, questions arise when studying a frame where the design should consider both system and member effects.

Today, *Eurocode 3* presents three main approaches for the use in global stability analysis of steel frames. The main difference between the approaches is the way imperfections and second-order effects are considered and implemented. The approaches originally presented in Section 5.2.2 in EN1993-1-1 (2005) are summarised as follows:

- (a) Totally global approach with cross-section check.
- (b) Partially global approach with individual stability checks.
- (c) Approach for basic cases by individual stability checks using appropriate buckling lengths in line with the global buckling mode of the structure.

Not only is the application of these approaches unclear but also how they differ in result. *Approach (b)* is considered the conventional approach (Shayan, 2013). Both *Approach (b)* and *(c)* consists of individual member stability checks which are performed analytically and considered to be time-consuming (Shayan, 2013). *Approach (a)* is an approach that considers all imperfections and second-order effects directly in the global analysis and is relatively new and unexplored within the industry (Dallemule, 2015). Therefore it is efficiently implemented in Finite Element Analysis (FEA), which is a numerical technique widely used in structural design. In order to avoid unsafe or highly conservative designs, it is of importance to clarify how these approaches differ from each other and how to apply and implement them in practical situation.

1.1 Aim and objectives

The aim of the thesis is to compare and establish guidelines for the different approaches presented in EN1993-1-1 (2005) regarding global stability. This, to be able to understand the differences and similarities between them and thereby provide recommendations for the designer regarding implementation.

To achieve the general aim, the following objectives needs to be met:

- Compare different methods to implement imperfections for a pinned column to understand the implementation on a member level.
- Compare different methods to consider both imperfections and second-order effects for a pinned column to be able to see the coherence before advancing to more complex structures.
- Study the design code in Eurocode 3 regarding global stability design of steel frames.
- Perform a parametric study on a frame by varying the slenderness and comparing the current approaches, regarding consideration of imperfections and second-order effects.

1.2 Limitations

The project was limited to global buckling, which implies that local and distortional buckling were not investigated. Within global buckling, only the strong buckling direction was studied for in-plane behaviour. The considered profile for all members was the flanged steel standard hot-rolled profile IPE300, which is double symmetric. The studies have also been limited to only consider elastic analyses and elastic cross-sectional response.

The boundary condition for the studied column was limited to pinned and it was loaded with an axial compressive force. When the cross-section was modelled in Abaqus, the rolled parts were neglected. The study of the frame was limited to only consider a frame sensitive to sway. The studied frame had rigid connections between the members and the boundary conditions at the supports were considered to be pinned. The cross-section in this study was modelled as "generalized cross-section" (SIMULIA, 2014), which implies that the rolled parts were considered. For all members within the frame, the lengths were set to be equal.

1.3 Method

Initially, a literature study was performed to both investigate the instability of steel elements and also to investigate the design process and regulations according to Eurocode 3 concerning the design of steel frames with respect to instability.

Before starting to analyse a frame structure, a column was studied. For a pinned column, an overview and comparison of different modelling techniques to implement imperfections was executed to comprehend the main differences in a basic practical application. The comparison was performed using the FE-software Abaqus provided by SIMULIA. During the thesis project, the software version was of year 2017 and the user's guide used was of version 6.14 (SIMULIA, 2014). The same column was then studied analytically regarding the consideration of both imperfections and second-order effects which can be executed in different ways for a simple member.

Finally, a frame was studied for three approaches to consider imperfections and second-order effects. A parametric study was performed where the slenderness was varied, and the analysis was executed by using both Abaqus and analytical calculations. The comparison between the approaches were made to be able to establish recommendations regarding global stability design of steel frames that could be used in practice.

As a complement to the studies, two benchmarks were analysed. The first benchmark was a pinned column, which was made to confirm the application procedure in the first study. The second benchmark considered was a fixed-pinned column subjected to axial compression, which had the purpose to confirm the computation and implementation of the imperfection in a totally global approach.

2

Instability of Steel Elements

The design procedures that exist today in EN1993-1-1 (2005) regarding structural stability of steel is a product of many years of research. To be able to apprehend these design procedures, it is of importance to understand the theory behind instability of steel elements. The chapter begins with a description of the definition of structural instability followed by the buckling behaviour of a member under applied load. Then, imperfections are presented along with the different types of sectional response.

2.1 Structural instability

The definition of instability is that a small increase in load will develop a large increase in displacement (Galambos and Surovek, 2008). When studying a structural system, it could be enough with local instability of separate members to cause collapse of the total structure.

Buckling is a type of instability, often referred to as structural or geometrical instability that is caused by a compression force on the member which leads to a failure mode. When the applied force on the member becomes large enough it will cause a sudden deflection. By increasing the load further it can lead to a total loss of the load-carrying capacity of the structural member.

When it comes to buckling of a member, it is of importance to distinguish between the different directions a member may buckle in. Eurocode 3 (2005) differentiate between three different member axis (Gardner and Nethercot, 2011):

- x-x along the member
- y-y axis of the cross-section
- z-z axis of the cross-section

A member with an I-profile, which is studied in this thesis project, have a so called strong and weak direction of buckling. The directions are shown in Figure 2.1, where strong respectively weak direction are referring to the stiffness of the member in different directions. In the strong direction, the member has a larger stiffness and can thereby endure larger exposure to external actions compared to the weak direction.

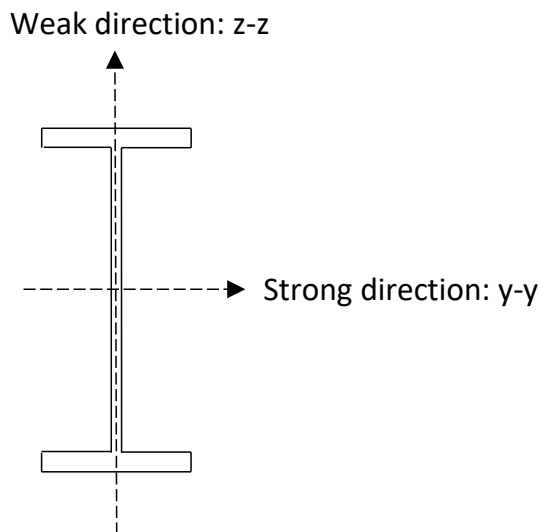


Figure 2.1: Strong and weak direction of buckling for an I-profile.

According to Schafer and Ádány (2005) buckling can be divided into three different main groups which are global, distortional and local buckling. Global buckling symbolises different types of buckling that influence the whole member, while local buckling only represent a number of small buckles in the compressed web or flange. Distorsional buckling is a buckling mode that mostly affect cold-rolled profiles with free edges and is characterised by rotation of the flange at the connection between flange and web. These three main groups of buckling modes are illustrated in Figure 2.2. But, as mentioned in Section 1.2, only global buckling was considered in this thesis.

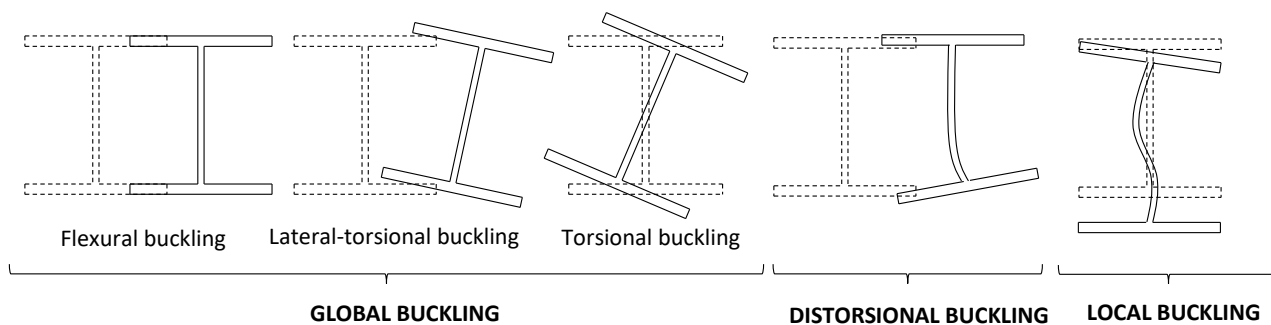


Figure 2.2: Main groups for different buckling modes.

2.2 Euler buckling

In theory, global buckling is often described by studying a column according to the classic buckling theory, so-called Euler buckling. According to Al-Emrani and Åkesson (2013) it is described as a linear elastic buckling problem that is dependent on the derivation of Euler's critical load, N_{cr} . The assumptions made for the derivation of this type of ideal column are the following ones:

- The column is initially “perfect” without any geometrical imperfections.
- Elastic material behaviour is assumed.
- No residual stresses or other inner stresses.
- Loading is applied in the centre of the column.
- The cross-section and its support conditions are such that only plane buckling in one direction is relevant.

The expression for the critical force is presented in Equation (2.1).

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2} \quad (2.1)$$

where:

N_{cr} - elastic critical buckling force

E - young's modulus

I - second moment of area

L_{cr} - critical buckling length

An illustration of such column is illustrated in Figure 2.3.

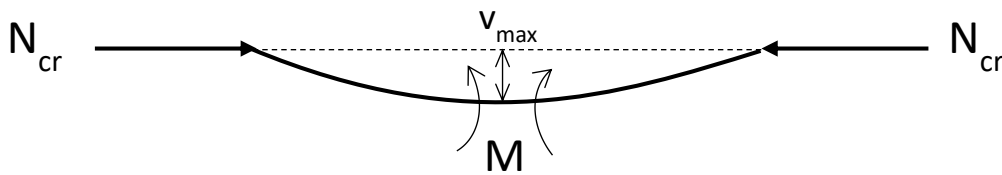


Figure 2.3: Euler buckling of a column where v_{max} represent the maximum deformation.

Due to different boundary conditions of a compressed column, there will be different types of buckling modes. For the different cases, the critical buckling length varies and can be described as in Equation (2.2).

$$L_{cr} = \beta \cdot L \quad (2.2)$$

Where β is a factor for each Euler buckling mode that provides the critical buckling length of the actual member length L . These different Euler buckling modes are shown in Figure 2.4 for four standard cases.

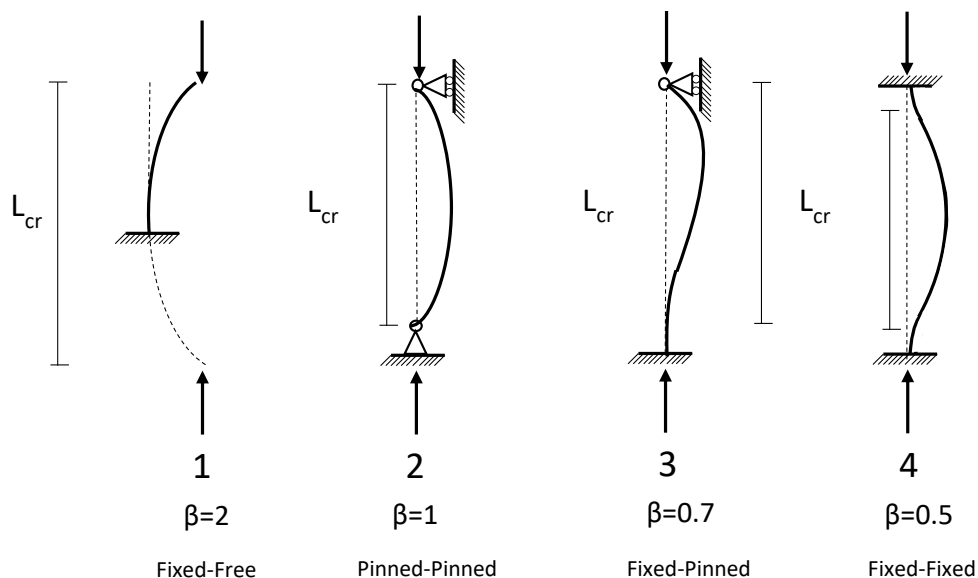


Figure 2.4: Euler buckling modes for different support conditions.

The critical buckling load can in it's own turn be described in terms of critical buckling stress, that are expressed according to Equation (2.3).

$$\sigma_{cr} = \frac{N_{cr}}{A} = \frac{n^2 \pi^2 EI}{(AL_{cr})^2} \quad (2.3)$$

Where:

- σ_{cr} - critical buckling stress
- A - cross-sectional area
- n - reflector of the buckling mode

This formula can be rewritten by introducing the so-called *radius of gyration*, $i = \sqrt{\frac{I}{A}}$, and by choosing the reflector of the buckling mode n to 1.0 (minimum value). The new formula is then expressed according to Equation (2.4).

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_{cr}}{i}\right)^2} \quad (2.4)$$

By lastly referring to L_{cr}/i as the *slenderness* of the column, λ , the final expression for the critical stress can be seen in Equation (2.5)

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \quad (2.5)$$

The relation between the slenderness and the critical load is illustrated in Figure 2.5 (a), while the right hand part, Figure 2.5 (b), illustrates the curve with an limitation for the maximum stress allowed.

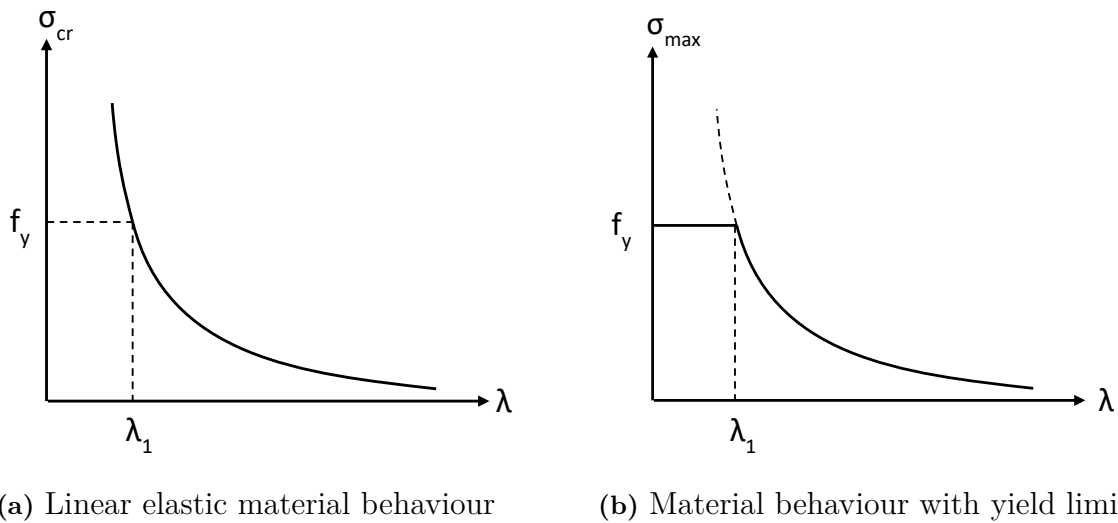


Figure 2.5: Relationship between the slenderness and the critical buckling stress of a column.

By allowing the critical buckling stress to be equal to the yield stress f_y of the steel quality, a new variable is defined as the reference slenderness, see Equation (2.6).

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} \quad (2.6)$$

To simplify the design, the column strength curve is normalised by expressing it in steel quality instead, see Figure 2.6, where $\bar{\lambda}$ is the non-dimensional slenderness and χ is the reduction factor due to buckling.

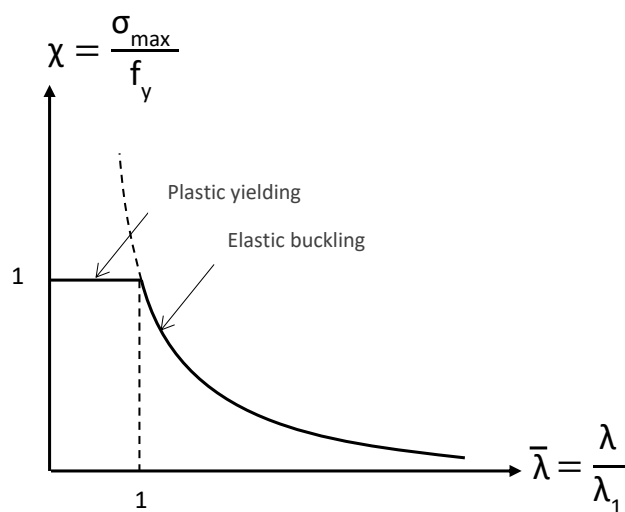


Figure 2.6: Relationship between the non-dimensional slenderness and the buckling reduction factor.

2.3 Real buckling behaviour

It is stated by Al-Emrani and Åkesson (2013) that real members, columns and beams, are usually not straight and does always consist of some geometrical and material deviations. So, the conditions assumed for both columns and beams based on the Euler theory does not comply with reality. In Figure 2.7 the difference between the Euler buckling curve and tests results on members (columns, beam-columns and beams) are shown for different slendernesses.

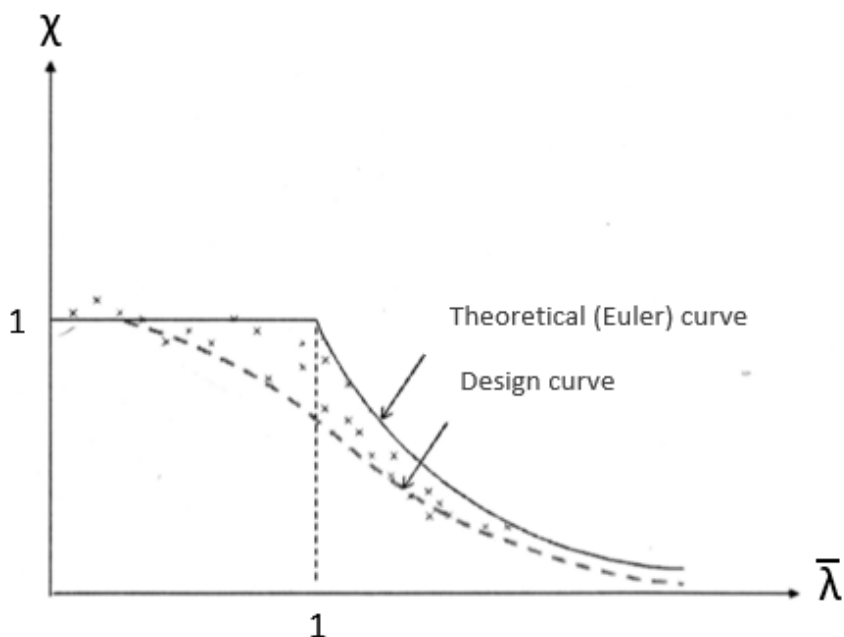


Figure 2.7: Relationship between test result of a real member and the buckling curve derived from the classic, Euler, buckling theory based on Al-Emrani and Åkesson (2013).

In Figure 2.7 it is possible to recognize that:

- A stocky member, $\lambda \ll 1$, have a higher load-carrying capacity than the capacity correlating to yielding. Column: $\sigma_{cr} \gg f_y$, Beam: $M_{cr} \gg M_{pl}$
- An intermediate slender member, $\lambda \approx 1$, shows a load-carrying capacity that is below the predicted one.
- A very slender member, $\lambda \gg 1$ have a capacity almost equal to the predicted one with the Euler theory. Column: $\sigma_{cr} \ll f_y$, Beam: $M_{cr} \ll M_{pl}$

If a member is governed by either yielding or buckling depends on the slenderness of the member, and this is illustrated in Figure 2.7. A very stocky member's load-carrying capacity is not governed by instability, but by yielding. While for a very slender member, the capacity is instead governed by buckling at a load far below the limit for yielding. Very slender members will fail in either elastic buckling for a column or pure lateral-torsional buckling for a beam. The load-carrying capacity for an intermediate slender member on the other hand, suffer the most reduction of capacity due to second-order effect.

For a beam, the critical buckling moment may be treated in the same way as for a column, described with Euler buckling theory. Instead of defining the critical axial force and by that expressing the relative slenderness, the elastic critical moment M_{cr} is introduced. So, the theory of Euler buckling are also adopted here:

- The beam is initially “perfect” without any geometrical imperfections.
- Elastic material behaviour is assumed.
- No residual stresses or other inner stresses.
- Loading is applied in the centre of the beam.

A beam’s non-dimensional slenderness is dependent on the elastic critical bending moment and the plastic bending moment M_{pl} , while the reduction factor due to buckling is dependent on the maximum bending moment M_{max} and the plastic moment. These relations are shown in Equation (2.7) and Equation (2.8).

$$\bar{\lambda} = \sqrt{\frac{M_{pl}}{M_{cr}}} \quad (2.7)$$

$$\chi = \frac{M_{max}}{M_{pl}} \quad (2.8)$$

As for "real" columns, these conditions do not apply in reality since there always exist some deviations within the member.

2.4 Imperfections

As described in the section above, imperfection always exist in real steel members to some degree and these imperfections can be of different kind (Al-Emrani and Åkesson, 2013). The imperfections can be for example geometric imperfections, variation in residual stresses and eccentricity of applied load.

Geometrical imperfections can be exemplified as lack of verticality, lack of straightness or lack of flatness. Variation in residual stress can be due to uneven cooling after the rolling or welding session. Another aspect is that the member never can be loaded exactly in the centroid of the section and thereby creates a minor eccentricity.

All these imperfections are included in the buckling curves given in 6.3 in EN1993-1-1 (2005), by the imperfection factor α , within the check of the structural stability. Based on the buckling curves, EN1993-1-1 (2005) introduces something called *equivalent initial imperfections*. The equivalent initial imperfections are expressed as initial geometrical imperfections that includes all imperfections presented above.

2.5 Sectional response

Depending on the sectional response of the studied member, it will behave different with respect to stability. The type of buckling is decided by the cross-sectional slenderness of the member and it will determine which type of analysis that needs to be performed concerning buckling. According to Al-Emrani et al. (2013) the cross-section response for an idealized steel member under pure bending is based on assumptions as follows:

- Plane sections remain plane.
- Ideal elastic-plastic stress-strain relation.
- Residual stresses can be neglected.
- The member is initially straight and the section remains perpendicular to its centre-line.

There are four different cross-section classes according to EN1993-1-1 (2005), class 1-4, depending on the sectional response. The cross-section classification is a categorisation to identify to what extent the local buckling limits the resistance and rotation capacity of the section. This is determined by identifying the sectional stiffness and thereby provides different sectional modulus, W , depending on the class. Where:

$$\begin{aligned} W &= W_{pl} \text{ (plastic)} && \text{for class 1 and 2} \\ W &= W_{el} \text{ (elastic)} && \text{for class 3} \\ W &= W_{eff} \text{ (effective)} && \text{for class 4} \end{aligned}$$

For profiles in class 1-3, the load-carrying capacity is reduced only in accordance to global buckling, while profiles in class 4 are reduced with reference to local buckling risk. The higher cross-section class, the more slender the member is and thereby experience higher risk for instability. The class determines the method to use in both local and global analysis and a summary of which class to use in each analysis is described in Table 2.1 (Al-Emrani and Åkesson, 2013).

Table 2.1: Type of analysis depending on cross-section class (Al-Emrani and Åkesson, 2013).

Cross-section class	Method for local analysis	Method for global analysis
1	plastic	plastic
2	plastic	elastic
3	elastic	elastic
4	effective	elastic

In EN1993-1-1 (2005), the difference between the classes are stated. For cross-section class 1, full plasticity is achieved due to the section's large rotation capacity (Al-Emrani and Åkesson, 2013). This enables a plastic hinge to develop in the most critical point, meanwhile the moment redistributes to regions that are less utilised. Then, plastic hinges will also occur in these regions and then the loading capacity is reached. Both the cross-section model and the global analysis model will then be of plastic behaviour. Sections in class 2 can also achieve their full plasticity, but has a limited rotation capacity due to local buckling. This means that no redistribution can take place and therefore must the global analysis be performed using elastic response. Class 3 cross-sections entails that the parts in pure compression can reach yield

stress, and due to the risk of local buckling the plastic capacity cannot be utilised. Lastly, cross-sections of class 4 are those where local buckling will occur before reaching yield stress in at least one part of the cross-section. This class was not investigated, the thesis project was limited to only study global buckling.

An illustration of these different classes is shown in Figure 2.8, where it is possible to recognise the difference in ability to reach plasticity.

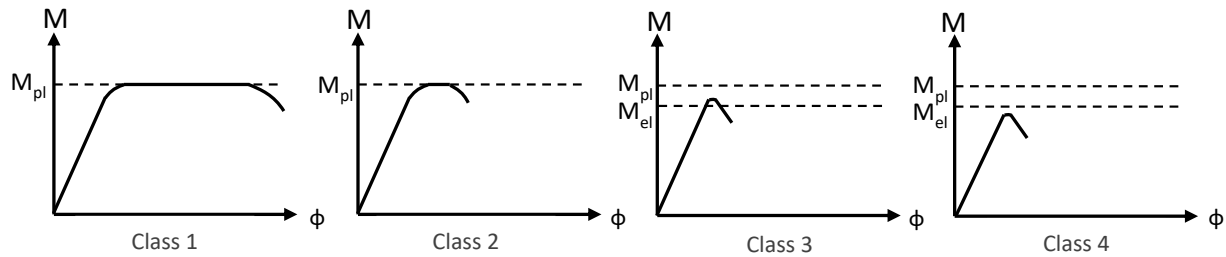


Figure 2.8: Relationship between moment and curvature for different cross-section classes.

Cross-sections can be considered to experience different responses, and for a cross-section to have a elastic response, the stresses in the most critical parts of the section may not exceed the yield stress of the steel. When the moment in the most critical area of the section has exceeded the elastic moment, a plastic response will be achieved. The strain distribution will remain linear while the stress distribution will change due to the fact that the stress exceeds the yield strength. The design moment resistance is expressed as in Equation (2.9), where the sectional modulus W is the part changing for different types of response.

$$M_{Rd} = f_y \cdot W_i \quad (2.9)$$

The studies were limited to only study elastic cross-sectional response and thereby was the elastic sectional modulus, W_{el} , used.

3

Frame Design Procedures

The design procedure to consider imperfections and second-order effects is more complex for a frame structure than individual elements. The design procedures for individual members are relatively well described in Chapter 6.3 in EN1993-1-1 (2005). However, questions arise when studying a frame where the design should include both system and member effects. Therefore, this chapter mainly focuses on the stability design of frame structures.

According to EN1993-1-1 (2005), there are three main approaches when designing a frame; totally global approach, partially global approach and an approach for basic cases. The second one, partially global, is considered the conventional (Shayan, 2013). The design code EN1993-1-1 (2005) Chapter 5.2.2 describing how to consider frame stability, only have guidelines for implementation described in a general way which leaves many questions to the designer. The purpose of this chapter is to clarify how to interpret the design code in *Eurocode 3* and what needs to be considered when dealing with frame stability.

An overview of the design procedures is presented in the flowchart illustrated in Figure 3.1. It is a summary of the work flow and provides guidance for interpreting the design code. The different parts of the flowchart is described in this chapter in purpose to give a more detailed perspective on how a global design of a steel frame is performed according to EN1993-1-1 (2005).

Frame design procedure

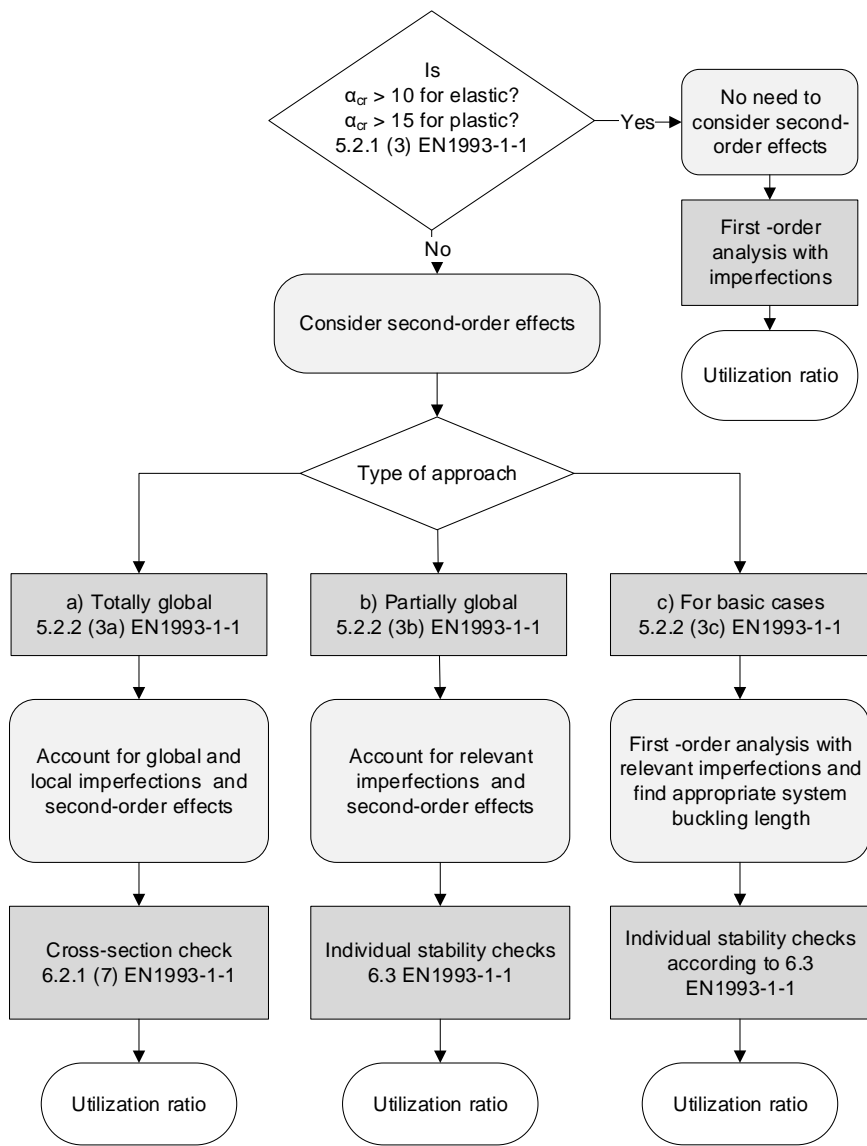


Figure 3.1: Overview of the design process for steel frames interpreted from Eurocode 3.

3.1 When to consider second-order effects

Initially, it is of importance to know whether or not it is required to consider second-order effects. According to EN1993-1-1 (2005), second-order effects should be considered when the effects of the deformed geometry is significant, otherwise first-order analysis can be used. To check whether first-order analysis is appropriate to use, the following criteria should be satisfied:

$$\begin{aligned}\alpha_{cr} &= \frac{F_{cr}}{F_{Ed}} \geq 10 \quad \text{for elastic analysis} \\ \alpha_{cr} &= \frac{F_{cr}}{F_{Ed}} \geq 15 \quad \text{for plastic analysis}\end{aligned}\tag{3.1}$$

where:

- α_{cr} - minimum force amplifier for the design load to cause elastic instability
- F_{Ed} - design load on the structure
- F_{cr} - elastic critical buckling load for global instability mode

For plane frames and portal frames sensitive to sway, the factor α_{cr} expressed in Equation (3.1) can be approximated by Equation (3.2) using frame geometry, applied loads and first-order sway displacement. For frames sensitive to sway, the sway mode is likely associated with the lowest critical buckling load which usually makes it governing.

$$\alpha_{cr} = \frac{H_{Ed}}{V_{Ed}} \frac{h}{\delta_{H.Ed}}\tag{3.2}$$

where:

- H_{Ed} - design value of the horizontal reaction at the bottom of the storey to the horizontal loads and fictitious horizontal loads
- V_{Ed} - total design vertical load on the structure at the bottom of the storey
- h - height of structure
- $\delta_{H.Ed}$ - horizontal displacement at the top of the storey relative to the bottom of the storey

To be able to use Equation (3.2), following conditions need to be met:

- the roof slope can not be steeper than 1:2, 26 degrees
- the axial compression in the beams may not be significant, which it is when $\bar{\lambda} \geq 0.3\sqrt{\frac{Af_y}{N_{Ed}}}$

If the criterion in Equation (3.1) is not fulfilled, then second-order effects have to be taken into account. This means that if the structure is not stiff enough the second-order effects will lead to a substantial increase in deflection and internal actions, and should therefore be considered in design.

Second-order effects can be considered in two general manners. Either, by performing a second-order analysis where the load is step-wise increased, which can preferably be done by using an appropriate FE-software. For each load proportion factor an equilibrium is found based on the deformed structure. Or, it can be performed by doing an approximate second-order

analysis using first-order analysis with allowance for second-order effects, preferably done with hand calculations.

For single storey frame design, the amplification factor for treating second-order sway effects due to vertical loads in an approximate second-order analysis may be determined by Equation (3.3) (EN1993-1-1, 2005).

$$K = \frac{1}{1 - \frac{1}{\alpha_{cr}}} \quad (3.3)$$

This expression applies as long as $\alpha_{cr} \geq 3.0$ and if not, more thorough second-order analysis needs to be performed. To be able to use the amplifier, the design should be based on elastic global analysis and the second-order effects are then considered by increasing the horizontal loads and/or the equivalent loads with the amplification factor.

So, for analyses based on elasticity, the limits for when second-effects need to be considered and how they can be considered are summarized in Table 3.1.

Table 3.1: Actions for elastic analysis after determined α_{cr} .

