

# Single Event Upset Behavior of CMOS Static RAM cells

Kjell O. Jeppson, Udo Lieneweg and Martin G. Buehler

## Abstract

An improved state-space analysis of the CMOS static RAM cell is presented. Introducing the concept of the dividing line, the critical charge for heavy-ion-induced upset of memory cells can be calculated considering symmetrical as well as asymmetrical capacitive loads. From the critical charge, the upset-rate per bit-day for static RAMs can be estimated.

## Introduction

To predict the heavy-ion-induced upset rate of static random access memory (SRAM) cells, Buehler and Allen [1] developed an analytical method based on state-space analysis [2]. Cell upsets are eventually caused if the hole-electron pairs generated along the track of an alpha particle hitting the memory cell, are collected by the reverse-biased pn-junction of an output node. A 5 MeV alpha particle generates, approximately, an estimated one million hole-electron pairs corresponding to a charge of 0.16 pC. If this charge is collected by the reverse-biased pn-junction of an output node, this node will be charged or discharged depending on its state. If the current pulse during the alpha hit is short compared to the response time of the cell, the node set and release approach [1] can be used. In this approach, the output node voltage is set by the alpha hit, whereafter the released cell is analyzed to see if the alpha hit causes an upset or not. For 5-MeV alpha particles, the node set and release approach is justified by the fact that, even if the current pulse is best approximated by a decaying exponential with a time constant of 1 ns [3], most of the charges are collected within 200 ps [4].

In this paper, an improved analysis of the static RAM-cell in the release mode is presented which yields better understanding of the RAM cell behavior and more accurate expressions of the critical upset charge. The analysis is based on the cell behavior close to the meta-stable state rather than on empirical observations of the initial slopes of the node voltage curves.

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## State-space analysis

The core of a CMOS static RAM cell is the bistable latch, or flip-flop, consisting of two cross-coupled inverters as shown in Fig. 1. The two coupling nodes, N1 and N2, have effective capacitances to ground,  $C_1$  and  $C_2$ , respectively, and a mutual capacitance  $C_m$ . The state of the flip-flop is described by the two node voltages,  $V_1$  and  $V_2$ . The bistable flip-flop has three steady states: the one-state (0,  $V_{DD}$ ), the zero-state ( $V_{DD}$ , 0) and the unstable state ( $V_{M1}, V_{M2}$ ), usually known as the metastable state.

The dynamic behaviour of the flip-flop is described by the current equations of the two nodes, i.e.:

$$i_1 = C_1 \frac{dV_1}{dt} + C_m \left( \frac{dV_1}{dt} - \frac{dV_2}{dt} \right). \quad (1)$$

$$i_2 = C_2 \frac{dV_2}{dt} + C_m \left( \frac{dV_2}{dt} - \frac{dV_1}{dt} \right). \quad (2)$$

where  $i_1$  and  $i_2$  are the currents flowing into the two nodes N1 and N2 .

The two equations closing the system are governed by Kirchhoff's current law and gives

$$i_1 = i_{p1} - i_{n1}. \quad (3)$$

$$i_2 = i_{p2} - i_{n2}. \quad (4)$$

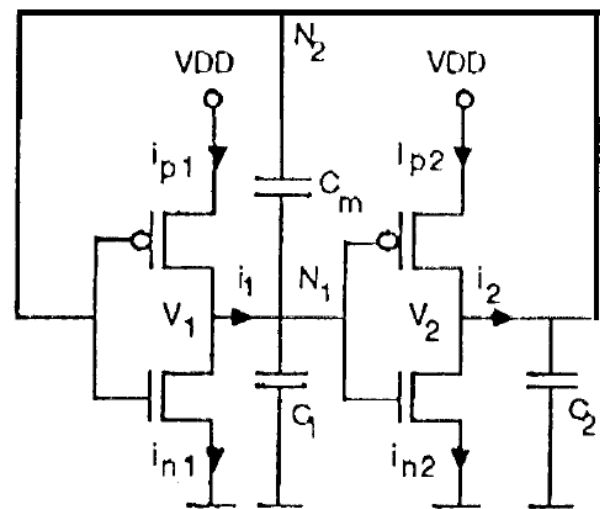


Fig. 1. Two cross-coupled inverters are used to design a bistable flip-flop.

where  $i_{p1}(V_2, V_1)$  and  $i_{p2}(V_1, V_2)$  are the currents through the two P-channel transistors and  $i_{n1}(V_2, V_1)$  and  $i_{n2}(V_1, V_2)$  are the currents through the two N-channel transistors, respectively.

