THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Structural Analysis Methods for the Assessment of Reinforced Concrete Slabs

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Department of Architecture and Civil Engineering

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Concrete Structures

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Cover:

[Finite element model and results, field test and results of a tested span of the Gruvvägsbron in Kiruna, a prestressed bridge with a RC bridge deck slab. The Photo was photographed by Niklas Bagge in June 2014.]

Chalmers Reproservice

Gothenburg, Sweden 2017

To my father, mother and

Xu Fang

谨以此书献给我的父亲，母亲和

徐方女士

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ABSTRACT

Reinforced concrete (RC) slabs are among the most critical parts of the load-carrying capacity of such structures as bridges and parking decks. Previous research indicated that the assessment methods used in current practices largely underestimated the load-carrying capacity. The objective of the study reported in this thesis is to develop and calibrate improved methods for the assessment of load-carrying capacity and the response of RC slabs.

A Multi-level Assessment Strategy has been proposed. The strategy is based on the principle of successively improved evaluation in structural assessment. The strategy includes simplified analysis, linear finite element (FE) analysis and non-linear shell FE analysis, as well as non-linear continuum FE analysis with and without consideration of the interaction between reinforcement and surrounding concrete.

According to the Multi-level Assessment Strategy, enhanced FE analyses have shown to possess great possibilities for achieving a better understanding of the structural response and revealing higher load-carrying capacity of existing structures. However, non-linear 3D continuum FE analysis, at the highest level of the proposed strategy, is demanding and an analysis involves many modelling choices that are decisive for results. For the purpose of mapping the influence of different modelling choices on the structural behaviour of the FE model of RC slabs, sensitivity analyses have been conducted for RC slabs subjected to bending and especially to shear and punching failure. The selected modelling choices, within five major categories are: geometric non-linearity, element properties, modelling of concrete and reinforcement, as well as modelling of supports. The results show the possibility of accurately reflecting the experimental results concerning load-carrying capacity, load-deflection response, crack pattern and load distribution, given that proper modelling choices are used. Thereafter, the selected modelling choices were applied in FE analyses to investigate the load distribution and several influencing factors, including cracking, flexural reinforcement and the geometry of slabs and supports. The effect of flexural reinforcement and the size of specimens on structural response were also studied.

To examine the previously developed enhanced analysis approach, the Multi-level Assessment Strategy was applied to several laboratory tests and to a 55-year-old field-tested existing RC bridge deck slab, and results were compared to the experiments. The difference between assessment methods at different levels of detail was discussed. The results show that in general, advanced models are more capable of demonstrating load-carrying capacity that better reflects reality. The high-level continuum FE analysis and shell FE analysis coupled with a mechanical model, such as the Critical Shear Crack Theory (CSCT) are capable of predicting the shear and punching behaviour of RC slabs with reasonable accuracy. In addition, the influence of parameters such as boundary conditions, the location of concentrated loads and shear force distribution were found to affect the shear capacity of the field-tested bridge deck slab.

Key words: Reinforced Concrete Slabs, Multi-level Assessment Strategy, Finite Element (FE) Analysis, Shear and Punching, Structural Response, Bridge Deck

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PREFACE

This thesis is based on a research project entitled “Load-Carrying Capacity of Existing Bridge Deck Slabs”, initiated in September 2012 at the Division of Structural Engineering, Department of Civil and Environmental Engineering, Chalmers University of Technology. The research presented was mainly funded by the Swedish Transport Administration (Trafikverket). However, the part of the project connected to the slab tests conducted at Chalmers was funded by the European Community Seventh Framework Programme, under grant agreement NMP2-LA-2009-228663 (TailorCrete).

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I am grateful to Professor Aurelio Muttoni at The École Polytechnique Fédérale de Lausanne (EPFL) for hosting me in Lausanne and for supporting me with his deep knowledge and experience. Associate Professor Beatrice Belletti and PhD student Matteo Scolari at The University of Parma have a great impact on my work and their contributions are very valuable. I would also like to thank Dr. David Fall, a former PhD student at Chalmers for leading the experimental studies and for sharing data of two-way slab tests; Niklas Bagge, PhD student at LTU, for sharing the data of field testing and his opinion for analysis; Assistant Professor Yuguang Yang at TU Delft, for those valuable discussions and suggestions. Thanks also to Dr. Jürgen Einpaul, Post Doctor at The Imperial College London, Dr. Rui Vaz Rodrigues, Assistant Professor at The Technical University of Lisbon, Dr. Juan Sagaseta, Lecturer at The University of Surrey and Dr. Luca Tassinary, a former PhD student at EPFL for sharing experiment data and ideas.

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J. Shu

Gothenburg, January 2017

LIST OF PUBLICATIONS

This thesis consists of an extended summary and the following appended publications, referred to by Roman numerals in the text:

Journal Papers

I. Plos, M., Shu, J., Zandi, K., & Lundgren, K. (2016): A Multi-level Structural Assessment Proposal for Reinforced Concrete Bridge Deck Slabs. *Structure and Infrastructure Engineering*, 13 (2), 223–241.

II. Fall, D., Shu, J., Rempling, R., Lundgren, K., & Zandi, K. (2014). Two-way Slabs: Experimental Investigation of Load Redistributions in Steel Fibre Reinforced Concrete. *Engineering Structures*, *80*, 61–74.

III. Shu, J., Fall, D., Plos, M., Zandi, K., & Lundgren, K. (2015). Development of Modelling Strategies for Two-way RC Slabs. *Engineering Structures*, 101, 439–449.

IV. Shu, J., Plos, M., Johansson, M., Zandi, K., & Nilenius, F. (2016). Prediction of Punching Behaviour of RC Slabs Using Continuum Non-linear FE Analysis. *Engineering Structures*, 125, 15-25.

V. Shu, J., Belletti, B., Muttoni, A., Plos, M., & Scolari, M., (2016). Internal Force Distribution in RC Slabs Subjected to Punching Shear: Shell and Continuum Non-linear FE Analyses. *Engineering Structures.* (Submitted)

VI. Shu, J., Bagge, N., Plos, M., Johansson, M., Yang, Y., & Zandi, K. (2016). Shear and Punching Capacity of a Field Failure Tested RC Bridge Deck Slab. *ASCE Journal of Structural Engineering*. (Submitted)

AUTHOR'S CONTRIBUTION TO JOINTLY PUBLISHED PAPERS

The appended papers were prepared in collaboration with the co-authors. In the following, the contribution by the author of this doctoral thesis to the appended papers is described.

I. The author assumed main responsibility for conducting the case study analyses, evaluating results and writing the parts of the paper describing the case studies. The author also participated in planning and writing the remained part of the paper.

II. The author participated in the execution of the experiment, assumed responsibility for the analyses of the supported rollers under the tested slab.

III. The author was responsible for the literature study, carrying out the analyses, evaluating the results, planning and writing the paper.

IV. The author was responsible for the literature study, carrying out the analyses, evaluating the results, planning and writing the paper.

V. The author was responsible for the literature study, carrying out the FE analyses using continuum elements, evaluating the results, planning and writing the paper.

VI. The author was involved in evaluating the results of the experiment, was responsible for the literature study, carrying out the analyses, evaluating of analysis results, planning and writing the paper.

OTHER PUBLICATIONS RELATED TO THIS THESIS:

In addition to the appended papers, the author of this thesis has also contributed to the following publications.

Licentiate Thesis

Shu, J. (2015). *Structural Analysis of Existing RC Bridge Deck Slabs*. Chalmers University of Technology, Licentiate Thesis, Gothenburg, ISSN 1652-9146.

Conference Papers

I. Shu, J., Fall, D., Plos, M., Zandi, K., & Lundgren, K. (2014). Finite Element Modelling of Two-way RC Slabs with Varying Modelling Choices. In *The 10th fib International PhD Symposium in Civil Engineering* (pp. 589–594). Quebec, Canada.

II. Shu, J., Plos, M., Lundgren, K., & Zandi, K. (2014). Structural Assessment of Bridge Deck Slabs. In *XXII Nordic Concrete Research Symposium* (pp. 453–456). Reykavik, Iceland.

III. Shu, J., Plos, M., Zandi, K., & Lundgren, K. (2015). A Multi-level Structural Assessment Proposal for Reinforced Concrete Bridge Deck Slabs. In *Nordic Concrete Research: Residual Capacity of Deteriorated Concrete Structures* (pp. 53-56). Oslo, Norway.

IV. Bagge, N., Shu, J., Plos, M., & Elfgren, L. (2015). Punching Capacity of a Reinforced Concrete Bridge Deck Slab Loaded to Failure. In *Nordic Concrete Research: Residual Capacity of Deteriorated Concrete Structures* (pp. 57–60). Oslo, Norway.

V. Shu, J., Plos, M., Zandi, K., & Johansson, M. (2016). Numerical Prediction of Punching Behaviour of RC Slabs Using 3D Nonlinear FE analysis. In *The 8th International Conference on Bridge Maintenance, Safety and Management, IABMAS2016*, (pp. 421–422). Iguaçu, Brazil.

VI. Shu, J., Plos, M., Zandi, K., Johansson, M. & Nilenius, Filip. (2016). Prediction of Punching Behaviour of RC Slabs Using Continuum Nonlinear FE Analysis. In *19th IABSE Congress Stockholm* (pp. 1625–1631). Stockholm, Sweden.

VII. Plos, M., Shu, J., & Lundgren, K. (2016). A Multi-level Structural Assessment Strategy for Analysis of RC Bridge Deck Slabs. In *19th IABSE Congress Stockholm* (pp. 1559–1566). Stockholm, Sweden.

VIII. Shu, J., Plos, M., Zandi, K., Johansson, M., & Nilenius, F. (2016). Shear Force Distribution in RC Slabs Subjected To Punching: Solid Nonlinear FE Analyses. In *fib Symposium 2016, Cape Town* (p. 55-58). Cape Town, South Africa.

Technical Report

Fall, D., Shu, J., Rempling, R., Lundgren, K., & Zandi, K. (2014). Steel Fibre in Reinforced Concrete Slabs: Experimental Results, Report 2014:4.

NOTATIONS

The following notations are used in this thesis:

a_s	unitary flexural reinforcement area
a_v	free shear span (distance between the edge of the support and the edge of the loading plate)
b_0	length of control perimeter
b_w	effective width
B	width of slabs
c	width of supportive column
c_{min}	minimum width of supportive column
c_{max}	maximum width of supportive column
d	flexural effective depth
d_c	diameter of supportive column
d_g	maximum aggregate size
d_{g0}	reference aggregate size
E_s	Young's modulus of steel
f_c	compressive strength of concrete measured in cylinders
f_{ct}	tensile strength of concrete
f_y	yield strength of steel reinforcement
f_u	ultimate strength of steel reinforcement
F_{sE}	acting anchorage force
F_{sR}	resistance of anchorage
G_f	Mode I fracture energy of concrete
h	thickness of slabs
h_b	crack bandwidth
k	size effect factor
k_v	a parameter depends on strain and the maximum aggregate size
k_ψ	a parameter depends on the deformations (rotations) of the slabs
k_{dg}	a parameter depends on the maximum aggregate size
R_u	ultimate resistance
l	length of yield line
$l_{element}$	length of elements
l_x	span length in *x* direction
l_y	span length in *y* direction
m	unitary acting moment
m_E	unitary moment for calculation of the flexural reinforcement in the support strip
m_R	the average flexural strength per unit length in the support strip
m_x	bending moment per unit width in x direction
m_y	bending moment per unit width in y direction
m_{xy}	torsional moment per unit width
M_{ACI}	factored moment at sections according to ACI 318-14
N	concentrated load
q	distributed load
Q_E	acting of concentrated loads on slabs
Q_R	resistance of slabs
r_s	the position where the radial bending moment is zero
S_{rm}	mean crack distance
v_E	unitary acting shear force
v_R	shear strength per unit length
v_{min}	minimum shear strength per unit length
$V_{R,c}$	shear resistance of RC slabs
V_{ACI}	factored shear force at sections according to ACI 318-14

V_R	shear resistance
$V_{R.EXP}$	shear resistance of RC slabs according to experiments
$V_{R.EC2}$	shear resistance of RC slabs calculated according to Eurocode 2
$V_{R.FEA}$	shear resistance of RC slabs calculated using FE analysis
w	crack width
w_u	ultimate crack width
z	effective shear depth
β	shear retention factor
β_v	reduction factor based on shear span ratio
δ	vertical displacement
ε	strain
ε_c	crack strain
ε_u	ultimate strain
ε_x	reference strain at mid-depth
σ	normal stress
σ_c	normal stress of concrete
τ_c	shear stress
ρ	flexural reinforcement ratio
ψ	slab rotation angle in radians

1 INTRODUCTION

Reinforced concrete (RC) slabs are among the most critical parts of the load-carrying capacity of structures such as bridges, parking garages and harbour structures. Based on the study presented in this thesis, a Multi-level Assessment Strategy, including simplified analytical methods, as well as the finite element (FE) method has been developed to increase the understanding of structural behaviour of RC slabs and provide a better estimation of their load-carrying capacity.

1.1 Background

The bridge network is very important to society (SB-LRA, 2007). Many bridges in Sweden are more than 50 years old (Pantura, 2012) and current loads are greater than the loads they were designed for (SB-LRA, 2007). RC slabs without shear reinforcement, which are commonly used in structures such as bridges, are among the most critical part concerning the load-carrying capacities. In addition, RC slabs such as bridge deck slabs are subjected to degradation since they are exposed to harsh environments, e.g. water, ice and de-icing salts. In such conditions, sudden collapses may occur to the slab structures, causing a huge loss of human life and properties, e.g. see Figure 1 (Johnson, 2007). However, in many cases, structures are repaired, strengthened, or even replaced before necessary, causing high costs for society.

Figure 1. The western part of the collapsed overpass in Quebec, from the eastern abutment, Johnson (2007).

In engineering practice, simplified methods according to building codes, such as Eurocode 2 (CEN, 2004), ACI 318-14 (ACI, 2014) and MC2010 (fib, 2013), are commonly used for the assessment of RC slabs. However, several full-scale tests on real bridges show large load overcapacities for both bending and shear, compared to simplified assessments, e.g. Plos (1995), Sas *et al*. (2012) and Bagge (2014). In a preliminary study, the Structural Engineering Group within Swedish Universities of the Building Environment investigated and realized the need for research and development to achieve more robust bridge deck slabs (Sundquist, 2011). Miller *et al.* (1994) carried out a study involving destructive testing of a decommissioned concrete slab bridge, which indicated that simple modelling methods, such as strip models, frequently overestimate structural demand and underestimate structural resistance due to the lack of consideration for additional load-carrying mechanisms, such as membrane action. Therefore, if improved assessment methods were developed and applied, possibilities would likely open up to use these reserves.

The development of an assessment strategy for RC slabs is based on the principle of successively improved evaluations in structural assessment (SB-LRA, 2007). It has been stated that enhanced analysis, such as non-linear FE analysis, is the method that has the greatest possibility to reveal higher load-carrying capacity in existing bridges (SB-4.5, 2007). Research has been carried out to study the possibility of using enhanced assessment methods. Several publications have investigated the behaviour up to the failure of RC slabs using non-linear FE analyses, including comparisons of experimental test data and analytical formulations, e.g. see Shahrooz (1994) and Amir (2014). For the more critical problem of shear failure, Critical Shear Crack Theory (CSCT) (Muttoni, 2009) has already been shown effective in calculating the punching shear capacity of RC slabs and has accordingly been included in MC2010 (fib, 2013). Non-linear three-dimensional (3D) FE analyses were also carried out and tested to be valid to investigate the structural behaviour of RC slabs; see Zheng *et al.* (2009), Eder *et al.* (2010) and Belleti *et al.* (2014). However, the enhanced assessment methods have not been widely used in engineering practice due to the lack of modelling recommendations. Consequently, they need to be further developed and improved.

1.2 Aim, scope and limitations

The overall aim of this research is to develop improved methods for the assessment of the load-carrying capacity and response of RC slabs. In order to achieve higher detectable load-carrying capacity using enhanced assessment methods, the objective can be expanded:

- To propose a methodology for successively improved structural analysis for the assessment of RC slabs, with specified analysis approaches at different levels of detailing and accuracy.
- To develop modelling strategies for the structural analysis of RC slabs subjected to bending and shear failure on enhanced assessment levels.
- To contribute to the realisation of tests on two-way slabs, including the study of load distribution, to use as benchmark tests for the development of modelling strategies.
- To investigate the structural behaviour of RC slabs such as shear force distribution using a developed modelling strategy.
- To examine the proposed multi-level structural assessment approach through case study analyses of previously tested RC slabs, including laboratory tests and a field test.