Limits on α_{cr}	Action
$\alpha_{cr} > 10$	first-order analysis
$10 > \alpha_{cr} > 3$	second-order analysis or first-order analysis with amplification factor
$\alpha_{cr} < 3$	second-order analysis

3.2 How to consider second-order effects and imperfections

If the condition in Equation (3.1) is not fulfilled, second-order effects need to be considered in the verification of the structural stability. When checking the structural stability, not only second-order effects needs to be considered, but also imperfections. According to Section 5.2.2 (3) in EN1993-1-1 (2005), the imperfections and second-order effects may be taken into account either by performing a:

- (a) Totally global analysis with cross-section check.
- (b) Partially global analysis with individual stability checks.
- (c) Analysis for basic cases by individual stability checks with appropriate buckling lengths corresponding to the global buckling mode of the structure.

In accordance with *Approach (a)* it is further described in Section 5.2.2 (7a) in EN1993-1-1 (2005) that if the second-order effects and relevant member imperfection in a global analysis are fully accounted for, it is not necessary to perform individual stability checks. This means that if both global and local imperfections are accounted for in a second-order global analysis, then a cross-section check as in Section 3.6 is sufficient.

In accordance with *Approach (b)*, Section 5.2.2 (7b) in EN1993-1-1 (2005) describes that if relevant member imperfection and second-order effects in individual members are not totally accounted for in the global analysis, then the individual stability checks of members should be performed considering effects not included in the global analysis. These individual stability checks are presented in Section 3.7.

Approach (c) is only valid for basic cases according to EN1993-1-1 (2005). It describes that the actions from a first-order analysis with relevant imperfections can be used in the individual stability checks, together with the global critical buckling length of the system. In this way, both global and local second order effects are considered within the individual stability checks. However, the phrase "basic cases" are not defined and therefore it is unclear when it is appropriate to use this approach.

3.3 Imperfection magnitude and shape

The imperfections presented in EN1993-1-1 (2005) are in the form of *equivalent initial imperfections*. An equivalent imperfection takes into account geometrical imperfections, residual stresses as well as eccentricities and are calibrated to provide conservative results (EN1993-1-1, 2005). The type of imperfections to use is dependent on if the frame is sensitive to sway buckling or not. For non-sway frames, equivalent imperfections should be designed for in the form of initial bow imperfections. For sway frames, initial sway imperfection, and in some cases initial bow imperfection, should be accounted for in the design. According to 5.3.2 (6) in EN1993-1-1 (2005) the member's bow imperfection needs to be included in the global analysis if the following criteria are fulfilled:

- $\bar{\lambda} \leq 0,5\sqrt{\frac{Afy}{N_{Ed}}}$
- at least one moment resistant joint at one member end

An alternative, presented in EN1993-1-1 (2005), to applying global and local imperfections separately is to use an Equivalent Unique Global and Local Initial imperfection, which will further be referred to as an EUGLI imperfection (Dallemler, 2015).

These imperfections are described more under each respective section.

3.3.1 Global initial sway imperfection

Initial sway imperfection is a global imperfection that is taken into account when having a system of members, for example a frame. This type of global imperfection is illustrated in Figure 3.2.

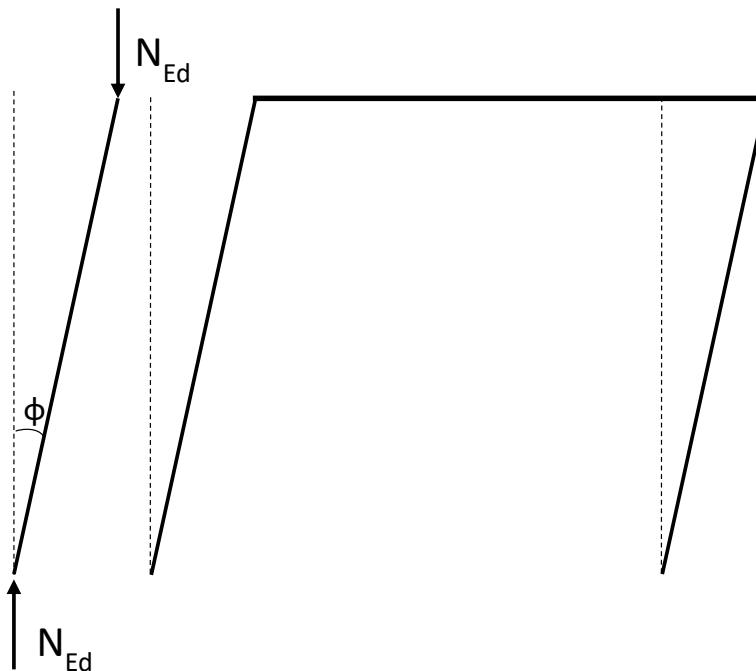


Figure 3.2: Sway imperfection.

The angle of the global initial sway imperfection is calculated according to Equation (3.4).

$$\phi = \phi_0 \cdot \alpha_h \cdot \alpha_m \quad (3.4)$$

where:

$$\phi_0 = 1/200$$

$$\alpha_h = \frac{2}{\sqrt{h}} \quad \text{but} \quad \frac{2}{3} \leq \alpha_h \leq 1$$

$$\alpha_m = \sqrt{0.5(1 + \frac{1}{m})}$$

h - height of structure [m]

m - number of columns in a row [no]

3.3.2 Local initial bow imperfection

The imperfection used for local geometrical imperfections is called initial bow imperfection. The equivalent initial bow imperfection, e_0 , of members exposed to flexural buckling is determined by Equation (3.5).

$$\frac{e_0}{L} \quad (3.5)$$

where L is the actual length of the member. The design values for this type of imperfection are given in Table 3.2.

Table 3.2: Imperfection values for initial bow imperfection (EN1993-1-1, 2005).

Buckling curve	elastic analysis	plastic analysis
	e_0/L	e_0/L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

An illustration of a local bow imperfection is shown in Figure 3.3.

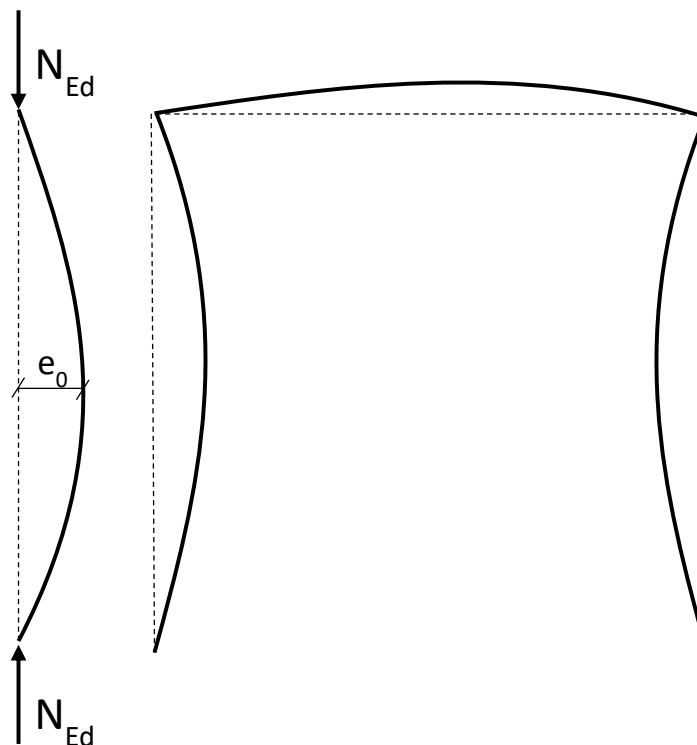


Figure 3.3: Bow imperfection.

3.3.3 EUGLI imperfection

An alternative to using imperfections presented in Section 3.3.1 and 3.3.2, is to use the EUGLI imperfection as mentioned before. As the name indicates, Equivalent Unique Global and Local Initial imperfection, it takes into account both local and global imperfection. The shape is the one of the first elastic critical buckling mode and the amplitude can be determined from Equation (3.6) which can be found in Section 5.3.2(11) in EN1993-1-1 (2005).

$$\eta_{init} = e_0 \cdot \frac{N_{cr}}{EI\eta_{cr,max}} \cdot \eta_{cr} = \frac{e_0}{\bar{\lambda}^2} \cdot \frac{N_{Rk}}{EI\eta_{cr,max}} \cdot \eta_{cr} \quad (3.6)$$

where:

$$e_0 = \alpha \cdot (\bar{\lambda} - 0.2) \cdot \frac{M_{Rk}}{N_{Rk}} \cdot \frac{1 - \chi\bar{\lambda}^2}{1 - \chi\bar{\lambda}^2} \quad for \quad \bar{\lambda} > 0.2 \quad (3.7)$$

and:

$\bar{\lambda} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}}$ is the non-dimensional slenderness

α - imperfection factor

χ - reduction factor due to buckling

γ_{M1} - partial factor for resistance of members to instability assessed by member checks

α_{cr} - minimum force amplifier for the design load to cause elastic instability

$\alpha_{ult,k}$ - minimum force amplifier to reach the characteristic resistance without taking buckling into account

M_{Rk} - characteristic moment resistance

N_{Rk} - characteristic normal force resistance

N_{cr} - critical elastic buckling force

η_{cr} - shape of the elastic critical buckling mode

$EI\eta_{cr,max}$ - bending moment due to η_{cr} at the critical cross-section

Generally, the method is an iterative process because of the critical cross-section that needs to be found. However, it should be noted that if both the cross-section and loading are uniform, no iterations are needed. This type of imperfection used in a second-order analysis can be seen as *Approach (a)*, totally global approach. As mentioned in Section 3.2, a cross-section check is sufficient since both local and global imperfections are considered. The calculation procedure to find the magnitude of the EUGLI imperfection can be separated into following steps (Brodnianski et al., 2014):

1. Find the shape of the first buckling mode, η_{cr} , and the eigenvalue of that mode.
2. Find the second derivative of the buckling mode, $\eta_{cr,m}''$, by using numerical analysis.
3. Estimate the critical cross-section m .
4. Calculate e_0 and η_{init} .
5. Calculate the internal forces and bending moments using second-order analysis. The utilisation closest to one provides the new position of the critical cross-section.
6. Iterate to find the exact location of the critical cross-section.
7. Calculate the final and actual internal forces and bending moments.
8. The utilisation at the critical cross-section is found.

3.4 EUGLI Background

In previous section, the EUGLI imperfection is presented as given in EN1993-1-1 (2005). In this section the theoretical background and derivation of the imperfection is presented. According to Dallemule (2015) the EUGLI method is relatively new and is not that often used in design practice. It is therefore of importance to clarify the principles and underlying assumptions made throughout the determination of the EUGLI imperfection.

The principle of the EUGLI imperfection is that it has the shape of the first elastic critical buckling mode. This shape is the most effective one when it comes to stability loss compared to other shapes (Dallemule, 2015). This means that it takes the least energy to develop. Today, most FE-software are provided with stability analyses which results in an eigenvalue α_{cr} and also the shape of the elastic critical buckling mode. However, finding the amplitude to scale the buckling mode with is the main difficulty. Chladný and Stujberová (2013) states that the amplitude of the imperfection should in first hand be determined from experiments. But as Dallemule (2015) mentions, no experiments of such structures are available today and will probably not be established due to the complexity of frame structures. Because of this, the amplitude in the EUGLI method is based on the requirement that the buckling resistance of an axially loaded frame shall be equal to the flexural buckling resistance of an equivalent member (Chladný and Stujberová, 2013). This approach is developed by Prof. Chladný (Brodniński et al., 2014) and is presented below.

In the first part of the derivation, as described shortly above, an equivalent member is used. The equivalent member has the same cross-section and axial force as in the critical cross-section, m , of the real member or frame. It is considered pinned at the supports with a length equal to $L_{cr,m}$ and slenderness $\bar{\lambda}_m$. For a member within cross-section class 1-3 and by using linear global analysis, the utilisation U_m at the critical cross-section can be expressed as in Equation (3.8), with the maximum value of the utilisation $U_m = 1$.

$$U_m = \frac{N_{Ed,m}^{II}}{N_{Rd,m}} + \frac{M_{\eta^{II}init,m,max}^{II}}{M_{Rd,m}} \quad (3.8)$$

The bending moment $M_{\eta^{II}init,m,max}^{II}$ for the equivalent member with initial imperfection, can be expressed as in Equation (3.9).

$$M_{\eta^{II}init,m,max}^{II} = -EI \left(e_{0d,m} \sin \left(\frac{\pi x}{L_{cr,m}} \right) \right) \cdot \frac{1}{\alpha_{cr} - 1} = N_{Ed,m} e_{0d,m} \cdot \frac{1}{1 - \frac{1}{\alpha_{cr}}} = N_{Ed,m} e_{0d,m} \cdot K \quad (3.9)$$

where K is the amplification factor expressed as:

$$K = \frac{1}{1 - \frac{1}{\alpha_{cr}}} = \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} = \frac{N_{cr}}{N_{cr} - N_{Ed}}$$

For the equivalent member, the equivalent geometrical initial imperfection e_{0d} can be divided into two parts, see Equation (3.10). The first part consists of the characteristic $e_{0,k}$, as expressed in Equation (3.11) and this part is derived in Section 3.7.1. The second part is a factor for transforming it to its design value, δ_e , as expressed in Equation (3.12). Note that when the partial factor $\gamma_{M1}=1$, then $\delta_e=1$.

$$e_{0,d} = e_{0,k} \cdot \delta_e \quad (3.10)$$

$$e_{0,k} = \alpha \cdot (\bar{\lambda}_m - \bar{\lambda}_0) \cdot \frac{M_{Rk,m}}{N_{R,mk}} \quad \text{for} \quad \bar{\lambda}_m > \bar{\lambda}_0 \quad (3.11)$$

$$\delta_e = \frac{1 - \chi \bar{\lambda}_m^2}{1 - \chi \bar{\lambda}^2} \quad (3.12)$$

The second part of the derivation takes the verified member or frame into account. The verified member or frame also has to satisfy $U_m = 1$, and the terms in Equation (3.8) should be identical.

For the verified member or frame, the EUGLI imperfection can be defined as in Equation 3.13.

$$\eta_{init,m}(x) = \eta_{init,m,max} \cdot \eta_{cr}(x) \quad (3.13)$$

where:

$\eta_{cr}(x)$ - first elastic critical buckling mode, with $|\eta_{cr}(x)|_{max} = 1$

$\eta_{init,m,max}$ - the amplitude of the EUGLI imperfection depending on the critical cross-section m

By knowing this, the bending moment $M_{\eta_{init,m}}^{II}$ for the verified member or frame, which is bent into the shape of $\eta_{init,m}(x)$, can be expressed as in Equation (3.14).

$$M_{\eta_{init,m,max}}^{II} = \frac{\eta_{init,m,max} |EI_m \eta_{cr}''|_m}{(\alpha_{cr} - 1)} \quad (3.14)$$

Since $M_{\eta_{init,m}}^{II}$ should be identical between the equivalent member (Equation (3.9)) and the verified member or frame (Equation (3.14)), the amplitude $\eta_{0,eugli,m}$ can be solved for. It is expressed in Equation (3.15) and can be presented in several ways knowing that:

$$\alpha_{cr} = \frac{N_{cr}(x)}{N_{Ed}(x)} \quad \text{and} \quad N_{cr} = \frac{N_{Rk}}{\bar{\lambda}^2}$$

$$\eta_{0,eugli,m} = \alpha_{cr} \cdot \frac{N_{Ed,m} \cdot e_{0,d,m}}{EI_m |\eta_{cr,m}''|} = \frac{N_{cr} \cdot e_{0,d,m}}{|M_{\eta_{cr,m}}|} = \frac{N_{Rk} \cdot e_{0,d,m}}{\bar{\lambda}^2 |M_{\eta_{cr,m}}|} \quad (3.15)$$

By applying Equation (3.15) to Equation (3.13), the full expression for the EUGLI imperfection is found, as presented in Equation (3.6).

To get a visual understanding how the method work, three graphical interpretations are presented in Figure 3.4 - 3.6. The figures represents a pinned column, fixed-pinned column and a sway frame.

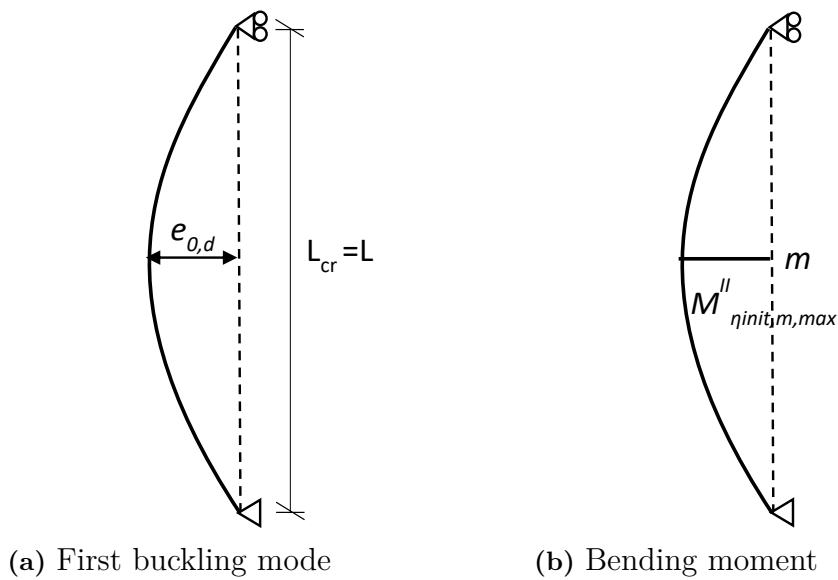


Figure 3.4: Shape of the elastic buckling mode, bending moments and location of the critical cross-section due to applied EUGLI imperfection for a pinned column.

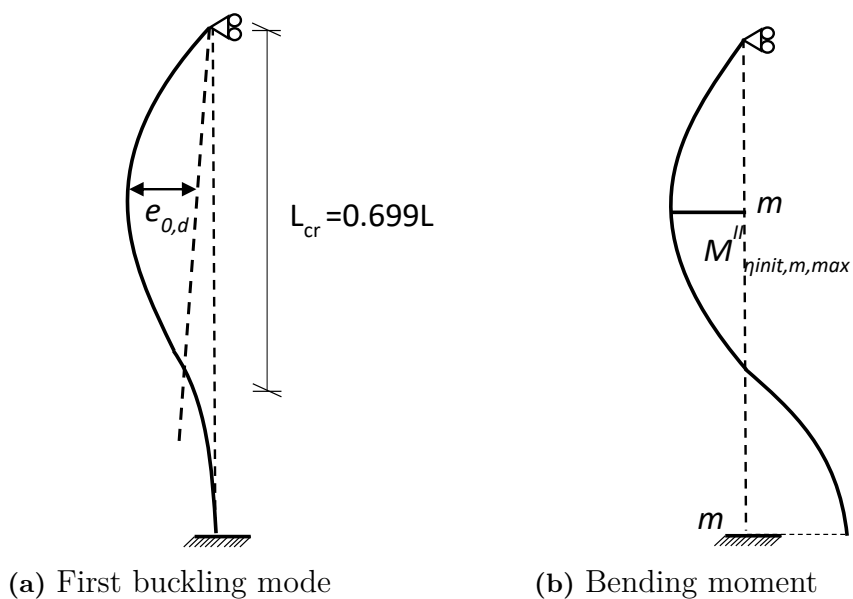


Figure 3.5: Shape of the elastic buckling mode, bending moments and location of the critical cross-section due to applied EUGLI imperfection for a fixed-pinned column.

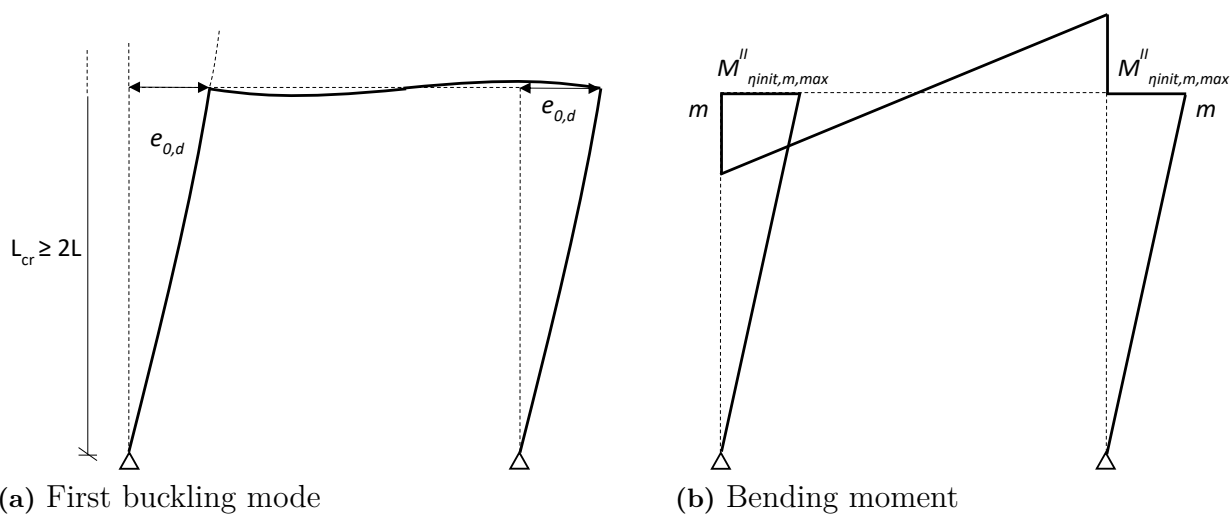


Figure 3.6: Shape of the elastic buckling mode, bending moments and location of the critical cross-section due to applied EUGLI imperfection for a sway frame.

3.5 Global analysis

To perform a global analysis where imperfections and second-order effects are considered, some decisions have to be made. Firstly, the choice of analysis type which is connected to how the second-order effects are accounted for need to be made. Secondly, the method on how the imperfections should be included in the analysis needs to be decided. The internal actions can then be used in either a cross-section check, for a totally global approach, or in the individual stability checks, for a partially global approach and for basic cases.

3.5.1 Analysis methods to determine internal actions

To determine the internal forces and moments in the structure from the FE-model, the analysis methods could be divided into four main methods (Gardner and Nethercot, 2011):

- First-order elastic analysis
- Second-order elastic analysis
- First-order plastic analysis
- Second-order plastic analysis

The difference between the methods in terms of a load-deformation curve is illustrated in Figure 3.7.

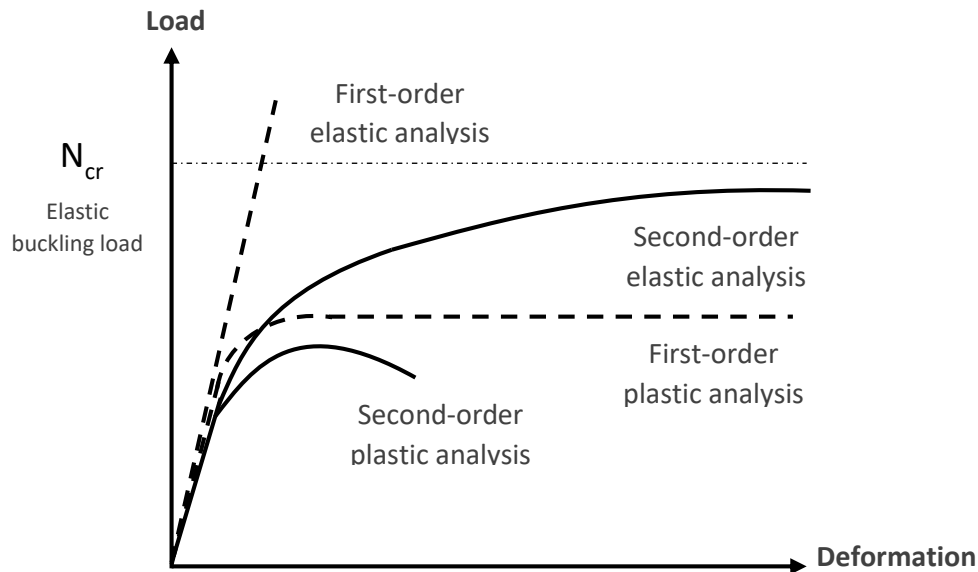


Figure 3.7: Load-deformation curve for the different analysis methods based on Gardner and Nethercot (2011).

The methods which are based on fulfilling equilibrium of the undeformed shape are denoted first-order, while the methods which are based on fulfilling equilibrium condition on the deformed shape are referred as second-order. The first-order analysis can also take second-order effects into account by amplification factors depending on N_{Ed} and N_{cr} , as mentioned in Section 3.1. Further on, the analysis methods are denoted whether plasticity is considered or not. The internal forces and moments can either be determined by using elastic or plastic analysis (EN1993-1-1, 2005). It is further stated that elastic analysis is always possible to use, while a plastic analysis only may be used when the member/structure achieves enough rotation capacity at the plastic hinges. In accordance with this thesis project limitations, has only elastic analyses been performed.

3.5.2 Methods to implement imperfections

The choice of how to implement the initial imperfections can have an effect on the results. It is a balance between the need for accuracy, a convenient procedure and verification feasibility. There are several different approaches to introduce the imperfections to be used in the FE-analysis. The main approaches to be found in EN1993-1-1 (2005) are presented below.

Explicit modelling

This is done by modelling imperfections directly in the model, either by modelling the residual stress and the initial geometric imperfections or by modelling the equivalent geometric imperfections. The explicit modelling will result in a geometrical imperfection.

Equivalent load method

One method that is mentioned briefly in EN1993-1-1 (2005) is the equivalent load method, which is also referred to as either equivalent notional load method or the notional horizontal force approach (Chan et al., 2005). The purpose of the method is to replace the equivalent geometrical imperfections with equivalent loads that are a fraction of the applied loads on the structure (EN1993-1-1, 2005), see Figure 3.8. The imperfection is transformed to an additional lateral load that is applied on the undeformed structure. The equivalent loads presented are made to give the same moment as explicitly implementing the initial imperfections would give.

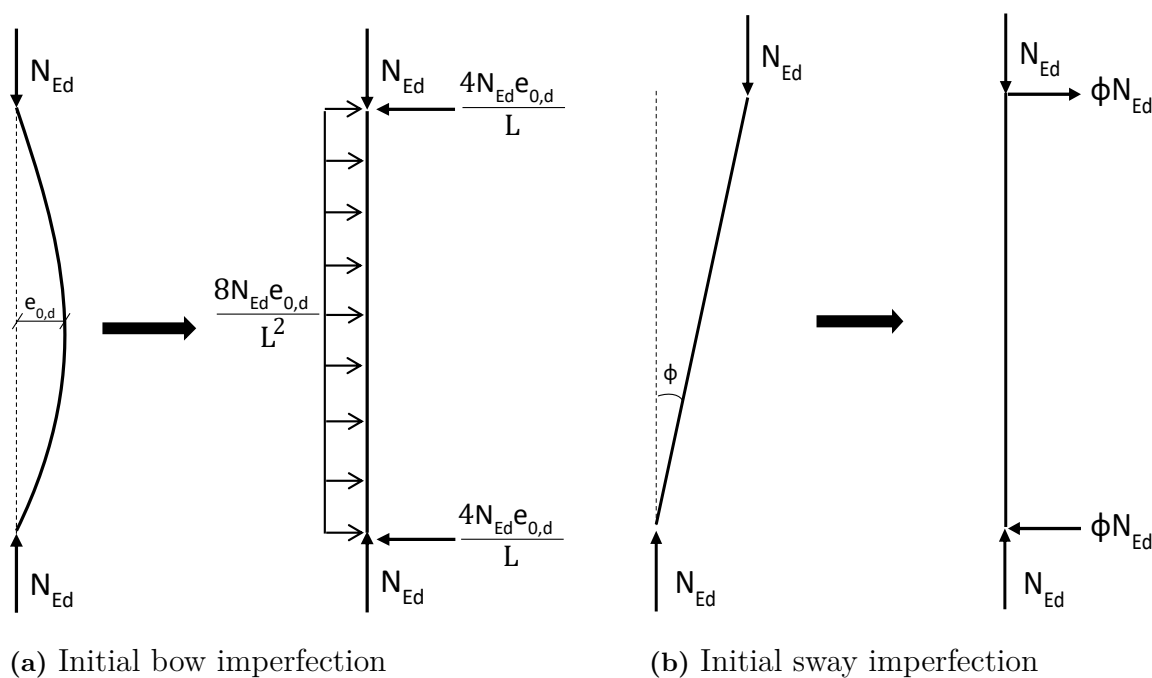


Figure 3.8: Replacement of imperfections with equivalent loads.

Scaling of first buckling mode

The first buckling mode is the one which requires the least energy to develop and thereby are the most effective shape regarding loss of stability (Dallemler, 2015). To scale the first buckling mode is a convenient way to present the worst case instability behaviour of the structure. EN1993-1-1 (2005) presents one way to find an appropriate scaling factor for the first mode shape, which is to use the EUGLI imperfection as described in Section 3.3.3. The scaling of the first buckling mode will, as for explicit modelling, result in a geometrical imperfection.

3.6 Cross-section check

If both second-order effects and imperfections are fully considered in the global analysis it is then enough to verify the structure with a cross-section check instead of individual stability checks. For members subjected to a combination of axial force and bending moment in cross-section class 1-3, this is done by using Equation (3.16) according to EN1993-1-1 (2005).

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1 \quad (3.16)$$

where:

N_{Ed} and $M_{i,Ed}$ - design axial force and bending moment around y- or z-axis

N_{Rd} and $M_{i,Rd}$ - design normal force and bending moment resistance around y- or z-axis

Equation (3.16) represent the sum of the utilization ratio for each stress resultant in the structure.

3.7 Individual stability checks

This section is about the individual stability checks that needs to be performed if the global analysis does not fully account for all second-order effects and relative member imperfections. This section represents the current design process of steel members/systems where the process is divided into two steps; (i) partially global analysis performed to obtain internal actions and (ii) stability checks to confirm adequate strength capacity within the members.

The stability checks depends on the loading situation of the studied member and there are different parts in Section 6.3 in EN1993-1-1 (2005) that deals with this. This thesis project has studied both columns subjected only to an axial force and a frame subjected to axial forces. As mentioned in Section 1.2, the thesis project is limited to a frame with rigid corner connections and thereby will all individual members be subjected to a combination of bending and axial compression. An introduction to the stability check for uniform compressed members is made followed by a chapter regarding members subjected to a combination of axial compression and bending.

3.7.1 Uniform members in compression

The most commonly used method to check the member stability for members subjected to uniform axial compression is the buckling curve method. The foundation of the method lies in calculating a reduction factor due to buckling, that will reduce the ultimate strength of the member and provide a design buckling resistance. The background for the reduction factor goes back to 1978, when the Ayrton-Perry formula was introduced to *Eurocode 3* (Jönsson and Stan, 2017). This approach contained both verification of simple members as well as numerical verification using $1/1000$ of the buckling length as initial bow imperfection.

The Ayrton-Perry formula was initially adopted for geometrically imperfect, elastic pinned columns loaded by uniform compression (Szalai and Papp, 2010). This is described by the differential equation in Equation (3.17).

$$v''(x) + \frac{N_{Ed}}{EI} \cdot (v_0(x) + v(x)) = 0 \quad (3.17)$$

where:

$v_0(x)$ - initial geometrical imperfection

$v(x)$ - lateral deflection

Accordinging Boissonnade et al. (2006), the initial geometrical imperfections is assumed to be half a sine-wave which gives the expression in Equation (3.18).

$$v_0(x) = e_0 \cdot \sin \frac{\pi x}{L} \quad (3.18)$$

where e_0 represent the maximum value of the additional imperfection at the centre of the member ($x=L/2$).

When the compressive axial load is applied to the member it will experience an additional deflection related to instability, defined as v for pinned members in Equation (3.19).

$$v(x) = a \cdot \sin \frac{\pi x}{L} \quad (3.19)$$

The lateral deflection, v , is found by solving the differential equation in Equation (3.17). First, by inserting both equations expressing v_0 and v into Equation (3.17) it will provide an expression for the amplitude, a , as follows in Equation (3.20).

$$a = \frac{N_{Ed}}{N_{cr} - N_{Ed}} \cdot e_0 \quad (3.20)$$

where N_{cr} is the elastic critical buckling load. Then the total, and maximum, second-order lateral deflection for a simply supported column can be expressed as in Equation (3.21). Note that the factor in front of e_0 is the amplification factor K , see Equation (3.4).

$$v_{max} = \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} \cdot e_0 \quad (3.21)$$

This can be illustrated through Figure 3.9 where e_0 stands for the initial bow imperfection, while v_{max} stands for the maximum deformation.

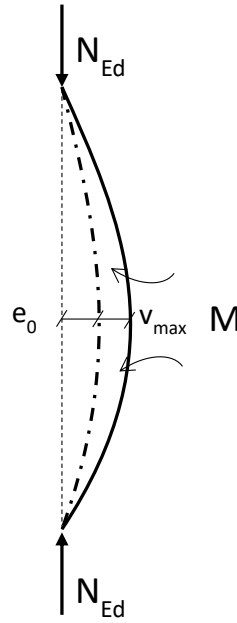


Figure 3.9: Bow imperfection and deflection illustrated for a member.