The three steady state solutions of the system are given by

$$i_1 = i_{p1} - i_{n1} = 0, \quad (5)$$

$$i_2 = i_{p2} - i_{n2} = 0. \quad (6)$$

where the two equations represent the transfer curves of the two inverters as shown in Fig. 2.

If the flip-flop, for any reason such as an alpha particle hit, is upset from its steady states, the "return-trajectory" from any given state,  $(V_{10}, V_{20})$ , to one of the steady states has to be derived numerically. This is because of the complicated non-linear voltage dependence of the transistor and capacitor models, which results in non-linear differential equations, and which generally cannot be solved analytically. The most convenient way to solve the problem is to use a circuit simulator such as SPICE. A typical example of the results of such simulations is shown in Fig. 3.

Equations (1) and (2) give directly the velocity in state space,

$$v = \left( \frac{dV_1}{dt}, \frac{dV_2}{dt} \right).$$

if the dc current-voltage characteristic of the latch,  $i_1(V_2, V_1)$ ,  $i_2(V_1, V_2)$ , and the node capacitances are

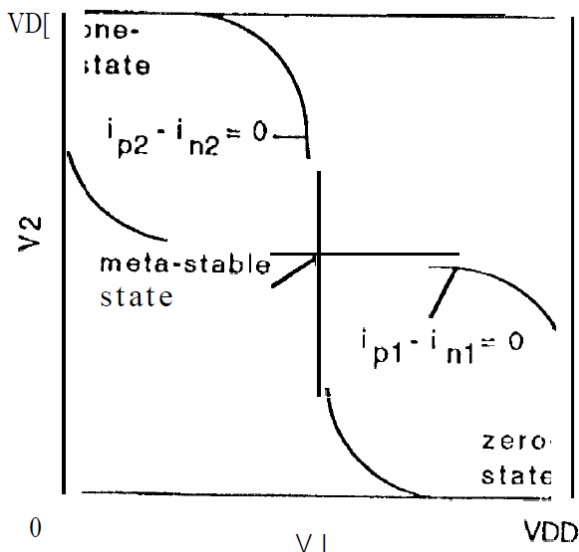


Fig. 2. The static transfer curves of the two inverters in a flip-flop are illustrated in the  $(V_1, V_2)$  state plane.

known. In Fig. 4 the two velocity components are shown plotted in the  $V_1, V_2$ -state plane, while Fig. 5 shows the corresponding vector field representation. As illustrated in Fig. 4 each velocity component is zero along the corresponding transfer curve (as long as the mutual capacitance can be neglected). For the steady state solutions both velocity components are zero. From the velocity vector field the slope,  $dV_2/dV_1$ , is known analytically in any point along each of the return trajectories as shown in Fig. 3. The velocity vector field also allows a graphical construction of the return trajectories by following the directions given by the vectors as illustrated in Fig. 5. Note that the return trajectories will always cross the static transfer curve characterized by  $i_2=0$  horizontally, and the other static transfer curve (characterized by  $i_1=0$ ) vertically [5]. Of particular interest with respect to single event upsets are the two trajectories leading to the metastable point. These two trajectories divide the state-plane into two halves and will serve as a "separatrix" [6] or "dividing line" during the alpha particle hit. If this dividing line is crossed during the hit, the cell will be upset and change its state during the following "release" mode, otherwise it will return to the same state as before. The next section will give an analytical expression for the dividing line as a guide for the RAM designer.