Within the scope of the work presented, the limitations of the study can be summarized as follows:

- The research is focused on enhanced analysis methods of existing RC slabs, with minor considerations of other assessment aspects, e.g. the inspection, monitoring and strengthening of existing structures.
- RC slabs with shear reinforcement are not included in the scope of this study.
- The research is aimed at locating the structural behaviour of existing RC slabs subjected to concentrated static loads, thereby not focusing on such factors as the influence of dynamic loads, fatigue problems, and the degradation of structures due to environmental impact.
- Due to the increased complexity of non-linear FE analysis, considerable model uncertainty may exist. However, the safety format concerning the modelling uncertainty is not included in this study.

1.3 Method and scientific approach

The scientific approach of this research encompasses literature studies, laboratory tests, analytical and FE analyses. In order to propose a methodology for successively improved structural analysis for assessment of RC slabs, a study including an investigation of previous assessment approaches in literature, including building codes, guidelines and scientific papers, was performed. A Multi-level Assessment Strategy specifically for RC slabs has been developed as a complement to existing guidelines.

To develop enhanced analysis methods, as part of the Multi-level Assessment Strategy for RC slabs subjected to different failure modes, two-way slab tests carried out at Chalmers University of Technology, shear and punching laboratory tests carried out in The École polytechnique fédérale de Lausanne (EPFL) were adopted as benchmark for analyses. Parameter studies have been carried out to investigate the influence of modelling choices for continuum FE analyses and a preliminary recommendation was proposed based on these studies.

The Multi-level Assessment Strategy was thereafter applied on extended laboratory tests. The results of analysis at different levels, including simplified analytical analyses, linear and non-linear shell and continuum FE analyses, were compared. The developed FE modelling methods were used to analyse RC slabs to investigate structural behaviour, e.g. load distribution and size effect.

To examine the Multi-level Assessment Strategy in engineering practice, an existing 55-year old RC bridge deck slab subjected to concentrated load near the girder was studied and compared to a field destructive test. The difference between assessment methods at different levels was discussed regarding the one-way and punching shear behaviour of slabs. The influence of parameters such as boundary conditions, location of concentrated loads and shear force distribution was investigated.

1.4 Original features

The new features of this study are summarized as follows:

- A Multi-level Assessment Strategy was proposed, including analytical methods and FE analyses with different levels of detail, to evaluate the load-carrying capacity of RC slabs.
- A preliminary recommendation was generated about how to make modelling choices when analysing RC slabs using continuum elements.
- The influence of support stiffness on the load distribution of two-way slabs was investigated.
- Load-distribution in RC slabs were investigated and such influencing factors as the geometry of slabs and supports, and layout of reinforcement were studied.
- The Multi-level Assessment Strategy and enhanced FE analysis methods were examined in a field destructive test of an existing RC bridge deck slab. The influence of boundary conditions and location of loads was investigated for modelling.

1.5 Outline of the thesis

The thesis consists of an extended summary and six appended papers. The summary provides the background of the research and summarizes the results and conclusions. Chapter 1 provides the aim, objectives and limitations of the study together with a general description of the methods used and the original features included. Chapter 2 presents the structural response of RC slabs subjected to different failure modes, as well as the existing mechanical and numerical models for RC slabs. In Chapter 3, experiments involved in this thesis are introduced, including laboratory tests and a field test (*Papers I - VI*). Chapter 4 presents the Multi-level Assessment Strategy for existing RC slabs (*Paper I*), the developed high level enhanced FE analysis methods (*Papers III*, *IV*) and their application on laboratory tests (*Paper V*). Chapter 5 presents the application of the Multi-level Assessment Strategy to a field-tested existing bridge deck slab (*Paper VI*). Conclusions and suggestions for future research are described in Chapter 6. At the end of the thesis, all journal papers have been appended and their relationship is presented in Figure 2.

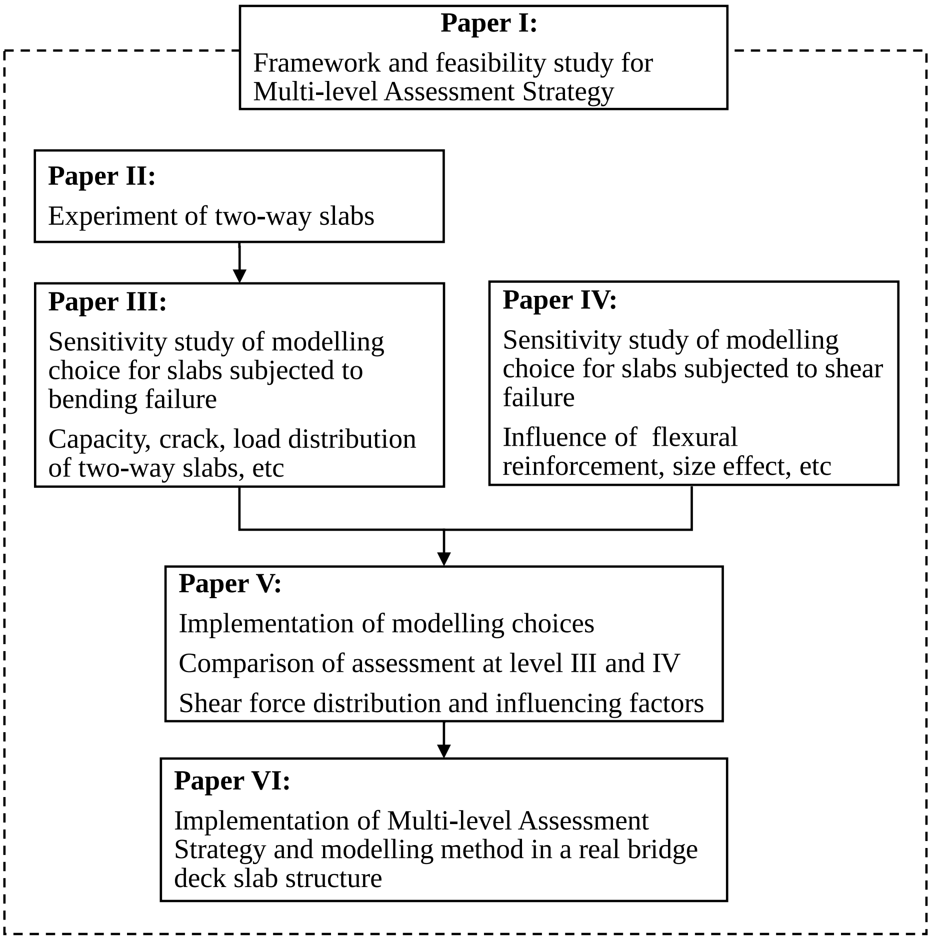


Figure 2. General information of all appended papers and their relationship.

2 STRUCTURAL RESPONSE OF REINFORCED CONCRETE SLABS

Since RC slabs have been widely used, a number of research projects have been carried out to understand the actual structural behaviour of RC slabs. Design and assessment methods for RC slabs subjected to different failure modes have been included in building codes for engineering practice. FE methods have also been increasingly used because of development of computers and software.

2.1 Failure modes

RC slabs without shear reinforcement subjected to a concentrated load can fail in bending, shear and anchorage failure; see Figure 3. Bending failure is a desired failure mode because it allows structures to have ductile deformation and redistribution of internal force before collapse. However, shear failure is an un-desired failure, because it is a brittle failure, resulting in a sudden collapse before ductile deformation. Therefore, shear failure is more dangerous since it more easily causes huge losses of human life and properties. There are two types of shear failure modes in literature, i.e. one-way shear failure and punching shear (also referred to as two-way shear) failure. One-way shear failure may occur when a slab component is subjected to a line load or is supported by line boundary condition, e.g. a slab-wall system; punching shear failure may occur when the slab is subjected to a concentrated load or support, e.g. a slab-column system. Anchorage failure considering bond-slip between reinforcement and concrete is another common failure mode for RC structures. However, this failure mode is not the major concern in this study because it is usually not a dominant failure mode in RC slab structures.

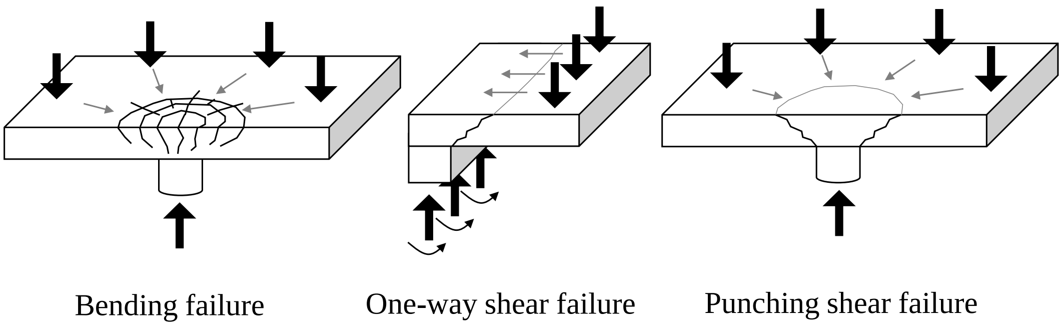


Figure 3. Three different failure modes in the scope of present research.

The earlier study of failure modes of RC slabs can be found in literature from the beginning of 20^{th} century (Talbot, 1913). In past decades, research has been carried out by using laboratory tests, field tests and numerical simulations to investigate the structural behaviour of RC slabs subjected to these failure modes, contributing to the development of empirical and mechanical models. Major contributions selected from literature on RC slabs subjected to the three different failure modes can be found in Table 1, Table 2 and Table 3, respectively. Some of them have been selected for use in building codes, including Eurocode 2 (CEN, 2004), ACI 318-14 (ACI, 2014) and MC2010 (fib, 2013).

Table 1. A review of studies and models for RC slabs subjected to bending failure.

Yield line method, Johansen (1943)	• Tested 134 rectangular slabs under different combination of loading and boundary conditions. • Formed the most commonly used upper bound yield line method, in which the external energy due to the loads should be balanced by the dissipated internal energy along the yield lines.
Strip method, Hillerborg (1956)	• Formed the most commonly used lower bound strip method, in which the slab moment capacity can be calculated by assuming a system of slab strips (in one or several directions) to carry the external loads.
Sawczuk & Jaeger (1963)	• Conducted extensive experiments to study the collapse of RC slab structures. • Investigated the upper bound solution for several different slab problems based on square yield conditions.
Taylor *et al*. (1966)	• Tested uniformly loaded simply supported square slabs. • Studied causes for the large enhancement of strength compared to strip method estimation, e.g. due to strain hardening of the reinforcement and tensile membrane action at large deflections.
Park (1965), Hopkins (1969), Park & Gamble (1999)	• Tested a series of uniformly loaded RC slabs with laterally restrained boundary conditions. • Studied the contribution of membrane action to the large enhancement of strength compared to strip method estimation, particularly when the boundary restraint was stiff, the span/depth ratio was low, and the reinforcement ratio was also low.

Table 2. A review of studies and models for RC slabs subjected to one-way shear failure.

Kani's Shear Valley (Kani, 1966)	• Proposed Kani's Shear Valley, in which shear capacity is dependent on the so-called "remaining arch" and "concrete teeth".
Regan (1982) Regan & Rezai-Jorabi (1988)	• Tested RC slabs with different shear spans, size of loading plates and simple/continuous supports. • Tested one-way RC slabs subjected to concentrated loads at great distance to the support, resulting in wide beam shear failure or punching shear failure.
Strut and tie model (Schlaich *et al*., 1987)	• Conducted study on lower bound plasticity theory, representing force flow using compressive struts and tension ties.
Furuuchi *et al*. (1998)	• Tested shear capacity of RC components in-between beams and slabs, with small shear span/depth ratio, called "deep slabs".
Critical Shear Crack Theory (CSCT), Muttoni (2003)	• Formed the Critical Shear Crack Theory based on 253 shear tests, with the assumption that shear strength depends on the width and roughness of "Critical Shear Crack".
Sherwood *et al*. (2006)	• Tested thick slabs and wide beams with observation that member width have only a minor effect on the shear stress at failure.
Simplified MCFT (Bentz *et al*., 2006)	• Summarized the results of over 100 pure shear tests on RC panels and formed the simplified version of Modified Compression Field Theory (MCFT).
Rombach & Latte (2008)	• Tested cantilever slabs (with and without haunches) to examine whether bridge deck slabs under concentrated loads have additional shear capacity compared to Eurocode 2 (CEN, 2004) estimation.
Lubell *et al*. (2009)	• Studied the influence of flexural reinforcement on one-way shear capacity of RC slabs and wide beams.

Reißen & Hegger (2013a, 2013b)	• Studied the influence of parameters on shear capacity, e.g. the compressive strength of concrete, the yield strength of reinforcement, detailing of reinforcement, shear span/depth ratio, as well as moment/shear force ratio.
Lantsoght (2013)	• Conducted a test series of 38 slabs under a concentrated load close to the support, with variable width, reinforcement layout, concrete strength and size of loading plates. • Developed Modified Bond Model based on Alexander and Simmonds (1992); studied effective width for load distribution.
Natário et al. (2014)	• Conducted series of 12 tests on 6 full scale slabs with variation of load location and presence of ducts. • Studied the role of shear crack on load distribution by tracing reaction force at the supportive line.

Table 3. A review of studies and models for RC slabs subjected to punching shear failure.

Talbot (1913)	• Tested approximately 200 footing of walls and columns, of which 20 failed in punching shear. • Proposed a simple model to calculate the critical shear stress around a fictitious circumference.
Kinnunen & Nylander (1960)	• Tested 61 circular slabs with circular column, varying flexural reinforcement ratio. • Formed a calculation method for punching assuming that punching capacity was reached for a given critical rotation.
Moe (1961)	• Tested square slabs to punching failure. • Proposed a limit state model (using mechanical and empirical relation) assuming a shear stress limitation at a critical perimeter with certain distance to the loading area. • Formed the basis for the ACI-Standard 318 of the year 1963.
Alexander and Simmonds (1986;1987) Alexander and Simmonds (1992)	• Proposed a 3D strut-and-tie model, with two types of compression struts (in-plane or anchoring struts and out-of-plane or shear struts). • Proposed a bond model, as a modification of the 3D strut-and-tie model, for concentrated loaded flat slab-column connections at failure, by a combination of radial arching action with the concept of a critical shear stress on a critical section.
Bažant & Cao (1987)	• Tested geometrically similar micro-concrete specimens in three different thicknesses. • Concluded that large specimens are less ductile than smaller ones, indirectly confirming the size-effect law.
Hallgren (1996)	• Conducted punching tests on high performance RC slabs. • Further developed the model by Kinnunen & Nylander (1960).
Menetrey (1997)	• Verified the analytical model (Menétrey & Willam, 1995), in which punching capacity was influence by bending capacity with shear crack inclination.
Staller (2000)	• Developed empirical models based on experimental data with the help of linear or non-linear, single or multiple, regression analysis.
Critical Shear Crack Theory (CSCT), Muttoni (2009)	• Proposed punching failure criterion by assuming punching strength is dependent on the slab rotation. • Compared results based on CSCT with 99 experimental results in literature.

2.2 Building code provisions

In engineering practice in building codes, structural analyses are performed to determine the load effects on the bridge deck slab, normally in terms of cross-sectional forces and moments. These forces are compared to corresponding (cross-sectional) capacities of the slab, determined by using local resistance models from code provisions. The following code provisions for bending, one-way and punching shear are studied: Eurocode 2 (CEN, 2004), ACI 318-14 (ACI, 2014) and MC2010 (fib, 2013). To determine the load effect for structural analysis, elastic analysis, elastic analysis with limited redistribution, plasticity analysis and non-linear analysis are recommended. To determine the corresponding (cross-sectional) capacities of the slab with respect to different failure modes, local cross-sectional resistance models are used. Calculation methods for bending are mainly based on either the strip method or the yield line method for the lower bound solution and the upper bound solution, respectively. Calculation methods for one-way and punching shear in Eurocode 2 and ACI 318-14 are developed on the basis of a (semi-) empirical model, but in MC2010 they are developed on the basis of the Modified Compression Field Theory (Bentz *et al.*, 2006) for one-way shear and Critical Shear Crack Theory (Muttoni, 2009) for punching shear, respectively.