The cross-section yields when following expression in Equation (3.22) is fulfilled (Szalai and Papp, 2010).

$$\frac{N_{Ed}}{A} + \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} \cdot \frac{N_{Ed} \cdot e_0}{W} = f_y \quad (3.22)$$

By expressing Equation (3.21) into (3.22), the original form of the Ayrton-Perry formula arises according to Equation (3.23) using the reduced slenderness definition that $\bar{\lambda}^2 = N_{Rk}/N_{cr}$.

$$(1 - \chi) \cdot (1 - \chi \bar{\lambda}^2) = e_0 \cdot \frac{A}{W} \cdot \chi = \eta \cdot \chi \quad (3.23)$$

In Equation (3.23), χ is the buckling reduction factor. As expressed in Boissonnade et al. (2006) the variable η represents the generalised initial imperfection and is presented in Equation (3.24).

$$\eta = \alpha \cdot (\bar{\lambda} - 0.2) \quad (3.24)$$

where $\bar{\lambda}$ is the non-dimensional slenderness. By finally inserting this expression for η into Equation (3.23), the reduction factor due to buckling can be found for the minimum solution. This expression for χ is the same that is used today in EN1993-1-1 (2005) for member instability due to compression.

Solving for e_0 gives Equation (3.25).

$$e_0 = \left(\frac{1}{\chi} - 1\right)(1 - \chi \bar{\lambda}^2) \frac{W}{A} \quad \text{or} \quad e_0 = \alpha \cdot (\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}} \quad (3.25)$$

According to EN1993-1-1 (2005), a uniform member subjected to compression should be verified as follows in Equation (3.26).

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1 \quad (3.26)$$

where $N_{b,Rd}$ is the design buckling resistance.

For cross-section class 1-3 the design resistance can be calculated according to Equation (3.27).

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad (3.27)$$

The reduction factor due to buckling, χ , that comes from the Ayrton-Perry formula and are used today in *Eurocode 3*, can be expressed as in Equation (3.28):

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}, \quad \text{but } \chi \leq 1 \quad (3.28)$$

where:

$$\phi = 0.5[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 for cross-sections}$$

where α is the imperfection factor.

The buckling curves are based on cross-section type, thickness of the plates within the cross-section, yield strength and buckling axis. These attributes implies different "imperfection classes" which symbolise grades of imperfection of the member. To find relevant buckling curve, see Figure 3.10. Eurocode presents five different buckling curves; a_0 , a , b , c and d . Buckling curve a_0 is the one which gives the lowest reduction due to imperfections while curve d provides the highest. These five curves are illustrated in Figure 3.11, where the reduction factor due to buckling also can be found instead of using Equation (3.28).

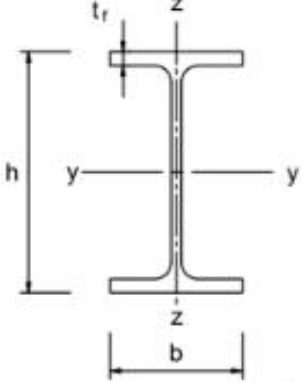

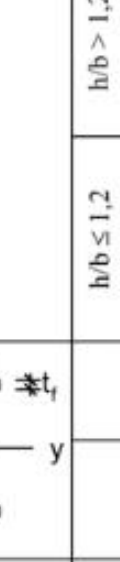
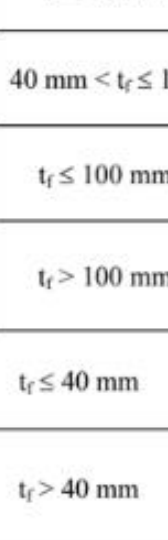
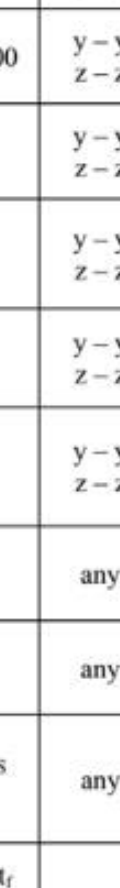
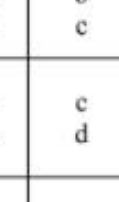
Cross section	Limits	Buckling about axis	Buckling curve		
			S 235 S 275 S 355 S 420	S 460	
Rolled sections 	$h/b > 1,2$	y-y z-z	$t_f \leq 40$ mm	a b	a ₀ a ₀
			$40 < t_f \leq 100$	b c	a a
	$h/b \leq 1,2$	y-y z-z	$t_f \leq 100$ mm	b c	a a
			$t_f > 100$ mm	d d	c c
Welded I-sections 	$t_f \leq 40$ mm	y-y z-z	b c	b c	
	$t_f > 40$ mm	y-y z-z	c d	c d	
Hollow sections 	hot finished	any	a	a ₀	
	cold formed	any	c	c	
Welded box sections 	generally (except as below)	any	b	b	
	thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c	
U-, T- and solid sections 		any	c	c	
L-sections 		any	b	b	

Figure 3.10: Determination of buckling curve (EN1993-1-1, 2005).

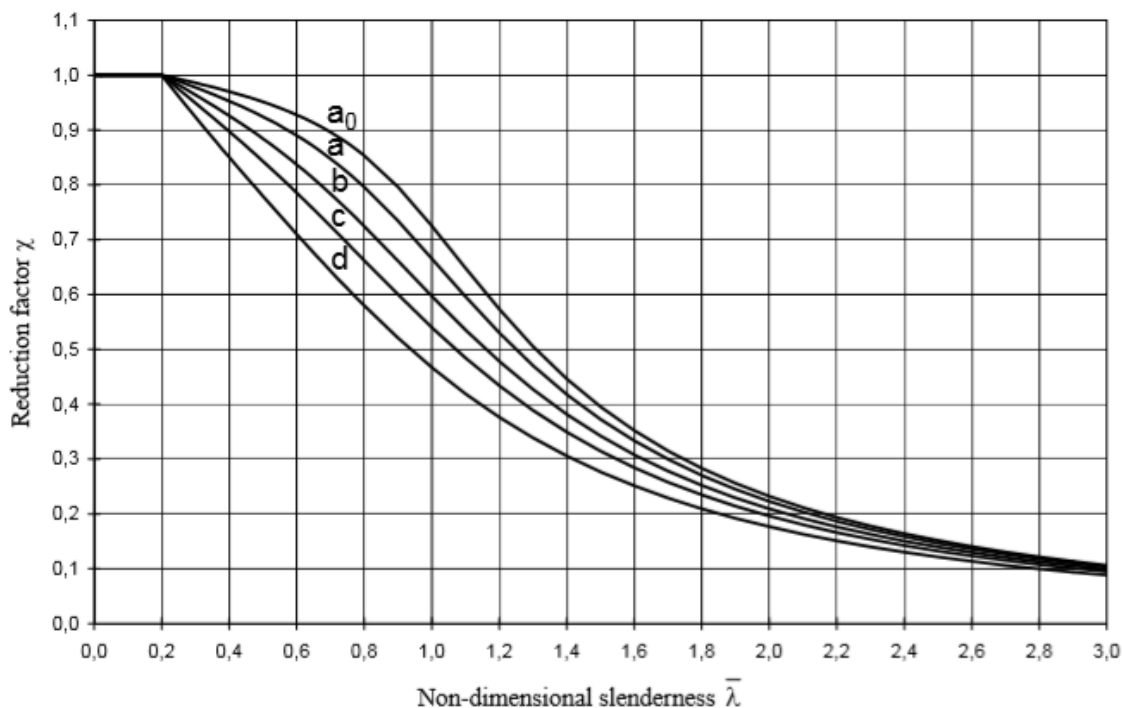


Figure 3.11: Relationship between non-dimensional slenderness and buckling reduction factor for the five different buckling curves (EN1993-1-1, 2005).

The imperfection factor that depends on the buckling curves is found in Table 3.3.

Table 3.3: Imperfection factors for buckling curves (EN1993-1-1, 2005).

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0.13	0.21	0.34	0.49	0.76

To check whether the control of buckling resistance is necessary, following conditions are used:

$$\bar{\lambda} \leq 0,2 \quad \text{or} \quad \frac{N_{Ed}}{N_{cr}} \leq 0,04$$

If these conditions are fulfilled, it means that there is no need for check of the buckling resistance and then only cross-sectional checks will be necessary. But if this is not the case, the buckling effects have to be considered.

3.7.2 Uniform members in bending and axial compression

A member subjected to uniform axial compression can be verified regarding stability according to Equation (3.22), allowing for an initial imperfection $e_{0,d}$ (Boissonnade et al., 2006). Another way of expressing this is according to Equation (3.29).

$$\frac{N_{Ed}}{N_{Rd}} + K \cdot \frac{N_{Ed}e_{0,d}}{M_{Rd}} \leq 1 \quad (3.29)$$

where K is the amplification factor.

If the member is subjected to an additional first-order moment, it can be extended to Equation (3.30). Also, the additional moment give cause to second-order effects.

$$\frac{N_{Ed}}{N_{Rd}} + K \cdot \frac{N_{Ed}e_{0,d}}{M_{Rd}} + \frac{M_{Ed,max}^{II}}{M_{Rd}} \leq 1 \quad (3.30)$$

where $M_{Ed,max}^{II}$ is the maximum second-order bending moment induced by the additional first-order bending moments

Equation (3.30) represent a second-order cross-section check to be used in the most critical section. However, because of the additional moment the most critical section may not be straightforward to find. Therefore, is $M_{Ed,max}^{II}$ approximated by a sinusoidal amplified equivalent moment, which is illustrated in Figure 3.12 and expressed as in Equation (3.31). The equation consists of the equivalent moment $C_m M_{Ed,max}$ that is a sinusoidal first-order bending moment.

$$M_{Ed,max}^{II} = K \cdot C_m M_{Ed,max} \quad (3.31)$$

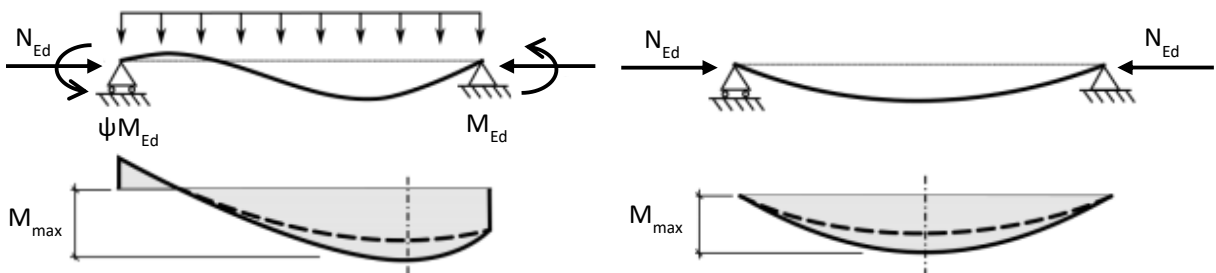


Figure 3.12: Actual second-order moment and corresponding equivalent sinusoidal one (Boissonnade et al., 2006).

This means that an elastic second-order check of the most loaded cross-section of the member can be expressed as in Equation (3.32) according to Boissonnade et al. (2006).

$$\frac{N_{Ed}}{N_{Rd}} + K \cdot \frac{N_{Ed}e_{0,d}}{M_{Rd}} + K \cdot \frac{C_m M_{Ed,max}}{M_{Rd}} \leq 1 \quad (3.32)$$

The basic buckling interaction formula can then be derived, by using Equation (3.32) together with Equation (3.25) and by multiplying both sides with $(1 - \frac{N_{Ed}}{N_{cr}})$. Then, a expression according to Equation (3.33) is achieved.

$$\left(1 - \frac{N_{Ed}}{N_{cr}}\right) \frac{N_{Ed}}{N_{Rd}} + \frac{N_{Ed} \left(\frac{1}{\chi} - 1\right) (1 - \chi \bar{\lambda}^2) \frac{W}{A}}{M_{Rd}} + \frac{C_m M_{Ed,max}}{M_{Rd}} \leq 1 - \frac{N_{Ed}}{N_{cr}} \quad (3.33)$$

Which can be rewritten as:

$$\left(1 - \frac{N_{Ed}}{N_{cr}}\right) \frac{N_{Ed}}{N_{Rd}} + \frac{N_{Ed} \left(\frac{1}{\chi} - 1\right) (1 - \chi \bar{\lambda}^2)}{N_{Rd}} \leq 1 - \frac{N_{Ed}}{N_{cr}} - \frac{C_m M_{Ed,max}}{M_{Rd}}$$

Inserting that $N_{cr} = \frac{N_{Rd}}{\bar{\lambda}^2}$ leads to:

$$\left(1 - \frac{N_{Ed} \bar{\lambda}^2}{N_{Rd}}\right) \frac{N_{Ed}}{N_{Rd}} + \frac{N_{Ed} \left(\frac{1}{\chi} - 1\right) (1 - \chi \bar{\lambda}^2)}{N_{Rd}} \leq 1 - \frac{N_{Ed} \bar{\lambda}^2}{N_{Rd}} - \frac{C_m M_{Ed,max}}{M_{Rd}}$$

Expanding the expression leads to:

$$\frac{N_{Ed}}{N_{Rd}} - \frac{N_{Ed}^2 \bar{\lambda}^2}{N_{Rd}^2} + \frac{N_{Ed}}{\chi N_{Rd}} - \frac{N_{Ed} \bar{\lambda}^2}{N_{Rd}} - \frac{N_{Ed}}{N_{Rd}} + \frac{N_{Ed} \bar{\lambda}^2 \chi}{N_{Rd}} - 1 + \frac{N_{Ed} \bar{\lambda}^2}{N_{Rd}} \leq - \frac{C_m M_{Ed,max}}{M_{Rd}}$$

Some terms cancel each other out and it is possible to write the expression as Equation (3.34).

$$\left(-1 + \frac{N_{Ed}}{\chi N_{Rd}}\right) \left(1 - \frac{N_{Ed} \bar{\lambda}^2 \chi}{N_{Rd}}\right) \leq - \frac{C_m M_{Ed,max}}{M_{Rd}} \quad (3.34)$$

Then, by defining:

$$k_c = \frac{1}{\left(1 - \frac{N_{Ed} \bar{\lambda}^2 \chi}{N_{Rd}}\right)}$$

The expression becomes:

$$\left(-1 + \frac{N_{Ed}}{\chi N_{Rd}}\right) \cdot \frac{1}{k_c} \leq - \frac{C_m M_{Ed,max}}{M_{Rd}}$$

Which can be rewritten into the final expression according to Equation (3.35), defining that $k = k_c \cdot C_m$.

$$\frac{N_{Ed}}{\chi N_{Rd}} + k_c \cdot \frac{C_m M_{Ed,max}}{M_{Rd}} = \frac{N_{Ed}}{\chi N_{Rd}} + k \cdot \frac{M_{Ed,max}}{M_{Rd}} \leq 1 \quad (3.35)$$

This equation represents the interaction formula for in-plane buckling in the strong direction. This ends up as a part of the final interaction formulas presented in Section 6.3.3 in EN1993-1-1 (2005). The final interaction formula can be divided into two equations, Equation (3.36) and (3.37), depending on the direction of the bending.

$$\frac{N_{Ed}}{\chi_y \frac{N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \quad (3.36)$$

$$\frac{N_{Ed}}{\chi_z \frac{N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \quad (3.37)$$

where:

k_{ii} - interaction factors

χ_{LT} - reduction factor due to lateral torsional buckling

$\Delta M_{i,Ed}$ - moments around y- or z-axis due to the shift of the centroidal axis for class 4

The characteristic normal force and moment resistance can be expressed according to Equations (3.38) and (3.39) respectively.

$$N_{Rk} = f_y A_i \quad (3.38)$$

$$M_{i,Rk} = f_y W_i \quad (3.39)$$

These sectional properties can be found in Table 3.4 along with the moments due to shift of the centroidal axis.

Table 3.4: Cross-sectional properties and moments depending on cross-section classes (EN1993-1-1, 2005).

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y} N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z} N_{Ed}$

This design procedure are based on linear-elastic conditions where the concept is that the axial force and the bending moment are being linearly summed, according to Boissonnade et al. (2006). Then, the non-linear effects are taken into account by the interaction factors that can be recognised as, k_{ij} , in the formulas in Equation (3.36) and (3.37). The index i and j represent the directions. The interaction factors can be calculated by two different approaches, often referred to as *Method 1* and *Method 2* (Boissonnade et al., 2006). In this thesis, only *Method 2* was treated and thereby was the formulas derived according to this method. According to Greiner and Lindner (2006) this method is derived to give higher accuracy for in-plane buckling while it approximates out-of-plane buckling. Therefore, this method was more suited since the analyses were limited to only study in-plane buckling behaviour. When determining the interaction factors there is a distinct difference between if elastic or plastic cross-sectional response are used in the model.

4

Models and Analyses

To be able to understand the design process for a system of members, it is of importance to first understand the theory and concept on a member level. Therefore has the analyses in this thesis been divided in to a member level and system level study. The first study is performed on a pinned column and compares application methods of imperfections as well as how to consider both imperfections and second order effects. The second study is performed on a sway-frame, where the design process presented in Chapter 3 is applied and *Approach (a)*, *(b)* and *(c)* are compared. The studies performed can be summarised as following:

- Study I - Pinned column
 - a) Comparison of different methods to implement initial equivalent imperfection, either by using geometric modelling or equivalent loads.
 - b) Comparison of different methods to consider both imperfections and second-order effects.
- Study II - Sway frame
Comparison of the different approaches on how to implement imperfections and second-order effects.

4.1 Input data for FE-modelling

The software that has been used to perform the Finite Element (FE) analyses is the commercially available Abaqus. Abaqus is made for FE-analysis and computer-aided engineering. To generate and reach the final model, routines have been written in the scripting language Python. The routines are implemented by using the Abaqus Scripting Interface which is an object oriented extension library. During the analysis, a user's guide have been used provided by SIMULIA (2014).

4.1.1 Elements

All models created, both the column and the frame, have been made with B21 beam elements. B21 represents that there is a 2 node linear beam in the plane. Beam elements were chosen mainly because of the ability to capture large displacements and rotations in a non-linear analysis, which was of interest in this thesis. One more advantage is also that it is more time efficient to use beam elements compared to shell elements or solids, which requires more

computational effort. By achieving a short computational time, it allows the designer to create the model by trial and error which could save a lot of time. Beam elements are a simplification or approximation of the actual model. Abaqus uses the theory of 3D continuum elements and replaces them with a 1D theory line. This is illustrated for both the column and the frame in Figure 4.1 and 4.2 respectively.

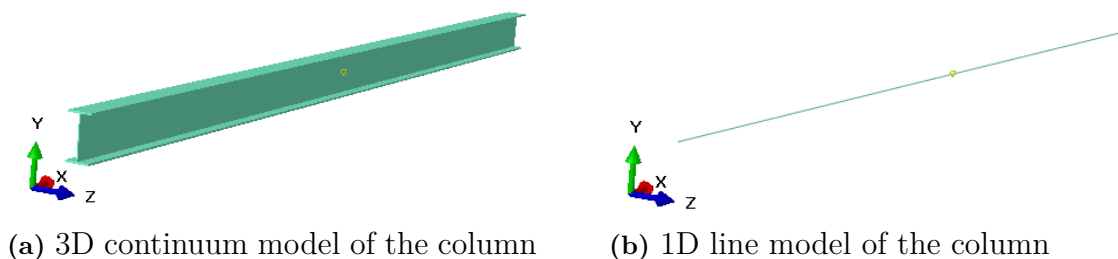


Figure 4.1: Approximation of a 3D continuum element to a 1D theory line model for a column.

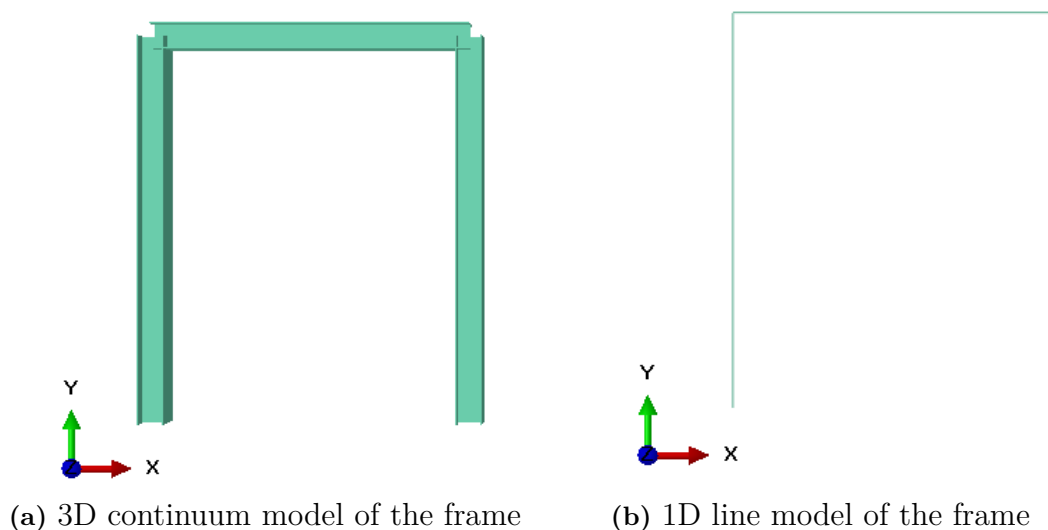


Figure 4.2: Approximation of a 3D continuum element to a 1D theory line model for a frame.

One of the first choices to make in Abaqus when creating a model is what type of part that the model should consist of. The models have all been based on 2D planar, deformable wires.

4.1.2 Material properties and geometry

The material properties introduced to the models are taken as characteristic values. Young's modulus, E , were set to 210 GPa and Poisson's ratio, ν , were set to 0.3. By this, isotropic linear-elastic material behaviour were modelled. Thereby, no plasticity was implemented in the analyses.

In this thesis, the standard profile IPE300 was used for both *Study I* and *Study II*. When modelling the cross-section in Abaqus there are two ways of doing this when using beam elements. Either, the cross-section is modelled as a generalised profile or as a specific chosen profile. The specific chosen profile would in this case be an I-section. The cross-section in *Study II* was modelled as generalised, which implies that the rolled parts were considered. This was chosen since the analytical calculations in this study uses sectional constants where the rolled parts are included. In *Study I* a specific chosen profile was used, which implies that the rolled parts of the sections were neglected. This effects the results from the FE-analysis in this study, but not the comparisons. That is because there was no comparison between the results obtained from FE-analysis with analytical calculations and it was therefore possible to use this type of profile which could appear easier to implement.

The cross-sections were then created by inserting corresponding values for an IPE300 profile as seen in Figure 4.3 for the analyses.

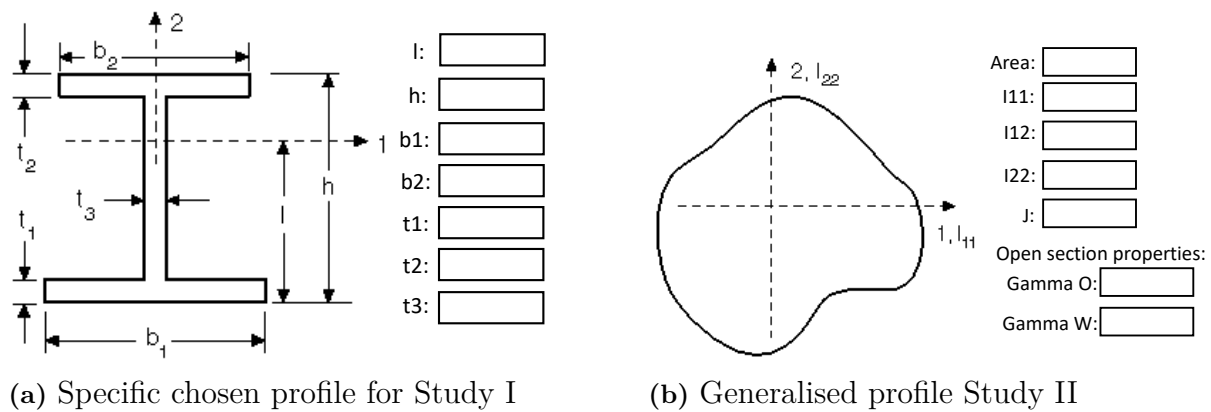


Figure 4.3: Input parameters for a specific chosen profile and a general profile in Abaqus.

The parameters visualised in Figure 4.3 are specified with numerical values in Table 4.1. Since the studies was limited to in-plane behaviour, only parameters relevant for this are presented.

Table 4.1: Section properties for an IPE300 profile.

h	b1	b2	t1	t2	t3	A	I11
[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm^2]	[mm^4]
300	150	150	10.7	10.7	7.1	5381	83.56

4.1.3 Loads and boundary conditions

The loading and boundary conditions that have been used for the two separate models are visualised in Figure 4.4(a) for the column and in 4.4(b) for the frame.

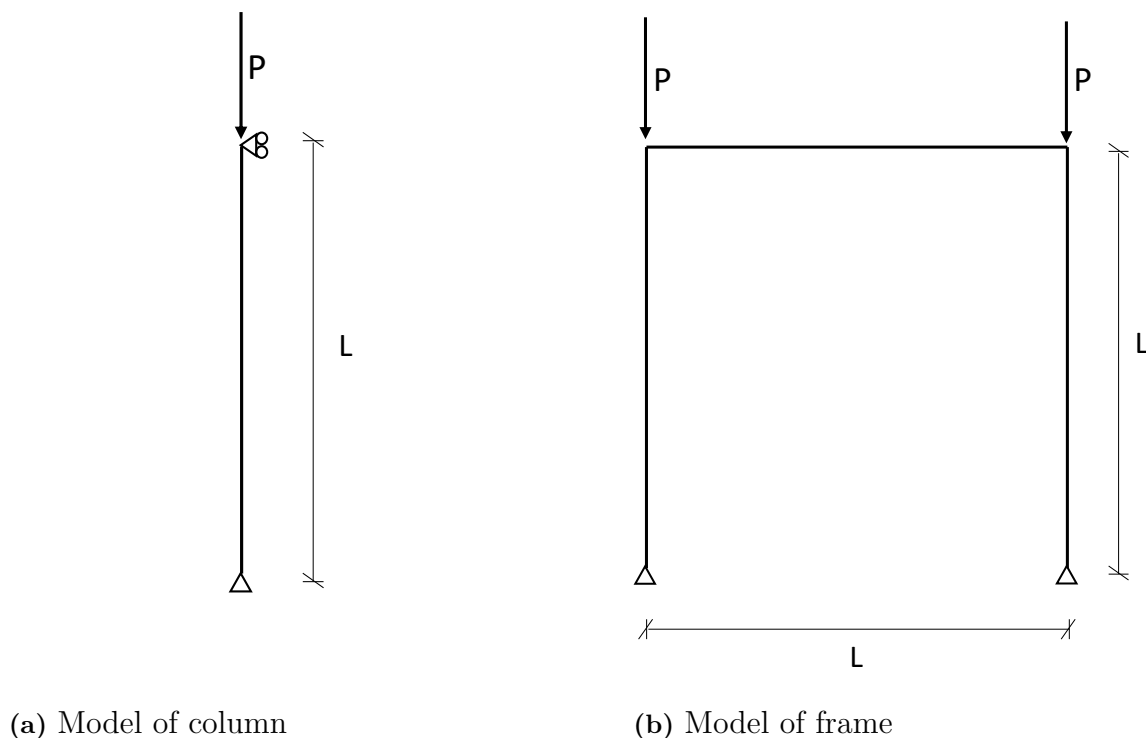


Figure 4.4: Models in Abaqus with corresponding support and loading conditions.

For the column the boundary conditions can be considered as pinned, keeping all rotations free and preventing one end to translate upwards and preventing both ends to translate sideways. The boundary conditions for the frame can also be considered pinned at the supports. However, the translations are prevented upwards and sideways for both supports. Just as for the column all rotations are free. The corners of the frame are modelled as fully moment resistant. The type of boundary condition that was chosen in Abaqus was named *displacement/rotations* and they were applied in the end-nodes of each column.

The loads that have been applied on both structures are axial loads. For the frame there are two concentrated axial loads on each column. When using beam elements, it is not possible to apply the loads to the whole surface of the cross-section, but instead the loads are applied to nodes. The magnitude of the applied point load P is step-wise increased, where a load proportionality factor of 1.0 corresponds to $P = 500kN$.

4.1.4 Mesh

A mesh convergence study was performed to find a sufficient mesh size that provides both efficient and converged results from the analyses. Non-linear analysis was executed for both the column and the frame where the deflection was studied in both cases. It was decided that the wanted mesh was found when the relative error between the finest studied mesh and the other meshes were smaller than 1.0%. The result showing the mesh convergence study can be found in Appendix A. The study showed that appropriate mesh size, which has been used throughout the analyses, for the column and for the frame, was:

$$\text{Mesh size} = \frac{\text{length of member}}{\text{number of elements}} = 0.05$$

4.1.5 Second-order analysis

In Abaqus there are two different ways to execute a second-order analysis. Either to use the step called *Static, General* or the one called *Static, Riks*. The one that has been used in these analyses is *Static, Riks*. A set value in the non-linear step called *NLGEOM* was activated to account for geometrical non-linearities in the analyses.

4.2 Study I

As mentioned, *Study I* can be divided into two substudies. The first one considers the evaluation and comparison on how to implement initial imperfections according to 5.3.2 in EN1993-1-1 (2005) in a numerical second-order analysis. The second one is a comparison of methods to consider both imperfections and second-order effects analytically. In Figure 4.4(a) the model over the studied column is shown.

In *Study I a)*, the two investigated methods for how to model imperfections were geometric modelling and equivalent loads. The length of the studied members have been determined by varying the slenderness of the structure. The focus in this study was to investigate members with intermediate slenderness, because that is when the second-order effects have the largest influence on the member behaviour. Therefore, studies were made for different slendernesses close to intermediate slenderness which is when $\bar{\lambda} = 1.0$. The chosen slendernesses to use in the the study then ended up with values of 0.8, 1.0 and 1.2. These values were then inserted into Equation (4.1) to obtain the lengths of each element to be used in the analysis.

$$L = \sqrt{\frac{\bar{\lambda}^2 \pi^2 E I_y}{A f_y}} \quad (4.1)$$

To geometrically model the initial bow imperfection, both magnitude and shape should be according to Figure 3.8(a). The magnitude of the initial bow imperfection was received from Table 3.2 in Section 3.3.2. Since the studied column is uniform and the boundary conditions are pinned, the bow shape of the imperfection happens to correspond to the first buckling mode of the column. A stability analysis was performed and as a result from that, the first buckling mode was saved. This was then used to model the imperfection by scaling the buckling mode with the magnitude of the imperfection, creating the correct shape and magnitude

as input data to the second-order analysis.

When the method of equivalent loads was used the implementation was made by introducing the fictitious loads, see Figure 3.8, along with the axial force loaded on the column. The fictitious loads are dependent on the load acting on the structure. Therefore should the fictitious loads also be step-wise increased just as the axial load acting on the structure, during the second-order analysis. The lengths used, the values of the equivalent loads and the initial bow imperfections were calculated with a software called Mathcad.

The analytical analysis in *Study I b)* for comparing different methods to consider both imperfections and second-order effects, was made completely analytical by using Mathcad. Lengths of the column that were studied here were lengths of 4, 8 and 10 m and these were chosen so that all methods was allowed to be used The utilisation ratio for three different lengths were calculated for the four different methods. The methods studied were; Buckling curves, Initial bow imperfection, Equivalent loads and EUGLI imperfection. When using the buckling curves, the imperfections are considered within the imperfection factor α that is a part of the final design buckling resistance, $N_{b,Rd}$, of the column. The utilisation was determined through Equation (4.2).

$$u = \frac{N_{Ed}}{N_{b,Rd}} \quad (4.2)$$

The utilisation ratio for the three other methods were determined by using Equation (4.3).

$$u = \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed}^{II}}{M_{Rd}} \quad (4.3)$$

where:

$$M_{Ed}^{II} = N_{Ed} \cdot \frac{e_0}{1 - \frac{N_{Ed}}{N_{cr}}}$$

The part that differs between the methods are the way e_0 is calculated. The magnitude of e_0 highly affects the first-order moment and thereby also the second-order moment M_{Ed}^{II} . When using initial bow and EUGLI imperfection, the magnitude of the imperfections were calculated as follows for both methods:

Initial bow imperfection:
$$e_0 = \frac{L}{300}$$

EUGLI imperfection:
$$e_0 = \alpha(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}}$$

When using equivalent loads, the procedure is made in another way by fictitious loads to the structure. The fictitious loads are dependent on the load that is acting on the structure and is determined according to Equation (4.4) which represent a uniform distributed load. These loads are made to give the same moment on the member as modelling an initial imperfection would give. Of course, these loads will create an deflection, which is not directly correlated to an imperfection.

$$Q_{eq} = \frac{8N_{Ed}e_0}{L^2} \quad (4.4)$$

In the analysis the load is then used to determine the deflection and bending moment that it causes, which are calculated as:

$$e_0 = \frac{5Q_{eq}L^4}{384EI}$$

$$M = \frac{Q_{eq}L^2}{8}$$

So, the final second-order moment for this method consists of one part that is created by a second-order moment due to the deformation caused by the loads, and another moment comes from the equivalent loads acting on the column.

The Mathcad code for the analytical calculations and the Python scripts used for Abaqus, can all be found in Appendix B for *Study I*.