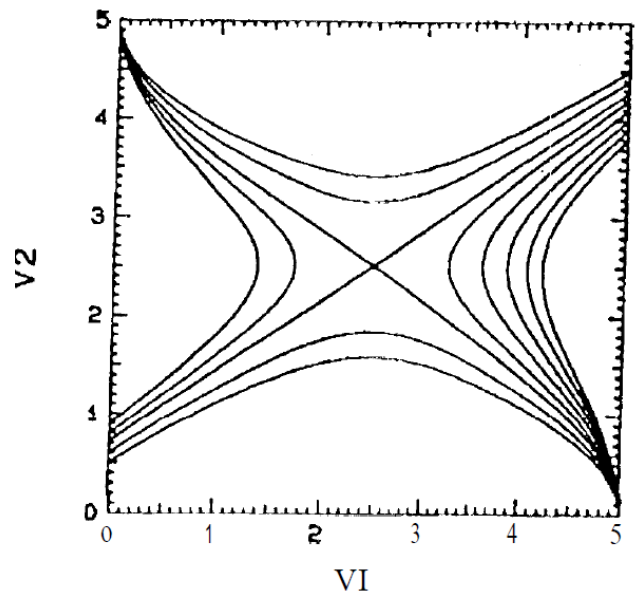


Fig. 3. SPICE-simulated "return-trajectories" to one of the stable states from an arbitrary point  $(V_{10}, V_{20})$ , in the  $(V_1, V_2)$  state plane.

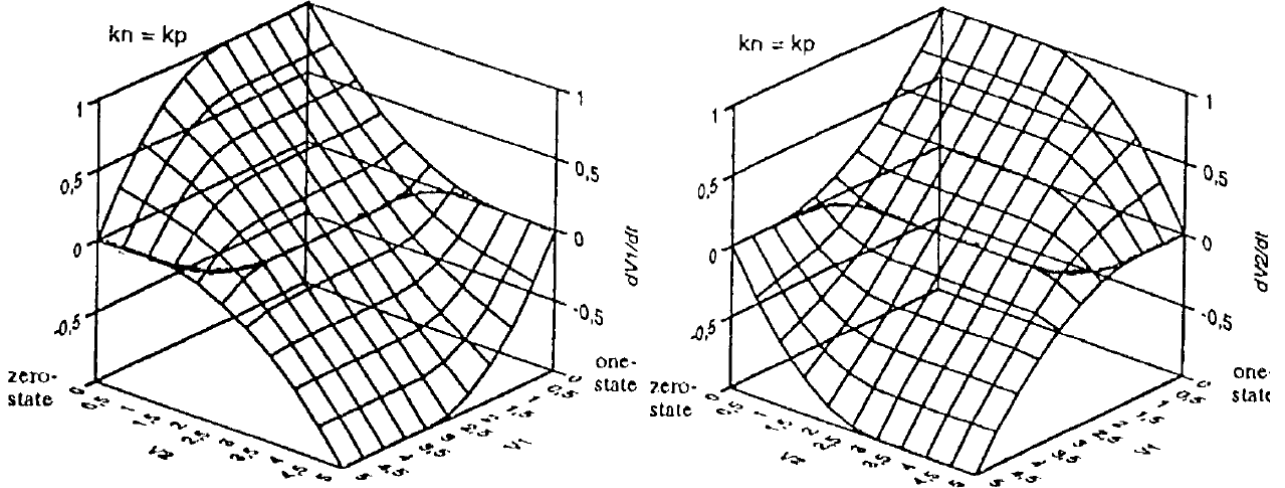


Fig. 4. Phase-space diagrams for the velocities  $dV_1/dt$  (left) and  $dV_2/dt$  (right).

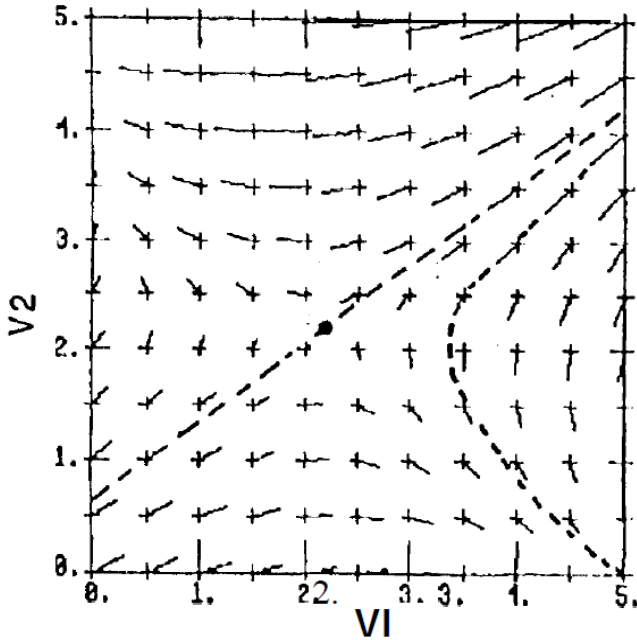


Fig. 5. The velocity vector field in the  $(V_1, V_2)$  state plane for  $C_1/C_2=0.5$  and  $\beta_n/\beta_p=3.3$ . Also shown is one of the possible return trajectories and the two halves of the separatrix ending in the metastable point.