One-way shear and punching shear are expressed by similar equations in building codes, where the shear resistance $V_{R,c}$ is calculated by multiplying a unitary shear strength $v_{R,c}$ (nominal shear strength) by an control length b, (effective width b_w for one-way shear and control perimeter b_0 for punching shear) and the effective depth d.

$$V_{R,c} = v_{R,c} \cdot b \cdot d \tag{1}$$

The equations for one-way shear and punching in Eurocode 2 (CEN, 2004), ACI 318-14 (ACI, 2014) and MC2010 (fib, 2013) are listed in Table 4. In Eurocode 2 (CEN, 2004), the values of $C_{R,c}$ and v_{min} may be chosen nationally and k is a factor for size effect. In ACI 318-14 (ACI, 2014), V_{ACI} is factored shear force at section and M_{ACI} is factored moment at section. In MC2010, k_v and k_ψ (factors related to strain and rotation for one-way shear and punching shear, respectively) has been calculated based on Level-of-approximation I and II. The level I equation is a simplification of level II equation; see MC2010 (fib, 2013).

Table 4. Equations for one-way shear and punching shear in building codes.

	One-way shear	Punching shear
Eurocode 2	$V_{R,c} = C_{R,c} \cdot k \cdot (100 \cdot \rho \cdot f_c)^{1/3} \cdot b_w \cdot d$ $V_{R,c} \geq (v_{\min}) \cdot b_w \cdot d$ $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$	$V_{R,c} = C_{R,c} \cdot k \cdot (100 \cdot \rho \cdot f_c)^{1/3} \cdot b_0 \cdot d$ $V_{R,c} \geq (v_{\min}) \cdot b_0 \cdot d$ $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$
ACI 318-14	$V_{R,c} = (0.16 \cdot \sqrt{f_c} + 17 \cdot \rho \cdot \frac{V_{ACI} \cdot d}{M_{ACI}}) b_w \cdot d$ $V_{R,c} \leq 0.29 \cdot \sqrt{f_c} \cdot b_w \cdot d$	$V_{R,c} = \frac{1}{3} \cdot \sqrt{f_c} \cdot b_0 \cdot d$
MC2010	$V_{R,c} = k_v \cdot \sqrt{f_c} \cdot z \cdot b_w \cdot d$ $k_v = \frac{180}{1000 + 1.25 \cdot z}$ (level I) $k_v = \frac{0.4}{1 + 1500 \cdot \varepsilon_x} + \frac{1300}{1000 + k_{dg} \cdot z}$ (level II) $k_{dg} = \frac{32}{16 + d_g} \geq 0.75$ $\varepsilon_x = \frac{1}{2 \cdot E_s \cdot a_s}(\frac{m}{z} + v)$	$V_{R,c} = k_\psi \cdot \sqrt{f_c} \cdot b_0 \cdot d$ $k_\psi = \frac{1}{1.5 + 0.9 \cdot k_{dg} \cdot \psi \cdot d} \leq 0.6$ $\psi = 1.5 \cdot \frac{r_s}{d} \cdot \frac{f_y}{E_s}$ (level I) $\psi = 1.5 \cdot \frac{r_s}{d} \cdot \frac{f_y}{E_s} \cdot (\frac{m_E}{m_R})^{1.5}$ (level II)

a_s	unitary flexural reinforcement area	b_0	length of control perimeter
b_w	effective width	$C_{R,c}$	A factor according to national annex
d	effective depth	f_y	yield strength of steel reinforcement
d_g	maximum aggregate size	d_{g0}	reference aggregate size
E_s	Young's modulus of steel	k	size effect factor
m	unitary acting moment	ρ	flexural reinforcement ratio
v	unitary acting shear force	v_R	shear strength per unit length
$V_{R,c}$	shear resistance of RC slabs	ε_x	reference strain at mid-depth
z	effective shear depth	ψ	slab rotation angle in radians
f_c	compressive strength of concrete measured in cylinders	r_s	the position where the radial bending moment is zero
k_v	a parameter depends on strain and the maximum aggregate size	k_{dg}	a parameter depends on the maximum aggregate size
m_R	the average flexural strength per unit length in the support strip	k_ψ	a parameter depends on the deformations (rotations) of the slabs
v_{min}	minimum shear strength per unit length	m_E	unitary moment for the flexural reinforcement in the support strip
V_{ACI}	factored shear force at sections according to ACI 318-14	M_{ACI}	factored moment at sections according to ACI 318-14

The method for determining the critical section and its effective width for one-way shear can be found in Figure 4. Critical section and effective width for evaluation of one-way shear resistance in building codes of practice. The critical shear stress should be checked at the face of the support, with an effective width using a 45-degree method, according to the Dutch (Lantsoght *et al.*, 2014) and French practices (Chauvel *et al.*, 2007), but the starting points for load distribution are different. In MC2010 (fib, 2013), the critical shear stress should be checked at a distance of $x \leq a_v/2$ (a_v is the free shear span) to the line support. The effective width b_{eff} is determined by the load distribution angle α = 45 degree for clamped slabs and α = 60 degrees for simply supported slabs.

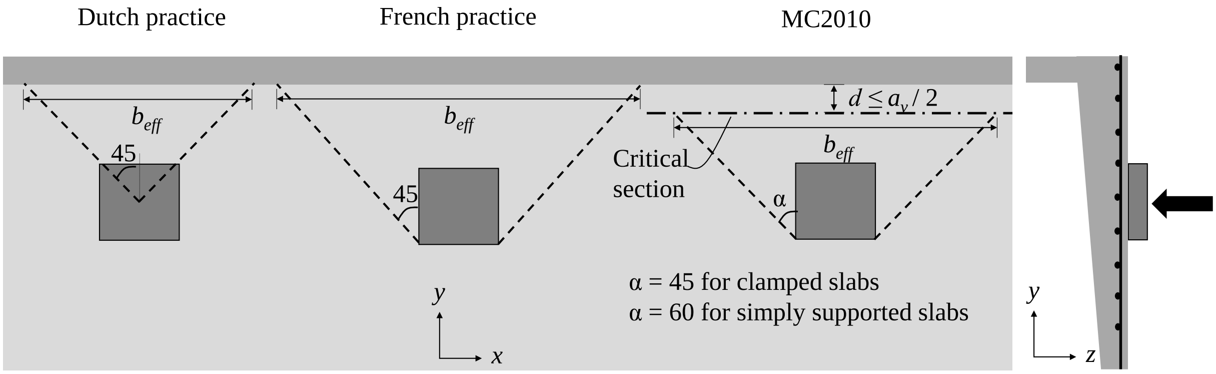


Figure 4. Critical section and effective width for evaluation of one-way shear resistance in building codes of practice.

The method to determine the location of the critical section and the length of control perimeter for punching shear can be found in Figure 5. The distance from the critical section to the edge of load may be d/2 or 2d, with rounded or shaped corners.

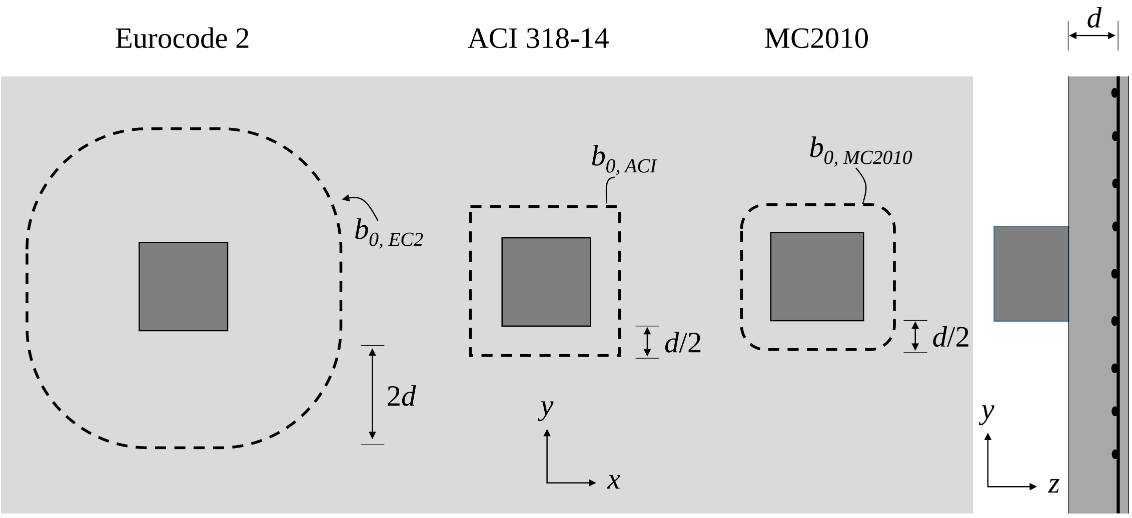


Figure 5. Control perimeters for punching verification in building codes of practice.

2.3 Finite element analysis

Linear FE analysis has become commonly used for the design of RC slabs. With such analysis, complicated slab geometries can be included in the structural analysis. There are several references available with recommendations for a 3D linear FE analysis of RC slabs, e.g., Rombach (2004) and Blaauwendraad (2010). Detailed recommendations for the modelling and designing of RC slabs based on a linear FE analysis are given in Pacoste *et al*. (2012).

Including the non-linear response of concrete and reinforcement, with cracking, crushing and plasticity, the non-linear FE method can be used to realistically analyse the response of RC structures. Existing recommendations for the modelling of RC structures based on a non-linear FE analysis can be found, e.g. fib Bulletin No. 45 (fib, 2008) and Hendriks *et al*. (2012).

2.3.1 Modelling of concrete

Concrete has a non-linear behaviour both in tension and compression; see Figure 6 for a typical uniaxial stress-strain relationship of a typical concrete material model. The material model can be described based on the theory of elasticity, plasticity, damage and fracture mechanics, or as a combination of them at different phases, e.g. fracture mechanics for tension and plasticity for compression. In general, the cracking of concrete can be modelled using three different crack approaches: the discrete crack approach, the smeared crack approach and the embedded crack approach. A summary of commonly used models to describe the cracking in concrete is illustrated in Table 5.

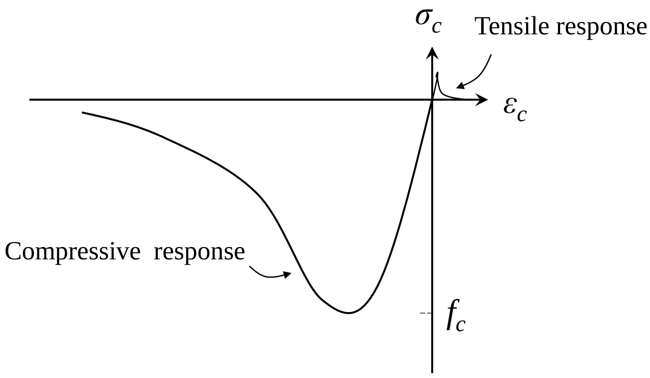


Figure 6. Typical non-linear uniaxial stress-strain relation for concrete.

Smeared crack approach

In the smeared crack approach, which is the approach used in the study of this thesis, cracks are described by using a stress-strain relationship in the element which contains cracks (Rashid, 1968). There are two types of models of the smeared crack approach, i.e. the fixed crack model (Rashid, 1968) and the rotating crack model (Cope *et al*., 1980). In the fixed crack model, the crack direction is assumed to be fixed after initiation and the decrease of shear stiffness is described by a shear retention factor. This model was further developed to a multi-directional fixed crack model, in which cracks with different orientations can coexist in one finite element (Borst & Nauta, 1985). In the rotating crack model, the crack direction is assumed to be constantly perpendicular to the principal strain and shear stiffness is not explicitly included (Cope *et al*., 1980). The rotating crack model is used commonly in commercial software, e.g. DIANA (TNO, 2015) and ATENA (Červenka *et al.* 2014), since it yields reasonably accurate results even though the shear behaviour is not described realistically (Rots & Blaauwendraad, 1989).

Table 5. A review of models to describe the cracking in concrete.

<table>
<tr><th colspan="2">Crack approaches</th><th>Features</th></tr>
<tr><td colspan="2">Discrete crack approach
Hillerborg et al. (1976)</td><td>• Cracks are modelled by means of a separation between elements.
• The possible crack and the FE mesh must be assumed in advance so that the crack path follows element boundaries.
• Techniques of adaptive re-meshing (Ingraffea & Saouma, 1985) and techniques which permit discrete cracks to extend through finite elements (Blaauwendraad & Henk, 1981; Blaauwendraad, 1985) have been developed to eliminate the drawback that cracks have to be predefined in advance.</td></tr>
<tr><td rowspan="4">Smeared crack approach
(Rashid, 1968)</td><td>Crack band model
Bažant and Oh (1983)</td><td>• The damage will localize in a band of finite elements.
• The model is simple but suffers from mesh dependency.</td></tr>
<tr><td>Nonlocal model
Pijaudier & Bažant (1987)
Ozbolt & Bažant (1996)</td><td>• Stress and deformation interaction within a distance in the continuum is taken into account, i.e. the strain depends on the entire strain field within vicinity of the crack.
• The model avoids mesh dependency but is more numerically expensive.</td></tr>
<tr><td>Damage-plasticity model
Grassl & Jirásek (2006)
Grassl et al. (2013)</td><td>• Combination of plasticity based on the effective stress and isotropic damage driven by the plastic strain.
• Able to predict the failure of concrete with varying loading cases from uniaxial tension to triaxial compression.
• Able to partially capture the reduction of shear stiffness.</td></tr>
<tr><td>Micro-plane model
Ozbolt & Bazant (1992)
Bažant & Prat (1988)
Ozbolt et al. (2001)</td><td>• The material properties are characterized separately on planes of various orientations, i.e. the microplanes.
• The state of each microplane is described by normal deviatoric and volumetric strains as well as shear strain.</td></tr>
<tr><td colspan="2">Embedded crack approach
Jirásek & Zimmermann (2001)</td><td>• Deformation due to the crack is treated as a strain or displacement discontinuity in a localisation band within an element.
• The response outside the localization band within the element will be elastic.</td></tr>
</table>

Localization of cracks

In the smeared crack approach, cracks are normally assumed to localize into crack bands, with crack bandwidths of h_b. Consequently, the crack bandwidth represents the width within which a crack localizes, and is typically given as input for a smeared crack FE analysis. Research by Borst (1995), Plos (1995) and Johansson (2000) indicated that the choice is dependent on the interaction between the reinforcement and the surrounding concrete elements. In regions without reinforcement, or if the reinforcement bond-slip relationship is included, cracks are believed to localize in one element row, thus, the element size may be used as the crack bandwidth. However, if the reinforcement is fully bonded to the surrounding concrete elements, the entire region is likely to become cracked, without localization into crack bands; in such cases a mean crack distance of s_m may be used as the crack bandwidth. A suggestion to calculate the mean crack distance is $s_m = s_{r,max}/1.7$, in which the maximum crack distance $s_{r,max}$ can be calculated according to Eurocode 2 (CEN, 2004).

Localization in compression can be included for the softening of the stress-strain relationship in the compression of cracked concrete. The softening behaviour of compression is related to the size of specimen in compressive tests (Mier, 1984) and the standard stress-strain relationship has been calibrated for cylinder specimens of 300 mm length. Therefore, the softening branch needs to be modified according to the size of the elements by assuming that the compressive failure occurs in one element row (Zandi *et al*., 2013).

Regarding localization, both in tension and compression, the result should be verified after the non-linear FE analysis because it is unknown in advance how many elements in which the cracks will localize.

2.3.2 Modelling of reinforcement

Reinforcement in RC slabs can be modelled at different levels of detailing. One alternative is to model reinforcement as fully bonded with surrounding concrete, also denoted as embedded reinforcement (TNO, 2015), i.e. the reinforcement is modelled by adding stiffness directly to the surrounding concrete elements, without attributing any separate degree of freedom to reinforcement elements. There are two approaches to model the fully bonded reinforcement for slabs: as a reinforcement bar or as a grid layer; see Figure 7. To model reinforcement as a grid layer, the thickness of each grid layer is calculated in the direction of the reinforcement as the total reinforcement cross-sectional area for a unit width of the slab divided by the unit width.