4.3 Study II

Study II is a study of a sway frame, where comparisons were made for the three approaches according to Section 5.2.2(3) in EN1993-1-1 (2005). The three approaches are a totally global approach, a partially global approach and an approach for basic cases. As can be seen in Figure 4.4(b), the boundary conditions are considered pinned i.e. preventing horizontal and vertical motion in both supports. The corners of the frame are considered fully moment resistant. The system is loaded with two concentrated loads and each member of the system has equal lengths L .

In the totally global approach, further referred to as *Approach (a)*, the method with EUGLI imperfection was used to scale the first buckling mode. The buckling mode was scaled in the same way as performed in *Study I*. This study was made by using both the FE-software Abaqus and the software Matlab by interacting the results.

A buckling analysis was initially made in Abaqus to obtain the shape of the first buckling mode, its eigenvalue and the maximum curvature. Then, the EUGLI imperfection was calculated and inserted into the FE-model to perform a second-order analysis. From the analysis, the internal actions were obtained for all elements in each load step. Then the critical element for a certain load step was found, where the utilisation first reached 1.0 (± 0.01). For the investigated case, where only in-plane buckling in the strong direction was considered, the Equation (3.16) for the cross-section check was simplified into Equation (4.5).

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} \leq 1 \quad (4.5)$$

For the corresponding increment the load proportionality factor was obtained. The internal actions were then obtained from that certain element. This approach is summarised in the flowchart shown in Figure 4.5.

Approach (a)

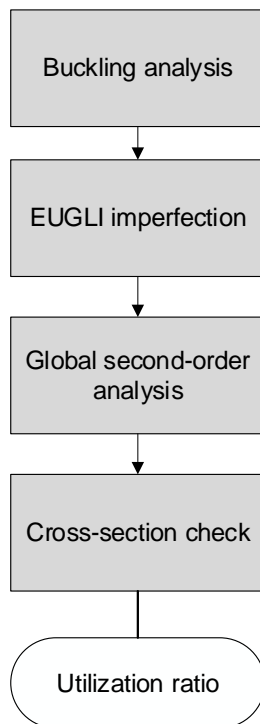


Figure 4.5: Overview of *Approach (a)* - Totally global.

In the next step a partially global approach, *Approach (b)*, was executed. This approach can also be done in two separate ways. Either by using a numerical second-order analysis or an approximate analytical second-order analysis using first order analysis with allowance for second-order effects.

In the numerical second-order analysis, the global sway imperfection was geometrically modelled to the frame. It should be noted that according to the criteria in Section 3.3 the bow imperfection did not need to be implemented.

Then, a global second-order analysis was performed on the frame. To be able to compare the results with *Approach (a)*, the same load proportionality factor was found and the internal actions were taken from the same critical cross-section. The internal actions were then used in the individual stability checks. As for the cross-section check used in *Approach (a)*, the expression in Equation (3.36) was simplified into Equation (4.6).

$$\frac{N_{Ed}}{\chi_y N_{Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rd}} \leq 1 \quad (4.6)$$

In the individual stability checks the equivalent uniform moment factor, C_m , is introduced. Both in EN1993-1-1 (2005) and in Boissonnade et al. (2006) it is expressed that for members

were sway buckling mode occurs, the moment factor should not be determined by the formula depending on the moment diagram but should instead be set to $C_m = 0.9$. The original formula was developed for a simply supported member but the approximate value of C_m is given since the columns within the sway frame have one end unrestrained. The larger the C_m -factor is, the more the moment will effect the utilisation.

In the approximate second-order analysis, the analysis was done in a similar manner, except that the global second-order effects were considered by using an amplification factor in the first-order analysis. The amplifier is used to increase the first-order moments and is the one expressed in Equation (3.3). These two approaches are summarised in Figure 4.6 in the form of a flowchart.

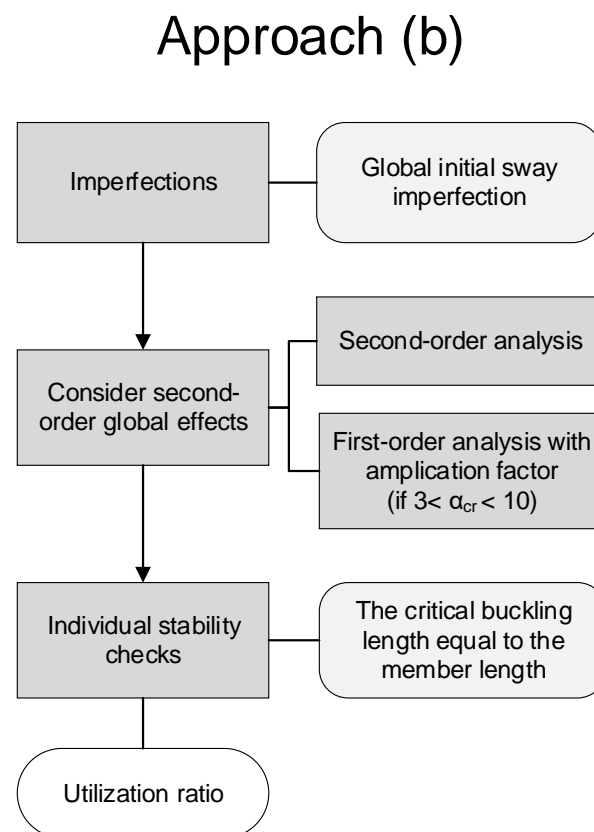


Figure 4.6: Overview of *Approach (b)* - Partially global.

Finally, the approach for basic cases were performed. This approach is referred to as *Approach (c)*. In this approach, a first-order analysis was made considering the sway imperfection of the frame. Just as for *Approach (b)* the bow imperfection could be neglected, according to criteria in 5.3.2(6) EN1993-1-1 (2005). Then a buckling analysis was performed to get the critical buckling length of the system. In the individual stability checks, the system buckling length were used instead of using the actual member length of the critical member as in *Approach (b)*.

In that way, the global and local second-order effects were taken into account in the individual stability checks. This approach is summarised in Figure 4.7.

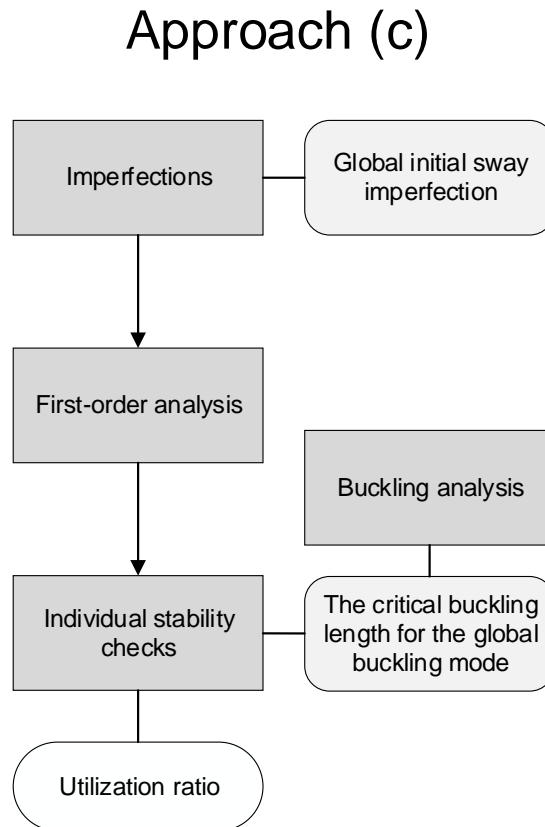


Figure 4.7: Overview of *Approach (c)* - For basic cases.

In Appendix C it is possible to find the Python scripts, the Matlab codes and the Mathcad codes used for *Study II*.

5

Benchmarks

Along with the studies, two benchmarks were used to act as support and to establish reference values. The first benchmark was used to verify the results provided in *Study I* while the second benchmark was used to act as a reference for the implementation of an EUGLI imperfection used in *Study II*. The two benchmarks that were studied and used were a pinned column and a fixed-pinned column. More about the benchmarks are described further under each section respectively.

5.1 Benchmark I: Pinned column

The column that was investigated is a pinned column that Björnsson (2017) studies in his thesis, see Figure 5.1. This column was studied to verify the implementation of equivalent loads as well as to act as a reference for considering both imperfections and second-order in different ways.

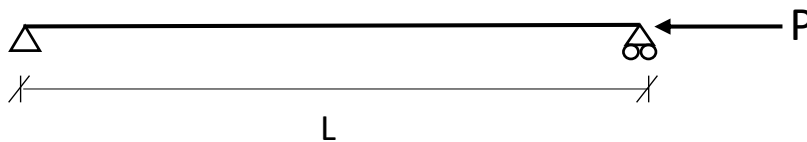


Figure 5.1: Benchmark I: Pinned column with loading and boundary condition.

To begin with, the implementation of equivalent loads was verified by comparing the results with Björnsson (2017). The profile used was the standard profile IPE300 with a steel quality of S235. The length of the column was $5m$. The equivalent loads were implemented as described as in Section 4.2, and the comparison with the results from the analysis by Björnsson (2017) did coincide with less than 1% difference. The verification of the results can be found in Appendix D.

After that, analytical calculations were made to compare the different ways to consider imperfections and second-order effects. The methods investigated was the same studied in *Study I*; buckling curves, initial bow imperfection, equivalent loads and EUGLI imperfection. It was the utilisation ratio that was compared, and the results did coincide with 100% accuracy. The results that was provided in the thesis were made partially made with numerical calculations in Abaqus where the second-order bending moment was calculated, and partially through analytical calculations in Mathcad where the utilisation was obtained. The results of the comparison can be found in Appendix D.

5.2 Benchmark II: fixed-pinned column

In this benchmark, a fixed-pinned column was studied, see Figure 5.2. The original study was made by Baláž and Koleková (2012) and it was used as a benchmark to achieve credibility for the design process of *Study II* concerning EUGLI imperfection.



Figure 5.2: Benchmark II: fixed-pinned column with loading and boundary condition.

The column was modelled with a cross-section of HEB260 profile and has a steel grade of S355. The length of the member is $L = 4.6m$ and the partial safety factor equals $\gamma_{M1} = 1.1$. Flexural buckling around the strong axis was studied and the relevant buckling curve for this column is curve b, with an imperfection factor of $\alpha = 0.34$. The value of Young's modulus of elasticity that was used in the benchmark was not stated and therefore it was assumed to $E = 210GPa$. The axial force, P , acting on the column was determined by setting the utilisation ratio equal to one and by that solve for N_{Ed} equal to the resistance, see Equation (5.1).

$$u = \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed}^{II}}{M_{Rd}} = 1.0 \quad (5.1)$$

It was of interest to study this type of column, not only to verify the design process, but also to investigate a case where the positions of the maximum bow imperfection $e_{0,d,m}$ and the critical cross-section m are not always obvious to find depending on the boundary conditions. Since this column is fixed in one end and pinned in the other end, a non-uniform shape is created when loading the column. By finding the maximum curvature of the first buckling mode, $EI\eta_{cr,max}^{II}$ was obtained and the magnitude of the EUGLI-imperfection could be solved for. However, since the magnitude of the imperfection is dependent on N_{Ed} , an iterative process was used to find when Equation 5.1 was reached. This type of column is also illustrated and discussed in Dallemule (2015).

When studying a pinned column, the critical cross-section is found in the middle of the column due to symmetry. This can be illustrated in Figure 5.3, where different parameters for the critical cross-section are shown.

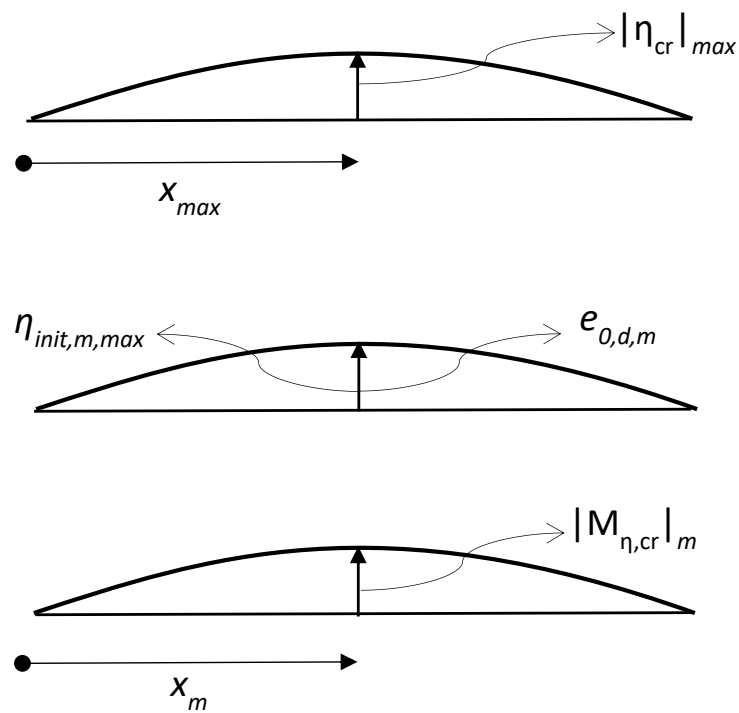


Figure 5.3: Illustration of the first buckling mode shape, its amplitude and the bending moment due to the imperfection shape for a pinned column.

Now, when a fixed-pinned column is studied, the location of $e_{0,d,m}$ is not obvious to find. The same parameters illustrated for the pinned column are now illustrated for the studied fixed-pinned column in Figure 5.4. The dashed vertical line is the location of the inflex point which has a big role in finding the maximum bow imperfection. The benchmark uses an analytical method for finding these parameters, but they can also be found by using FE-analysis as in the analysis made for the comparison with the benchmark.

The first buckling mode shape, $\eta_{cr}(x)$, sets the reference shape for the imperfection shape $\eta_{init,m}(x)$, where the index m stands for the critical cross-section (Dallemler, 2015). Further, the maximum amplitude, $\eta_{init,m,max}$ is dependent on the design value of the initial bow imperfection $e_{0,d}$ and the bending moment due to the imperfection shape, $M_{\eta,cr,m}$.

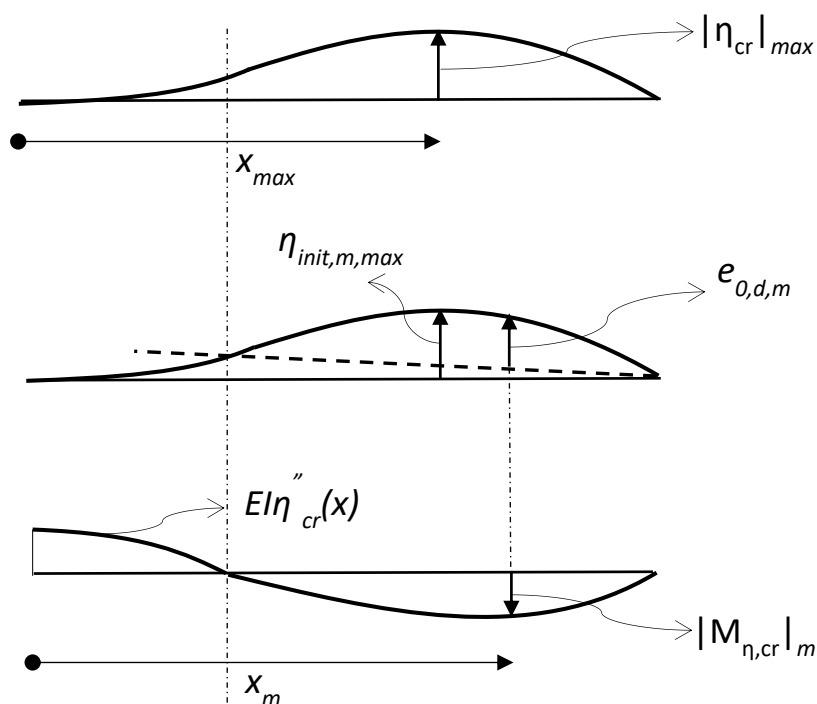


Figure 5.4: Illustration of the first buckling mode shape, its amplitude and the bending moment due to the imperfection shape for a fixed-pinned column.

The results from the study made by Baláž and Koleková (2012) and from the created model in this thesis project are presented in Appendix D. The deviations in the results may be explained by that Baláž and Koleková (2012) did their calculations completely analytically, while some parts conducted in this thesis was made with FE-analysis. The deviation is mainly due to calculation of the curvature $\eta_{cr,max}^{II}$, which was determined analytically by Baláž and Koleková (2012) and by FE-analysis in the thesis. Another possible reason to the deviations that occurred between the results may be because of the assumed Young's modulus.

6

Results

In this chapter, results from the FE-models and the analytical calculations are presented. First, the simply supported column was investigated in FE-analysis for different methods to consider imperfections, either by using initial bow imperfection or by using equivalent loads. Further, different methods to consider imperfections and second-order effects were studied analytically for the same column. Finally, the frame was studied. A parametric study was performed where the slenderness was varied and three different approaches to consider imperfections and second-order effects were compared. This chapter is divided into *Study I* and *Study II* with results corresponding to each study respectively.

6.1 Study I

In *Study I*, two separate approaches on how to introduce equivalent initial imperfections have been studied and compared. The first approach was performed by introducing initial imperfections on an axial loaded column with the shape of a bow imperfection. The second one was performed by introducing equivalent loads onto the column, which should create the same reactions as implementing a bow imperfection. Second-order analysis in the FE-software Abaqus was executed and the different methods have been compared by analysing the load proportionality factor (LPF) in relation to both the deformation and the sectional moment. The results from the analysis were obtained from the centre of the span of the column.

The methods have been analysed for three different slendernesses of 0.8, 1.0 and 1.2. An axial load was chosen so that all three column would be able to reach a load proportionality factor of 1.0. The load that has been used is of magnitude 500 kN. The results for the column with slenderness 0.8, 1.0 and 1.2 are illustrated in Figure 6.1-6.3.

6. Results

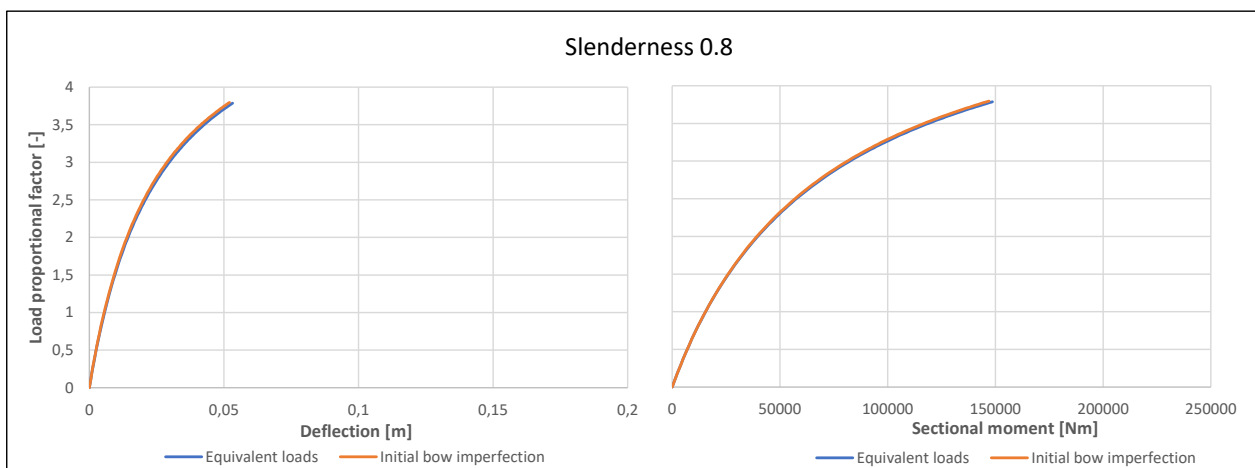


Figure 6.1: Load proportionality factor in relation to the deformation and the sectional moment for a column with non-dimensional slenderness of 0.8.

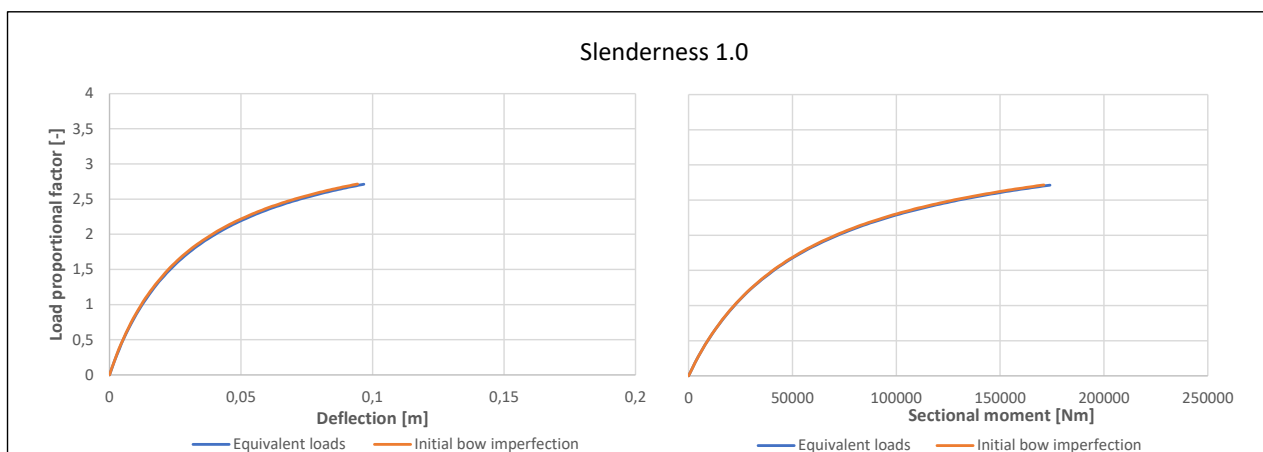


Figure 6.2: Load proportionality factor in relation to the deformation and the sectional moment for a column with non-dimensional slenderness of 1.0.

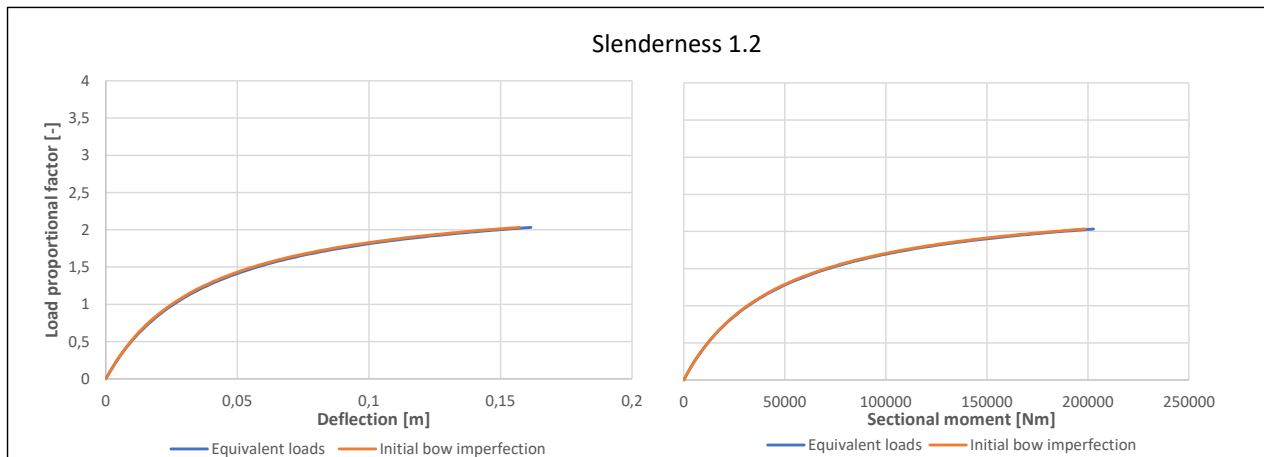


Figure 6.3: Load proportionality factor in relation to the deformation and the sectional moment for a column with non-dimensional slenderness of 1.2.

As visualised in Figure 6.1 - 6.3, it is possible to spot that the curve corresponding to initial bow imperfection is slightly above the curve corresponding to equivalent loads. The difference between the methods for all three columns are displayed in Table 6.1, where it is easier to see the slightly larger difference for the deflection. All values were taken for when the load proportionality factor was 1.0, meaning that the chosen load was fully applied.

Table 6.1: Difference in deflection and sectional moment for initial bow and equivalent load method when having a load proportionality factor equal 1.0.

	Difference: Deflection	Difference: Sectional moment
$\lambda=0.8$	2.69%	0.24%
$\lambda=1.0$	2.67%	0.56%
$\lambda=1.2$	2.72%	0.98%

In the second part of this study the utilisation ratio was compared for different methods to consider imperfections and second-order effects for a simply supported column according to EN1993-1-1 (2005). These comparisons have been made analytically for different length and the ones that have been compared are presented with corresponding result in Table 6.2.

Table 6.2: Comparison of utilisation ratio for different methods to consider second-order effects and imperfections for a column.

Method	L=4m	L=8m	L=10m
	Utilisation [-]	Utilisation [-]	Utilisation [-]
Buckling curves	0.276	0.339	0.415
Initial bow imperfection	0.297	0.344	0.380
Equivalent loads	0.297	0.345	0.381
EUGLI imperfection	0.274	0.305	0.327

The length $L = 4m$ correspond to a slenderness of 0.42, $L = 8m$ of 0.84 and $L = 10m$ to a slenderness of 1.05. The results from all approaches are pretty similar for stocky columns,

even though the methods with initial bow and equivalent loads have some higher utilisation ratio compared to the buckling curves and EUGLI imperfections. The method by using either initial bow imperfection or EUGLI imperfection are relatively similar procedures. Even though, a difference appear between the methods. The difference lies in the magnitude of the imperfection and can be seen Table 6.3.

Table 6.3: Comparison of imperfections depending on using an initial bow imperfection or an EUGLI imperfection.

Method	L=4m Imperfection [mm]	L=8m Imperfection [mm]	L=10m Imperfection [mm]
Initial bow imperfection	13.30	26.70	33.30
EUGLI imperfection	4.80	13.90	18.50

6.2 Study II

In *Study II* were *Approach (a)*, *(b)* and *(c)* analysed and compared. The results of the comparisons are presented in this section. The overview of the approaches, given in Section 4.3, are also presented below in Figure 6.4 for convenience. Firstly, the comparison between the two different ways of accounting for global second-order effects within *Approach (b)* is presented. Then, the main comparison of all the approaches is presented.

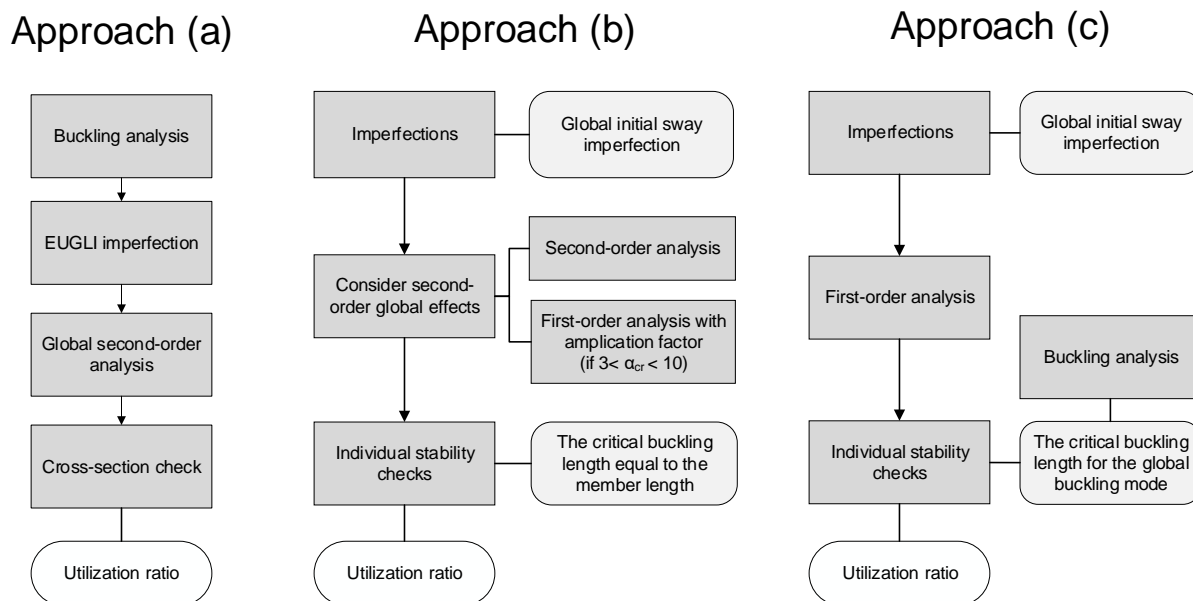


Figure 6.4: Overview of the investigated approaches for a frame, where:

Approach (a) - *Totally global*, *Approach (b)* - *Partially global*, *Approach (c)* - *For basic cases*

As mentioned in Section 4.3 and seen in Figure 6.4, there were two different methods investigated for finding the global second-order effects in *Approach (b)*. It was performed either by using an approximate analytical amplification factor on the first-order moment or by performing a numerical second-order analysis. A comparison of the different methods is presented by studying the internal actions in the critical cross-section, see Table 6.4.

Table 6.4: Comparison of internal actions after considering global imperfections and global second-order effects for the numerical second-order analysis and the approximate analytical second-order analysis.

			Approach (b) Numerical		Approach (b) Analytical	
Length	System slenderness	α_{cr}	N_{Ed} [kN]	M_{Ed} [kNm]	N_{Ed} [kN]	M_{Ed} [kNm]
2m	0.508	4.204	1780	19.6	1780	20.2

As seen in Table 6.4, the moments in the analytical approach are higher than those in the numerical approach. That is because in the analytical second order analysis the moment is

amplified by the amplification factor, which is a conservative factor. It should be noted that the analytical approach is not valid for $\alpha_{cr} \geq 3.0$. If that is the case, a more accurate analysis such as the numerical approach is needed according to 5.2.2 in EN1993-1-1 (2005).

The main comparison of all three approaches was between the utilisation ratio and the system slenderness. The result of the comparison is shown in Figure 6.5. In *Approach (a)* the utilisation of 1.0 (± 0.01) are found at a certain load proportionality factor, and the same load proportionality factor is then used for *Approach (b)* and *(c)* to find their utilisation ratios respectively. This means that only the difference between the approaches is displayed, and not what is on the safe or unsafe side.

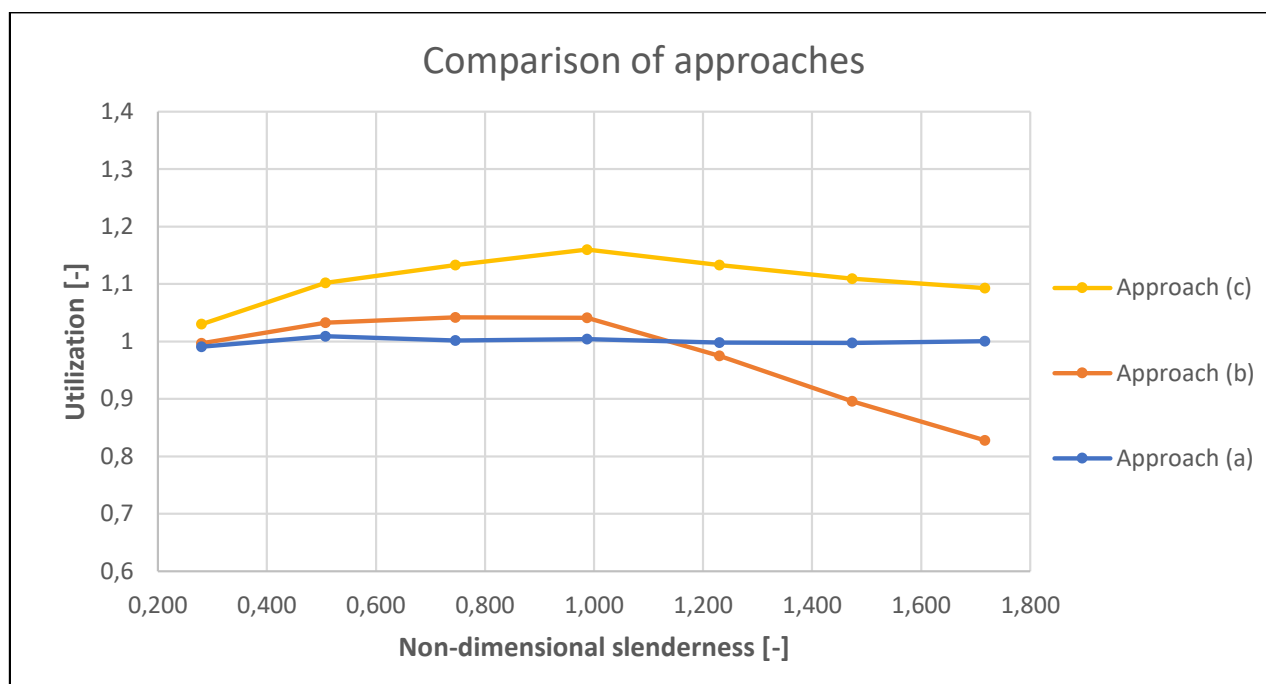


Figure 6.5: Comparison of the approaches presented in *Eurocode 3* to consider imperfections and second-order effects in frame design.

To be able to analyse the result in Figure 6.5 further, results for three different slenderness are evaluated. The chosen slendernesses are illustrated in Figure 6.6. Different parameters are presented for these three slendernesses in Table 6.5 for all three approaches.