#### Analytical Description of the Dividing Line

To derive analytical expressions for the dividing line, simplified transistor models must be used. Simulations using different transistor models suggest that the trajectories leading to the metastable state, i.e. the dividing line, can be approximated by a straight line with very good accuracy. To derive an expression for the slope of this line, transistor currents  $i_1$  and  $i_2$  are linearized around the metastable point. Assuming identical inverters (except for their capacitive loads), and neglecting the mutual

capacitance  $C_m$ , and the output conductances of the transistors, we obtain from Eqs (1) and (2),

$$\frac{dV_2}{dV_1} = \frac{C_1 (g_{mn1} + g_{mp1})(V_1 - V_{M1})}{C_2 (g_{mn2} + g_{mp2})(V_2 - V_{M2})} \quad (7)$$

where  $g_{mn}$  and  $g_{mp}$  are the transconductances in the metastable point  $(V_m, V_m)$  of the n- and p-channel transistors, respectively. Assuming a linear relationship between  $V_2$  and  $V_1$  along the dividing line,

$$V_2 - V_M = K(V_1 - V_M) \quad (8)$$

where  $K = dV_2/dV_1$ , we obtain

$$K = \pm \sqrt{\frac{C_1}{C_2}} \quad (9)$$

This result suggests that the RAM cell enters the metastable state along a straight line with slope  $\sqrt{C_1/C_2}$ , and leaves it along another straight line with slope  $-\sqrt{C_1/C_2}$ . The dividing line is therefore given by

$$V_2 - V_M = \sqrt{\frac{C_1}{C_2}}(V_1 - V_M) \quad (10)$$

Simulations show that this equation for the dividing line is a very good approximation of the simulated behavior. To examine closer the tification of assuming a constant transconductance in the saturation region, let us use a modified Shockley transistor model

$$i_n = k_n \frac{(V_{GS} - V_{TN})^2}{2(1 + \delta_n)} \quad (11)$$

where  $k_n$  is the transistor gain factor,  $V_{TN}$  the threshold voltage and  $\delta_n$  the Taylor series expansion coefficient of the bulk charge (in standard textbook equations, usually  $\delta_n=0$ ). In this model, the transistor is saturated for  $V_{DS} > V_{DSAT} = (V_{GS} - V_{TN}) / (1 + \delta_n)$ . The linear region drain current is given by

$$i_n = k_n \left[ (V_{GS} - V_{TN}) V_{DS} - (1 + \delta_n) \frac{V_{DS}^2}{2} \right]. \quad (12)$$

Using similar equations for the p-channel transistor, we can write the two node currents as

$$i_1(V_2) = \frac{\beta_p}{2} (V_2 - V_{DD} - V_{TP})^2 - \frac{\beta_n}{2} (V_2 - V_{TN})^2, \quad (13)$$

and

$$i_2(V_1) = \frac{\beta_p}{2} (V_1 - V_{DD} - V_{TP})^2 - \frac{\beta_n}{2} (V_1 - V_{TN})^2. \quad (14)$$

where for a p-type transistor  $\beta_p = k_p / (1 + \delta_p)$ .

For the special case when  $\beta_n = \beta_p = \beta$ , we obtain a constant transconductance,

$$g_m = g_{mn} + g_{mp} = -\beta(V_{DD} + V_{TP} - V_{TN}), \quad (15)$$

for the region where all four transistors are saturated. Hence, for this region<sup>1</sup> shown shaded in Fig. 6 the linear equation given by Eq. (10) is an exact solution for the dividing line.

For the more general case when  $\beta_n \neq \beta_p$ , we obtain from Eqs (1) and (2) neglecting  $C_m$

$$\frac{dV_2}{dV_1} = \frac{C_1 i_2(V_1)}{C_2 i_1(V_2)}. \quad (16)$$

Separating variables, we obtain after integration

$$C_1 \left[ \beta_p (V_{DD} + V_{TP} - V_1)^3 + \beta_n (V_1 - V_{TN})^3 - \beta_p (V_{DD} + V_{TP} - V_M)^3 - \beta_n (V_M - V_{TN})^3 \right] = \\ C_2 \left[ \beta_p (V_{DD} + V_{TP} - V_2)^3 + \beta_n (V_2 - V_{TN})^3 - \beta_p (V_{DD} + V_{TP} - V_M)^3 - \beta_n (V_M - V_{TN})^3 \right], \quad (17)$$

where the metastable point  $(V_M, V_M)$  is given by

$$V_M = \frac{V_{DD} + V_{TP} + x V_{TN}}{1 + x} \text{ with } x = \sqrt{\beta_n / \beta_p}. \quad (18)$$

In Fig. 6 the region when Eq. (17) is valid is shaded.