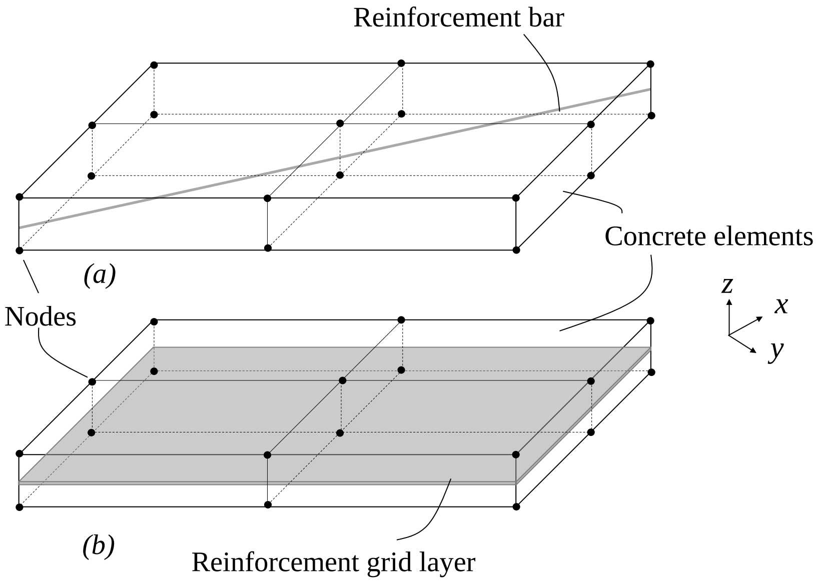


Figure 7. The reinforcing steel is modelled as fully bonded: (a) reinforcement bar and (b) reinforcement grid layer.

Another alternative is to include the bond-slip relationship between the reinforcement and the surrounding concrete. Model Code 1990 (fib, 1993) provided a simplified analytical 1D model, which has been further developed by Lundgren *et al.* (2012), considering corroded deformed reinforcement. A 3D friction model at a higher level of detailing was also formulated in Lundgren & Gylltoft (2000) and later modified in Lundgren (2005) for monotonic and cyclic loading.

2.3.3 Reinforced concrete slabs subjected to shear and punching

In previous years, several numerical investigations were carried out applying the FE method to the structural behaviour of RC slabs subjected to one-way shear and punching shear (summarized in Table 6), which is more challenging than the bending failure mode. Among those studies, 2D systems using rotationally symmetric elements, 3D systems using layered shell elements and 3D systems using continuum elements are the main categories.

Table 6. A review of studies with FE analyses of RC slabs subjected to shear and punching.

	State-of-the-art	
2D rotationally symmetric elements	Contributions by: Loseth *et al*. (1982), Borst & Nauta (1985), Gonzruez-Vidosa *et al*. (1989), Menetrey (1994), Hallgren (1996), etc.	
	Menetrey *et al*. (1994)	• Developed a model characterized by an efficient triaxial strength criterion for concrete (Menétrey & Willam, 1995) concerning the brittleness of concrete failure under various states of stress. • Conducted parameter studies concerning reinforcement ratio, size of specimens as well as tensile and compressive failure of concrete.
	Hallgren (1996)	• Studied high strength and normal strength concrete slabs, using the measured material strength and fracture mechanical properties. • Conducted parameter studies regarding tensile strength, the depth of compression zone and the ductility of concrete. • Improved mechanical model by Kinnunen and Nylander (1960).
3D systems layered shell elements	Contributions by: Marzouk & Chen (1993), Polak (1998), etc.	
	Polak (1998)	• Developed a quadratic, degenerated, isoperimetric shell element with the ability to take into account out-of-plane shear response. • Implemented 3D constitutive model for the evaluation of the stiffness matrix, with 3D states of strain and stress in each layer. • Validated applicability of the shell FE analyses for punching failure, requiring less computational effort than continuum FE analyses.
3D continuum elements	Contributions by: Ozbolt *et al*. (1999), Staller (2000), Zheng *et al*. (2009), Eder *et al*.(2010), Amir (2014), Belletti *et al*.(2014), etc.	
	Ozbolt *et al*. (1999)	• Developed a microplane material model and implemented it in the smeared crack approach for the modelling of RC slabs. • Conducted parameter study including the slab geometry, the fracture energy and the reinforcement ratio as well as concrete compressive and tensile strengths.
	Amir (2014)	• Implemented the composed elements (TNO, 2015) for the analysis of in-plane force. • Conducted FE analysis to study the in-plane forces and level of compressive membrane action in a laterally restrained bridge deck slab.
	Genikomsou & Polak (2015)	• Implemented a damaged-plasticity model in 3D FE analyses in ABAQUS. • Simulated the structural behaviour of interior RC slabs connected to column under static and reversed cyclic loadings. • Studied the effect of the compressive membrane action in isolated slab and continuous floor slab systems.

3 EXPERIMENTS OF REINFORCED CONCRETE SLABS

In order to develop improved analysis methods for the assessment of the load-carrying capacity and the response of RC slabs, several experimental tests have been reviewed and included in this study; see Table 7.

Table 7. A summary of experiments of RC slabs involved in this thesis.

<table>
<tr><td>i</td><td>CR1, CR2, CR3
(Paper I)</td><td>Two-way slabs subjected to bending failure</td></tr>
<tr><td>ii</td><td>DR1-a
(Vaz Rodrigues et al., 2008)</td><td>Cantilever slabs subjected to shear failure</td></tr>
<tr><td rowspan="4">iii</td><td>PG1~PG11
Guandalini et al. (2009)</td><td rowspan="4">Square slabs subjected to punching failure</td></tr>
<tr><td>PT32
Tassinari (2011)</td></tr>
<tr><td>AM04
Sagaseta et al. (2014)</td></tr>
<tr><td>PE7
Einpaul (2016)</td></tr>
<tr><td>iv</td><td>Kiruna bridge
(Paper VI)</td><td>Existing bridge deck slab subjected to shear failure</td></tr>
</table>

Test series i, ii and iii were selected because they represent the different failure modes of bending, shear and punching in RC slabs, respectively, and were well documented in details. Test series iv was selected to demonstrate the application of the developed analysis methods for practical implementation.

3.1 Bending tests of two-way slabs

To understand the behaviour of RC slabs subjected to bending failure, major contributions from different researchers have been studied and presented in Table 1, Section 2.1. In connection with the study reported in this thesis, a series of tests on two-way slabs (*Paper II*) was carried out at Chalmers University of Technology, see Figure 8(*a*). This test series was selected as a bench mark test for the development of modelling choices for RC slabs subjected to bending failure (*Paper III*). The tests included specimens containing traditional steel bar reinforcement in ordinary concrete and in fibre reinforced concrete. Three slabs with traditional reinforcement bars in ordinary concrete were selected for this study. Load distribution in these RC slab was of particular interest to this study. Thus, the supportive system was designed using a total of 20 high-tolerance steel pipes with strain gauges to measure the reaction force in two main directions and along supportive line; see Figure 8(*b*). Since the load distribution of RC slabs has been of interest to many researchers, other methods for measuring the reaction force have also been developed, e.g. putting loading cells along the support; see Lantsoght (2013) and Natario (2015).

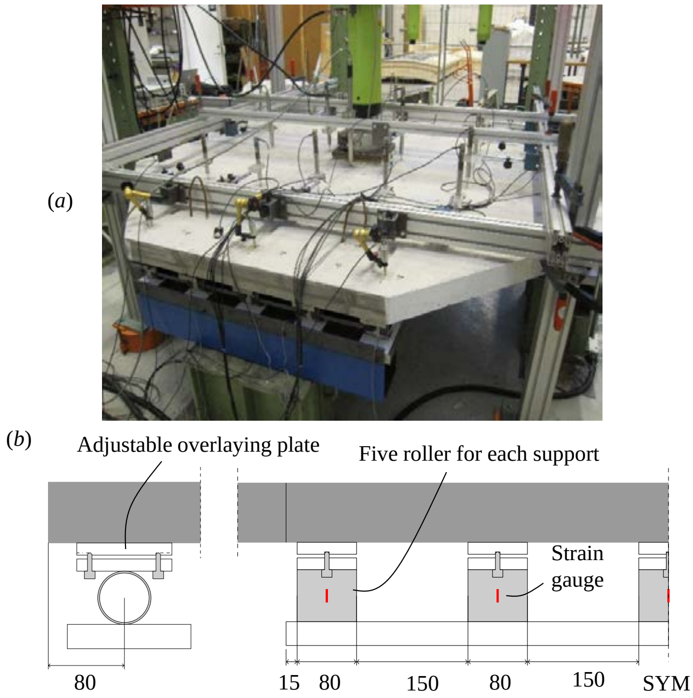


Figure 8. (a) Test set-up of two-way slabs and (b) layout of pipe roller supports (Paper II).

The results of the load-carrying capacity and load distribution of two-way slabs were obtained from these tests, about which more information can be found in *Paper II*. The tests were aborted at the rupture of reinforcement (see Figure 9), which indicated a clear bending failure. The results showed that the bending capacity of the two-way slabs obtained from the experiment was much higher than that calculated from the yield line method. The main reason is that the simplified method had neither included the strain hardening of the reinforcement bars nor the membrane effect of the slabs. An extended discussion on this issue can be found in *Papers I -III*. The load distribution in two main directions and along supportive lines were measured during the tests and compared with the FE analysis results obtained in Section 4.3.1.

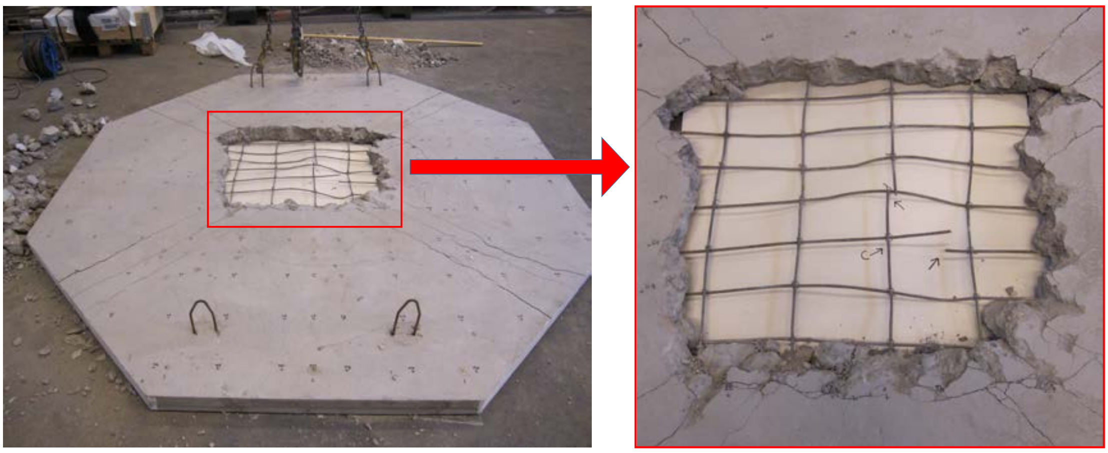


Figure 9. Failure of tested slab CR1 and illustration of rupture of flexural reinforcements after removing the cracked concrete, the slab upside down after completion of the test (Fall, 2013).

3.2 Shear tests of cantilever slabs

Major studies of RC slabs subjected to one-way shear have been reviewed and presented in Table 2, Section 2.1. In connection with the study in this thesis, experiments carried out by Vaz Rodrigues *et al*. (2008) were selected to investigate one-way shear failure mode of cantilever bridge deck slabs. A slab with four concentrated loads, slab DR1-a, was chosen for the study, see Figure 10. The tested specimen failed in brittle shear and the results verified that the shear capacity of the cantilever slab obtained from the experiment was much higher than that calculated based on Eurocode 2 (CEN, 2004). The failure of the tested cantilever slab showed great interaction between shear and bending failure modes. In addition, the failure mode of this test was a combination of one-way shear and semi-punching failure, a failure mode had not been clearly defined in Eurocode 2 (CEN, 2004) nor in MC2010 (fib, 2013). Extensive structural analyses based on the Multi-level Assessment Strategy and an investigation of the structural behaviour of this cantilever slab can be found in *Paper I*.

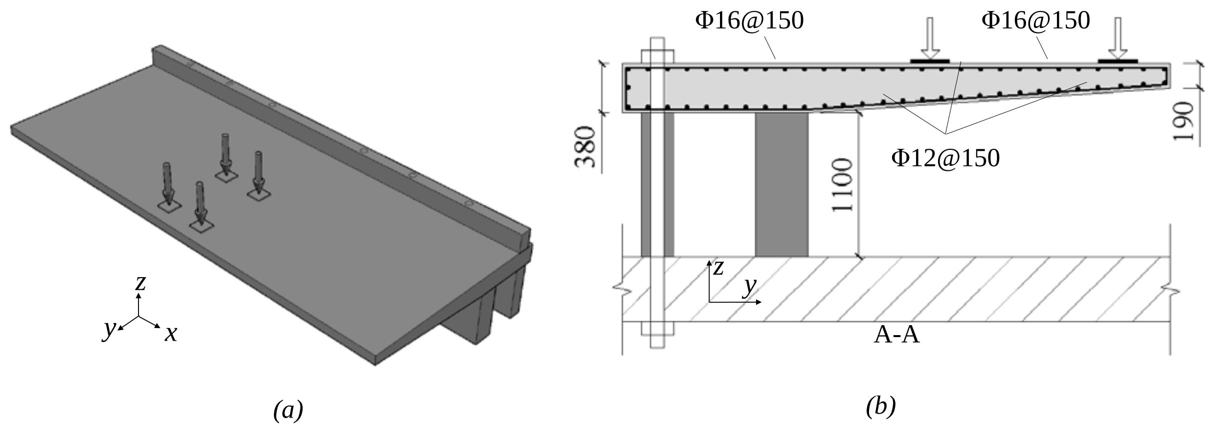


Figure 10. (a) Large scale model under loading patterns (b) and test DR1-a; adapted from Vaz Rodrigues et al.(2008).

3.3 Punching tests of slabs

Major studies of RC slabs subjected to punching shear have been presented in Table 3 in Section 2.1. In connection with this study, experiments PGs carried out by Guandalini *et al*. (2009), PTs by Tassinari (2011), AMs by Sagaseta *et al*. (2014) and PEs by Einpaul (2016) were selected. Experiments carried out by Guandalini *et al*. (2009) were adopted to investigate modelling choices for non-linear continuum FE analyses and to develop the modelling method for RC slabs with respect to punching failure (*Paper IV*). Experiments carried out by Guandalini *et al*. (2009), Tassinari (2011), Sagaseta *et al*. (2014) and Einpaul (2016) were selected to investigate the structural behaviour and punching failure mode of RC slabs. Figure 11 shows the set-up of one of the tests by Einpaul (2016).

Another study is regarding parameters that would influence shear force distributions (*Paper V*). A square slab with a square column (PG1) was selected as a reference slab and the other three were selected as comparative slabs to vary the parameters: PT32 because the reinforcement was not symmetrical; PE7 since the geometry of the slab was octagonal and the column was circular instead of square; AM04 because the column was rectangular with long side 3 times the short side. These comparative tests were also selected because they had high reinforcement ratios (ρ

= 0.75% - 1.5%) so that punching occurred before reaching the plastic plateau in the load-rotation curve of the slab related to flexural failure.

Figure 11. Punching test of an RC slab supported on a column in the centre, loaded at 8 points close to the perimeter (Einpaul, 2016).

3.4 Field test of a bridge deck slab

Only a limited number of field tests on real bridge deck slabs can be found in literature. A summary of field tests of RC bridge deck slabs subjected to shear is presented in Table 8.

Table 8. A summary of field tests of RC bridge deck slabs subjected to shear failure.

Miller *et al*. (1994)	• Tested a dismissed 38-year old concrete slab bridge to failure. • Concrete in the shoulder area was heavily deteriorated, with reinforcement bar completely exposed. • The bridge failed in shear, without reaching the theoretical bending capacity.
Pressley *et al*. (2004)	• Tested a 33-year old RC flat slab bridge “No 1049”. • Two destructive bending and two destructive punching shear tests. • Results showed that pile-soil interaction had to be modelled to obtain the correct load distribution and load-carrying capacity with respect to punching.
Lantsoght *et al*. (2016)	• Tested a 42-year old RC slab bridge, as a part of road “N924”. • Bridge failed in bending even though shear failure was predicted by Eurocode 2 estimation. • Tests results showed a conservative estimation of the capacity of existing slab bridges by the current method in building codes. • The bridge deck slabs were cut into beams and tested in the lab to study the difference between smooth bar reinforcement and ribbed bar reinforcement.