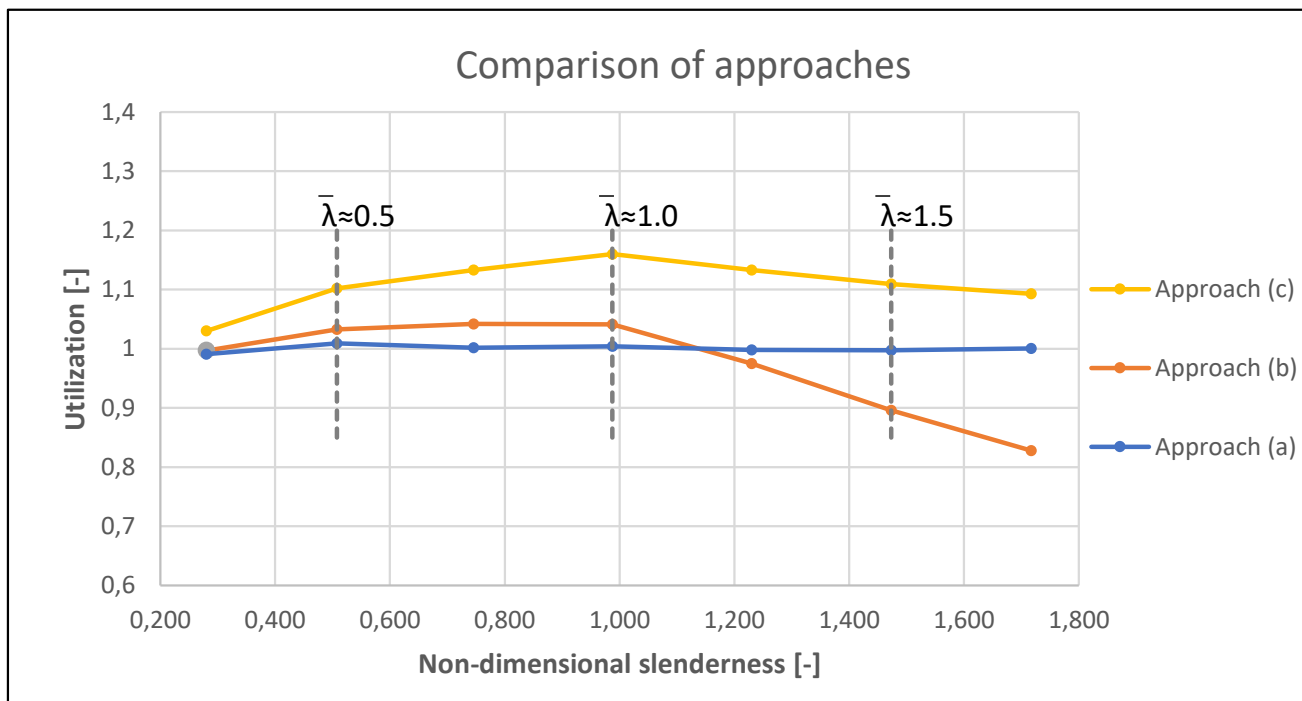


Figure 6.6: The evaluated slendernesses within the comparison of the approaches.

Table 6.5: Output values for the approaches, for three different slendernesses

For all approaches			
	$\bar{\lambda} \approx 0.5$	$\bar{\lambda} \approx 1.0$	$\bar{\lambda} \approx 1.5$
Load proportionality factor [-]	3.53	2.56	1.46
Member length [m]	2	4	6
Approach (a)			
	$\bar{\lambda} \approx 0.5$	$\bar{\lambda} \approx 1.0$	$\bar{\lambda} \approx 1.5$
Max magnitude of EUGLI imperfection [mm]	6.7	17.2	27.7
L_{cr} [m] based on global buckling	4.8	9.4	14.0
$\beta = L_{cr}/L$ [-]	2.42	2.35	2.34
N_{Ed} [kN] from totally global second-order analysis	1780	1310	770
M_{Ed} [kNm] from totally global second-order analysis	15.3	62.8	117.6
Utilisation ratio	1.00	1.00	1.00
Approach (b)			
	$\bar{\lambda} \approx 0.5$	$\bar{\lambda} \approx 1.0$	$\bar{\lambda} \approx 1.5$
Angle sway imperfection [-]	0.0043	0.0043	0.0035
Max magnitude of global sway imperfection [mm]	8.6	17.3	21.2
L_{cr} [m] only member buckling	2.0	4.0	6.0
N_{Ed} [kN] from global second-order analysis	1780	1310	760
M_{Ed} [kNm] from global second-order analysis	19.5	59.2	83.3
k_{yy}	1.01	1.06	1.05
χ	1.00	0.95	0.88
Utilisation ratio	1.03	1.04	0.90
Approach (c)			
	$\bar{\lambda} \approx 0.5$	$\bar{\lambda} \approx 1.0$	$\bar{\lambda} \approx 1.5$
Angle sway imperfection [-]	0.0043	0.0043	0.0035
Max magnitude of global sway imperfection [mm]	8.6	17.3	21.2
L_{cr} [m] based on global buckling	4.8	9.4	14.0
$\beta = L_{cr}/L$ [-]	2.42	2.35	2.34
N_{Ed} [kN] from first-order analysis	1780	1290	735
M_{Ed} [kNm] from first-order analysis	15.4	22.4	15.6
k_{yy}	1.18	1.43	1.44
χ	0.92	0.68	0.39
Utilisation ratio	1.10	1.16	1.11

7

Discussion

This thesis has been divided into two main studies. The first study was to investigate the difference in the methods for considering imperfections and second-order effects for a simply supported column. The second study was to compare different approaches to consider imperfections and second-order effects on a frame structure. This discussion chapter will discuss the simplifications and limitations made during the project, but mainly discuss the coherence between the obtained results from both studies.

7.1 Study I

The equivalent load method is designed to give the same sectional moment as the initial bow imperfection. In a first-order analysis it is therefore easy to see that the sectional moment will be the same in the two different methods. However, the deflection when using the equivalent load method should not be mixed up with the imperfection. The deflection caused by equivalent loads should not be compared to the initial bow imperfection. Instead, the deflection in equivalent load method should be compared to the deflection alone in the geometrical modelling of the initial bow. This can be illustrated as v_{max} in Figure 3.9 not including the initial bow imperfection e_0 .

In a second order analysis the results show that both methods produce similar sectional moment and deflection. It should be observed that in the Figures 6.1-6.3, showing the relation between load proportionality factor and deflection, the initial value of the initial bow imperfection is not included in the x-axis called deflection. Only the deflection when the load is increased is included. Therefore it should be noted that the actual displacement, if the real member has the imperfection designed for, is the initial imperfection added to the deflection.

As mentioned, the results from the methods are very similar, nonetheless it is possible to spot a small difference between them. The difference is slightly larger between the methods when studying the deflection compared to the sectional moment. This applies for all slendernesses that were studied. The difference could be because of that also the fictitious loads are applied in increments and in each step, a new equilibrium is found. It is an approximation of the continuous real imperfection that is applied in the geometrically modelled initial bow imperfection.

The results from comparing methods to consider both imperfections and second-order effects analytically, showed that the method with buckling curves and with EUGLI imperfection almost provides identical results for stocky columns. However, for more slender columns the difference gets larger and they don't correspond as well anymore. The reason for this is be-

cause that for stocky members, the reduction factor due to buckling, χ , becomes close to 1.0. This means that the design buckling resistance $N_{b,Rd} = \chi \cdot N_{Rd}$ almost becomes the design normal force resistance N_{Rd} . Also, the imperfection for a stocky member is relatively small which means that the bending moment will be small. So, the ratio between the second-order moment and the design bending moment resistance is very small for stocky columns, leaving the same expression as when using the buckling curves to determine the utilisation ratio.

When using initial bow imperfection and equivalent loads, the difference in utilisation ratio is negligible. These utilisation ratios are also very similar to the buckling curves and EUGLI for stocky columns. The difference that exist between initial bow and EUGLI is mainly caused by the difference in magnitude of imperfection. The imperfection when using local bow according to Table 3.2 becomes much larger than when using EUGLI imperfection. For a stocky column, $L = 4m$ for example, the imperfection when using EUGLI only becomes around $5mm$, while for the table with initial bow imperfection it becomes around $13mm$.

All the methods' fundamentals are based on the buckling curves. But, in all methods except the one when only using the buckling curves, imperfections are additionally added. The imperfections when using buckling curves are incorporated in the reduction factor for buckling, χ , while for the other methods they are embedded in the second-order bending moment. Both the method with the buckling curves and EUGLI imperfection are based on the parameters λ and α and probably therefore provide similar results. While the equivalent load and initial bow imperfection are based on the same constant value given in Table 3.2, which provides similar results.

It should also be noted that these methods only get similar results since the member studied is considered pinned. First of all, for other boundary conditions it is not possible to use the buckling curves since it is derived for pinned uniformed compressed members. Secondly, further studies needs to be conducted to investigate whether the difference between the results for rest of the methods stays consistent.

7.2 Study II

In the *Approach (b)* a comparison was made between two different ways of accounting for global second order effects. As can be seen in Table 6.4, the moment obtained in the approximated second-order analysis is an overestimation compared to the moment from the numerical second-order analysis. This is reasonable since the amplification factor should be designed to give results on the safe side. In a global elastic analysis the limit of $\alpha_{cr} \geq 3.0$ for sway buckling mode may be because the amplification factor is only applied to the moment, while the normal force is of first-order. However, for more slender frames the normal force will also increase with increased deflection and second-order effects. This might be why the method of using amplification factor is limited to $\alpha_{cr} \geq 3.0$.

The main comparison of *Approach (a)*, *(b)* and *(c)* can be seen in Figure 6.5. It should once again be noted that the results are for a single storey sway frame. It is clear that the approaches differ in result. The results show that for stocky frames the approaches give more similar results compared to the more slender frames. Up to a non-dimensional slenderness

of $\bar{\lambda} = 1.1$, *Approach (a)* is the least conservative one. For slendernesses higher than this, *Approach (b)* becomes the least conservative approach. *Approach (c)* on the other hand is consistently the most conservative approach, which at a intermediate slenderness $\bar{\lambda} \approx 1.1$ gives approximately 16% higher utilisation than *Approach (a)*.

At a non-dimensional slenderness of around 0.5, *Approach (a)* have an EUGLI imperfection with a magnitude of 6.7mm, which is less than the sway imperfection of 8.6mm applied in the other approaches. The EUGLI imperfection takes into account both global and local imperfections and when global and local second-order effects are considered it clearly gives a more favourable result than the other approaches.

At the non-dimensional slenderness around 1.0, the magnitude of the EUGLI imperfection of 17.2mm is similar to the magnitude of the sway imperfection of 17.3mm. This will give the same critical normal force in *Approach (a)* and *(b)*. However, since the shape of the imperfection differs, when using EUGLI imperfection the shape is the first critical buckling mode while for the sway imperfection it is geometrically offsetted. This means that the EUGLI imperfection will be influenced by local bow imperfection as well, and give a higher moment after the second-order analysis. In contrast to *Approach (b)*, only global imperfections and global second-order effects are considered after the second-order analysis. Then, the local imperfections and local second-order effects are considered within the individual stability checks.

At the non-dimensional slenderness of around 1.5 the magnitude of the EUGLI imperfection is larger than the magnitude of the sway imperfection, 27.7mm compared to 21.2mm. Then the moment in *Approach (a)* will be substantially larger than the moment after the global analysis in *Approach (b)*. Even after the local imperfection and second-order effects are considered within individual stability checks in *Approach (b)*, the approach gives a more favourable result.

An observation that was made is that within the calculation of the angle for the sway imperfection there is a height reduction factor, α_h , as presented in Section 3.3.1. The limit interval of the magnitude of the factor is between $2/3 \leq \alpha_h \leq 1.0$. For the study it means that the α_h is of a constant value of 1.0 up to a slenderness of 1.0. That in turn gives a constant imperfection angle of 0.0043 as can be seen in Table 6.5. For slendernesses larger than 1.0 the reduction factor leads to a decrease in angle. This could be a definite explanation to why both the utilisation of *Approach (b)* and *(c)*, decreases in relation to *Approach (a)* for slendernesses above 1.0.

One approximation that was made in the calculation procedure is that a small deviation between the LPF may occur between *Approach (a)* and *(b)*. The maximum error of the LPF is around 0.3%. In the analysis it means that, for the member length of 1.0m, each column in the frame in *Approach (a)* have been loaded with 1852kN but in *Approach (b)* with 1847kN. *Approach (c)* is made with the rounded values of LPFs presented in Table 6.5.

During the calculation process some advantages and disadvantages were discovered within each approach. *Approach (a)* is a straightforward procedure. However, finding the magnitude of the EUGLI imperfection can be difficult. In *Approach (b)* the main difficulty lies in understanding all the parameters within the individual stability check and to know what is considered in each step. On the other hand, the imperfection implemented in the global

analysis is straightforward to determine. As mentioned in Section 3.2, *Approach (c)* should only be used for basic cases. However, the phrase basic cases is not defined by Eurocode and thereby it is up to the designer to interpret the phrase. Within Study II, *Approach (c)* was the most simple to apply, yet gives the most conservative results.

Since it can be tricky to interpret the frame design process regarding stability design in EN1993-1-1 (2005) it is of importance to first create a platform where the most important parts are described and understood. Once the basic knowledge is mastered, then the details can be added. However, for example in the individual stability checks it is not clear what each factor considers and when too many factors are involved, it is easy to lose grip of the mechanics and theory behind the process and thereby may errors occur.

8

Conclusions

This chapter contains conclusions for both studies conducted, followed by general conclusions.

Study I

When using initial bow imperfection or equivalent loads in a second-order analysis it can be concluded that they provide almost identical results regardless of slenderness. For these methods, some application guidance can be given. In equivalent load method it is of importance to step-wise increase not only the applied load but also the fictitious loads. In both methods it is of importance to make sure that the imperfection or fictitious loads are applied in the most unfavourable direction.

From comparing different methods to consider both imperfections and second order effects for a pinned column, some conclusions can be drawn. For all methods it showed that all methods produced similar results for stocky columns, while the difference increases with increased slenderness. However, the methods when using initial bow imperfection and equivalent loads produced similar results regardless of the slenderness.

Study II

From this study, it is possible to conclude that the approaches give more similar results for stocky frames compared to slender ones. *Approach (a)* is considered to be the least conservative approach up to an intermediate slenderness around 1.0, and after that *Approach (b)* is the one that is least conservative. For *Approach (c)*, conclusion can be drawn that this approach is regardless of the slenderness the most conservative approach.

It is also possible to draw conclusion regarding the implementation of the different approaches. The procedure in *Approach (a)* can be seen as relatively clear while *Approach (b)* is divided into two steps which makes it less straightforward. The difficulty with *Approach (a)* is finding the magnitude of the EUGLI imperfection, while for *Approach (b)* the difficulty lies in interpreting and using the individual stability checks. For *Approach (b)*, there is a choice between two different ways to account for global second-order effects. This requires the designer to have knowledge of the main difference in results that these methods provide. The main conclusion between these methods is that for the approximated second-order analysis there is an overestimation of the bending moment compared to the moment from the numerical second-order analysis, which would lead to a more conservative result. The main difficulty with *Approach (c)* is to know when it is possible to apply, since it is only mentioned in *Eurocode 3* that it can be used for "basic cases". Besides that, it is a straightforward approach to use but it should be kept in mind that this approach produces the most conservative results.

General

As a general conclusion, it may be difficult to interpret the design process regarding stability design of steel structures in EN1993-1-1 (2005). Initially, it can be problematic to get a comprehensive impression and to apprehend the link between the parts within the code concerning stability analysis. This thesis has therefore performed a limited study, where only parts of the design code has been interpreted. Due to the chosen limitations for this thesis project, further studies need to be conducted within the subject to be able to draw more general conclusions.

9

Suggestions for Further Studies

The results obtained for the main study, *Study II*, can only be assumed to be valid for the chosen limitations of the frame. To be able to draw more general conclusion for all type of frames, there would need to be more analyses performed. It is preferable to study the approaches further in order to isolate specific effects and thereby be able to draw definite conclusions. In this chapter, studies where other aspects are covered are proposed.

Perform parametric studies:

In this thesis project, different slenderness were studied for the purpose to come up with some conclusions for the different approaches. However, there has not been any investigation of the impact of member profile, neither the type of loading or boundary condition. This means that the results only reflect the chosen case, and cannot be interpreted to all sway frames. Also a similar parametric study, as done in this thesis, except on a frame stabilised against sway would be of interest.

Include plasticity in the analyses:

In all studies, only elastic analyses were performed with elastic cross-sectional response. It would be interesting to know how the effect of plasticity would affect the results.

Study *Method 1* in the individual stability checks:

In the individual stability checks, there is a choice to either use *Method 1* or *Method 2* to determine the interaction factors k_{ii} . *Method 1* could then be used in the analytical analyses and then be compared to the results with *Method 2* to be able to draw conclusions regarding the influence of choice of method.

Study out-of-plane buckling:

This thesis only covered analyses regarding in-plane behaviour. When studying in-plane behaviour there are lot of simplifications made in the design process and the calculations. Therefore it would be good to perform a similar project as this, but studying the of out-of-plane behaviour, and then set up design guidelines for this more complex buckling type.

Find a existing frame to analyse and compare results:

If possible, it would be of interest if physical experiments were conducted on frames, where the initial imperfections were known. Then, more realistic profiles/cross-sections, load combinations and boundary conditions were to be examined.

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A

Mesh convergence study

This Appendix presents the results obtained from the mesh convergence study. The study was made for both the column analysed in *Study I* and for the frame analysed in *Study II*. The results are illustrated in Figure A.1-A.4.

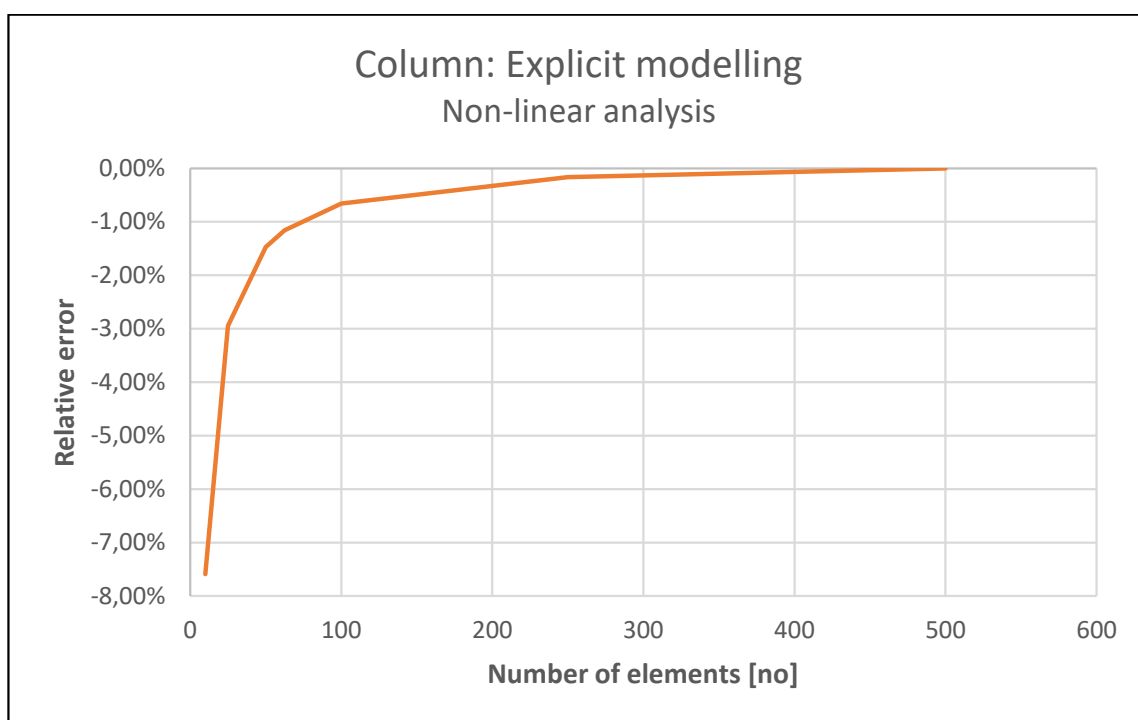


Figure A.1: Mesh convergence study for the column with initial bow imperfection.

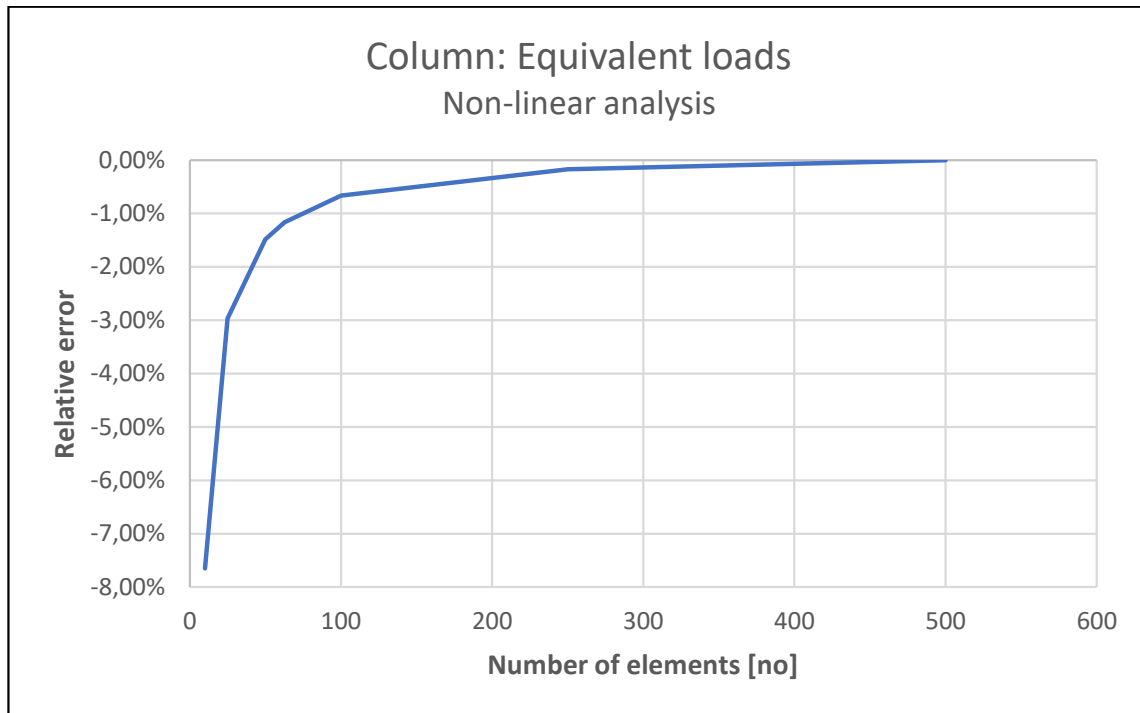


Figure A.2: Mesh convergence study for the column with equivalent load method.

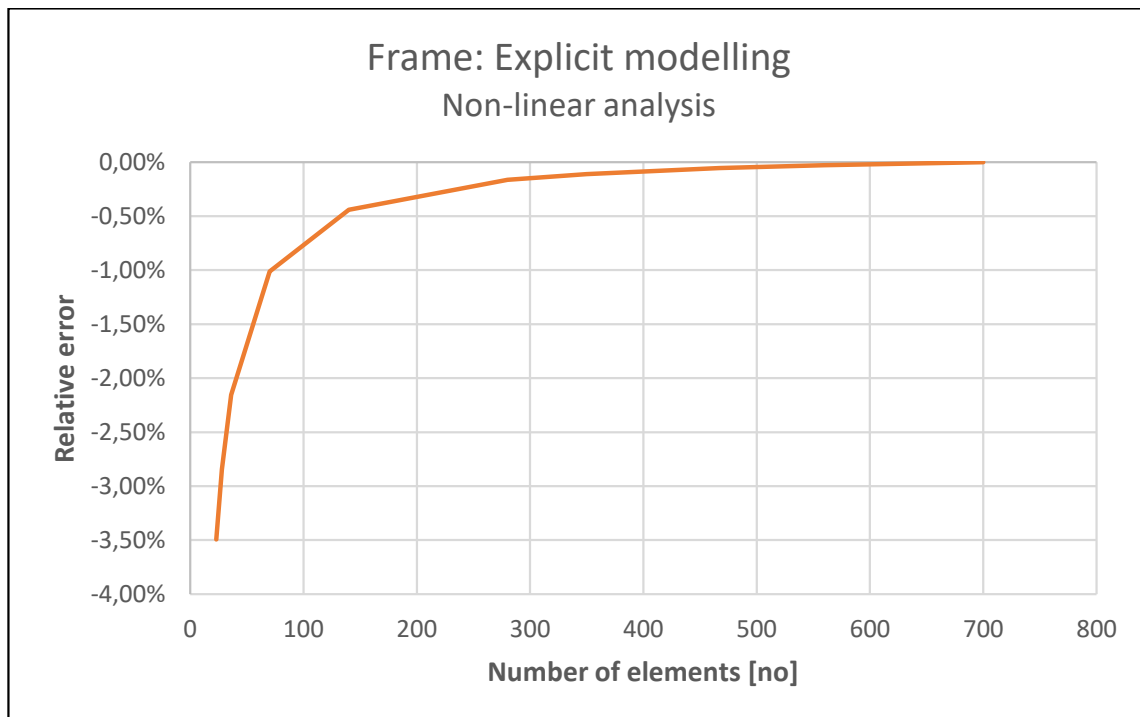


Figure A.3: Mesh convergence study for the frame with initial sway imperfection.

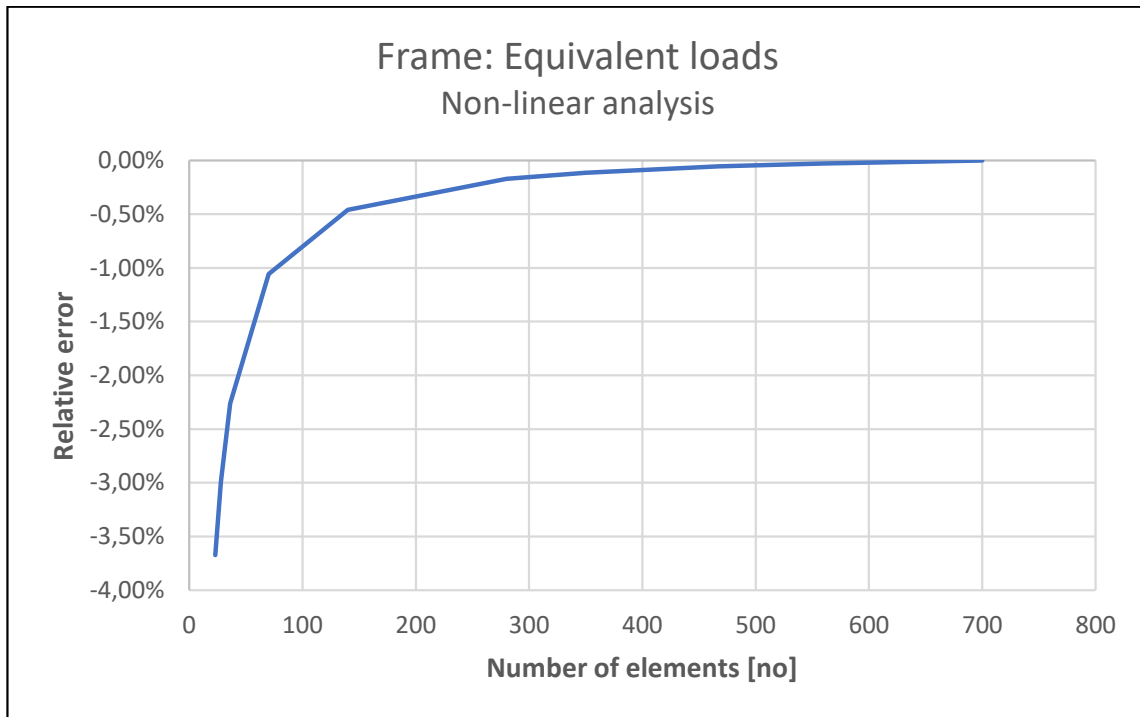


Figure A.4: Mesh convergence study for the frame with equivalent load method.

B

Study I

In this Appendix, all the results from *Study I* are presented. First, the Mathcad code is presented in Section B.1. This code includes the calculations of the imperfections to be used in Abaqus along with the calculation of the utilisation, by using different methods to consider imperfections and second-order effects. Then, two separate Python scripts are added in Section B.2 and B.3. These two scripts are a code to create a column in Abaqus which is exposed to either a initial bow imperfection or exposed to equivalent loads during a second-order analysis.

B.1 Mathcad code

Analyse I

The column is stabilised in the weak direction. Steel quality S355
Elastic cross-section analysis is to be used (assuming CSC 3)

INPUT DATA

IPE 300:

$$h := 300\text{mm}$$

$$t_f := 10.7\text{mm}$$

$$t_w := 7.1\text{mm}$$

$$b_f := 150\text{mm}$$

Steel S355:

$$E := 210\text{GPa}$$

$$\nu := 0.3$$

$$G := \frac{E}{2 \cdot (1 + \nu)} = 80.769 \cdot \text{GPa}$$

$$f_y := 355\text{MPa}$$

Sectional constants:

$$A := 0.005381\text{m}^2$$

y-y direction: STRONG DIRECTION

$$I_y := 0.00008356\text{m}^4$$

$$W_{el,y} := 0.000557\text{m}^3$$

Partial safety factors:

$$\gamma_{M0} := 1$$

$$\gamma_{M1} := 1$$

Axial load:

$$N_{Ed} := 500\text{kN}$$



Study I, part 1

Comparison of two different methods on how to implement initial imperfections.

- Initial bow imperfection
- equivalent loads

Three different slendernesses that were studied:

$$\lambda_1 := 0.8 \quad \lambda_2 := 1.0 \quad \lambda_3 := 1.2$$

$$L_1 := \sqrt{\frac{(\lambda_1^2 \cdot \pi^2 \cdot E \cdot I_y)}{A \cdot f_y}} = 7.617 \text{ m}$$

$$L_2 := \sqrt{\frac{(\lambda_2^2 \cdot \pi^2 \cdot E \cdot I_y)}{A \cdot f_y}} = 9.522 \text{ m}$$

$$L_3 := \sqrt{\frac{(\lambda_3^2 \cdot \pi^2 \cdot E \cdot I_y)}{A \cdot f_y}} = 11.426 \text{ m}$$

$$L := L_1 = 7.617 \text{ m}$$

Initial bow imperfection:

Table 5.1: Design values of initial local bow imperfection e_0 / L

Buckling curve acc. to Table 6.1	elastic analysis	plastic analysis
	e_0 / L	e_0 / L
a_0	1 / 350	1 / 300
a	1 / 300	1 / 250
b	1 / 250	1 / 200
c	1 / 200	1 / 150
d	1 / 150	1 / 100

For buckling curve "a"

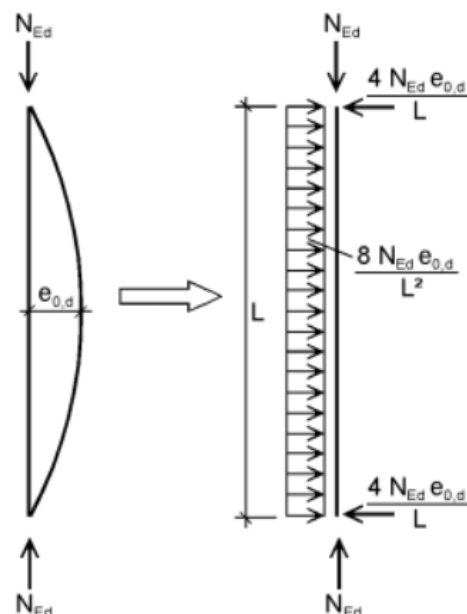
$$e_{0d} := \frac{L}{300} = 25.391 \cdot \text{mm}$$

according to 5.3.2(3b)

Equivalent loads:

$$N_{eq} := \frac{(4 \cdot N_{Ed} \cdot e_{0d})}{L} = 6.667 \cdot \text{kN}$$

$$Q_{eq} := \frac{(8 \cdot N_{Ed} \cdot e_{0d})}{L^2} = 1.75 \cdot \frac{\text{kN}}{\text{m}}$$



Study I, part 2

Study of different ways to consider imperfections and second order effects for a simply supported column.