<sup>1</sup> This region is defined by the following border line equations

$$\frac{V_1 - V_{TN}}{1 + \delta_n} \leq V_2 \leq V_{DD} + \frac{V_1 - V_{DD} - V_{TP}}{1 + \delta_p}, \quad \frac{V_2 - V_{TN}}{1 + \delta_n} \leq V_2 \leq V_{DD} + \frac{V_2 - V_{DD} - V_{TP}}{1 + \delta_p}, \quad V_{TN} \leq V_1 \leq V_{DD} - V_{TP}, \quad V_{TN} \leq V_2 \leq V_{DD} - V_{TP}$$

<sup>2</sup> For  $\beta_n = \beta_p$ , Eq. (17) yields  $C_1(V_1 - V_M)^2 = C_2(V_2 - V_M)^2$ , an equation from which Eq. (10) was obtained. In regions outside the shaded area where one of the transistors (for instance  $P_2$ ) is turned OFF, the dividing line only slightly deviates from a straight line as shown by the following equation obtained from Eq. (17)

$$C_1 \left[ (V_1 - V_{TN})^3 - (V_{DD} + V_{TP} - V_M)^3 - (V_M - V_{TN})^3 \right] = 3C_2 (V_{DD} + V_{TP} - V_{TN})(V_2 - V_M)^2.$$

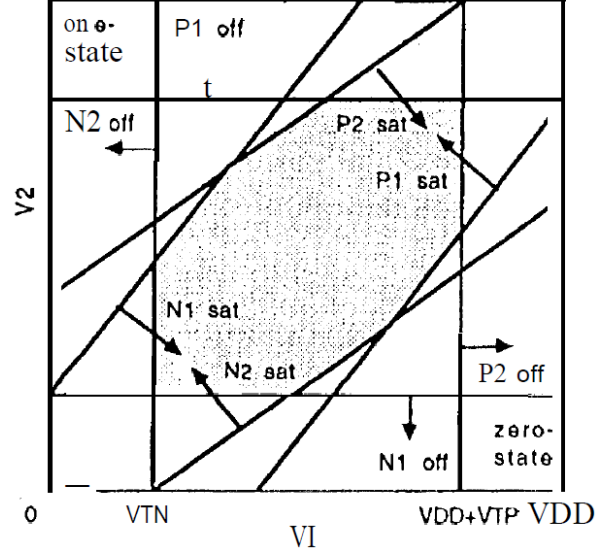


Fig. 6. All four transistors are saturated in the shadowed area.  $\delta_n = \delta_p = 0.3$ ,  $V_{TN} = -V_{TP} = V_{DD}/5$ .

The previously obtained straight line solution for the case of  $\beta_n = \beta_p$  given by Eq. (10) is simply a special case<sup>2</sup> of the general solution in Eq. (17).

A plot of the dividing line for the case of  $\beta_n / \beta_p = 3.3$  is shown in Fig. 7 using  $C_1 / C_2$  as the parameter ( $C_1 / C_2 = 1, 0.7, 0.5, 0.25$  and  $0.1$ ). As indicated by Fig. 7, the dividing line is very close to a straight line also for  $\beta_n \neq \beta_p$ . As an example, the second order deviation in  $V_2$  for  $V_1 = V_{DD}$  is less than 5% for  $\beta_n / \beta_p = 3.3$  and  $C_1 / C_2 > 0.1$ . For  $C_1 = C_2$  it is exactly a straight line.

Comparisons to SPICE simulations show that the solution of Eq. (17) can be extended with small errors also into regions of the state plane where one transistor is linear if the current through this transistor is small compared to the current through the other transistor of the same inverter.