The field test included in this study (see *Paper VI*) was intended to result in punching failure and was carried out by Bagge *et al*. (2014; 2015). It was a 55-year-old bridge, which had to be demolished due to urban transformation of the city Kiruna in northern Sweden. In order to develop methods for improved bridge assessment, a condition study and a destructive field tests were carried out on this bridge before demolishing. Investigations of this bridge have previously been reported by e.g. Nilimaa (2015), Nilimaa *et al*. (2016), Bagge & Elfgren (2016) and Huang *et al*. (2016). Configuration and dimensions of the bridge are also presented in Figure 12. Previous to the slab test, the bridge was tested to failure in two of the main girders. The slab was thereafter loaded in the middle of the previously tested span, close to the undamaged main girder, until a sudden shear type failure occurred without prior notice. An extended description of the bridge test and additional test results can be found in *Paper VI*.

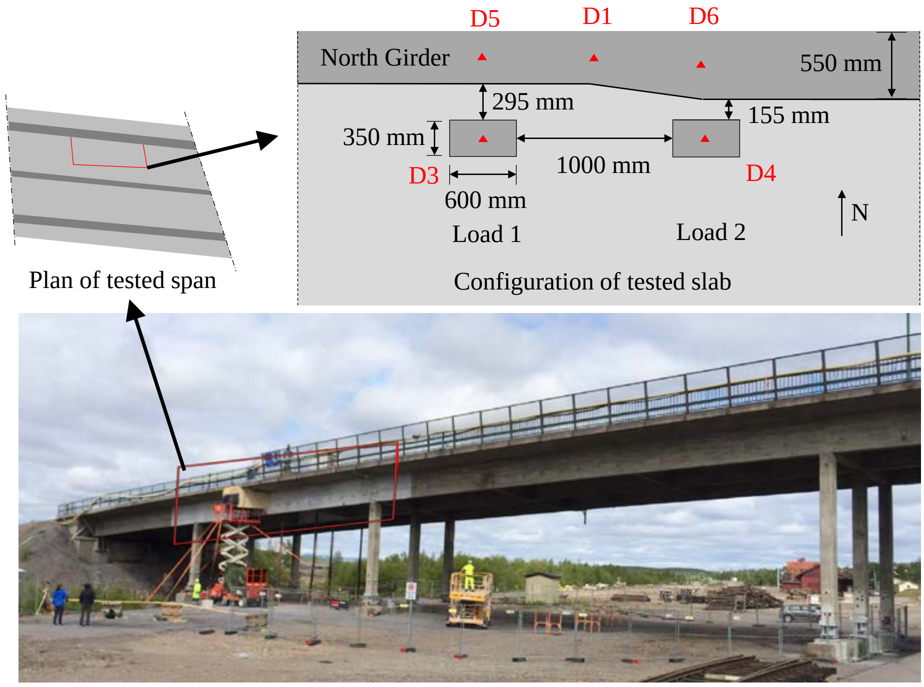


Figure 12. Photo, configuration and dimension of the tested bridge (Shu, 2014).

Figure 13 shows photographs after the test, illustrating the failure under loading plate 1. It was observed that a shear type failure occurred next to plate 1 only, but not at plate 2. By observation at the bottom of the tested slab, a semi-circular failure surface under plate 1 was detected, which means that shear failure only occurred on one side of the slab, i.e. the failure mode was not a pure punching failure. An in depth discussion of this failure mode is presented in *Paper VI*.

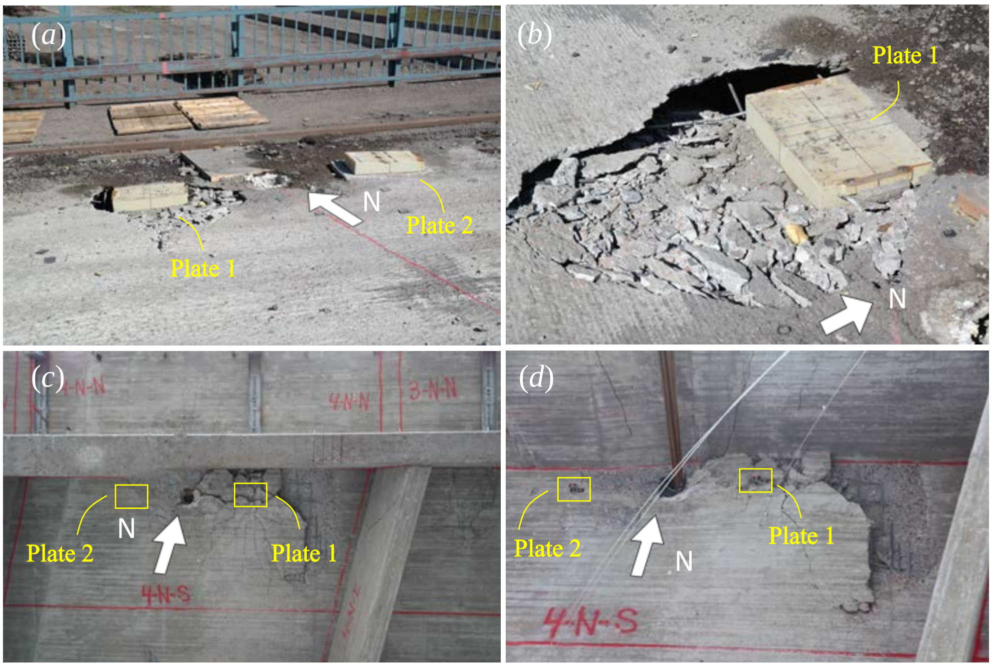


Figure 13. Photos of the failed bridge deck slab after shear failure: (a) & (b) are photos at top of the slab and (c) & (d) are photos at bottom of the slab (Bagge, 2014).

4 MULTI-LEVEL ASSESSMENT STRATEGY FOR CONCRETE SLABS

The development of an assessment strategy for RC bridge deck slabs is based on the principle of successively improved evaluations in structural assessment (SB-LRA, 2007). In Figure 14, a flow diagram of the assessment process is proposed. It starts with a need for an assessment due to changing requirements, deterioration of or damage to the structure. First of all, an initial assessment based on a site visit, a study of the documentation and an analysis using simplified methods should be carried out. If the requirements are not fulfilled, an economical and sustainability decision analysis should be carried out to determine if the assessment would continue. A continued assessment can include an enhanced examination with improved information (inspections, monitoring and testing), as well as an improved analysis (structural analysis, resistance models and reliability based assessment). If the assessment would not continue, the bridge may be demolished, strengthened or subjected to restricted loads for future use. An enhanced assessment may result in a decision whether it would be possible to continue using the bridge, possibly after strengthening or repair, or whether its use might be redefined under intensified monitoring.

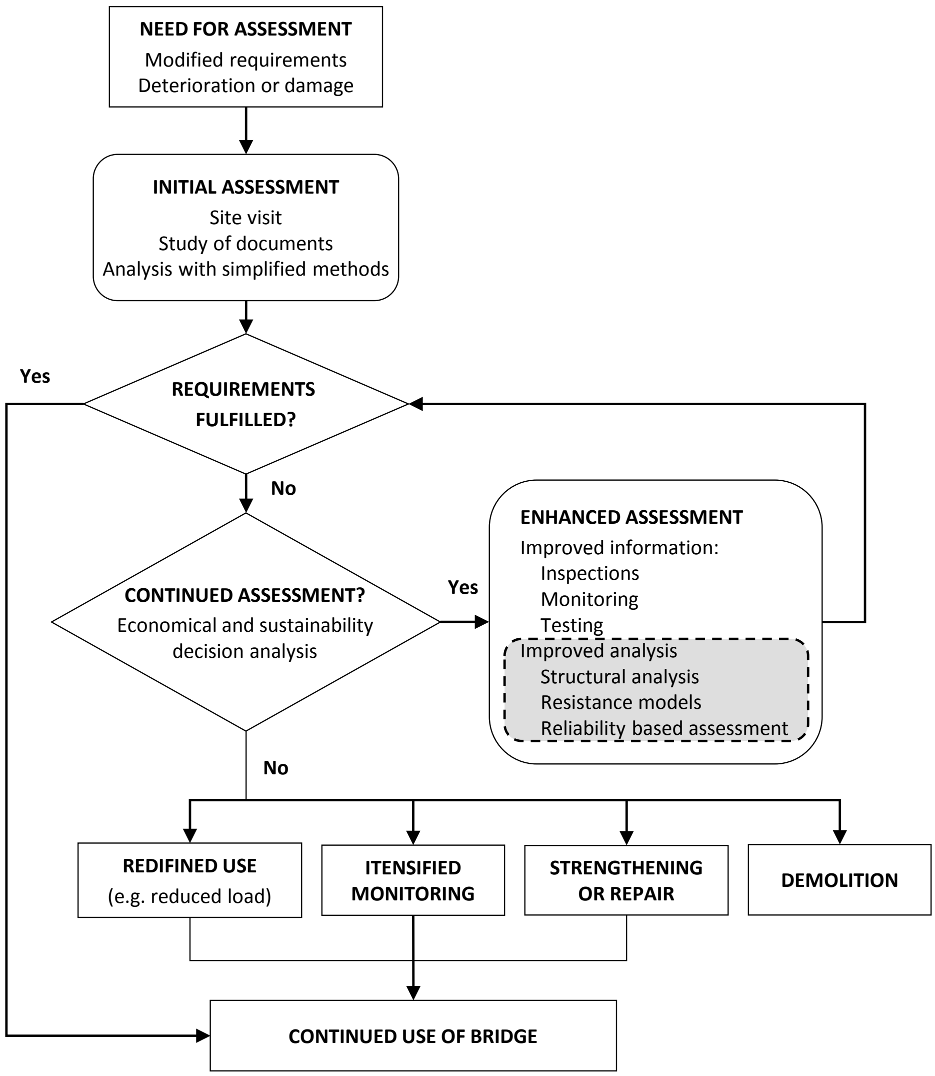


Figure 14. Flow diagram for structural assessment based on the principle of successively improved evaluation (Paper I).

Since the traditional structural analysis method may be too conservative in assessing the load-carrying capacity of bridge deck slabs, enhanced methods can preferably be used. Such methods might lead to a better understanding of the structural response and reveal higher load-carrying capacity. For the assessment of RC slabs, 3D linear FE analyses are already commonly used in engineering practice; see e.g. Rombach (2008b) and Blaauwendraad (2010). Shell elements taking into account non-linear response have been used in research; see e.g. Marzouk & Chen (1993) and Polak (1998). Research using continuum elements to represent RC slabs has also been carried out; see e.g. Amir (2014) and Belletti *et al*.(2014).

4.1 Description of Multi-level Assessment Strategy

To provide a clear strategy for enhance the assessment, a Multi-level Assessment Strategy has been proposed (*Paper I*), as seen in Figure 15. The proposed assessment strategy focuses on an enhanced assessment through improved structural analyses and resistance evaluations; see Figure 14. The general idea is based on the principle of successively improved evaluations in structural assessment (SB-LRA, 2007) and the level-of-approximation approach in MC2010 (fib, 2013). Higher level methods can be used in cases where higher accuracy would be required; for example, Belletti *et al*. (2014) indicated that by increasing the level of approximation, the design load obtained would increase as well. Such higher levels generally require greater effort but have shown to be economically advantageous in many cases (Plos, 2002; SB-4.5, 2007).

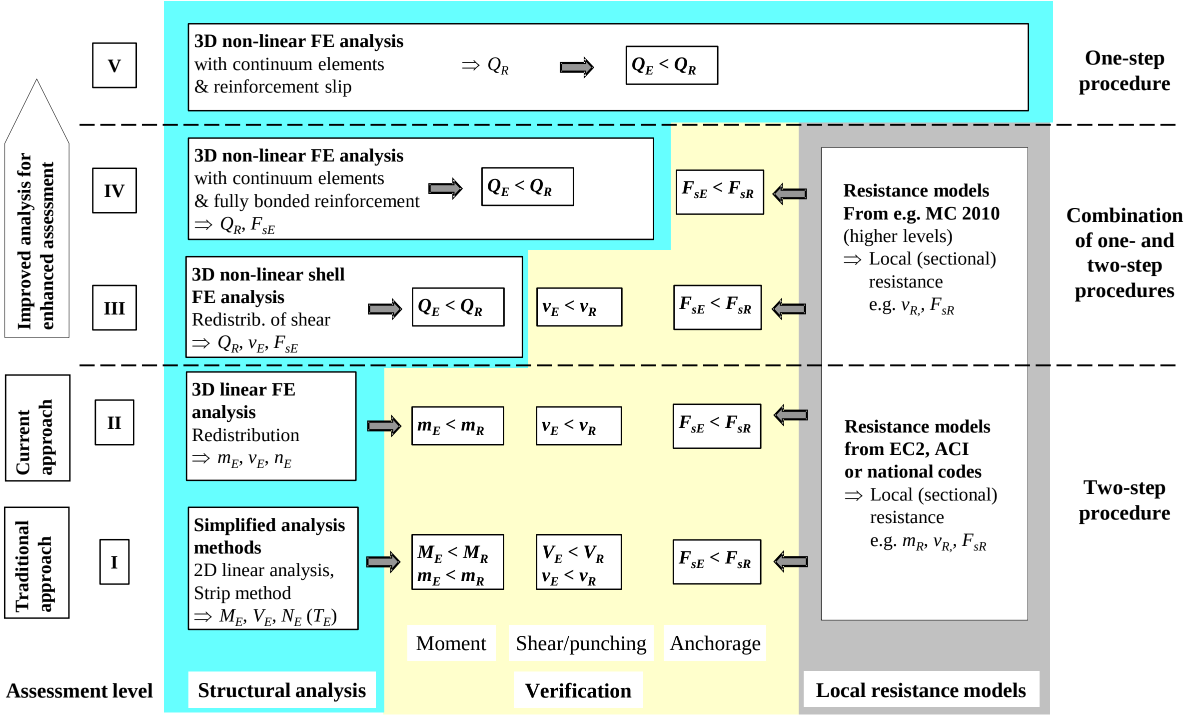


Figure 15. Scheme for Multi-level Assessment Strategy of RC bridge deck slabs (Paper I).

For RC slabs, assessments of load-carrying capacity with associated responses can be conducted through the following levels and methods:

- Level I: Simplified analysis methods are used (typically, code provisions or simplified mechanical models).
- Level II: 3D linear FE analysis is performed assuming linear elastic behaviour to be able to superimpose the effect of different loads, in order to achieve the maximum internal forces throughout the structure for all possible load combinations. The internal forces (axial forces,

shear forces and flexural moments) are then compared to the corresponding resistance determined by local models for bending, shear, punching and anchorage of reinforcement.

- Level III: Non-linear shell FE analysis with fully bonded reinforcement is used with the capability of reflecting the flexural strength of RC slabs directly in the FE analysis. However, at level III, the out-of-plane shear strength usually need to be determined using separate mechanical or local resistance models.
- Level IV: Both bending and shear type failures including punching can be reflected by performing a non-linear analysis using 3D continuum elements coupled with fully bonded reinforcement. However, at level IV, bond strength and its effect on shear type failure has to be verified separately.
- Level V: This is a refinement of level IV, where the bond-slip behaviour of the interface between the reinforcement and the concrete is included. With this level of accuracy in the structural analysis, no failure modes need to be checked separately using resistance models. Thus, the load-carrying capacity at the structural level V can be determined using a one-step procedure.

The Multi-level Assessment Strategy differs from the Level-of-Approximation concept (Muttoni & Ruiz, 2012a, 2012b) in MC2010 (fib, 2013) in that MC2010 focuses on resistance models for different failure modes whereas this approach focuses on the structural analysis of the slab and connects this structural analysis with resistance models at different levels in Eurocode 2 (CEN, 2004) and MC2010 (fib, 2013). Thus Multi-level Assessment Strategy can be seen as a complement to the Level-of-Approximation concept in MC2010 for assessing RC slabs.

4.2 Modelling choices for continuum finite element analysis

In the scope of the Multi-level Assessment Strategy, it is already known that the 3D non-linear continuum FE analysis at levels VI and V has the highest potential for discovering any additional sources of load-carrying capacity in RC slabs, as discussed in *Paper I*. Since anchorage failure, which is accounted for in the level V analysis, is usually not the critical failure mode for RC slabs, analysis at level IV is of paramount interesting in this study. At this level, both bending and shear type failures can be reflected in a one-step procedure. Sustainable Bridge (2007b) also ranks the non-linear analysis at the highest level in terms of successively improved analysis. However, non-linear continuum FE analyses are usually demanding and require skills and experience necessary to assess modelling choices that exert considerable influence on results. Therefore, modelling strategies for this type of analysis are needed and consequently, the aim of this study is to contribute to the development of such strategies and provide recommendations on modelling choices.