Different lengths that were studied:

$$L_1 := 4\text{m} \quad L_2 := 8\text{m} \quad L_3 := 10\text{m}$$

$$L := L_1 = 4\text{m}$$

1. Solution according to Eurocode 3 [1993-1-1,6.3] - Buckling curves

The theoretical critical buckling load for the column is:

$$N_{cr} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} = 1.0824 \times 10^4 \cdot \text{kN}$$

This gives a slenderness ratio:

$$\lambda := \sqrt{\frac{A \cdot f_y}{N_{cr}}} = 0.4201$$

The reduction factor with respect to buckling can be calculated using appropriate buckling curve

$BC_{check} :=$	$"bc1" \text{ if } \left(\frac{h}{b_f}\right) > 1.2 = "bc1"$ $"bc2" \text{ otherwise}$
-----------------	---

$b_{curve.y1} :=$	$"a" \text{ if } t_f \leq 40\text{mm} = "a"$ $"b" \text{ if } 40\text{mm} < t_f \leq 100\text{mm}$	bc1
-------------------	---	------------

$b_{curve.y2} :=$	$"b" \text{ if } t_f \leq 100\text{mm} = "b"$ $"d" \text{ otherwise}$	bc2
-------------------	--	------------

For:

- Hot-rolled standard section
- buckling in the strong direction → Curve a
- $h/b > 1.2$
- $t < 40\text{mm}$
- steel quality S355

Table 6.2: Selection of buckling curve for a cross-section

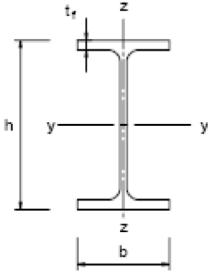
Cross section	Limits	Buckling about axis	Buckling curve	
			S 235 S 275 S 355 S 420	S 460
Rolled sections 	$h/b > 1,2$	y-y z-z	$t_f \leq 40 \text{ mm}$	a a ₀
			$40 \text{ mm} < t_f \leq 100$	b c
	$h/b \leq 1,2$	y-y z-z	$t_f \leq 100 \text{ mm}$	b c
			$t_f > 100 \text{ mm}$	d c

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a ₀	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

The imperfection factor is:

$$\alpha := 0.21$$

$$\Phi := 0.5 \cdot [1 + \alpha \cdot (\lambda - 0.2) + \lambda^2] = 0.611$$

$$\chi := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} = 0.95$$

The load carrying capacity of the column is:

$$N_{bRd} := \chi \cdot A \cdot f_y = 1.81 \times 10^3 \cdot \text{kN}$$

The utilization ratio then becomes:

$$u := \frac{N_{Ed}}{N_{bRd}} = 0.276$$

2. Solution according to 2nd-order theory using Bow imperfection, either according to 5.3.2(3b) or 5.3.2(11):

The magnitude of equivalent initial imperfection depends on:
 - the length of the column
 - the buckling curve

It can be obtained from table 5.1 in 1993-1-1

Table 5.1: Design values of initial local bow imperfection e_0 / L

Buckling curve acc. to Table 6.1	elastic analysis	plastic analysis
	e_0 / L	e_0 / L
a_0	1 / 350	1 / 300
a	1 / 300	1 / 250
b	1 / 250	1 / 200
c	1 / 200	1 / 150
d	1 / 150	1 / 100

$$M_{Rk} := W_{el,y} \cdot f_y \qquad M_{Rd} := W_{el,y} \cdot \frac{f_y}{\gamma_{M1}} = 197.735 \cdot \text{kN} \cdot \text{m}$$

$$N_{Rk} := A \cdot f_y \qquad N_{Rd} := A \cdot \frac{f_y}{\gamma_{M1}} = 1.91 \times 10^3 \cdot \text{kN}$$

For buckling curve "a"

$$e_{0d} := \frac{L}{300} = 13.333 \cdot \text{mm} \qquad \text{OBS!! compare to} \qquad e_0 := \alpha \cdot (\lambda - 0.2) \cdot \frac{M_{Rk}}{N_{Rk}} = 4.784 \cdot \text{mm}$$

according to 5.3.2(3b)

according to 5.3.2(11)

Amplification factor to account for second order effects:

$$\alpha_{cr} := \frac{N_{cr}}{N_{Ed}} = 21.648 \qquad k := \frac{1}{1 - \left(\frac{1}{\alpha_{cr}}\right)} = 1.048$$

According to 5.3.2(3b)

First order moment

$$M_I := N_{Ed} \cdot e_{0d} = 6.667 \cdot \text{kN} \cdot \text{m}$$

Second order moment

$$M_{Ed,II} := N_{Ed} \cdot \frac{e_{0d}}{1 - \frac{N_{Ed}}{N_{cr}}} = 6.99 \cdot \text{kN} \cdot \text{m}$$

$u := \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed,II}}{M_{Rd}} = 0.297$

According to 5.3.2(11)

First order moment $M_I := N_{Ed} \cdot e_0 = 2.392 \cdot \text{kN} \cdot \text{m}$

Second order moment $M_{Ed,II} := N_{Ed} \cdot \frac{e_0}{1 - \frac{N_{Ed}}{N_{cr}}} = 2.508 \cdot \text{kN} \cdot \text{m}$

$$u := \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed,II}}{M_{Rd}} = 0.274$$

3. Solution according to 2nd-order theory using Equivalent loads

$$Q_{eq} := \frac{(8 \cdot N_{Ed} \cdot e_{0d})}{L^2} = 3.333 \cdot \frac{\text{kN}}{\text{m}}$$

$$e_{loads} := \frac{(5 \cdot Q_{eq} \cdot L^4)}{384 \cdot E \cdot I_y} = 0.633 \cdot \text{mm}$$

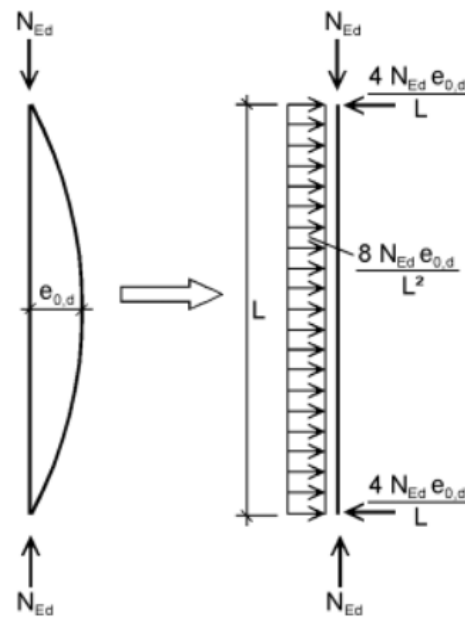
$$M_{loads} := \frac{(Q_{eq} \cdot L^2)}{8} = 6.667 \cdot \text{kN} \cdot \text{m}$$

$$M_{II} := N_{Ed} \cdot \frac{e_{loads}}{1 - \frac{N_{Ed}}{N_{cr}}} = 0.332 \cdot \text{kN} \cdot \text{m}$$

Second order moment

$$M_{Ed,II} := M_{II} + M_{loads} = 6.999 \cdot \text{kN} \cdot \text{m}$$

$$u := \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed,II}}{M_{Rd}} = 0.297$$



B.2 Python code - Equivalent loads

```

1 from abaqus import *
2 from abaqusConstants import *
3 import __main__
4
5 import section
6 import regionToolset
7 import displayGroupMdbToolset as dgm
8 import part
9 import material
10 import assembly
11 import step
12 import interaction
13 import load
14 import mesh
15 import optimization
16 import job
17 import sketch
18 import visualization
19 import xyPlot
20 import displayGroupOdbToolset as dgo
21 import connectorBehavior
22
23 #####
24 ##                               INPUT DATA
25 #####
26 ##Column IPE300
27 L=11.426                               #[m]
28 htot=0.3                               #[m]
29 h_2=htot*(2)**(-1)                     #[m]
30 bf1=0.15                               #[m]
31 bf2=0.15                               #[m]
32 tf1=0.0107                             #[m]
33 tf2=0.0107                             #[m]
34 tw=0.0071                              #[m]
35
36 ##Load
37 F1=500000                              #[N]
38
39 ##Initial bow impefection
40 e0el=L*(300)**(-1)                     #[m]
41
42 ##Equivalent loads
43 Neq=(4*F1*e0el)*L**(-1)                #[N]
44 Qeq=(8*F1*e0el)*(L**2)**(-1)           #[N/m]
45
46
47 #####
48 ##                               GEOMETRY
49 #####
50 mdb.models.changeKey(fromName='Model-1', toName='BeamColumn')
51 s = mdb.models['BeamColumn'].ConstrainedSketch(name='__profile__',
52 sheetSize=20.0)
53 g, v, d, c = s.geometry, s.vertices, s.dimensions, s.constraints
54 ##Setting geometry for an I-profile
55 s.Line(point1=(0.0, 0.0), point2=(L*(2)**(-1), 0.0))
56 s.Line(point1=(L*(2)**(-1), 0.0), point2=(L, 0.0))
57 s.HorizontalConstraint(entity=g.findAt((L*(2)**(-1), 0.0)), addUndoState=False)
58 p = mdb.models['BeamColumn'].Part(name='BeamColumn',
59 dimensionality=TWO_D_PLANAR, type=DEFORMABLE_BODY)
60 p = mdb.models['BeamColumn'].parts['BeamColumn']
61 p.BaseWire(sketch=s)
62 s.unsetPrimaryObject()
63 p = mdb.models['BeamColumn'].parts['BeamColumn']
64 del mdb.models['BeamColumn'].sketches['__profile__']
65 #####
66 ##                               MATERIAL
67 #####

```

```

68 mdb.models['BeamColumn'].Material(name='Steel')
69 # Elastic
70 mdb.models['BeamColumn'].materials['Steel'].Elastic(table=((210000000000.0,
71 0.3),))
72 #####
73 ## PROFILE
74 #####
75 mdb.models['BeamColumn'].IPProfile(name='IPE220', l=h_2, h=htot, b1=bf1,
76 b2=bf2, t1=tf1, t2=tf2, t3=tw)
77 #####
78 ## CREATE SECTION
79 #####
80 mdb.models['BeamColumn'].BeamSection(name='beamcolumn',
81 integration=DURING_ANALYSIS, poissonRatio=0.0, profile='IPE220',
82 material='Steel', temperatureVar=LINEAR, consistentMassMatrix=False)
83 #####
84 ## ASSIGN SECTION
85 #####
86 p = mdb.models['BeamColumn'].parts['BeamColumn']
87 e = p.edges
88 edges = e.getSequenceFromMask(mask=('[#3 ]',),)
89 region = p.Set(edges=edges, name='Set-1')
90 p = mdb.models['BeamColumn'].parts['BeamColumn']
91 p.SectionAssignment(region=region, sectionName='beamcolumn', offset=0.0,
92 offsetType=MIDDLE_SURFACE, offsetField='',
93 thicknessAssignment=FROM_SECTION)
94 #####
95 ## SECTION ORIENTATION
96 #####
97 p = mdb.models['BeamColumn'].parts['BeamColumn']
98 e = p.edges
99 edges = e.getSequenceFromMask(mask=('[#3 ]',),)
100 region=p.Set(edges=edges, name='Set-2')
101 p = mdb.models['BeamColumn'].parts['BeamColumn']
102 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
103 -1.0))
104 #####
105 ## ASSEMBLY
106 #####
107 a = mdb.models['BeamColumn'].rootAssembly
108 a.DatumCsysByDefault(CARTESIAN)
109 p = mdb.models['BeamColumn'].parts['BeamColumn']
110 a.Instance(name='BeamColumn-1', part=p, dependent=ON)
111 #####
112 ## CREATE STEP: STATIC RIKS
113 #####
114 mdb.models['BeamColumn'].StaticRiksStep(name='Staticriks', previous='Initial',
115 initialArcInc=0.1, minArcInc=1e-08, maxArcInc=0.1, nlgeom=ON)
116 #####
117 ## BOUNDARY CONDITIONS
118 #####
119 a = mdb.models['BeamColumn'].rootAssembly
120 v1 = a.instances['BeamColumn-1'].vertices
121 verts1 = v1.findAt(((0.0, 0.0, 0.0),))
122 region = a.Set(vertices=verts1, name='Set-3')
123 mdb.models['BeamColumn'].DisplacementBC(name='BC-1', createStepName='Initial',
124 region=region, u1=SET, u2=SET, ur3=UNSET, amplitude=UNSET,
125 distributionType=UNIFORM, fieldName='', localCsys=None)
126 a = mdb.models['BeamColumn'].rootAssembly
127 v1 = a.instances['BeamColumn-1'].vertices
128 verts1 = v1.findAt(((L, 0.0, 0.0),))
129 region = a.Set(vertices=verts1, name='Set-4')
130 mdb.models['BeamColumn'].DisplacementBC(name='BC-2', createStepName='Initial',
131 region=region, u1=UNSET, u2=SET, ur3=UNSET, amplitude=UNSET,
132 distributionType=UNIFORM, fieldName='', localCsys=None)
133 p1 = mdb.models['BeamColumn'].parts['BeamColumn']
134 #####
135 ## LOADS
136 #####
137 v1 = a.instances['BeamColumn-1'].vertices

```

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```
138 verts1 = v1.findAt(((0.0, 0.0, 0.0), ), ((L, 0.0, 0.0), ))
139 region = a.Set(vertices=verts1, name='Set-5') #set 6
140 mdb.models['BeamColumn'].ConcentratedForce(name='N_eq',
141       createStepName='Staticriks', region=region, cf2=-Neq,
142       distributionType=UNIFORM, field='', localCsys=None)
143 #
144 e1 = a.instances['BeamColumn-1'].edges
145 edges1 = e1.getSequenceFromMask(mask=('[#3 ]', ), )
146 region = a.Set(edges=edges1, name='Set-6')
147 mdb.models['BeamColumn'].LineLoad(name='Q_eq', createStepName='Staticriks',
148       region=region, comp2=Qeq)
149 mdb.models['BeamColumn'].loads['Q_eq'].setValues(comp2=Qeq)
150
151 # AXIAL FORCE: N
152 v1 = a.instances['BeamColumn-1'].vertices
153 verts1 = v1.findAt(((L, 0.0, 0.0), ))
154 region = a.Set(vertices=verts1, name='Set-7') #set 3
155 mdb.models['BeamColumn'].ConcentratedForce(name='AxialForce',
156       createStepName='Staticriks', region=region, cf1=-F1,
157       distributionType=UNIFORM, field='', localCsys=None)
158 #####
159 ###                               MESH
160 #####
161 a1 = mdb.models['BeamColumn'].rootAssembly
162 a1.makeIndependent(instances=(a1.instances['BeamColumn-1'], ))
163 elemType1 = mesh.ElemType(elemCode=B21, elemLibrary=STANDARD)
164 a = mdb.models['BeamColumn'].rootAssembly
165 e1 = a.instances['BeamColumn-1'].edges
166 edges1 = e1.getSequenceFromMask(mask=('[#3 ]', ), )
167 pickedRegions =(edges1, )
168 a.setElementType(regions=pickedRegions, elemTypes=(elemType1, ))
169 a = mdb.models['BeamColumn'].rootAssembly
170 e1 = a.instances['BeamColumn-1'].edges
171 pickedEdges = e1.getSequenceFromMask(mask=('[#3 ]', ), )
172 # MESH SIZE
173 a.seedEdgeBySize(edges=pickedEdges, size=0.05, deviationFactor=0.1,
174       constraint=FINER)
175 a = mdb.models['BeamColumn'].rootAssembly
176 partInstances =(a.instances['BeamColumn-1'], )
177 a.generateMesh(regions=partInstances)
178 #####
179 ###                               FIELD OUTPUT
180 #####
181 mdb.models['BeamColumn'].fieldOutputRequests['F-Output-1'].setValues(variables=(
182     'S', 'PE', 'PEEQ', 'PEMAG', 'LE', 'TE', 'TEEQ', 'TEVOL', 'U', 'RF',
183     'RM', 'CF', 'SF', 'CSTRESS', 'CDISP', 'MVF'))
184 #####
185 ###                               CREATE SET FOR NODE OF INTEREST
186 #####
187 p = mdb.models['BeamColumn'].parts['BeamColumn']
188 v = p.vertices
189 verts = v.getSequenceFromMask(mask=('[#2 ]', ), )
190 p.Set(vertices=verts, name='MidP')
191 #####
192 ###                               JOB: NON-LINEAR
193 #####
194 mdb.Job(name='NonlinearEQloads', model='BeamColumn')
195 mdb.jobs['NonlinearEQloads'].setValues(description='', memoryUnits=PERCENTAGE,
196     memory=50, numCpus=1, numDomains=1)
197 mdb.jobs['NonlinearEQloads'].submit(datacheckJob=False)
198 mdb.jobs['NonlinearEQloads'].waitForCompletion()
```


B.3 Python code - Initial bow imperfection

```

1 from abaqus import *
2 from abaqusConstants import *
3 import __main__
4
5
6 import section
7 import regionToolset
8 import displayGroupMdbToolset as dgm
9 import part
10 import material
11 import assembly
12 import step
13 import interaction
14 import load
15 import mesh
16 import optimization
17 import job
18 import sketch
19 import visualization
20 import xyPlot
21 import displayGroupOdbToolset as dgo
22 import connectorBehavior
23
24 #####
25 ##                               DEFINE FUCTIONS
26 #####
27
28 import string
29 def GetBlockPosition(modelName, blockPrefix):
30     if blockPrefix == '':
31         return len(mdb.models[modelName].keywordBlock.sieBlocks)-1
32     pos = 0
33     for block in mdb.models[modelName].keywordBlock.sieBlocks:
34         if string.lower(block[0:len(blockPrefix)])==string.lower(blockPrefix):
35             return pos
36         pos=pos+1
37     return -1
38
39 #####
40 ##                               INPUT DATA
41 #####
42 ##Column IPE300
43 L=11.426                                #[m]
44 htot=0.3                                #[m]
45 h_2=htot*(2)**(-1)                      #[m]
46 bf1=0.15                                 #[m]
47 bf2=0.15                                 #[m]
48 tf1=0.0107                              #[m]
49 tf2=0.0107                              #[m]
50 tw=0.0071                               #[m]
51
52 ##Load
53 F1=500000                                #[N]
54
55 ##Initial bow imperfection
56 bow=L*(300)**(-1)                        #[m]
57 #####
58 ##                               GEOMETRY
59 #####
60 mdb.models.changeKey(fromName='Model-1', toName='BeamColumn')
61 s = mdb.models['BeamColumn'].ConstrainedSketch(name='__profile__',
62     sheetSize=20.0)
63 g, v, d, c = s.geometry, s.vertices, s.dimensions, s.constraints
64 # SETTING THE GEOMETRY
65 s.Line(point1=(0.0, 0.0), point2=(L*(2)**(-1), 0.0))
66 s.Line(point1=(L*(2)**(-1), 0.0), point2=(L, 0.0))
67 s.HorizontalConstraint(entity=g.findAt((L*(2)**(-1), 0.0)), addUndoState=False)

```

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```
68 p = mdb.models[ 'BeamColumn' ]. Part (name='BeamColumn' ,
69     dimensionality=TWO_D_PLANAR, type=DEFORMABLE_BODY)
70 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
71 p.BaseWire(sketch=s)
72 s.unsetPrimaryObject()
73 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
74 del mdb.models[ 'BeamColumn' ]. sketches[ '__profile__' ]
75
76 #####
77 ##                               MATERIAL
78 #####
79 mdb.models[ 'BeamColumn' ]. Material(name='Steel')
80 # Elastic
81 mdb.models[ 'BeamColumn' ]. materials[ 'Steel' ]. Elastic(table=((21000000000.0,
82     0.3), ))
83 #####
84 ##                               PROFILE
85 #####
86 mdb.models[ 'BeamColumn' ]. IProfile(name='IPE220' , l=h_2, h=htot, b1=bf1,
87     b2=bf2, t1=tf1, t2=tf2, t3=tw)
88 #####
89 ##                               CREATE SECTION
90 #####
91 mdb.models[ 'BeamColumn' ]. BeamSection(name='beamcolumn' ,
92     integration=DURING_ANALYSIS, poissonRatio=0.0, profile='IPE220' ,
93     material='Steel' , temperatureVar=LINEAR, consistentMassMatrix=False)
94 #####
95 ##                               ASSIGN SECTION
96 #####
97 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
98 e = p.edges
99 edges = e.getSequenceFromMask(mask=('[#3 ]' , ), )
100 region = p.Set(edges=edges, name='Set-1')
101 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
102 p.SectionAssignment(region=region, sectionName='beamcolumn' , offset=0.0,
103     offsetType=MIDDLE_SURFACE, offsetField='',
104     thicknessAssignment=FROM_SECTION)
105 #####
106 ##                               SECTION ORIENTATION
107 #####
108 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
109 e = p.edges
110 edges = e.getSequenceFromMask(mask=('[#3 ]' , ), )
111 region=p.Set(edges=edges, name='Set-2')
112 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
113 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
114     -1.0))
115 #####
116 ##                               ASSEMBLY
117 #####
118 a = mdb.models[ 'BeamColumn' ]. rootAssembly
119 a.DatumCsysByDefault(CARTESIAN)
120 p = mdb.models[ 'BeamColumn' ]. parts[ 'BeamColumn' ]
121 a.Instance(name='BeamColumn-1' , part=p, dependent=ON)
122
123 #####
124 ##                               CREATE STEP: STATIC
125 #####
126 mdb.models[ 'BeamColumn' ]. StaticStep(name='Static' , previous='Initial')
127 #####
128 ##                               BOUNDARY CONDITIONS
129 #####
130 a = mdb.models[ 'BeamColumn' ]. rootAssembly
131 v1 = a.instances[ 'BeamColumn-1' ]. vertices
132 verts1 = v1.findAt(((0.0, 0.0, 0.0), ))
133 region = a.Set(vertices=verts1, name='Set-1')
134 mdb.models[ 'BeamColumn' ]. DisplacementBC(name='BC-1' , createStepName='Initial' ,
135     region=region, u1=SET, u2=SET, ur3=UNSET, amplitude=UNSET,
136     distributionType=UNIFORM, fieldName='', localCsys=None)
137 a = mdb.models[ 'BeamColumn' ]. rootAssembly
```

```

138 v1 = a.instances['BeamColumn-1'].vertices
139 verts1 = v1.findAt(((L, 0.0, 0.0), ))
140 region = a.Set(vertices=verts1, name='Set-2')
141 mdb.models['BeamColumn'].DisplacementBC(name='BC-2', createStepName='Initial',
142     region=region, u1=UNSET, u2=SET, ur3=UNSET, amplitude=UNSET,
143     distributionType=UNIFORM, fieldName='', localCsys=None)
144 p1 = mdb.models['BeamColumn'].parts['BeamColumn']
145 #####
146 ##                                LOADS
147 #####
148 v1 = a.instances['BeamColumn-1'].vertices
149 verts1 = v1.findAt(((L, 0.0, 0.0), ))
150 region = a.Set(vertices=verts1, name='Set-3')
151 # AXIAL FORCE: N
152 mdb.models['BeamColumn'].ConcentratedForce(name='AxialForce',
153     createStepName='Static', region=region, cf1=F1,
154     distributionType=UNIFORM, field='', localCsys=None)
155 #####
156 ###                                MESH
157 #####
158 elemType1 = mesh.ElemType(elemCode=B21, elemLibrary=STANDARD)
159 p = mdb.models['BeamColumn'].parts['BeamColumn']
160 e = p.edges
161 edges = e.findAt(((L*(4)**(-1), 0.0, 0.0), ))
162 pickedRegions =(edges, )
163 p.setElementType(regions=pickedRegions, elemTypes=(elemType1, ))
164 p = mdb.models['BeamColumn'].parts['BeamColumn']
165 e = p.edges
166 pickedEdges = e.findAt(((L*(4)**(-1), 0.0, 0.0), ))
167 # SELECTING MESH SIZE
168 p.seedEdgeBySize(edges=pickedEdges, size=0.05, deviationFactor=0.1, minSizeFactor=0.1,
169     constraint=FINER)
170 p = mdb.models['BeamColumn'].parts['BeamColumn']
171 p.generateMesh()
172 #####
173 ##                                FIELD OUTPUT
174 #####
175 mdb.models['BeamColumn'].fieldOutputRequests['F-Output-1'].setValues(variables=(
176     'S', 'PE', 'PEEQ', 'PEMAG', 'LE', 'TE', 'TEEQ', 'TEVOL', 'U', 'RF',
177     'RM', 'CF', 'SF', 'CSTRESS', 'CDISP', 'MVF'))
178 #####
179 ##                                JOB
180 #####
181 mdb.Job(name='Static_Analysis', model='BeamColumn')
182 mdb.jobs['Static_Analysis'].setValues(description='', memoryUnits=PERCENTAGE,
183     memory=50, numCpus=1, numDomains=1)
184 mdb.jobs['Static_Analysis'].submit(datacheckJob=False)
185 #####
186 ##                                CREATE STEP: BUCKLE
187 #####
188 mdb.models['BeamColumn'].BuckleStep(name='Static', previous='Initial',
189     maintainAttributes=True, numEigen=10, vectors=18, maxIterations=100)
190 mdb.models['BeamColumn'].steps.changeKey(fromName='Static', toName='BUCKLE')
191 import job
192 #####
193 ##                                KEYWORDS 1
194 #####
195 mdb.models['BeamColumn'].keywordBlock.synchVersions(storeNodesAndElements=False)
196 mdb.models['BeamColumn'].keywordBlock.insert(GetBlockPosition('BeamColumn', '*End Step')-1, ""
197 *NODE FILE, GLOBAL=YES, LAST MODE=1
198 U"")
199
200 #####JOB1 BUCKLE
201 mdb.Job(name='BUCKLE', model='BeamColumn')
202 mdb.jobs['BUCKLE'].setValues(description='', memoryUnits=PERCENTAGE, memory=50,
203     numCpus=1, numDomains=1)
204 mdb.jobs['BUCKLE'].submit(datacheckJob=False)
205
206 #####step Non-linear
207 mdb.models['BeamColumn'].StaticRiksStep(name='BUCKLE', previous='Initial',