For the example shown in Fig. 7 where  $\beta_n / \beta_p = 3.3$ , transistor P1 becomes linear along the dividing line when  $C_1 / C_2 \leq 0.39$ . However, the error is negligible as long as  $i_{p1}$  is less than one tenth of  $i_{n1}$  (which is true for  $C_1 / C_2 \geq 0.07$ ).

For the limiting case of very small  $C_1/C_2$  capacitance ratios, the dividing line becomes a horizontal line through the metastable state. The return track from the metastable state to one of the stable states then coincides with the static transfer curve (obtained for  $i_1=0$ ). For very large  $C_1/C_2$  capacitance ratios the limiting dividing line is a vertical line through the metastable state. The return track from the metastable state to one of the stable states now coincides with the other static transfer curve ( $i_2=0$ ). See Appendix.

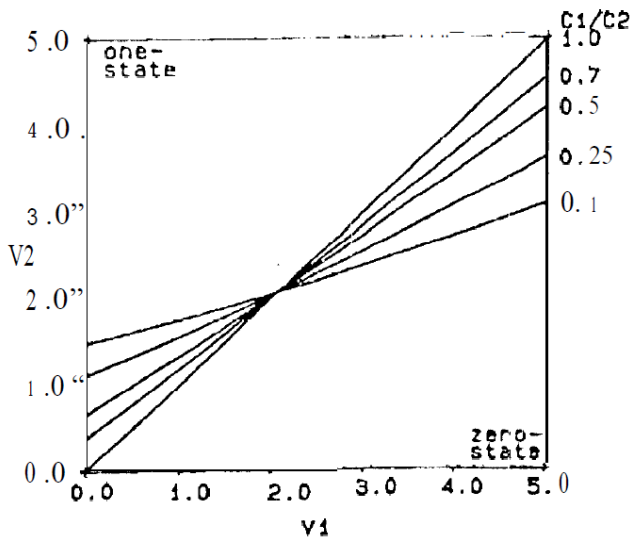


Fig. 7. The dividing line for  $\beta_n/\beta_p=3.3$  with parameter  $C_1/C_2=1, 0.7, 0.5, 0.25$  and  $0.1$ .

### Critical Charge Expressions

To calculate the upset-rate of a static RAM in space, Buehler and Allen used the Petersen equation [7] which assumes a 10-percent worst case differential cosmic-ray spectrum. According to the Petersen equation, the upset rate (in upsets per bit-day) for a heavy-ion hit on a q-type reverse-biased pn-junction of node  $i$  is given by equation

$$R_i = 5 \times 10^{-10} AJ_{qi} \left( \frac{X_{qi}}{QC_{qi}} \right)^2. \quad (16)$$

where  $AJ_{qi}$  is the area of the reverse-biased pn-junction of node  $i$  (in  $\mu\text{m}^2$ ),  $X_{qi}$  is the carrier collection depth of the same junction (in  $\mu\text{m}$ ), and  $QC_{qi}$  is the critical charge (in pC).

The critical charge for a heavy-ion-induced upset can be calculated for each reverse-biased pn-junction of the RAM cell using the results from the previous analysis.

For  $C_1/C_2$  the critical charges for a hit of inverter 2 is calculated as follows. With the RAM cell being in the zero-state,  $V_2=0$  and the drain of transistor P2 is reverse-biased. The critical voltage,  $VC_{p2}$ , for a P2 heavy-ion hit is obtained from the dividing line at  $V_1=V_{DD}$ , [1]. In the one-state,  $V_2=V_{DD}$  and the drain of transistor N2 is reverse-biased. The critical voltage,  $VC_{n2}$ , for an N2 heavy-ion hit is obtained from the dividing line at  $V_1=0$  [1]. Therefore, the critical voltages can be obtained from the straight line equation (10) as

$$VC_{p2} = V_M + \sqrt{\frac{C_1}{C_2}} (V_{DD} - V_M). \quad (20)$$

and

$$VC_{n2} = V_M + \sqrt{\frac{C_1}{C_2}} V_M. \quad (21)$$

respectively.

The expression for  $VC_{p2}$  of Eq. (20) can be compared to the expression derived by Buehler and Allen. They based their analysis on the empirical observation that

$$\frac{dV_2}{dV_1} = \frac{VC_{p2}}{V_{DD}}. \quad (22)$$

at  $(V_{DD}, VC_{p2})$  in the state-plane. After some approximations their derivation yielded

$$VC_{p2} = V_{TN} + \sqrt{\frac{C_1}{C_2}} (V_{DD} - V_{TN}). \quad (23)$$

Here, it can be seen that while their expression for  $VC_{p2}$  only depends on the threshold voltage of the n-channel transistors, the new expression in Eq. (20) depends on the switching voltage of the inverter. Thereby, the influences of both the p- and n-channel transistors are considered.