For the purpose of identifying important factors regarding modelling choices for the structural behaviour of the FE model of RC slabs, sensitivity analyses have been conducted using slab CR1 in Section 3.1 and slab PG1 in Section 3.3 with analyses at level IV. In previous research conducted by e.g. Eder *et al*. (2010), Belletti *et al.* (2014) and Amir (2014), some modelling choices were already studied, e.g. tension softening diagram, crack models, Poisson's ratio, reductions in compressive strength due to lateral cracking, fracture energy in tension and compression, as well as the shear-retention factor. Eder *et al*. (2010) indicated that tension softening, which is related to effective fracture energy, was found to have a greater effect than tensile strength on the predicted load-deflection response. However, the shape of stress-strain diagram (i.e. linear or exponential) was not found to have a significant influence on the predicted performance. Belletti *et al*. (2014) showed that the structural response of the slabs was largely affected by such parameters as shear retention factor, the compressive fracture energy and reduction in compressive strength due to lateral cracking. In general, these studies revealed the impact of modelling choices on the structural behaviour of the specified model of RC slabs. In this study, the selected modelling choices, including the five major categories: geometry non-linearity, element properties, modelling of concrete and reinforcement, as well as modelling of supports were investigated; see Figure 16. Details of the sensitivity analyses can be found in *Paper III* and *Paper IV*. Parameters governing these aspects were selected because they were suspected to have a decisive influence on the outcome of FE analyses. In the study, one alternative of each parameter was selected to be included a reference model (marked with *) for the benchmark analysis and the rest was used for comparative analyses.

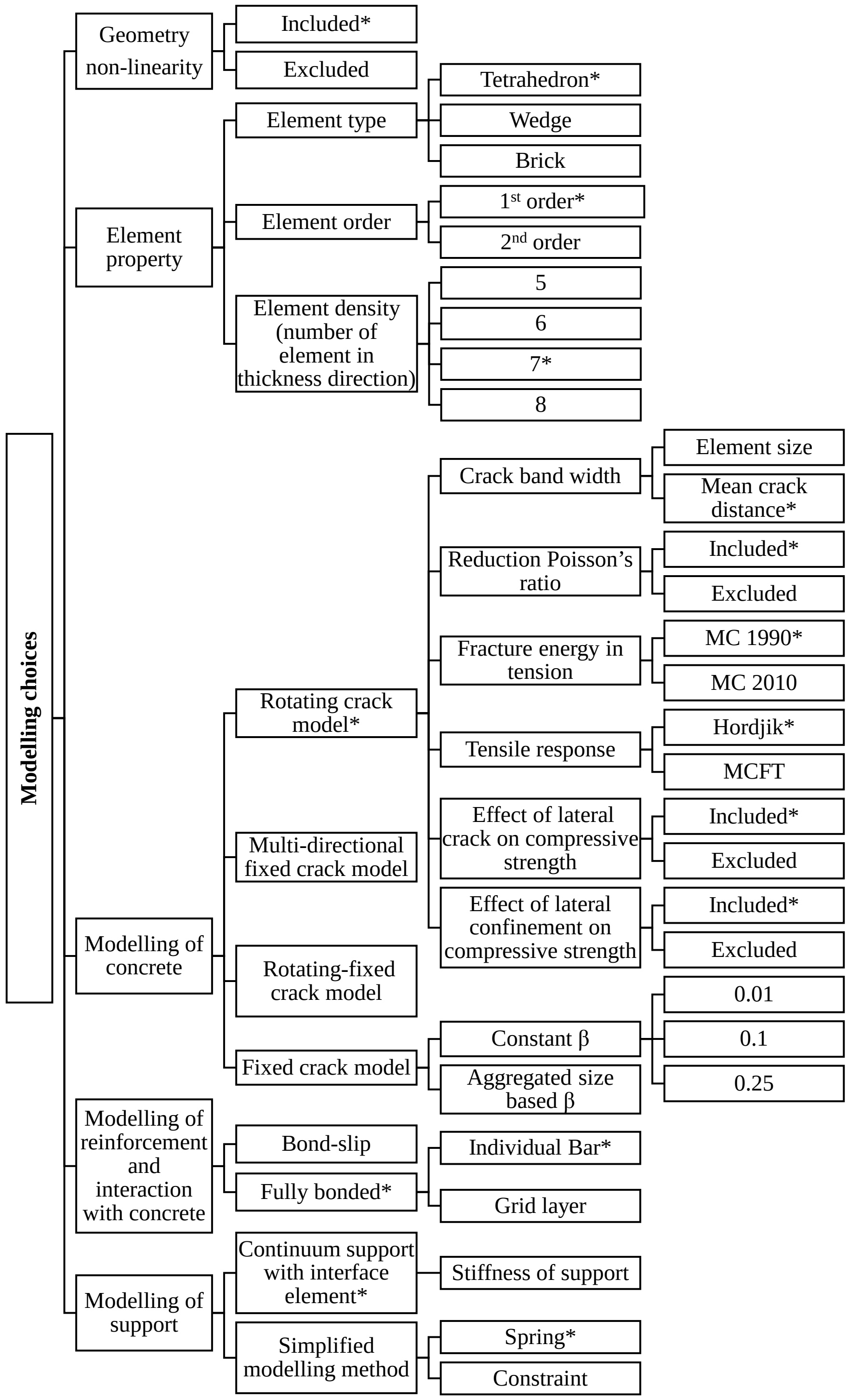


*Figure 16. Sensitivity analyses of modelling choices for assessment at level V using slab CR1 and PG1; the selections as reference models are marked with *.*

Geometry non-linearity was included in the model of two-way slabs because, for the structure studied, the displacement in the centre of the slab was "large", i.e. exceeding the thickness of the slab. Finite element properties, including element type, order and density have been studied for the general modelling of RC slabs using continuum elements. Material modelling of concrete, including various modelling choices for different crack models were studied as well. Some modelling choices were studied specifically for shear behaviour of RC slabs, e.g. reduction in the compressive strength of concrete due to lateral cracking and confinement effects. In most cases, the studies were carried out by using the Total Strain (TS) rotating crack model (TNO, 2015) since it was proven to have better numerical performance. However, the rotating crack model is not capable of reflecting the real behaviour of shear cracks (Rots & Blaauwendraad, 1989); therefore, standard fixed crack model, multi-directional fixed crack model and rotating-fixed crack model (TNO, 2015) were investigated to find a better solution. The modelling of reinforcement needs to be studied for the higher requirement of modelling detailing, e.g. when interaction between concrete and reinforcement should be considered. The modelling of support has been studied to illustrate the modelling variations and reach reasonably simplified methods.

The influence of different modelling choices on the analysis results such as the load-carrying capacity, load-deflection response, crack patterns and load distribution of RC slabs subjected to bending and shear have been investigated and compared to corresponding experimental data in *Papers III* & *IV*. The major results are summarized and presented in Table 9, and based on these results, a preliminary modelling strategy, together with modelling recommendations for 3D FE analyses with continuum elements, are also presented in *Papers III* & *IV*. An example is the influence of stiffness of line support on load distribution. A sensitivity analysis regarding the relationship between stiffness of support and load distribution was carried out and details of the study can be found in *Paper III*. It shows that the reaction force distribution was found to be highly influenced by the stiffness of supports. To estimate the support reaction distribution, the stiffness of the "supportive system" in reality, including such items as columns, edge beams and transversal beams must be taken into account. In the future, these results together with recommendations for the choice of modelling alternatives and other research works, e.g. Engen *et al.* (2015) and Belletti *et al.* (2014), should complement more general guidelines already available in the literature, such as fib: bulletin 12 (fib, 2001), fib: bulletin 45 (fib, 2008) and Hendriks *et al*. (2012).

Table 9. Results of sensitivity analyses of modelling choices using slab CR1 and PG1.

Modelling choices	Reference choice	Comparative choice	Results of using comparative choice
Element type	Tetrahedron	Wedge	No significant influence
		Brick	No significant influence
Element order	1st order	2nd order	Less clear crack pattern (major)
Crack bandwidth (full-bond reinforcement)	Mean crack distance	Element size	Overestimation of load-carrying capacity (major)
Crack bandwidth (bond-slip reinforcement)		Element size	More accurate in load-carrying capacity (major)
Reduction of Poisson's ratio	Include	Exclude	More accurate load-carrying capacity but higher stiffness (minor)
Mode I Fracture energy	MC1990	MC2010	Overestimation of load-carrying capacity (major)
Compressive strength due to lateral crack	Include	Exclude	Overestimation of load-carrying capacity (minor)
Compressive strength due to lateral confinement	Include	Exclude	Overestimation of load-carrying capacity (minor)
Crack model	Rotating	Standard fixed	Overestimation of load-carrying capacity (major)
		Multi-directional fixed	Poor numerical performance (major)
		Rotating-fixed	Lack of knowledge on threshold for switch from rotating to fixed model (major)
Shear retention factor for fixed crack model	= 0.01	≥ 0.1	Overestimation of load-carrying capacity (major)
		Aggregated size based	Lack knowledge on shear stress-strain relationship (major)
Interaction between reinforcement and concrete	Full-bond	Bond-slip	More clear crack pattern (major)
Modelling of reinforcement (full-bond)	Rebar	Grid layer	Less clear crack pattern (major)
Simplification for modelling of support	Spring	Constraint	Underestimation of load-carrying capacity (major)
Stiffness of support	Low	High	Less accurate load distribution (major)

Note: Effect due to change of modelling choice from reference model to comparative model is marked as “major” or “minor”.

4.3 Non-linear continuum finite element analyses

After the sensitivity study of modelling choices, non-linear continuum FE analyses have been applied on all slabs subjected to bending, shear and punching, which were tested in laboratories, as described in Sections 3.1, 3.2 and 3.3. The overall aim of this study was to investigate the feasibility of predicting the response of RC slabs using continuum elements in non-linear FE analysis. A major concern of this study was about the load-distribution in RC slabs subjected to concentrated loads. Two other aspects studied were the effect of flexural reinforcement and size of the slab on punching capacity. These aspects were investigated using continuum FE analyses and the results were compared to those of experiment. More details can be found in *Papers III - V*.

4.3.1 Load distribution

Load distribution is very important for the structural response of a RC slab and has to be understood for RC slabs subjected to concentrated loads. The load-distribution can be affected by several factors, such as the development of cracks (including bending and shear cracks), the layout of reinforcement, as well as the geometry of the slabs and supportive columns. In order to understand the distribution of loads and phenomenon of redistribution, several researchers have carried out studies, e.g. Sagaseta *et al*. (2011; 2014) investigated the effect of shear deformation of the slabs and geometry irregularity, and Einpaul (2016) studied the influence of flexural deformation of continuous slabs.

Effect of cracking on load distribution

The load distribution of RC slabs is influenced by the development of bending cracks and stiffness of cracked sections. This was investigated using a FE analysis of two-way RC slabs subjected to a concentrated load in *Paper III*. In the experiment, load distribution and redistribution were studied through the measurement of reaction force, as described in Section 3.1. The supportive system was designed using 20 high-tolerance steel pipes to measure the reaction force of slab CR1 along the supportive lines; see Figure 8(*b*). Two aspects were investigated: the distribution of reaction force in strong (more reinforcement) and weak (less reinforcement) directions due to different reinforcement ratios and the distribution of reaction forces along the supportive lines due to the crack development. The corresponding results were obtained from the FE analysis and compared to those obtained from the experiment; see Figure 17. The reaction force distribution along the support edges could be correctly described when the vertical stiffness of the supports were correctly modelled; see sensitivity analyses of modelling choices in *Paper III*.

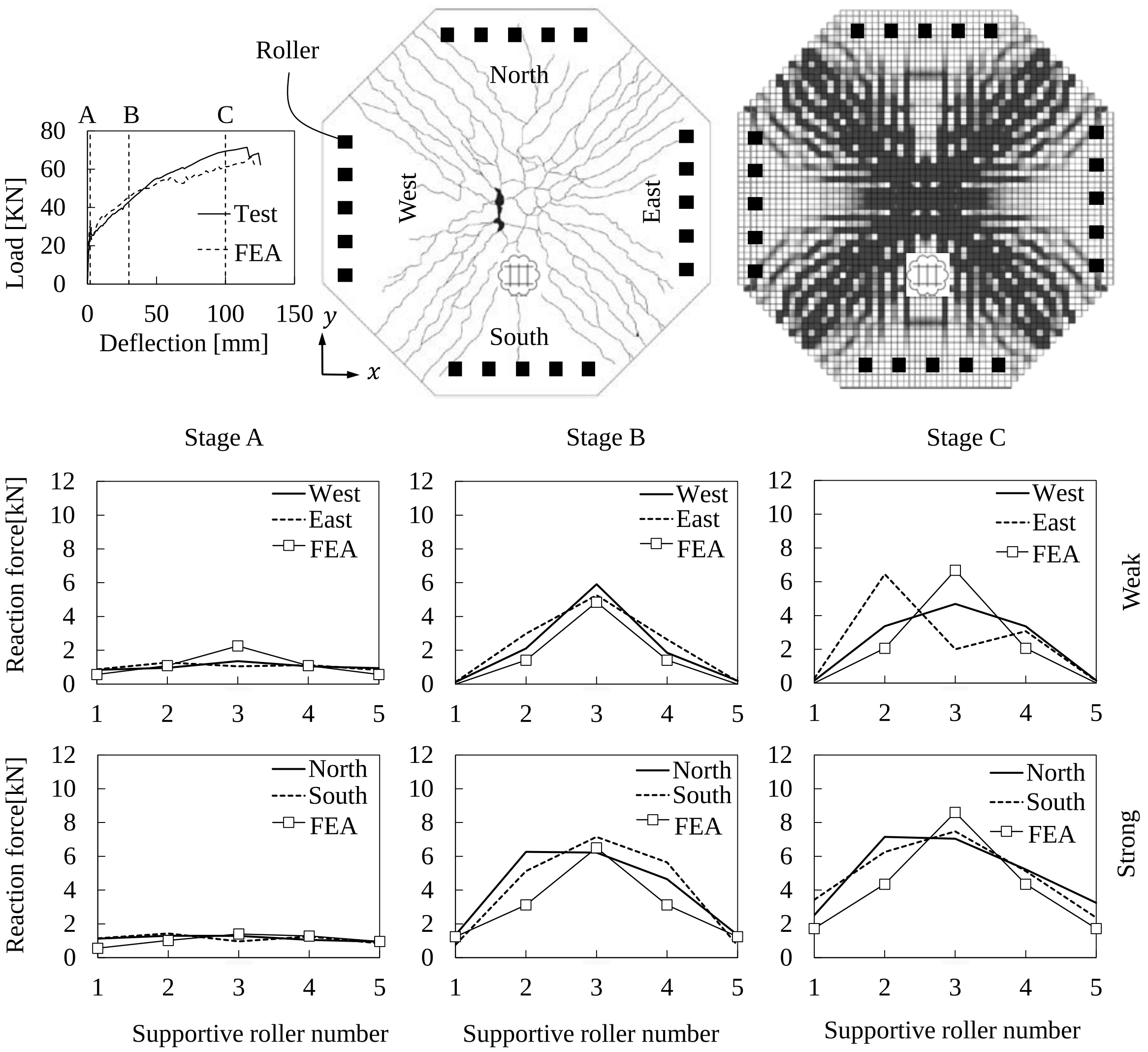


Figure 17. Crack pattern and reaction force distribution (of CR1) along the support edge, obtained from test and FEA: "West", "East", "North" and "South" are results from tests; adapted from Paper III.

Not only the reaction force distribution in slabs subjected to bending may be affected by the formation of bending cracks, but the shear force distribution in slabs subjected to punching can also be affected by the formation of bending and shear cracks. This was investigated by applying a non-linear continuum FE analysis on slab PG1; details of this analysis can be found in *Paper V*. Figure 18 presents the shear force distribution along the symmetry line in x direction at different load levels ($V/V_{R.EXP}$ = 0.1, 0.2…0.9), from the FE analysis of PG1. It becomes clear that, along with increasing load levels, shear force, within the region of the critical shear crack, increases much faster than that outside this region. The shear force outside the region of a critical shear crack increases very slowly when $V/V_{R.EXP}$ > 0.6, due to the formation of the critical shear crack.