```

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```
208     maintainAttributes=True, initialArcInc=0.1, minArcInc=1e-08,
209     maxArcInc=0.1)
210
211 mdb.models['BeamColumn'].steps.changeKey(fromName='BUCKLE', toName='Static riks')
212 mdb.models['BeamColumn'].steps['Static riks'].setValues(nlgeom=ON)
213 #####
214 ###                               KEYWORDS 2
215 #####
216 mdb.models['BeamColumn'].keywordBlock.synchVersions(storeNodesAndElements=False)
217 mdb.models['BeamColumn'].keywordBlock.setValues(edited = 0)
218 mdb.models['BeamColumn'].keywordBlock.synchVersions(storeNodesAndElements=False)
219 mdb.models['BeamColumn'].keywordBlock.insert(GetBlockPosition('BeamColumn', '*Step, name="Static riks", nl
220 *IMPERFECTION, FILE=Buckle, STEP=1
221 1, ""+str(bow)
222 )
223 #####
224 ###                               CREATE SET FOR NODE OF INTEREST
225 #####
226 p = mdb.models['BeamColumn'].parts['BeamColumn']
227 v = p.vertices
228 verts = v.getSequenceFromMask(mask=('[#2 ]', ), )
229 p.Set(vertices=verts, name='MidP')
230 #####
231 ###                               JOB: NON-LINEAR
232 #####
233 mdb.Job(name='NonlinearBow', model='BeamColumn', description='', type=ANALYSIS,
234     atTime=None, waitMinutes=0, waitHours=0, queue=None, memory=90,
235     memoryUnits=PERCENTAGE, getMemoryFromAnalysis=True,
236     explicitPrecision=SINGLE, nodalOutputPrecision=SINGLE, echoPrint=OFF,
237     modelPrint=OFF, contactPrint=OFF, historyPrint=OFF, userSubroutine='',
238     scratch='', resultsFormat=ODB)
239 mdb.jobs['NonlinearBow'].submit(consistencyChecking=OFF)
240 mdb.jobs['NonlinearBow'].waitForCompletion()
```

C

Study II

This appendix provide extra material behind the comparison between *Approach (a)*, *(b)* and *(c)* made for the sway frame. In Section C.1-C.3 the Mathcad codes used for the individual stability checks are presented. In Section C.4-C.6 the Python codes used to create models and perform analyses in Abaqus are presented. In Section C.7-C.8 the Matlab codes are presented that was used in combination with Abaqus to, for example, calculate the EUGLI-imperfection or interact results between *Approach (a)* and *(b)*.

C.1 Mathcad code - Approach (b)

Analyse II Approach (b): Partially global

The frame is stabilised in the weak direction. Steel quality S355
Elastic cross-section analysis is to be used (assuming CSC 3)

INPUT DATA

Length:

$$L_{\text{member}} := 2\text{m}$$

IPE 300:

$$h := 300\text{mm}$$

$$t_f := 10.7\text{mm}$$

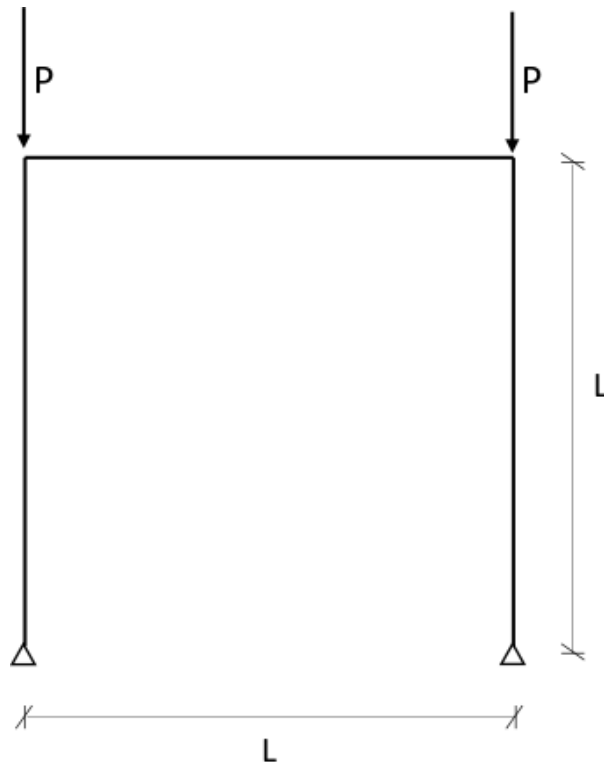
$$t_w := 7.1\text{mm}$$

$$b_f := 150\text{mm}$$

Steel S355:

$$E := 210\text{GPa}$$

$$f_y := 355\text{MPa}$$



Sectional constants:

$$A := 0.00538\text{m}^2$$

y-y direction: STRONG DIRECTION

$$I_y := 0.0000835\text{m}^4$$

$$W_{el,y} := 0.000557\text{m}^3$$

Partial safety factors:

$$\gamma_{M0} := 1$$

$$\gamma_{M1} := 1$$

Axial load:

$$P := 500\text{kN}$$

Load capacity:

$$N_{Rk} := A \cdot f_y = 1.91 \times 10^3 \cdot \text{kN} \qquad N_{Rd} := A \cdot \frac{f_y}{\gamma_{M1}} = 1.91 \times 10^3 \cdot \text{kN}$$

$$M_{y,Rk} := W_{el,y} \cdot f_y = 197.735 \cdot \text{kN} \cdot \text{m} \qquad M_{y,Rd} := W_{el,y} \cdot \frac{f_y}{\gamma_{M1}} = 197.735 \cdot \text{kN} \cdot \text{m}$$

Critical buckling length:

$$L_{cr} := L_{\text{member}} = 2 \text{ m}$$

DESIGN ACTIONS

$$N_{Ed} := 1780290 \text{ N}$$

$$M_{y,Ed} := 19478 \text{ N} \cdot \text{m}$$

Individual stability checks**BUCKLING**

Buckling curve for torsional buckling is determined by:

y-y direction

BC _{check} :=	"bc1" if $\left(\frac{h}{b_f}\right) > 1.2$ = "bc1"
	"bc2" otherwise

$$b_{\text{curve.y1}} := \begin{cases} \text{"a"} & \text{if } t_f \leq 40 \text{ mm} \\ \text{"b"} & \text{if } 40 \text{ mm} < t_f \leq 100 \text{ mm} \end{cases} = \text{"a"} \qquad \text{bc1}$$

$$b_{\text{curve.y2}} := \begin{cases} \text{"b"} & \text{if } t_f \leq 100 \text{ mm} \\ \text{"d"} & \text{otherwise} \end{cases} = \text{"b"} \qquad \text{bc2}$$

For:

- Hot-rolled standard section
 - buckling in the strong direction
 - $h/b > 1.2$
 - $t < 40 \text{ mm}$
 - steel quality S355
- > Curve a

Table 6.2: Selection of buckling curve for a cross-section

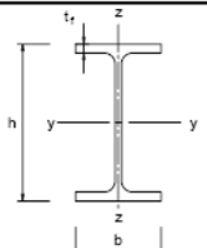
Cross section	Limits	Buckling about axis	Buckling curve	
			S 235 S 275 S 355 S 420	S 460
Rolled sections 	$h/b > 1,2$	y-y z-z	$t_f \leq 40 \text{ mm}$	a a ₀
			$40 \text{ mm} < t_f \leq 100$	b c a
	$h/b \leq 1,2$	y-y z-z	$t_f \leq 100 \text{ mm}$	b c a
			$t_f > 100 \text{ mm}$	d d c

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a ₀	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

The imperfection factor is:

$$\alpha_y := 0.21$$

$$N_{cr,y} := \frac{(\pi^2 E \cdot I_y)}{L_{cr}^2} = 4.33 \times 10^4 \cdot \text{kN}$$

$$\lambda_{bar,y} := \sqrt{\frac{N_{Rd}}{N_{cr,y}}} = 0.21$$

$$\Phi_y := 0.5 \cdot [1 + \alpha_y \cdot (\lambda_{bar,y} - 0.2) + \lambda_{bar,y}^2] = 0.523$$

$$\chi_y := \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \lambda_{bar,y}^2}} = 0.998$$


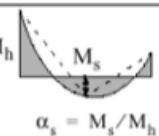
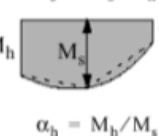
$$N_{bRd,y} := \frac{(\chi_y \cdot N_{Rk})}{\gamma_{M1}} = 1.906 \times 10^3 \cdot \text{kN}$$

IF RESTRAINT AGAINST LT-BUCKLING:

$$\chi_{LT} := 1.0$$

METHOD 2: Interaction factors

Tables from EN1993-1-1 Appendix B

Moment diagram	range		C_{my} and C_{mz} and $C_{m1,T}$	
			uniform loading	concentrated load
	$-1 \leq \psi \leq 1$		$0,6 + 0,4\psi \geq 0,4$	
	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi \leq 1$	$0,2 + 0,8\alpha_s \geq 0,4$	$0,2 + 0,8\alpha_s \geq 0,4$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi \leq 1$	$0,1 - 0,8\alpha_s \geq 0,4$	$-0,8\alpha_s \geq 0,4$
	$0 \leq \alpha_h \leq 1$	$-1 \leq \psi \leq 1$	$0,1(1-\psi) - 0,8\alpha_s \geq 0,4$	$0,2(-\psi) - 0,8\alpha_s \geq 0,4$
		$0 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \leq \alpha_h < 0$	$-1 \leq \psi < 0$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
		$-1 \leq \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$
For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0,9$ or $C_{mz} = 0,9$ respectively.				
C_{my} , C_{mz} and $C_{m1,T}$ should be obtained according to the bending moment diagram between the relevant braced points as follows:				
moment factor	bending axis	points braced in direction		
C_{my}	y-y	z-z		
C_{mz}	z-z	y-y		
$C_{m1,T}$	y-y	y-y		

Not susceptible to torsional deformations:

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my} \left(1 + 0,6\bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{my} \left(1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0,6 k_{zz}$
k_{zy}	I-sections RHS-sections	$0,8 k_{yy}$	$0,6 k_{yy}$
k_{zz}	I-sections	$C_{mz} \left(1 + 0,6\bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,6 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$	$C_{mz} \left(1 + (2\bar{\lambda}_z - 0,6) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 1,4 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections	$C_{mz} \left(1 + 0,6\bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,6 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$	$C_{mz} \left(1 + (\bar{\lambda}_z - 0,2) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,8 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.			

Critical member: $C_{my} := 0.9$ Due to sway bucklingmode

$$n_y := \frac{N_{Ed}}{N_{bRd,y}} = 0.934$$

$$k_{yy} := \begin{cases} \lceil C_{my} \cdot [1 + 0.6(\lambda_{bar,y}) \cdot n_y] \rceil & \text{if } C_{my} \cdot [1 + 0.6(\lambda_{bar,y}) \cdot n_y] \leq C_{my} \cdot (1 + 0.6 \cdot n_y) \\ \lceil C_{my} \cdot (1 + 0.6 \cdot n_y) \rceil & \text{otherwise} \end{cases} = 1.006$$

VERIFICATION**Bending & axial compression: N + My**

$$u := \frac{N_{Ed}}{N_{bRd,y}} + k_{yy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot M_{y,Rd}} = 1.033$$

C.2 Mathcad code - Approach (b) considering second order effects analytically

Analyse II Approach (b): Partially global - Approximate

The frame is stabilised in the weak direction. Steel quality S355
Elastic cross-section analysis is to be used (assuming GSC 3)

INPUT DATA

Length:

$$L_{\text{member}} := 2\text{m}$$

IPE 300:

$$h := 300\text{mm}$$

$$t_f := 10.7\text{mm}$$

$$t_w := 7.1\text{mm}$$

$$b_f := 150\text{mm}$$

Steel S355:

$$E := 210\text{GPa}$$

$$f_y := 355\text{MPa}$$

Sectional constants:

$$A := 0.005381\text{m}^2$$

y-y direction: STRONG DIRECTION

$$I_y := 0.00008356\text{m}^4$$

$$W_{el,y} := 0.000557\text{m}^3$$

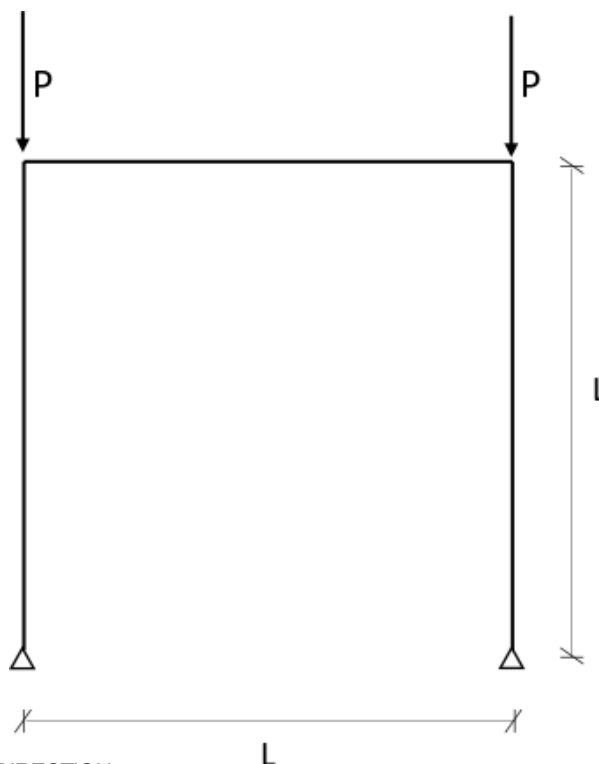
Partial safety factors:

$$\gamma_{M0} := 1$$

$$\gamma_{M1} := 1$$

Axial load:

$$P := 500\text{kN}$$



Load capacity:

$$N_{Rk} := A \cdot f_y = 1.91 \times 10^3 \cdot \text{kN}$$

$$N_{Rd} := A \cdot \frac{f_y}{\gamma_{M1}} = 1.91 \times 10^3 \cdot \text{kN}$$

$$M_{y,Rk} := W_{el,y} \cdot f_y = 197.735 \cdot \text{kN} \cdot \text{m}$$

$$M_{y,Rd} := W_{el,y} \cdot \frac{f_y}{\gamma_{M1}} = 197.735 \cdot \text{kN} \cdot \text{m}$$

Imperfection - Sway:

$$m_{\text{columns}} := 2$$

$$\text{phi}_0 := \frac{1}{200} = 5 \times 10^{-3}$$

$$\alpha_h := \left(\frac{2}{\sqrt{L_{\text{member}} \cdot \frac{1}{m}}} \right) = 1.414$$

$$\alpha_h := \begin{cases} \frac{2}{3} & \text{if } \alpha_h \leq \frac{2}{3} \\ 1 & \text{if } \alpha_h \geq 1 \\ \left(\left(\frac{2}{\sqrt{L_{\text{member}} \cdot \frac{1}{m}}} \right) \right) & \text{otherwise} \end{cases} = 1$$

$$\alpha_m := \sqrt{0.5 \cdot \left[1 + \left(\frac{1}{m_{\text{columns}}} \right) \right]} = 0.866$$

$$\text{phi} := \text{phi}_0 \cdot \alpha_h \cdot \alpha_m = 4.33 \times 10^{-3}$$

$$\text{imp}_{\text{sway}} := \text{phi} \cdot L_{\text{member}} = 8.66 \times 10^{-3} \text{ m}$$

Indata:

$$L_{cr} := L_{member} = 2 \text{ m}$$

$$LPF := 3.53$$

$$eig := 14.828$$

$$\alpha_{cr,global} := \frac{eig}{LPF} = 4.201$$

$$N_{cr,y} := \frac{(\pi^2 \cdot E \cdot I_y)}{L_{cr}^2} = 4.33 \times 10^4 \cdot \text{kN}$$

$$\text{LoadPerColumn} := P \cdot LPF = 1.765 \times 10^3 \cdot \text{kN}$$

$$N_{Ed} := \frac{[\text{LoadPerColumn} \cdot \text{imp}_{sway} + \text{LoadPerColumn} \cdot (\text{imp}_{sway} + L_{member})]}{L_{member}} = 1.78 \times 10^6 \text{ N}$$

Amplification factor that makes sure that second-order effects are considered:

$$\text{amp} := \frac{1}{1 - \left(\frac{1}{\alpha_{cr,global}} \right)} = 1.312$$

significant _{check} :=	"NOT" if $N_{Ed} \leq 0.09 \cdot N_{cr,y}$ = "NOT"
	"YES" otherwise

Moment due to sway:

$$M_{sway} := N_{Ed} \cdot \text{imp}_{sway} = 15.418 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{max.2ndorder}} := M_{sway} \cdot \text{amp} = 20.235 \cdot \text{kN} \cdot \text{m}$$

DESIGN ACTIONS

$$N_{Ed} = 1.78 \times 10^6 \cdot \text{N}$$

$$M_{y,Ed} := M_{\text{max.2ndorder}} = 2.023 \times 10^4 \cdot \text{N} \cdot \text{m}$$

C.3 Mathcad code - Approach (c)

Analyse II Approach (c)

The frame is stabilised in the weak direction. Steel quality S355
Elastic cross-section analysis is to be used (assuming CSC 3)

INPUT DATA

Length:

$$L_{\text{member}} := 2\text{m}$$

IPE 300:

$$h := 300\text{mm}$$

$$t_f := 10.7\text{mm}$$

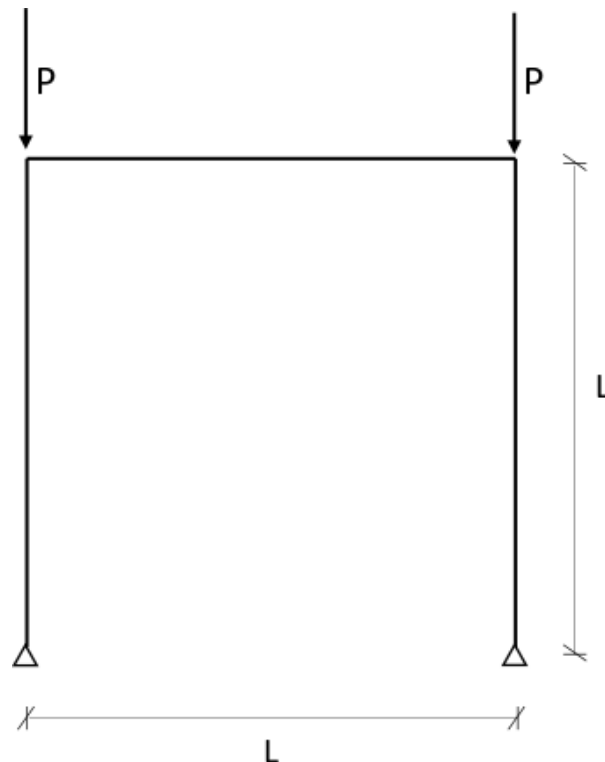
$$t_w := 7.1\text{mm}$$

$$b_f := 150\text{mm}$$

Steel S355:

$$E := 210\text{GPa}$$

$$f_y := 355\text{MPa}$$



Sectional constants:

$$A := 0.005381\text{m}^2$$

y-y direction: STRONG DIRECTION

$$I_y := 0.00008356\text{m}^4$$

$$W_{el,y} := 0.000557\text{m}^3$$

Partial safety factors:

$$\gamma_{M0} := 1$$

$$\gamma_{M1} := 1$$

Axial load:

$$P := 500\text{kN} \quad \text{Later adjusted by LPF}$$

Load capacity:

$$N_{Rk} := A \cdot f_y = 1.91 \times 10^3 \cdot \text{kN}$$

$$N_{Rd} := A \cdot \frac{f_y}{\gamma_{M1}} = 1.91 \times 10^3 \cdot \text{kN}$$

$$M_{y,Rk} := W_{el,y} \cdot f_y = 197.735 \cdot \text{kN} \cdot \text{m}$$

$$M_{y,Rd} := W_{el,y} \cdot \frac{f_y}{\gamma_{M1}} = 197.735 \cdot \text{kN} \cdot \text{m}$$

Imperfection - Sway:

$$m_{\text{columns}} := 2$$

$$\phi_{i0} := \frac{1}{200} = 5 \times 10^{-3}$$

$$\alpha_h := \left(\frac{2}{\sqrt{L_{\text{member}} \cdot \frac{1}{\text{m}}}} \right) = 1.414$$

$$\alpha_m := \begin{cases} \frac{2}{3} & \text{if } \alpha_h \leq \frac{2}{3} \\ 1 & \text{if } \alpha_h \geq 1 \\ \left(\left(\frac{2}{\sqrt{L_{\text{member}} \cdot \frac{1}{\text{m}}}} \right) \right) & \text{otherwise} \end{cases} = 1$$

$$\alpha_m := \sqrt{0.5 \cdot \left[1 + \left(\frac{1}{m_{\text{columns}}} \right) \right]} = 0.866$$

$$\phi := \phi_{i0} \cdot \alpha_h \cdot \alpha_m = 4.33 \times 10^{-3}$$

$$\text{imp}_{\text{sway}} := \phi \cdot L_{\text{member}} = 8.6603 \times 10^{-3} \text{ m}$$

Indata:

$$\text{LPF} := 3.53$$

$$\text{eig} := 14.828$$

$$\alpha_{\text{cr}} := \frac{\text{eig}}{\text{LPF}} = 4.201$$

$$\text{LoadPerColumn} := P \cdot \text{LPF} = 1.765 \times 10^3 \cdot \text{kN}$$

$$R_2 := \frac{[\text{LoadPerColumn} \cdot \text{imp}_{\text{sway}} + \text{LoadPerColumn} \cdot (\text{imp}_{\text{sway}} + L_{\text{member}})]}{L_{\text{member}}} = 1.78 \times 10^3 \cdot \text{kN}$$

$$R_1 := \frac{[\text{LoadPerColumn} \cdot (L_{\text{member}} - \text{imp}_{\text{sway}}) - \text{LoadPerColumn} \cdot \text{imp}_{\text{sway}}]}{L_{\text{member}}} = 1.75 \times 10^3 \cdot \text{kN}$$

$$N_{\text{Ed}} := \max(R_2, R_1) = 1.78 \times 10^6 \text{ N}$$

$$N_{\text{cr.y}} := \alpha_{\text{cr}} \cdot N_{\text{Ed}} = 7.478 \times 10^3 \cdot \text{kN}$$

Critical buckling length:

$$L_{\text{cr}} := \sqrt{\frac{(\pi^2 E \cdot I_y)}{N_{\text{cr.y}}}} = 4.812 \text{ m}$$

Moment due to sway:

$$M_{\text{sway}} := N_{\text{Ed}} \cdot \text{imp}_{\text{sway}} = 15.418 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{max.2ndorder}} := M_{\text{sway}} = 15.418 \cdot \text{kN} \cdot \text{m}$$

DESIGN ACTIONS

$$N_{\text{Ed}} = 1.78029 \times 10^3 \cdot \text{kN}$$

$$M_{y,\text{Ed}} := M_{\text{sway}} = 15.418 \cdot \text{kN} \cdot \text{m}$$

Individual stability checks**BUCKLING**

Buckling curve for torsional buckling is determined by:

y-y direction

$BC_{\text{check}} :=$	"bc1" if $\left(\frac{h}{b_f}\right) > 1.2$ = "bc1"
	"bc2" otherwise

$$b_{\text{curve.y1}} := \begin{cases} \text{"a"} & \text{if } t_f \leq 40\text{mm} \\ \text{"b"} & \text{if } 40\text{mm} < t_f \leq 100\text{mm} \end{cases} = \text{"a"}$$

bc1

$$b_{\text{curve.y2}} := \begin{cases} \text{"b"} & \text{if } t_f \leq 100\text{mm} \\ \text{"d"} & \text{otherwise} \end{cases} = \text{"b"}$$

bc2

For:

- Hot-rolled standard section
- buckling in the strong direction
- $h/b > 1.2$
- $t < 40$ mm
- steel quality S355

---> Curve a

Table 6.2: Selection of buckling curve for a cross-section

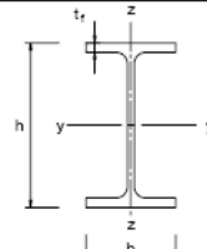
Cross section	Limits	Buckling about axis	Buckling curve	
			S 235 S 275 S 355 S 420	S 460
Rolled sections 	$h/b > 1.2$	y-y z-z	$t_f \leq 40$ mm	a a ₀
			$40 \text{ mm} < t_f \leq 100$	b c
	$h/b \leq 1.2$	y-y z-z	$t_f \leq 100$ mm	b c
			$t_f > 100$ mm	d c

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a ₀	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

The imperfection factor is:

$$\alpha_y := 0.21$$

$$\lambda_{\text{bar.y}} := \sqrt{\frac{N_{Rd}}{N_{cr.y}}} = 0.505$$

$$\Phi_y := 0.5 \cdot \left[1 + \alpha_y \cdot (\lambda_{\text{bar.y}} - 0.2) + \lambda_{\text{bar.y}}^2 \right] = 0.66$$

$$\chi_y := \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \lambda_{\text{bar.y}}^2}} = 0.923$$


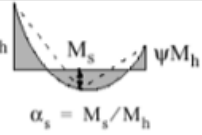
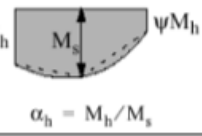
$$N_{bRd,y} := \frac{(\chi_y \cdot N_{Rk})}{\gamma_{M1}} = 1.762 \times 10^3 \cdot \text{kN}$$

IF RESTRAINT AGAINST LT-BUCKLING:

$$\chi_{LT} := 1.0$$

METHOD 2: Interaction factors

Tables from EN1993-1-1 Appendix B

Moment diagram	range		C_{my} and C_{mz} and C_{mLT}	
			uniform loading	concentrated load
	$-1 \leq \psi \leq 1$		$0,6 + 0,4\psi \geq 0,4$	
 $\alpha_s = M_s/M_h$	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi \leq 1$	$0,2 + 0,8\alpha_s \geq 0,4$	$0,2 + 0,8\alpha_s \geq 0,4$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi \leq 1$	$0,1 - 0,8\alpha_s \geq 0,4$	$-0,8\alpha_s \geq 0,4$
		$-1 \leq \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \geq 0,4$	$0,2(-\psi) - 0,8\alpha_s \geq 0,4$
 $\alpha_h = M_h/M_s$	$0 \leq \alpha_h \leq 1$	$-1 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \leq \alpha_h < 0$	$0 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
		$-1 \leq \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$
For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0,9$ or $C_{mz} = 0,9$ respectively.				
C_{my} , C_{mz} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:				
moment factor	bending axis	points braced in direction		
C_{my}	y-y	z-z		
C_{mz}	z-z	y-y		
C_{mLT}	y-y	y-y		

Not susceptible to torsional deformations:

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my} \left(1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{my} \left(1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0,6 k_{zz}$
k_{zy}	I-sections RHS-sections	$0,8 k_{yy}$	$0,6 k_{yy}$
k_{zz}	I-sections	$C_{mz} \left(1 + 0,6 \bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,6 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$	$C_{mz} \left(1 + (2\bar{\lambda}_z - 0,6) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 1,4 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections		$C_{mz} \left(1 + (\bar{\lambda}_z - 0,2) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,8 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$

For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.

Critical member:

$C_{my} := 0.9$ Due to sway bucklingmode

$n_y := \frac{N_{Ed}}{N_{bRd,y}} = 1.01$

$k_{yy} := \begin{cases} \left[C_{my} \left[1 + 0.6(\lambda_{bar,y}) \cdot n_y \right] \right] & \text{if } C_{my} \left[1 + 0.6(\lambda_{bar,y}) \cdot n_y \right] \leq C_{my} \cdot (1 + 0.6 \cdot n_y) \\ \left[\left[C_{my} \cdot (1 + 0.6 \cdot n_y) \right] \right] & \text{otherwise} \end{cases} = 1.176$

VERIFICATION

Since k_{yz} and k_{zz} is equal to zero it gives:

Bending & axial compression: N + My

$u := \frac{N_{Ed}}{N_{bRd,y}} + k_{yy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot M_{y,Rd}} = 1.102$

C.4 Python code - Approach (a) part 1

```

1 from abaqus import *
2 from abaqusConstants import *
3 import __main__
4
5 import section
6 import regionToolset
7 import displayGroupMdbToolset as dgm
8 import part
9 import material
10 import assembly
11 import step
12 import interaction
13 import load
14 import mesh
15 import optimization
16 import job
17 import sketch
18 import visualization
19 import xyPlot
20 import displayGroupOdbToolset as dgo
21 import connectorBehavior
22
23 ##Remove old files in both work directories
24 import os, re, os.path
25 mypath = 'C:/Arbete_ram/IN_PROGRESS_eugli'
26 for root, dirs, files in os.walk(mypath):
27     for file in files:
28         try:
29             os.remove(os.path.join(root, file))
30         except OSError:
31             pass
32
33 mypath = 'C:/Arbete_ram/IN_PROGRESS_explicit'
34 for root, dirs, files in os.walk(mypath):
35     for file in files:
36         try:
37             os.remove(os.path.join(root, file))
38         except OSError:
39             pass
40
41 ##Set work directory
42 import os
43 os.chdir('C:/Arbete_ram/IN_PROGRESS_eugli')
44 print os.getcwd() # Prints the current working directory
45
46 #####
47 ##DEFINE FUCTIONS
48 #####
49 import string
50 def GetBlockPosition(modelName, blockPrefix):
51     if blockPrefix == '':
52         return len(mdb.models[modelName].keywordBlock.sieBlocks)-1
53     pos = 0
54     for block in mdb.models[modelName].keywordBlock.sieBlocks:
55         if string.lower(block[0:len(blockPrefix)])==string.lower(blockPrefix):
56             return pos
57         pos=pos+1
58     return -1
59 #####
60 ##ENDATA
61 #####
62 ##Beam IPE300
63 L_b=3
64 h_b=0.3
65 b1_b=0.15
66 b2_b=b1_b
67 t1_b=0.0107

```



```

138 #####
139 ##COLUMN
140 s1 = mdb.models['Model-1'].ConstrainedSketch(name='__profile__',
141     sheetSize=L_c*2)
142 g, v, d, c = s1.geometry, s1.vertices, s1.dimensions, s1.constraints
143 s1.setPrimaryObject(option=STANDALONE)
144 s1.Line(point1=(0.0, 0.0), point2=(0.0, L_c))
145 s1.VerticalConstraint(entity=g.findAt((0.0, L_c*2**(-1))), addUndoState=False)
146 p = mdb.models['Model-1'].Part(name='Column', dimensionality=TWO_D_PLANAR,
147     type=DEFORMABLE_BODY)
148 p = mdb.models['Model-1'].parts['Column']
149 p.BaseWire(sketch=s1)
150 s1.unsetPrimaryObject()
151 p = mdb.models['Model-1'].parts['Column']
152 session.viewports['Viewport: 1'].setValues(displayedObject=p)
153 del mdb.models['Model-1'].sketches['__profile__']
154 #####
155 ##Section orientation
156 #####
157 p = mdb.models['Model-1'].parts['Column']
158 e = p.edges
159 edges = e.findAt(((0.0, L_c*4**(-1), 0.0), ))
160 region=p.Set(edges=edges, name='Set-1')
161 p = mdb.models['Model-1'].parts['Column']
162 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
163     -1.0))
164 p1 = mdb.models['Model-1'].parts['Beam']
165 p = mdb.models['Model-1'].parts['Beam']
166 e = p.edges
167 edges = e.findAt(((L_b*4**(-1), 0.0, 0.0), ))
168 region=p.Set(edges=edges, name='Set-1')
169 p = mdb.models['Model-1'].parts['Beam']
170 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
171     -1.0))
172 #####
173 ##MATERIAL
174 #####
175 mdb.models['Model-1'].Material(name='Steel')
176 mdb.models['Model-1'].materials['Steel'].Elastic(table=((E, vPo),
177     ))
178 #####
179 ##I-profiles
180 #####
181 ##Generalized
182 mdb.models['Model-1'].GeneralizedProfile(name='G_C_HEA300', area=A_c,
183     i11=I11_c, i12=0, i22=I11_c, j=J_c, gammaO=0.0,
184     gammaW=0.0)
185 mdb.models['Model-1'].GeneralizedProfile(name='G_B_IPE550', area=A_b,
186     i11=I11_b, i12=0, i22=I11_b, j=J_b, gammaO=0.0,
187     gammaW=0.0)
188 #####
189 ##Create section
190 #####
191 mdb.models['Model-1'].BeamSection(name='Column_section',
192     integration=BEFORE_ANALYSIS, poissonRatio=vPo, beamShape=CONSTANT,
193     profile='G_C_HEA300', thermalExpansion=OFF, temperatureDependency=OFF,
194     dependencies=0, table=((E, G), ),
195     alphaDamping=0.0, betaDamping=0.0, compositeDamping=0.0, centroid=(0.0,
196     0.0), shearCenter=(0.0, 0.0), consistentMassMatrix=False)
197
198 mdb.models['Model-1'].BeamSection(name='Beam_section',
199     integration=BEFORE_ANALYSIS, poissonRatio=vPo, beamShape=CONSTANT,
200     profile='G_B_IPE550', thermalExpansion=OFF, temperatureDependency=OFF,
201     dependencies=0, table=((E, G), ),
202     alphaDamping=0.0, betaDamping=0.0, compositeDamping=0.0, centroid=(0.0,
203     0.0), shearCenter=(0.0, 0.0), consistentMassMatrix=False)
204 #####
205 ##Assign section
206 #####
207 p = mdb.models['Model-1'].parts['Beam']

```

```

208 e = p.edges
209 edges = e.findAt(((L_b*4**(-1), 0.0, 0.0), ))
210 region = p.Set(edges=edges, name='Set-1')
211 p = mdb.models['Model-1'].parts['Beam']
212 p.SectionAssignment(region=region, sectionName='Beam_section', offset=0.0,
213     offsetType=MIDDLE_SURFACE, offsetField='',
214     thicknessAssignment=FROM_SECTION)
215 p = mdb.models['Model-1'].parts['Column']
216 e = p.edges
217 edges = e.findAt(((0.0, L_c*4**(-1), 0.0), ))
218 region = p.Set(edges=edges, name='Set-1')
219 p = mdb.models['Model-1'].parts['Column']
220 p.SectionAssignment(region=region, sectionName='Column_section', offset=0.0,
221     offsetType=MIDDLE_SURFACE, offsetField='',
222     thicknessAssignment=FROM_SECTION)
223 #####
224 ##Assembly
225 #####
226 a = mdb.models['Model-1'].rootAssembly
227 a.DatumCsysByDefault(CARTESIAN)
228 p = mdb.models['Model-1'].parts['Beam']
229 a.Instance(name='Beam-1', part=p, dependent=OFF)
230 a = mdb.models['Model-1'].rootAssembly
231 p = mdb.models['Model-1'].parts['Column']
232 a.Instance(name='Column-1', part=p, dependent=OFF)
233 a = mdb.models['Model-1'].rootAssembly
234 a.translate(instanceList=('Column-1', ), vector=(L_b, 0.0, 0.0))
235 a = mdb.models['Model-1'].rootAssembly
236 p = mdb.models['Model-1'].parts['Column']
237 a.Instance(name='Column-2', part=p, dependent=OFF)
238 a = mdb.models['Model-1'].rootAssembly
239 a.translate(instanceList=('Beam-1', ), vector=(0.0, L_c, 0.0))
240 #####
241 ##MERGE WITHOUT RETAINING BOUNDARIES
242 a = mdb.models['Model-1'].rootAssembly
243 a.InstanceFromBooleanMerge(name='Frame', instances=(a.instances['Beam-1'],
244     a.instances['Column-1'], a.instances['Column-2'], ),
245     originalInstances=SUPPRESS, domain=GEOMETRY)
246 #####
247 ##Orientation frame
248 #####
249 p = mdb.models['Model-1'].parts['Column']
250 session.viewports['Viewport: 1'].setValues(displayedObject=p)
251 p = mdb.models['Model-1'].parts['Frame']
252 session.viewports['Viewport: 1'].setValues(displayedObject=p)
253 p = mdb.models['Model-1'].parts['Frame']
254 e = p.edges
255 edges = e.findAt(((0.0, L_c*4**(-1), 0.0), ), ((L_b*4**(-1), L_c, 0.0), ), ((L_b, L_c*4**(-1), 0.0),
256     ))
257 region=p.Set(edges=edges, name='Set-2')
258 p = mdb.models['Model-1'].parts['Frame']
259 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
260     -1.0))
261 #####
262 ##Create Step: Static
263 #####
264 # mdb.models['Model-1'].StaticRiksStep(name='Static', previous='Initial', ##byt till static riks
265     # maxNumInc=100, initialArcInc=0.1, minArcInc=1e-08, maxArcInc=0.1,
266     # totalArcLength=0.1)
267 a = mdb.models['Model-1'].rootAssembly
268 mdb.models['Model-1'].StaticStep(name='Static', previous='Initial',
269     initialInc=0.1, minInc=1e-08, maxInc=0.1)
270 session.viewports['Viewport: 1'].assemblyDisplay.setValues(step='Static')
271 #####
272 ##BOUNDARY CONDITIONS
273 #####
274 ##Left side boundary
275 a = mdb.models['Model-1'].rootAssembly
276 v1 = a.instances['Frame-1'].vertices
277 verts1 = v1.findAt(((0.0, 0.0, 0.0), ))

```



```

278 region = a.Set(vertices=verts1, name='Set-1')
279 mdb.models['Model-1'].DisplacementBC(name='BC-1_stod_i_origo',
280     createStepName='Initial', region=region, u1=SET, u2=SET, ur3=UNSET,
281     amplitude=UNSET, distributionType=UNIFORM, fieldName='',
282     localCsys=None)
283 ##Right side boundary
284 a = mdb.models['Model-1'].rootAssembly
285 v1 = a.instances['Frame-1'].vertices
286 verts1 = v1.findAt(((L_b, 0.0, 0.0), ))
287 region = a.Set(vertices=verts1, name='Set-2')
288 mdb.models['Model-1'].DisplacementBC(name='BC-rightside',
289     createStepName='Initial', region=region, u1=SET, u2=SET, ur3=UNSET,
290     amplitude=UNSET, distributionType=UNIFORM, fieldName='',
291     localCsys=None)
292 #####
293 ##LOADS
294 #####
295 ##Vertical 1
296 a = mdb.models['Model-1'].rootAssembly
297 v1 = a.instances['Frame-1'].vertices
298 verts1 = v1.findAt(((L_b, L_c, 0.0), ))
299 region = a.Set(vertices=verts1, name='Set-4')
300 mdb.models['Model-1'].ConcentratedForce(name='Vertikal-1',
301     createStepName='Static', region=region, cf2=-V,
302     distributionType=UNIFORM, field='', localCsys=None)
303 ##Vertical 2
304 a = mdb.models['Model-1'].rootAssembly
305 v1 = a.instances['Frame-1'].vertices
306 verts1 = v1.findAt(((0.0, L_c, 0.0), ))
307 region = a.Set(vertices=verts1, name='Set-5')
308 mdb.models['Model-1'].ConcentratedForce(name='Vertikal-2',
309     createStepName='Static', region=region, cf2=-V,
310     distributionType=UNIFORM, field='', localCsys=None)
311 ## extra set
312 a = mdb.models['Model-1'].rootAssembly
313 v1 = a.instances['Frame-1'].vertices
314 verts1 = v1.findAt(((0, L_c, 0.0), ))
315 region = a.Set(vertices=verts1, name='Set-8')
316 #####
317 ##MESH
318 #####
319 p = mdb.models['Model-1'].parts['Frame']
320 p.seedPart(size=0.05, deviationFactor=0.1, minSizeFactor=0.1)
321 p = mdb.models['Model-1'].parts['Frame']
322 p.generateMesh()
323 #####
324 ##step buckle
325 #####
326 mdb.models['Model-1'].BuckleStep(name='Static', previous='Initial',
327     maintainAttributes=True, numEigen=5, vectors=18)
328 mdb.models['Model-1'].steps.changeKey(fromName='Static', toName='Buckle')
329 import job
330 #####Request output
331 mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(
332     sectionPoints=(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13))
333 a = mdb.models['Model-1'].rootAssembly
334 mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(variables=(
335     'U', 'SE'))
336 #####
337 ##Keywords 1
338 #####
339 mdb.models['Model-1'].keywordBlock.synchVersions(storeNodesAndElements=False)
340 mdb.models['Model-1'].keywordBlock.insert(GetBlockPosition('Model-1', '*End Step')-1, ""
341 *NODE FILE, GLOBAL=YES, LAST MODE=1
342 U"")
343 #####JOB1 BUCKLE
344 mdb.Job(name='Buckle', model='Model-1')
345 mdb.jobs['Buckle'].setValues(description='', memoryUnits=PERCENTAGE, memory=50,
346     numCpus=1, numDomains=1)
347 mdb.jobs['Buckle'].submit(datacheckJob=False)

```