From the critical voltages,  $VC_{p2}$  and  $VC_{n2}$ , Buehler and Allen [1] defined the corresponding critical charges for memory upset by a P2- and N2-hit as

$$QC_{p2} = C_2 \times VC_{p2}. \quad (24)$$

and

$$QC_{n2} = C_2 \times (V_{DD} - VC_{n2}). \quad (25)$$

respectively.

The critical charge is the minimum charge needed for memory upset, assuming that the charge is collected so rapidly that the voltage on the other node does not change. This is true for most memory



cells since they, typically, have a slow response time ( $>500$  ps) and most of the charge is collected within 200 ps as shown by [4]. However, if charge is lost during the alpha hit more charge must be collected to upset the cell. To simulate the cell during the alpha hit, one must be concerned with the detailed nature of the current pulse. For 5 MeV alpha particles, the current pulse can be approximated by a decaying exponential with a time constant of 1 ns [3]. However, in most cases, as in the simulated cases shown in Fig. 8, a simple square-wave current pulse is a good enough first-order approximation [8]. The corresponding set and release trajectories simulated for five different current sources are shown in Fig. 9.

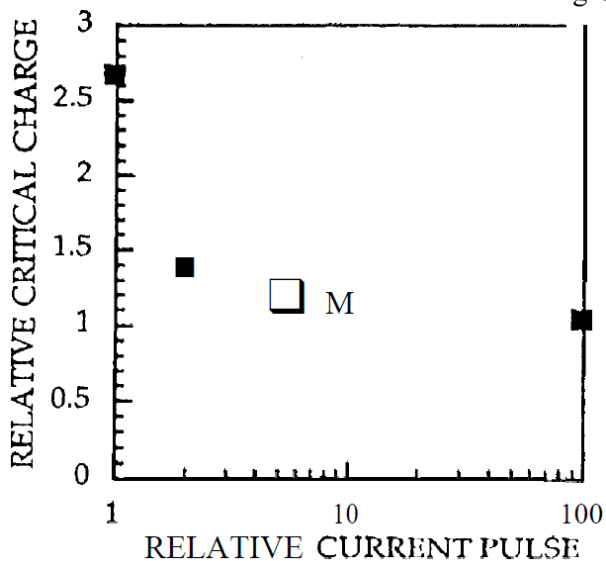


Fig. 8. The relative critical charge for memory upset versus the relative current pulse (normalized to the transistor saturation current).

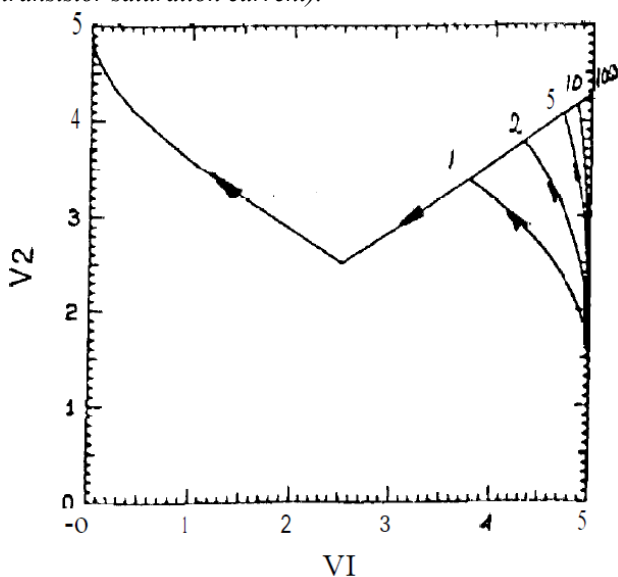


Fig. 9. The set and release trajectories for five different current pulses during the alpha hit. The current pulse is normalized to the transistor saturation current.