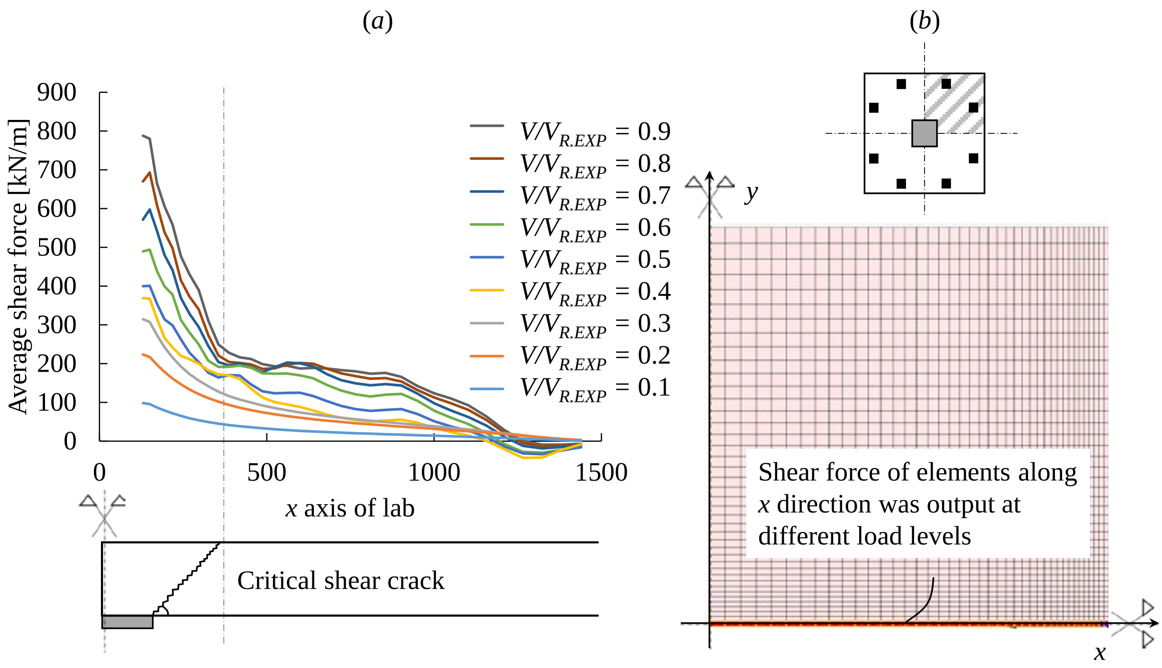


Figure 18. (a) Shear force distribution along x axis, at different load levels; (b) a quarter of slab model PG1 and the elements for obtaining shear force.

Effect of reinforcement layout on load distribution

The influence of different reinforcement ratios on the reaction forces in the two main directions of a two-way slab subjected to bending failure was studied for CR1; see *Paper III*. In Figure 19, the total reaction force at the supports in the strong and weak directions from the test as well as analysis is presented. Comparing the reaction force carried by the supports in each directions, it is observed that the load carried in the strong direction continued to increase upon cracking, whereas only a minor increase in support reaction in the weak direction is observed. In the FE analysis, two-thirds of the total reaction force are carried in the strong direction at full crack stage. This result corresponds to the difference in reinforcement amounts; the reinforcement ratio in the strong direction is twice as high as in the weak direction.

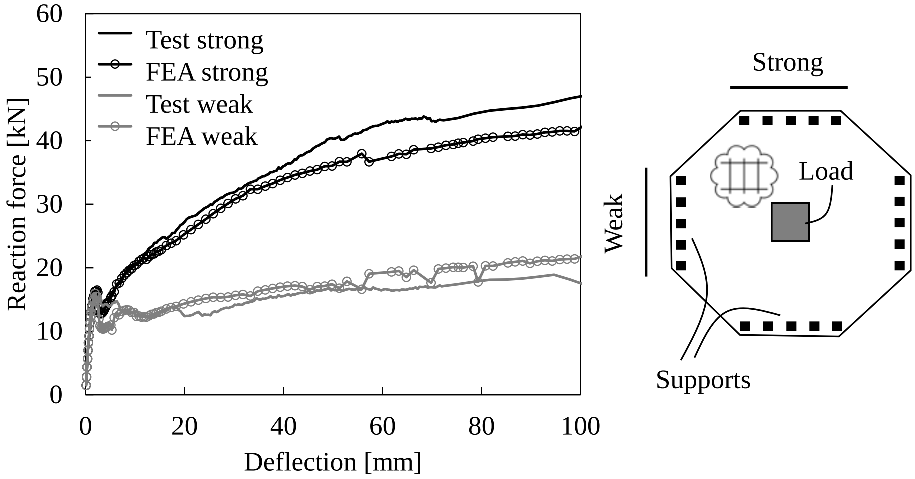


Figure 19. Reaction force distribution in two main directions of CR1, based on Test and FE analyses, adapted from Paper III.

Shear force redistribution due to the layout of reinforcement in slabs subjected to punching failure was investigated by applying continuum FE analyses on slabs PT32 and PG1; see *Paper V*. Reinforcement ratios in the x and y directions were $\rho_x = \rho_y$ for PG1 but $\rho_x = 2\rho_y$ for PT32. By comparing the average shear force along the elements located outside of the critical shear crack (the blue lines) versus the applied loads of slab PG1 and PT 32 obtained from continuum FE analyses (see Figure 20), it clearly illustrates that before the formation of the shear crack, the shear force distribution in the X and Y regions is very close. However, after the formation of a critical shear crack at a load level of $V/V_{R.EXP} \approx 0.5$ for PG1 and $V/V_{R.EXP} \approx 0.6$ for PT32, the redistribution is affected by the layout of reinforcement; the average shear force increases slowly in the Y region but much faster in the X region for PT 32, but different for PG1.

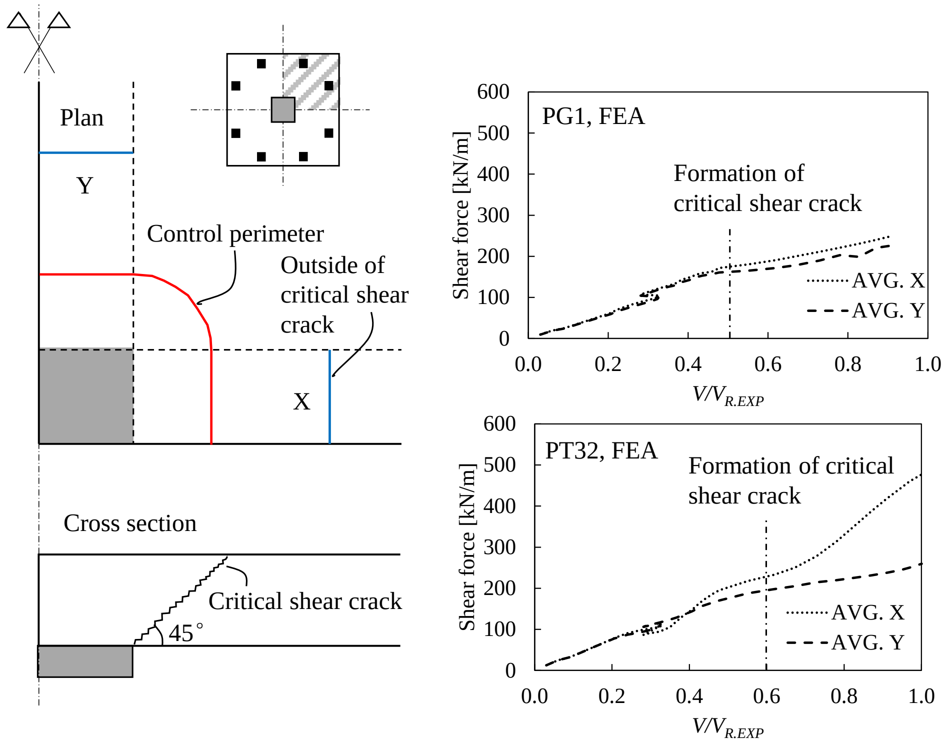


Figure 20. Average shear force variation just outside of the critical shear crack versus applied load in X and Y regions in the continuum FE analysis of slabs PG1 and PT32; adapted from Paper V.

Effect of column geometry on load distribution

The effect of column geometry on load distribution was studied by applying FE analyses to slabs PG1 and AM04; see *Paper V*. PG1 was supported by a square column but AM04 was supported by a rectangular column with a cross-section of $c_1 \times c_2$, where $c_1 = 3c_2$. As illustrated in Figure 21, the shear force was determined along a control perimeter around the column at a distance of $d/2$ from the column face, for a quarter of each slab. The studied control perimeter was divided into three different parts: regions X, Y and the diagonal region. The shear force distribution along the control perimeter at different loading stages ($V/V_{R.EXP}$) from the continuum FE analyses is presented in Figure 21. The horizontal axis shows the distance along the control perimeter; the vertical axis shows the shear force per unit length [kN/m]. By comparing different load stages for all slabs, it is evident that at lower load levels, i.e. $V/V_{R.EXP} = 0.1$ and 0.3, the shear force is distributed slightly unevenly along the control perimeter, with only minor discrepancies. At higher loading stages, i.e. $V/V_{R.EXP}$ = 0.6 and 0.9, the shear force of PG1 is distributed with limited fluctuations because of the square column shape, whereas the shear force of AM04 is considerably lower in region Y than in region X due to the rectangular Shape of the column.

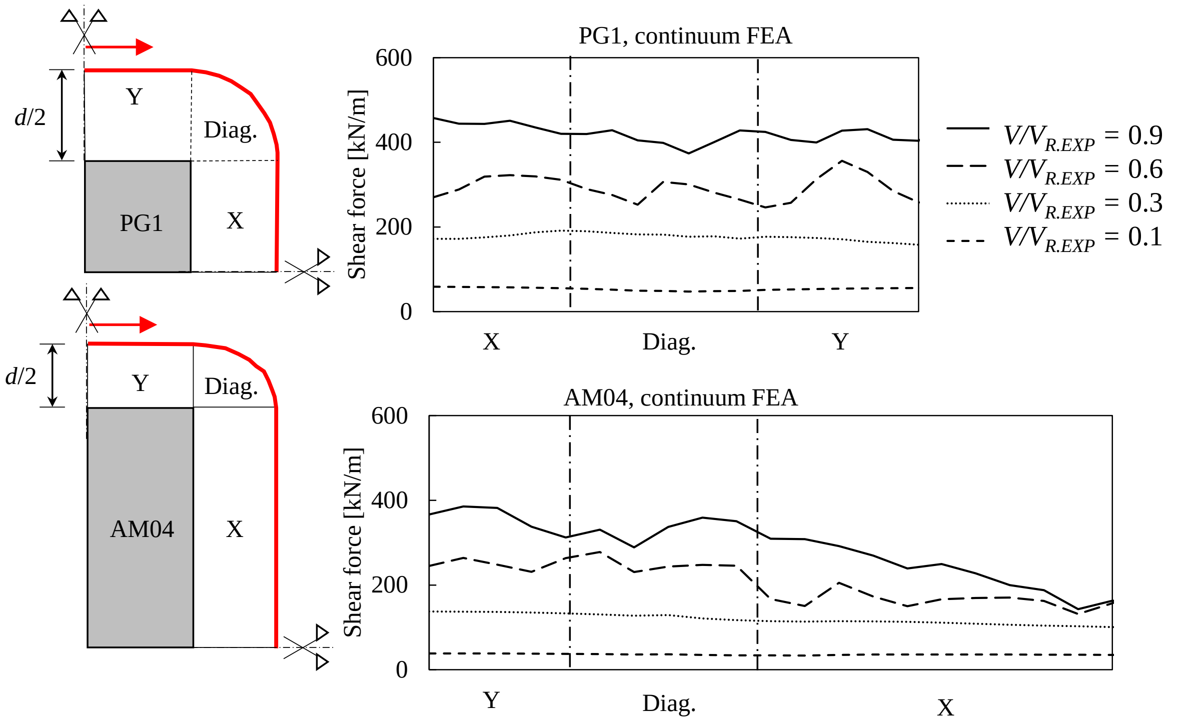


Figure 21. Shear force distribution along the control perimeters of slabs PG1 and AM04 at four loading stages calculated using continuum FE analyses; the horizontal axis shows the distance along the control perimeter (read line); the vertical axis shows shear force per unit length.

The study shows that load distribution can be greatly influenced by the formation of both bending and shear cracks. The layout of reinforcement and geometry of column support also affects the load distribution but the effect is not evident until the formation of cracks appears; the influence is more pronounced when a major shear crack is formed.

4.3.2 Effect of flexural reinforcement on punching strength

The effect of flexural reinforcement on punching strength, which has been studied in in the literature, e.g. by Kinnunen and Nylander (1960), Menetrey (1996) as well as Muttoni (2009), has been included in Eurocode 2 (CEN, 2004) using a factor k; see Table 4. Some conclusions regarding RC slabs failed in punching can be summarized as:

- For RC slabs with low reinforcement ratio, punching failure may occur after large plastic deformations with yielding of flexural reinforcement.
- For RC slabs with intermediate reinforcement ratio, punching failure may occur without large plastic deformations but yielding of part flexural reinforcement still occurred.
- For RC slabs with high reinforcement ratio, punching failure may occur without plastic deformations and yielding of flexural reinforcement.

However, the limits of reinforcement ratio (low, intermediate and high) are not clear yet and would differ from case to case. To further investigate the effect of flexural reinforcement on punching, non-linear continuum FE analyses have been applied to all laboratory tested slabs (described in Section 3.3) in *Paper IV*. The relation between nominal punching strength and deflection was studied in four experiments with different reinforcement ratios; see Figure 22 (*a*). The nominal punching strength was used to exclude the influence of specimen size and the compressive strength of concrete from the load-deflection response. It shows that the FE analyses qualitatively reflect the same change in structural behaviour as in experiments. When the reinforcement ratio increases from 0.25% (PG2b) to 1.5% (PG1), both the stiffness and punching strength increase while the ductility decreases. Figure 22 (*b*) presents the relation between the flexural reinforcement ratio and the accuracy of the punching capacity as predicted by Eurocode 2 (CEN, 2004) and FE analyses. It is observed that the scatter in the prediction increases as the flexural reinforcement ratio increases.

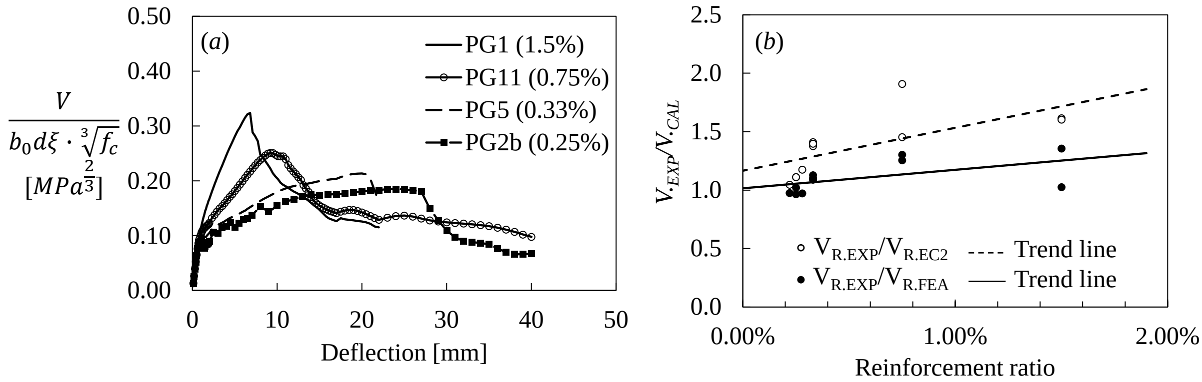


Figure 22. (a) Comparison of nominal load-deflection curve of four slabs with varying reinforcement ratios, obtained from FE analysis; (b) the relation between $V_{R.EXP}/V_{R.FEA}$ and reinforcement ratio.