```
348 mdb.jobs['Buckle'].waitForCompletion()
349 #####
350 ##Find eigenvalue & Curvature
351 #####
352 from odbAccess import *
353 odb = openOdb(path1)
354 STEP = odb.steps.values()[0]
355 mode=0
356 for i_frame in STEP.frames:
357     print i_frame.description
358     if mode==1:
359         f1=i_frame.description
360         MinEigen=float(f1[28:48])
361         fileS = open(fid2+str(mode), 'w')
362         fileS.write('%1.0e\n' % (MinEigen))
363         lastFrame = odb.steps['Buckle'].frames[mode]
364         Curvature=lastFrame.fieldOutputs['SK']
365         fieldValues=Curvature.values
366         fileS = open(fid2+str(mode), 'w')
367         fileS.write('%10.0f\t%10.7f\n' % (mode, MinEigen))
368         for v in fieldValues:
369             fileS.write('%10.0f\t%10.2e\n' % (v.elementLabel, v.data[0]))
370         mode=mode+1
371
372 fileS.close()
373
374 odb.close()
375 ## extra set
376 a = mdb.models['Model-1'].rootAssembly
377 n1 = a.instances['Frame-1'].nodes
378 nodes1 = n1.getSequenceFromMask(mask=(['#2'],), )
379 a.Set(nodes=nodes1, name='Set-8')
```

C.5 Python code - Approach (a) part 2

```

1  ##Set work directory
2  import os
3  os.chdir('C:/Arbete_ram/IN_PROGRESS_eugli')
4  print os.getcwd() # Prints the current working directory
5  #####
6  ##PART 2
7  #####
8  filehandle = open(path2, 'r')
9  eugli = filehandle.readline()
10 print ('EUGLI VALUE FROM FILE\t'+eugli)
11 #####step Non-linear
12 a = mdb.models['Model-1'].rootAssembly
13 mdb.models['Model-1'].StaticRiksStep(name='Buckle', previous='Initial',
14     maintainAttributes=True, initialArcInc=0.1, minArcInc=1e-08,
15     maxArcInc=0.1)
16 mdb.models['Model-1'].steps.changeKey(fromName='Buckle', toName='Staticriks')
17 mdb.models['Model-1'].steps['Staticriks'].setValues(nlgeom=ON)
18 #####
19 ##Keywords 2
20 #####
21 mdb.models['Model-1'].keywordBlock.synchVersions(storeNodesAndElements=False)
22 mdb.models['Model-1'].keywordBlock.setValues(edited = 0)
23 mdb.models['Model-1'].keywordBlock.synchVersions(storeNodesAndElements=False)
24 mdb.models['Model-1'].keywordBlock.insert(GetBlockPosition('Model-1', '*Step', name=Staticriks, nlgeom=YES
25 *IMPERFECTION, FILE=Buckle, STEP=1
26 1, ""+str(eugli)
27 )
28 #####Job non linear riks##2, 0.0001""##)
29 mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(variables=(
30 'COORD', 'S', 'SE', 'U', 'RF', 'RM', 'SF'))
31
32 mdb.Job(name='Nonlinear', model='Model-1')
33 mdb.jobs['Nonlinear'].setValues(description='', memoryUnits=PERCENTAGE,
34     memory=50, numCpus=1, numDomains=1)
35 mdb.jobs['Nonlinear'].submit(datacheckJob=False)
36 mdb.jobs['Nonlinear'].waitForCompletion()
37 #####
38 ##Find SM & SF, & LPF
39 #####
40 from odbAccess import *
41 odb = openOdb(path3)
42 STEP = odb.steps.values()[0]
43 inc=0
44 for frame_i in STEP.frames:
45     #####SEC. MOMENTS SAVE
46     Frame = odb.steps['Staticriks'].frames[inc]
47     SectMom=Frame.fieldOutputs['SM']
48     fieldValues=SectMom.values
49     fileS = open('eu_SM_'+str(inc), 'w')
50     #fileS.write('%10.0f\n' % (inc))
51     for v in fieldValues:
52         fileS.write('%10.0f\t%10.6e\n' % (v.elementLabel, v.data[0]))
53     fileS.close()
54     #####SEC. FORCES SAVE
55     Frame2 = odb.steps['Staticriks'].frames[inc]
56     SectFor=Frame2.fieldOutputs['SF']
57     fieldValues=SectFor.values
58     fileS = open('eu_SF_'+str(inc), 'w')
59     #fileS.write('%10.0f\n' % (inc))
60     for v in fieldValues:
61         fileS.write('%10.0f\t%10.6e\t%10.6e\n' % (v.elementLabel, v.data[0], v.data[1]))
62     inc=inc+1
63     fileS.close()
64 odb.close()
65
66 ##LPF
67 odb = openOdb(path3)

```

```
68 odb = session.odbs[ 'C:/Arbete_ram/IN_PROGRESS_eugli/Nonlinear.odb' ]
69 xy_result = session.XYDataFromHistory( name='Denna', odb=odb,
70     outputVariableName='Load proportionality factor: LPF for Whole Model',
71     steps=('Staticriks', ), __linkedVpName__='Viewport: 1' )
72 x0 = session.xyDataObjects[ 'Denna' ]
73 session.writeXYReport( fileName='C:/Arbete_ram/IN_PROGRESS_eugli/B3_loadprop.fil', xyData=(x0, ) )
74 session.odbs[ 'C:/Arbete_ram/IN_PROGRESS_eugli/Nonlinear.odb' ].close()
```

C.6 Python code - Approach (b)

```

1 from abaqus import *
2 from abaqusConstants import *
3 import __main__
4
5 import section
6 import regionToolset
7 import displayGroupMdbToolset as dgm
8 import part
9 import material
10 import assembly
11 import step
12 import interaction
13 import load
14 import mesh
15 import optimization
16 import job
17 import sketch
18 import visualization
19 import xyPlot
20 import displayGroupOdbToolset as dgo
21 import connectorBehavior
22 import os, re, os.path
23 ##Set work directory
24 os.chdir('C:/Arbete_ram/IN_PROGRESS_explicit')
25 print os.getcwd() # Prints the current working directory
26 #####
27 ##                               INDATA
28 #####
29 ##Beam IPE300
30 L_b=2
31 h_b=0.3
32 b1_b=0.15
33 b2_b=b1_b
34 t1_b=0.0107
35 t2_b=t1_b
36 t3_b=0.0071
37 ##properties in generalised section
38 A_b=0.005381
39 I11_b=0.00008356
40 I12_b=0.000011520
41 I22_b=0.000007881
42 J_b=0.0000001975
43 ##Column IPE300
44 L_c=L_b
45 h_c=0.3
46 b1_c=0.15
47 b2_c=b1_c
48 t1_c=0.0107
49 t2_c=t1_c
50 t3_c=0.0071
51 ##properties in generalised section
52 A_c=0.005381
53 I11_c=0.00008356
54 I12_c=0.000011520
55 I22_c=0.000007881
56 J_c=0.0000001975
57 ##Loads
58 V=500000
59 H=0
60 ##Other
61 E=210E9
62 G=80000000000.0
63 vPo=0.3
64 fy=355E6
65 bucklingcoefficient=0.21 ##Only used in matlab,bucklingcurve a
66 #####OBS
67 ##Imperfection angle

```

```

68 ##
69 m=2 ##no of columns in a row
70 phinoll=0.005
71 alphah=2*((L_c)**(0.5))**(-1)#NOTE:Should be checked manually so that 2/3<alphah<1
72 alpham=(0.5*(1+(m)**(-1)))**0.5)
73 phi=phinoll*alphah*alpham ##multipliceras sedan med L_c
74 imperfectionen_sway=phi*L_c
75 imp=imperfectionen_sway
76 #####
77 ##                                GEOMETRY
78 #####
79 ##BEAM
80 s = mdb.models[ 'Model-1' ]. ConstrainedSketch(name='__profile__', sheetSize=20.0)
81 g, v, d, c = s.geometry, s.vertices, s.dimensions, s.constraints
82 s.setPrimaryObject(option=STANDALONE)
83 s.Line(point1=(0.0, 0.0), point2=(L_b, 0.0))
84 s.HorizontalConstraint(entity=g.findAt((L_b*2**(-1), 0.0)), addUndoState=False)
85 p = mdb.models[ 'Model-1' ]. Part(name='Beam', dimensionality=TWO_D_PLANAR,
86     type=DEFORMABLE_BODY)
87 p = mdb.models[ 'Model-1' ]. parts[ 'Beam' ]
88 p.BaseWire(sketch=s)
89 s.unsetPrimaryObject()
90 p = mdb.models[ 'Model-1' ]. parts[ 'Beam' ]
91 del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
92 #####
93 ##COLUMN
94 s = mdb.models[ 'Model-1' ]. ConstrainedSketch(name='__profile__', sheetSize=20.0)
95 g, v, d, c = s.geometry, s.vertices, s.dimensions, s.constraints
96 s.setPrimaryObject(option=STANDALONE)
97 s.Line(point1=(0.0, 0.0), point2=(imp, L_c))
98 p = mdb.models[ 'Model-1' ]. Part(name='Column', dimensionality=TWO_D_PLANAR,
99     type=DEFORMABLE_BODY)
100 p = mdb.models[ 'Model-1' ]. parts[ 'Column' ]
101 p.BaseWire(sketch=s)
102 s.unsetPrimaryObject()
103 p = mdb.models[ 'Model-1' ]. parts[ 'Column' ]
104 session.viewports[ 'Viewport: 1' ]. setValues(displayedObject=p)
105 del mdb.models[ 'Model-1' ]. sketches[ '__profile__' ]
106 #####
107 ## Section orientation
108 #####
109 p = mdb.models[ 'Model-1' ]. parts[ 'Column' ]
110 e = p.edges
111 edges = e.findAt(((0.0, 0.0, 0.0), ))
112 region=p.Set(edges=edges, name='Set-1')
113 p = mdb.models[ 'Model-1' ]. parts[ 'Column' ]
114 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
115     -1.0))
116 p1 = mdb.models[ 'Model-1' ]. parts[ 'Beam' ]
117 p = mdb.models[ 'Model-1' ]. parts[ 'Beam' ]
118 e = p.edges
119 edges = e.findAt(((L_b*4**(-1), 0.0, 0.0), ))
120 region=p.Set(edges=edges, name='Set-1')
121 p = mdb.models[ 'Model-1' ]. parts[ 'Beam' ]
122 p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
123     -1.0))
124 #####
125 ##MATERIAL
126 #####
127 mdb.models[ 'Model-1' ]. Material(name='Steel')
128 mdb.models[ 'Model-1' ]. materials[ 'Steel' ]. Elastic(table=((210000000000, 0.3),
129     ))
130 #####
131 ##I-profiles
132 #####
133 ##Generalized
134 mdb.models[ 'Model-1' ]. GeneralizedProfile(name='G_C_HEA300', area=A_c,
135     i11=I11_c, i12=0, i22=I11_c, j=J_c, gammaO=0.0,
136     gammaW=0.0)
137 mdb.models[ 'Model-1' ]. GeneralizedProfile(name='G_B_IPE550', area=A_b,

```

```

138     i11=I11_b, i12=0, i22=I11_b, j=J_b, gammaO=0.0,
139     gammaW=0.0)
140 #####
141 ##create section
142 #####
143 mdb.models['Model-1'].BeamSection(name='Column_section',
144     integration=BEFORE_ANALYSIS, poissonRatio=vPo, beamShape=CONSTANT,
145     profile='G_C_HEA300', thermalExpansion=OFF, temperatureDependency=OFF,
146     dependencies=0, table=((E, G), ),
147     alphaDamping=0.0, betaDamping=0.0, compositeDamping=0.0, centroid=(0.0,
148     0.0), shearCenter=(0.0, 0.0), consistentMassMatrix=False)
149
150 mdb.models['Model-1'].BeamSection(name='Beam_section',
151     integration=BEFORE_ANALYSIS, poissonRatio=vPo, beamShape=CONSTANT,
152     profile='G_B_IPE550', thermalExpansion=OFF, temperatureDependency=OFF,
153     dependencies=0, table=((E, G), ),
154     alphaDamping=0.0, betaDamping=0.0, compositeDamping=0.0, centroid=(0.0,
155     0.0), shearCenter=(0.0, 0.0), consistentMassMatrix=False)
156 #####
157 ##assign section
158 #####
159 p = mdb.models['Model-1'].parts['Beam']
160 e = p.edges
161 edges = e.findAt(((L_b*4**(-1), 0.0, 0.0), ))
162 region = p.Set(edges=edges, name='Set-1')
163 p = mdb.models['Model-1'].parts['Beam']
164 p.SectionAssignment(region=region, sectionName='Beam_section', offset=0.0,
165     offsetType=MIDDLE_SURFACE, offsetField='',
166     thicknessAssignment=FROM_SECTION)
167 p = mdb.models['Model-1'].parts['Column']
168 e = p.edges
169 edges = e.findAt(((0.0, 0.0, 0.0), ))
170 region = p.Set(edges=edges, name='Set-1')
171 p = mdb.models['Model-1'].parts['Column']
172 p.SectionAssignment(region=region, sectionName='Column_section', offset=0.0,
173     offsetType=MIDDLE_SURFACE, offsetField='',
174     thicknessAssignment=FROM_SECTION)
175 #####
176 ##Assembly
177 #####
178 a = mdb.models['Model-1'].rootAssembly
179 a.DatumCsysByDefault(CARTESIAN)
180 p = mdb.models['Model-1'].parts['Beam']
181 a.Instance(name='Beam-1', part=p, dependent=OFF)
182 a = mdb.models['Model-1'].rootAssembly
183 p = mdb.models['Model-1'].parts['Column']
184 a.Instance(name='Column-1', part=p, dependent=OFF)
185 a = mdb.models['Model-1'].rootAssembly
186 a.translate(instanceList=('Column-1', ), vector=(L_b, 0.0, 0.0))
187 a = mdb.models['Model-1'].rootAssembly
188 p = mdb.models['Model-1'].parts['Column']
189 a.Instance(name='Column-2', part=p, dependent=OFF)
190 a = mdb.models['Model-1'].rootAssembly
191 a.translate(instanceList=('Beam-1', ), vector=(imp, L_c, 0.0))
192 #####
193 ##MERGE WITHOUT RETAINING BOUNDARTIES
194 a = mdb.models['Model-1'].rootAssembly
195 a.InstanceFromBooleanMerge(name='Frame', instances=(a.instances['Beam-1'],
196     a.instances['Column-1'], a.instances['Column-2'], ),
197     originalInstances=SUPPRESS, domain=GEOMETRY)
198 #####
199 ##Orientation frame
200 #####
201 p = mdb.models['Model-1'].parts['Column']
202 session.viewports['Viewport: 1'].setValues(displayedObject=p)
203 p = mdb.models['Model-1'].parts['Frame']
204 session.viewports['Viewport: 1'].setValues(displayedObject=p)
205 p = mdb.models['Model-1'].parts['Frame']
206 e = p.edges
207 edges = e.findAt(((0.0, 0.0, 0.0), ), ((imp, L_c, 0.0), ), ((L_b, 0.0, 0.0),

```

```
208     ))
209     region=p.Set(edges=edges, name='Set-2')
210     p = mdb.models['Model-1'].parts['Frame']
211     p.assignBeamSectionOrientation(region=region, method=N1_COSINES, n1=(0.0, 0.0,
212         -1.0))
213     #####
214     ##Create Step: Non linear riks static
215     #####
216     mdb.models['Model-1'].StaticRiksStep(name='Staticriks', previous='Initial',
217         maxNumInc=100, initialArcInc=0.1, minArcInc=1e-08, maxArcInc=0.1,
218         nlgeom=ON)
219     #####
220     ##BOUNDARY CONDITIONS
221     #####
222     ##Left side boundary
223     a = mdb.models['Model-1'].rootAssembly
224     v1 = a.instances['Frame-1'].vertices
225     verts1 = v1.findAt(((0.0, 0.0, 0.0), ))
226     region = a.Set(vertices=verts1, name='Set-1')
227     mdb.models['Model-1'].DisplacementBC(name='BC-1_stod_i_origo',
228         createStepName='Initial', region=region, u1=SET, u2=SET, ur3=UNSET,
229         amplitude=UNSET, distributionType=UNIFORM, fieldName='',
230         localCsys=None)
231     ##Right side boundary
232     a = mdb.models['Model-1'].rootAssembly
233     v1 = a.instances['Frame-1'].vertices
234     verts1 = v1.findAt(((L_b, 0.0, 0.0), ))
235     region = a.Set(vertices=verts1, name='Set-2')
236     mdb.models['Model-1'].DisplacementBC(name='BC-rightside',
237         createStepName='Initial', region=region, u1=SET, u2=SET, ur3=UNSET,
238         amplitude=UNSET, distributionType=UNIFORM, fieldName='',
239         localCsys=None)
240     #####
241     ##LOADS
242     #####
243     ##Vertikal 1
244     a = mdb.models['Model-1'].rootAssembly
245     v1 = a.instances['Frame-1'].vertices
246     verts1 = v1.findAt(((L_b+imp, L_c, 0.0), ))
247     region = a.Set(vertices=verts1, name='Set-4')
248     mdb.models['Model-1'].ConcentratedForce(name='Vertikal-1',
249         createStepName='Staticriks', region=region, cf2=-V,
250         distributionType=UNIFORM, field='', localCsys=None)
251     ##Vertikal 2
252     a = mdb.models['Model-1'].rootAssembly
253     v1 = a.instances['Frame-1'].vertices
254     verts1 = v1.findAt(((imp, L_c, 0.0), ))
255     region = a.Set(vertices=verts1, name='Set-5')
256     mdb.models['Model-1'].ConcentratedForce(name='Vertikal-2',
257         createStepName='Staticriks', region=region, cf2=-V,
258         distributionType=UNIFORM, field='', localCsys=None)
259     ## extra set
260     a = mdb.models['Model-1'].rootAssembly
261     v1 = a.instances['Frame-1'].vertices
262     verts1 = v1.findAt(((imp, L_c, 0.0), ))
263     region = a.Set(vertices=verts1, name='Set-8')
264     #####
265     ##MESH
266     #####
267     p = mdb.models['Model-1'].parts['Frame']
268     p.seedPart(size=0.05, deviationFactor=0.1, minSizeFactor=0.1)
269     p = mdb.models['Model-1'].parts['Frame']
270     p.generateMesh()
271     #####
272     ##JOB
273     #####
274     #####Request output
275     mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(
276         sectionPoints=(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13))
277     a = mdb.models['Model-1'].rootAssembly
```



```

278 mdb.models['Model-1'].fieldOutputRequests['F-Output-1'].setValues(variables=(
279     'COORD', 'S', 'SF', 'PE', 'PEEQ', 'PEMAG', 'LE', 'TE', 'TEEQ', 'TEVOL',
280     'U', 'RF',
281     'RM', 'CF', 'SF', 'CSTRESS', 'CDISP', 'MVF'))
282 mdb.Job(name='Nonlinear', model='Model-1')
283 mdb.jobs['Nonlinear'].setValues(description='', memoryUnits=PERCENTAGE,
284     memory=50, numCpus=1, numDomains=1)
285 mdb.jobs['Nonlinear'].submit(datacheckJob=False)
286 mdb.jobs['Nonlinear'].waitForCompletion()
287 ##check no of elem
288 elemArr = mdb.models['Model-1'].rootAssembly.instances['Frame-1'].elements;
289 noelem=len(elemArr);
290 #####
291 ##Export results
292 #####
293 from odbAccess import *
294 from abaqusConstants import *
295 import cmath
296 odb = openOdb(path='C:/Arbete_ram/IN_PROGRESS_explicit/Nonlinear.odb')
297 ns = odb.rootAssembly.nodeSets['SET-8']
298 disp = odb.steps['Staticriks'].frames[-1].fieldOutputs['U'].getSubset(region=ns,
299     position=NODAL).values[0].data[0]
300 absdisp2=abs(disp)
301 #####
302 ##Export results
303 #####
304 ##LPF
305 odb = session.odbs['C:/Arbete_ram/IN_PROGRESS_explicit/Nonlinear.odb']
306 session.viewports['Viewport: 1'].odbDisplay.basicOptions.setValues(
307     averagingThreshold=100)
308 xy_result = session.XYDataFromHistory(name='LPF Whole Model-1', odb=odb,
309     outputVariableName='Load proportionality factor: LPF for Whole Model',
310     steps=('Staticriks', ), __linkedVpName__='Viewport: 1')
311 x0 = session.xyDataObjects['LPF Whole Model-1']
312 session.writeXYReport(fileName='Ex_loadprop.fil', xyData=(x0, ))
313 session.odbs['C:/Arbete_ram/IN_PROGRESS_explicit/Nonlinear.odb'].close()
314 #####
315 ##Find SM & SF
316 #####
317 from odbAccess import *
318 odb = openOdb('C:/Arbete_ram/IN_PROGRESS_explicit/Nonlinear.odb')
319 STEP = odb.steps.values()[0]
320 inc=0
321 for frame_i in STEP.frames:
322     ####SEC. MOMENTS SAVE
323     Frame = odb.steps['Staticriks'].frames[inc]
324     SectMom=Frame.fieldOutputs['SM']
325     fieldValues=SectMom.values
326     fileS = open('ex_SM_'+str(inc), 'w')
327     #fileS.write('%10.0f\n' % (inc))
328     for v in fieldValues:
329         fileS.write('%10.0f\t%10.6e\n' % (v.elementLabel, v.data[0]))
330     fileS.close()
331     ####SEC. FORCES SAVE
332     Frame2 = odb.steps['Staticriks'].frames[inc]
333     SectFor=Frame2.fieldOutputs['SF']
334     fieldValues=SectFor.values
335     fileS = open('ex_SF_'+str(inc), 'w')
336     #fileS.write('%10.0f\n' % (inc))
337     for v in fieldValues:
338         fileS.write('%10.0f\t%10.6e\t%10.6e\n' % (v.elementLabel,
339             v.data[0], v.data[1]))
340     inc=inc+1
341     fileS.close()
342 odb.close()

```

C.7 Matlab code - Approach (a)

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2                                     % 2019-03-25
3  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4  %%
5  clc
6  close all
7  clear all
8  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
9                                     % EUGLI
10 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
11 %Iterate NEd
12 NEd=1778940
13 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
14 addpath C:\Arbete_ram\IN_PROGRESS_eugli
15 %Input constants
16 inc=100;
17 gamma=1.0;
18 Indata = Fil2str('B1_Inputdata');
19 Indata = strsplit(Indata);
20 Indata=str2double(Indata);
21 ad=1%0 %Factor to fit the file from Abaqus, set to 0 or 1
22 In_b=Indata(1+ad:5+ad);%L_b,h_b,b1_b,t1_b,t3_b
23 In_c=Indata(6+ad:10+ad);%L_c,h_c,b1_c,t1_c,t3_c
24 Loads=Indata(11+ad:12+ad);
25 Imperfection_sway=Indata(13+ad);
26 E=Indata(14+ad);
27 fy=Indata(15+ad);
28 alp=Indata(17+ad)
29 In_b(2,:)=Indata(18+ad:22+ad);
30 In_c(2,:)=Indata(23+ad:27+ad);
31
32 Wely=557.1*10^(3)/(10^3)^3;
33 A=In_c(2,1)
34 Iy=In_c(2,2)
35
36 %Input from buckle
37 buck = Fil2str('B2_buckout1');
38 buck = strsplit(buck);
39 buck=str2double(buck);
40 ModEig=buck(2:3);                                     %Mode and eigenvalue
41 Codd = buck(5:2:end);                                 %Curvature (Odd-Indexed)
42 Even = buck(4:2:end);                                 %Element no (Even-Indexed)
43 ElCurv=[Even' Codd'];                               %Element no and curvature (Even-Indexed)
44
45 format long
46
47 NRk=fy*A;
48 MyRk=fy*Wely;
49
50 [maxcurv, elatmax]=max(abs(ElCurv(:,2)));
51
52 eta_crmax=maxcurv;
53 Ncry=ModEig(2)/(NEd/Loads(2))*NEd;
54
55 alph_cr=ModEig(2)*Loads(2)/(NEd);
56 alph_ult=NRk/NEd;
57
58 l_bar=sqrt(alph_ult/alph_cr);
59 o_y=0.5*(1+alp*(l_bar-0.2)+l_bar^2);
60 Xy=1/(o_y+(o_y^2-l_bar^2)^(1/2));
61
62 M_c=E*Iy*eta_crmax;
63
64 e_0_utantabell=alp*(l_bar-0.2)*(MyRk/NRk)*(1-Xy*l_bar^2/gamma)/...
65 (1-Xy*l_bar^2);
66 eta_init_utantabell=(e_0_utantabell)*(Ncry/M_c);
67

```

```

68 A0output_eugli=[1_bar ModEig(2) Ncry NRk/1_bar^2];
69 A1_output_eugli=[NEd eta_init_utantabell e_0_utantabell maxcurv M_c]
70
71 %PRINT EUGLI IMP
72 fileID = fopen(fullfile('C:\Arbete_ram\IN_PROGRESS_eugli', 'EUGLI_value.dat')...
73     , 'w');
74 fprintf(fileID, '%10.7f', eta_init_utantabell);
75 fclose(fileID);
76
77 %%
78 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
79                               %POST PROCESSING
80 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
81 inc=100
82 hold on
83 NRd=NRk/gamma;
84 MRd=MyRk/gamma;
85 %GET SF AND SM
86 for i=0:inc
87     fid = sprintf('%s%d', 'eu_SF', i);
88     SFraw = str2double(strsplit(Fil2str(fid)));
89     SF=[SFraw(2:3:end) SFraw(3:3:end) SFraw(4:3:end)]; %Element no and section forces (Even-Indexed)
90     fid = sprintf('%s%d', 'eu_SM', i);
91     SMraw = str2double(strsplit(Fil2str(fid)));
92     SM=[SMraw(2:2:end) SMraw(3:2:end)]; %Element no and section moments (Even-Indexed)
93
94     u_per_el_inc=(abs(SF(1:end,2))/NRd)+(abs(SM(1:end,2))/MRd);
95     [u_max_inc, pos] = max(u_per_el_inc);
96     sd(i+1,:)=i, pos, u_max_inc, SF(pos,2), SM(pos,2)];
97
98 end
99
100 %Get LPF
101 indat= str2double(strsplit(Fil2str('B3_loadprop.fil')));
102
103 LPF=[ indat(4:2:end-1) indat(5:2:end) ];
104 inc=[0:length(LPF)-1];
105 LPF=[inc LPF];
106 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
107 %FIND position & N & M where utilization ratio = 1
108 RefUtilization = 1.0; %Value to compare the calculated utilization values
109 for ii = 1:length(sd(:,3));
110     DifCalc(ii) = abs(sd(ii,3) - RefUtilization);
111 end
112 pU1=find(DifCalc == min(DifCalc));
113 LPFU1=LPF(pU1,:);
114 A2_outputinfoU1=[sd(pU1,:) LPFU1(3)];
115 LoadE=A2_outputinfoU1(4:5);
116
117 %PRINT LPF to Explicit
118 fileID = fopen(fullfile('C:\Arbete_ram\IN_PROGRESS_explicit', ...
119     'LPF_eugli.dat'), 'w');
120 fprintf(fileID, '%10.7f\t', A2_outputinfoU1);
121 fclose(fileID);
122
123 %PLOT
124 figure(1)
125 hold on
126 scatter(LPF(:,1), LPF(:,3), '. ')
127 xlabel('increment')
128 ylabel('LPF')
129
130 figure(2)
131 scatter(sd(:,1), sd(:,2), '. ');
132 xlabel('increments')
133 ylabel('Element')
134
135 figure(3)
136 hold on
137 plot([0 MRd*0.8], [NRd NRd])

```

```
138 plot([MRd MRd], [0 0.5*NRd])
139 scatter(abs(sd(:,5)),abs(sd(:,4)),'.')
140 xlabel('M [Nm]')
141 ylabel('N [N]')
142 plot(abs(A2_outputinfoU1(5)),abs(A2_outputinfoU1(4)), 'o')
143 legend('EUGLI')
144
145 figure(4)
146 hold on
147 xlabel('increments')
148 ylabel('Utilization ratio')
149 scatter(sd(:,1),sd(:,3),'.');
150 legend('EUGLI')
```

C.8 Matlab code - Interacting results from approach (a) and (b)

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  % 2019-03-25
3  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4
5  % clc
6  % close all
7  % clear all
8
9  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
10 % LPF handling
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12 addpath('C:\Arbete_ram\IN_PROGRESS_explicit');
13 indat= str2double(strsplit(Fil2str('Ex_loadprop.fil')));
14
15 LPF_ex=[indat(6:2:end-1)' indat(7:2:end)'];
16 inc=[0:length(LPF_ex)-1]';
17 LPF_ex=[inc LPF_ex];
18
19 figure(1)
20 hold on
21 scatter(LPF_ex(:,1),LPF_ex(:,3),'.')
22 %scatter(LPF_sta(:,1),LPF_sta(:,2),'x')%att tas bort
23 xlabel('increment')
24 ylabel('LPF')
25 %LPF_ex=[LPF_sta(:,1) LPF_sta(:,1) LPF_sta(:,2)]%att tas bort
26
27 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
28
29 LPF_eugli= str2double(strsplit(Fil2str('C:\Arbete_ram\IN_PROGRESS_explicit\LPF_eugli.dat')));
30 LPF_eugli=LPF_eugli(1:6);
31
32 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
33
34 %FIND position & LPF where utilization ratio = 1
35 RefLPF = LPF_eugli(6); %Value to compare the LPF of explicit to
36 for ii = 1:length(LPF_ex);
37     DifCalc(ii) = abs(LPF_ex(ii,3) - RefLPF);
38 end
39 pLPF=find(DifCalc == min(DifCalc));
40 pLPF=pLPF(1)
41 infoLPF=[LPF_ex(pLPF,1) LPF_eugli(6) LPF_ex(pLPF,3)];
42
43 plot(LPF_eugli(1),LPF_eugli(6),'o')
44 plot(infoLPF(1),infoLPF(3),'o')
45 legend('EUGLI','EXPLICIT','data eugli','data explicit')
46 fprintf('At increment %1.0f the LPF of explicit (LPF=%1.3f) matches the LPF of eugli (LPF=%1.3f). Differen
47
48 %GET SF AND SM %%BYGG OM
49 inc=100;
50 for i=0:inc
51     fid = sprintf('%s%d','ex_SF',i);
52     SFraw = str2double(strsplit(Fil2str(fid)));
53     SF=[SFraw(2:3:end)' SFraw(3:3:end)' SFraw(4:3:end)'];%Element no and
54     fid = sprintf('%s%d','ex_SM',i); %section forces
55     SMraw = str2double(strsplit(Fil2str(fid))); % (Even-Indexed)
56     SM=[SMraw(2:2:end)' SMraw(3:2:end)'];%Element no and
57     if i==infoLPF(1) %section moments(Even-Indexed)
58         info_krafter=[infoLPF(1) infoLPF(2) infoLPF(3) SF(LPF_eugli(2),2)...
59             SM(LPF_eugli(2),2)];
60     end
61 end
62
63 A3_outputInfoEx=[infoLPF(1) 0 0 info_krafter(4) info_krafter(5)...
64     info_krafter(2) info_krafter(3)]
65

```

```
66 %save work space
67 baseFileName = sprintf('figure_%d.mat',In_c(1));
68 fullFileName = fullfile('C:\Arbete_ram\EXPLICIT\Data', baseFileName);
69 save(fullFileName)
70
71 fprintf('Increment: %1.0f\n Load factor eu: %1.3f \n Load factor ex: %1.3f\n Sectional force ex: %10.1f N
72
73 AAtot=zeros(12,7)
74 AAtot(1,1:4)=A0output_eugli
75 AAtot(5,1:5)=A1_output_eugli
76 AAtot(7,1:6)=A2_outputinfoU1
77 AAtot(12,1:7)=A3_outputInfoEx
```

D

Benchmarks

For the two studied benchmarks, the results for the verification are presented in this Appendix.

D.1 Benchmark I

In this benchmark, the implementation of equivalent loads in FE-analysis was initially verified. The results from the verification is presented in Table D.1.

Table D.1: Verification of the implementation of equivalent loads.

FE-analysis	Benchmark results	Thesis results	Difference
N_{Ed} [kN]	1000	993.9	0.61%
M_{Ed} [kNm]	19.60	19.66	0.31%
Utilisation [-]	0.940	0.936	0.43%

Then, four different methods to consider imperfections and second-order effects for a column were investigated. The obtained results was compared with the ones obtained from the benchmark, and the comparison is shown in Table D.2.

Table D.2: Comparison of the results on the consideration of imperfections and second-order effects.

Analytical analysis	Benchmark results Utilisation	Thesis results Utilisation
Buckling curves	0.837	0.837
Initial bow imperfection	0.940	0.940
Equivalent loads	0.940	0.940
EUGLI imperfection	0.835	0.835

These comparisons with this benchmark, was used a for *Study I* to verify the implementation of the imperfections and the methods concerning imperfections and second-order effects.

D.2 Benchmark II

This benchmark concerned a column exposed to an EUGLI imperfection. It was used as a benchmark for the design procedure concerning EUGLI imperfection that been used in *Study II*. The results from the comparison is presented in Table D.3.

Table D.3: Comparison of the results on the design procedure concerning an EUGLI imperfection.

	Benchmark results	Thesis results	Difference
N_{cr} [kN]	29897	28894	3.35%
α_{cr} [-]	8.36	8.08	3.35%
λ [-]	0.375	0.381	1.6%
ϕ [-]	0.60	0.60	0%
χ [-]	0.936	0.933	0.32%
$e_{0,d,m}$ [mm]	5.847	6.07	3.81%
x_m [m]	2.99	2.98	0.40%
$EI\eta''_{cr}(x)$ [kNm]	23751	21180	10.82%
$\eta_{init,m,max}$ [-]	7.98	8.27	3.63%