The critical charges for a hit of inverter 1 can be calculated similarly, at least for capacitively symmetric RAM cells or when  $C_1/C_2 > 1$ . In order to estimate the critical charges for N1 or P1 hits when  $C_1/C_2 < 1$ , one has to know how the dividing line continues into the regions where  $V_1 < 0$  and  $V_1 > V_{DD}$ . Due to the forward biasing of the drain diodes N1 or P1, respectively, large restoring currents  $i_1$  develop, bending the dividing line almost horizontal outside the frame of Fig. 7. Consequently, the dividing line will be impossible to reach by N1 or P1 hits, i. e.  $QC_{n1}$  and  $QC_{p1}$  become very large and the corresponding upset rates can be neglected.

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## APPENDIX

Generally, Eqs (1) and (2) can be rewritten as

$$\frac{dV_1}{dt} = \frac{i_1}{C_{11}} + \frac{i_2}{C_{12}}. \quad (\text{A1})$$

$$\frac{dV_2}{dt} = \frac{i_1}{C_{21}} + \frac{i_2}{C_{22}}. \quad (\text{A2})$$

where

$$C_{11} = C_1 + \frac{C_m}{1 + \frac{C_m}{C_2}}. \quad (\text{A3})$$

$$C_{12} = C_{21} = C_1 + C_2 + \frac{C_1 C_2}{C_m}. \quad (\text{A4})$$

$$C_{22} = C_2 + \frac{C_m}{1 + \frac{C_m}{C_1}}. \quad (\text{A5})$$

Using the following linear approximation of the transistor currents,

$$i_1 = g_m(V_2 - V_m) + g_0(V_1 - V_m). \quad (\text{A6})$$

$$i_2 = g_m(V_1 - V_m) + g_0(V_2 - V_m). \quad (\text{A7})$$

where  $g_m = g_{mn} + g_{mp}$  is the sum of the n- and p-channel transconductances in the metastable point  $(V_M, V_M)$  and  $g_o = g_{on} + g_{op}$  is the the sum of the output conductances, we obtain

$$\frac{dV_2}{dV_1} = \frac{a_2(V_2 - V_M) + b_2(V_1 - V_M)}{a_1(V_2 - V_M) + b_1(V_1 - V_M)}, \quad (\text{A8})$$

where

$$a_1 = \frac{g_m}{C_{11}} + \frac{g_o}{C_{12}} \quad a_2 = \frac{g_m}{C_{21}} + \frac{g_o}{C_{22}} \quad (\text{A9})$$

$$b_1 = \frac{g_o}{C_{11}} + \frac{g_m}{C_{12}} \quad b_2 = -\frac{g_o}{C_{21}} + \frac{g_m}{C_{22}}. \quad (\text{A10})$$

Assuming a linear relationship between  $V_2$  and  $V_1$  along the dividing line, i.e.

$$V_2 - V_m = K(V_1 - V_m), \quad (\text{A11})$$

we obtain the slope

$$K = -\frac{1}{2} \left( \frac{b_1}{a_1} - \frac{a_2}{a_1} \right) \pm \sqrt{\frac{1}{4} \left( \frac{b_1}{a_1} - \frac{a_2}{a_1} \right)^2 + \frac{b_2}{a_1}}. \quad (\text{A12})$$

If the mutual capacitance can be neglected compared to the load capacitances, the expression for the slope reduces to

$$K = \pm \sqrt{\frac{C_1}{C_2} + c^2} - c, \quad (\text{A13})$$

where

$$c = \frac{1}{2} \frac{g_o}{g_m} \left( 1 - \frac{C_1}{C_2} \right).$$

As expected, this expression reduces further to  $K = \pm \sqrt{C_1/C_2}$  while assuming  $g_o = 0$ . Assuming  $g_o \neq 0$  and letting  $C_2 \rightarrow \infty$ , we obtain from Eq. (A13)

$$K = \begin{cases} 0 \\ -\frac{g_o}{g_m} \end{cases}. \quad (\text{A14})$$

which confirms an incoming horizontal line to the metastable state and an outgoing line with a slope given by the reciprocal gain of the inverter. Similarly, letting  $C_1 \rightarrow \infty$ , we obtain

$$K = \begin{cases} \infty \\ -\frac{g_m}{g_o} \end{cases}. \quad (\text{A15})$$

which confirms an incoming vertical line to the metastable state and an outgoing line with a slope given by the gain of the inverter.

Finally, if the mutual capacitances cannot be neglected, we obtain assuming  $g_o = 0$ .

$$K = \pm \sqrt{\frac{C_{11}}{C_{22}}}. \quad (\text{A16})$$