4.3.3 Size effect on punching strength

Size effect should also be considered for the punching capacity of RC slabs. It is believed that nominal shear capacity decreases when the size of a specimen increases (Bažant & Cao, 1987). Such an effect has been later studied by researchers, e.g. Hallgren (1996) and Muttoni (2009). In addition, size effect is considered in Eurocode 2 (CEN, 2004) using a factor *k*, see equations in Table 4 in Section 2.2. In this study, non-linear continuum FE analyses have been applied to all laboratory tested slab specimens with varying thickness: 0.125 m, 0.25 m and 0.5 m (here represented by PG6, PG1 and PG3, respectively) in *Paper IV*. The study shows how well the size effect can be reflected using FE analysis compared to experiments and Eurocode 2 (CEN, 2004). By comparing nominal punching strength, excluding the influence of the reinforcement ratio and compressive strength of concrete, see Figure 23, it is shown that the influence of the size effect can be reflected in FE analyses; however, in Eurocode 2 (CEN, 2004), the size effect is smaller due to the limitation $k \leq 2.0$ but closer to the experiments.

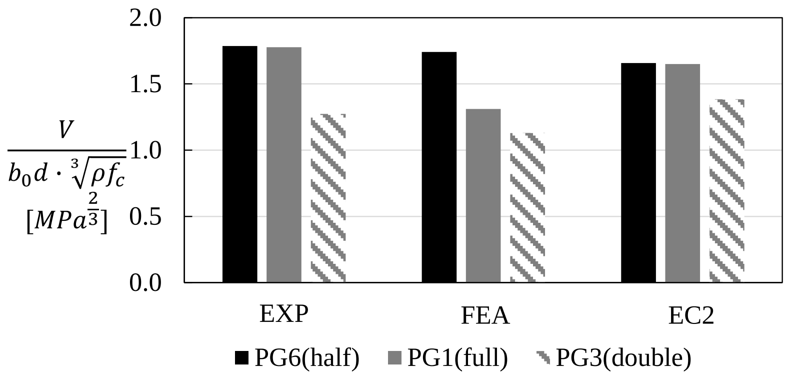


Figure 23. Comparison of nominal punching strength of three slabs with varying slab thickness, obtained from experiment, FE analysis and Eurocode 2 (Paper IV).

4.4 Non-linear shell finite element analyses

At level III, according to the Multi-level Assessment Strategy, non-linear shell finite elements can be used to analyse RC slabs. The structural behaviour with respect to bending can be reflected using a non-linear shell FE model within a one-step procedure. With respect to shear and punching, one alternative is to use degenerated layered shell elements with the ability to take into account out-of-plane shear response, e.g. Polak (1998); another alternative is to use a shell FE model to analyse the structural behaviour such as load effect, and to use a separate local resistance model to calculate shear and punching capacity. With this level of accuracy in structural analysis, resistance models at higher levels of approximation according to MC2010 (fib, 2013) are preferably used.

In *Paper I*, the specimen of a two-way slab CR1 (see Section 3.2) subjected to bending was analysed using a shell FE model and the specimen of cantilever slab DR1-a (see Section 3.2) subjected to shear was analysed using a shell FE model coupled with the Critical Shear Crack Theory (CSCT) by Muttoni (2009); see Figure 24. The punching shear failure loads were determined at the intersection point between the relation of the load versus the deflection obtained from the non-linear FE analysis and the corresponding failure criterion. The obtained load-carrying capacity was compared to results calculated from the higher level analyses and the experimental tests.

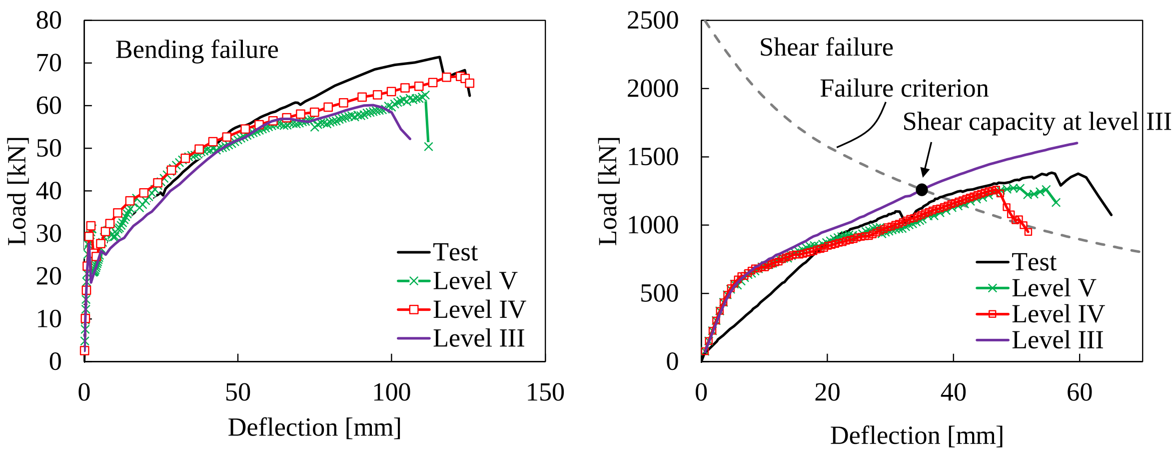


Figure 24. Comparison of load-deflation relation obtained from non-linear FE analysis for assessment at levels III-V and tests.

On the one hand, it is evident that from this observation that a shell FE analysis is able to predict the bending capacity very well, close to the results obtained using continuum FE analyses and test. On the other hand, the shell FE analysis coupled with CSCT (Muttoni, 2009) is also able to calculate the punching shear capacity, as well as continuum FE analyses. A similar observation are also gained in Figure 25, in which the punching capacity of four slabs (PG1, PT32, PE7 and AM04) calculated based on shell FE analyses and CSCT (Muttoni, 2009) are compared to that from continuum FE analyses and experiments. More information regarding the comparison can be found in *Paper V*. This outcome is very important because shell FE analysis requires lower computational efforts and analysis time than continuum FE analysis. In addition, the former approach is sufficiently robust as the non-linear FE analyses are used only for investigating the flexural behaviour and the shear strength is assessed on the basis of a mechanical model.

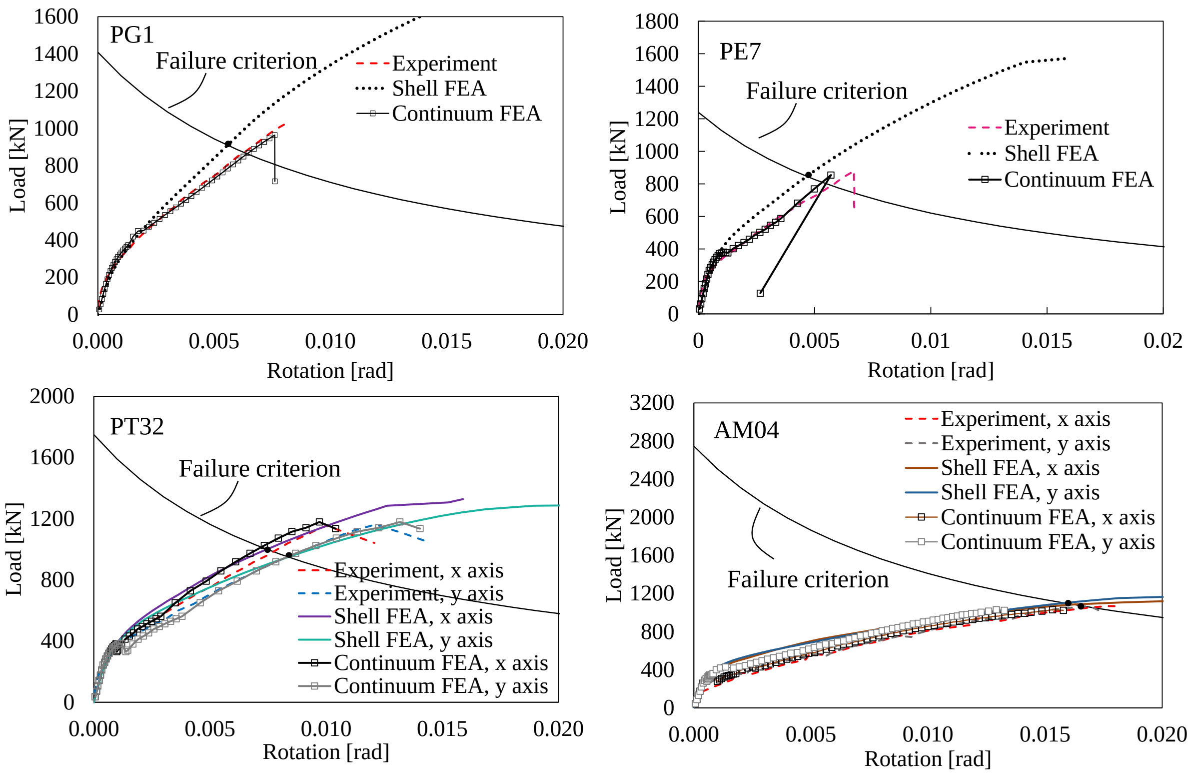


Figure 25. The load-rotation relation obtained from non-linear shell FE analyses, CSCT failure criterion, continuum FE analyses and compared to the experiments (Paper V).

4.5 Results of Multi-level Assessment

The Multi-level Assessment Strategy has been exemplified by two case studies, i.e. two-way slabs subjected to bending (Section 3.1) and a cantilever slab subjected to shear (Section 3.2) using the calculation method listed in Table 10. At level I, Eurocode 2 (CEN, 2004) was used both for the analysis of bending and shear failure. However, at levels II and III, MC2010 (fib, 2013) was used to calculate shear because the resistance model was believed to be more advanced.

Table 10. Applied analysis methods for two-way and cantilever slabs at different assessment levels.

	Two-way slab	Cantilever slab	Procedure
Experiment			
Level IV	Non-linear continuum FE analysis with bond-slip reinforcement		One-step
Level IV	Non-linear continuum FE analysis with fully-bonded reinforcement		One-step
Level III	Non-linear shell FEA	Non-linear shell FEA + MC2010	Two-step
Level II	Linear shell FEA + + MC2010	Linear shell FEA + MC2010	Two-step
Level I	Eurocode 2	Eurocode 2	Two-step

Figure 26 summarizes the bending and shear capacity from FE analyses at different assessment levels and the experimental tests of the two-way and cantilever slab, respectively. It is obvious that generally the detectable load-carrying capacity increased for higher levels of assessment but is always less than the experimental value. Similar results also has been obtained by Belleti *et al*. (2014). However, this may not always be the case. For example, for the two-way slab CR1, the load-carrying capacity obtained at level IV is higher than at level V.

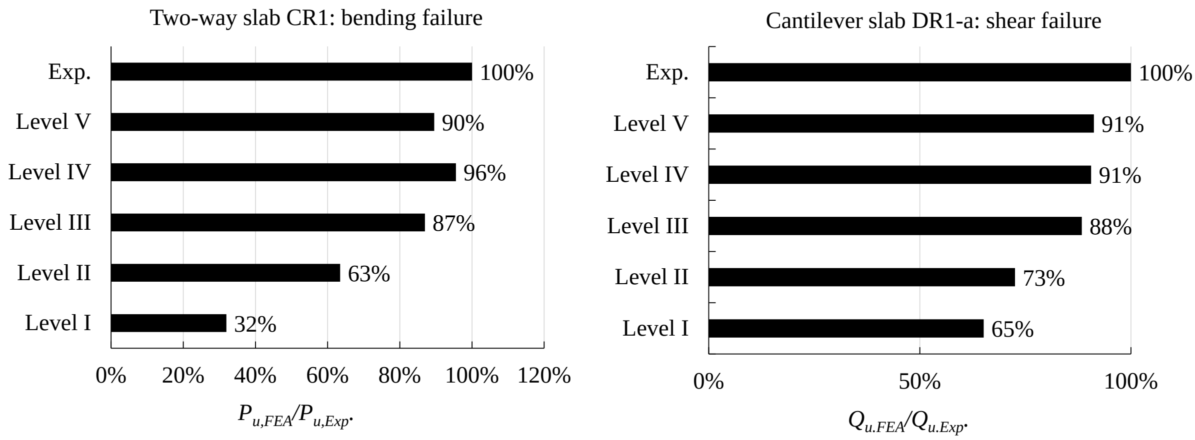


Figure 26. Load-carrying capacity of a two-way and a cantilever slab at different levels.

5 ASSESSMENT OF AN EXISTING BRIDGE DECK SLAB

Previous case studies presented in this thesis have shown that, according to the Multi-level Assessment Strategy, higher level methods based on non-linear FE analysis normally yield an improved understanding of the structural response and are capable of demonstrating higher, yet conservative, predictions of load-carrying capacity; see Chapter 4. However, those studies were carried out based on laboratory experiment. Therefore, the aim of this study was to examine the Multi-level Assessment Strategy and investigate the response of a real structure in engineering practice. A full-scale field test was carried out on a 55-year-old existing RC bridge deck slab under concentrated load near the girder, leading to a shear type failure of the slab; see Section 3.4. The Multi-level Assessment Strategy was used on the bridge deck slab to check the shear and punching capacity. The difference between assessment methods at different levels was discussed. Furthermore, the failure modes of one-way shear and punching were discussed and the influencing factors (e.g. boundary conditions, arching action and load distribution) were investigated.

5.1 Non-linear continuum finite element analyses

Since conducting field tests in existing bridge structures is of high economic cost, FE modelling of bridge structures can be a useful method for such assessment. Case studies have shown that it is possible to use non-linear FE analysis to assess existing bridge structures through full-scale bridge models, e.g. Plos (1995), Broo *et al*. (2009) and Huang *et al*. (2016).

5.1.1 Modelling of the bridge

In the Multi-level Assessment of the exiting bridge deck slab, level IV analysis is of interest because the slab was tested for shear failure. Therefore, the modelling method developed in Chapter 4 was applied to the analysis of the bride deck slab. The tested bridge (in Section 3.4) was modelled, using continuum elements for the tested second span and beam elements of the remaining parts of the bridge, in the software TNO DIANA (TNO, 2015); see Figure 27. Details of the model can be found in *Paper VI*.

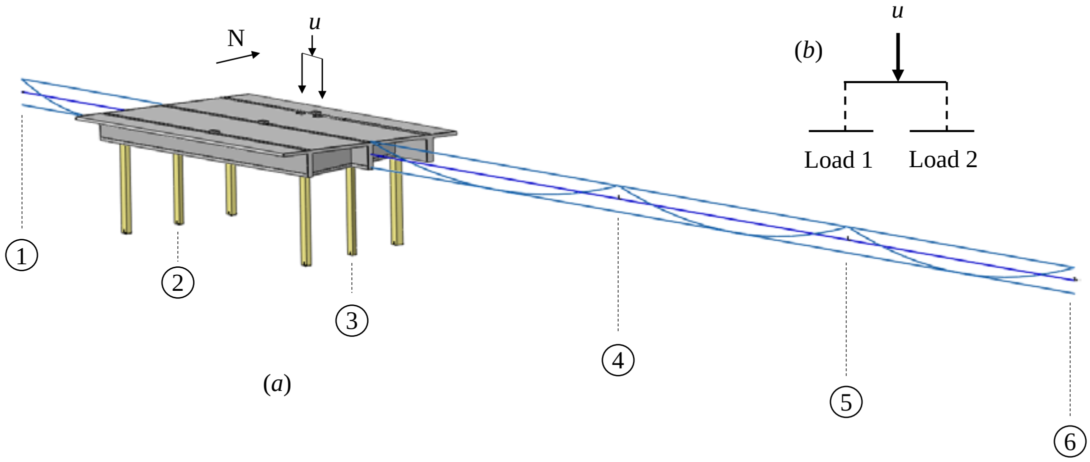


Figure 27. Level IV analysis: (a) non-linear continuum FE model of tested slab with support 1-6 (b) displacement control loading system for loading on the girder (Paper VI).

5.1.2 Calibration of the bridge model

The shear test of the bridge deck slab was to be simulated, but prior to this procedure, the global response of the bridge model was calibrated by loading on the girders instead. In the field test, the total loads were added to 13.4 MN before shear failure happened to the south girder. However, the investigated part of the deck slab remained intact since the north girder was not loaded up to failure.

In order to avoid numerical un-convergence due to failure of the girders, a total load of 12 MN instead of 13.4MN, was added to the girders of the model. Thereby the girders were damaged in the same way as in the test but the numerical model can still be used to continue the simulation of the slab test. The result of the FE analysis and comparison with the test are illustrated in Figure 28. Both the load-deflection curve and crack pattern predicted for the girder are close to that of the experiment. This indicates that the model had been calibrated relatively well to continue the simulation of the slab test.

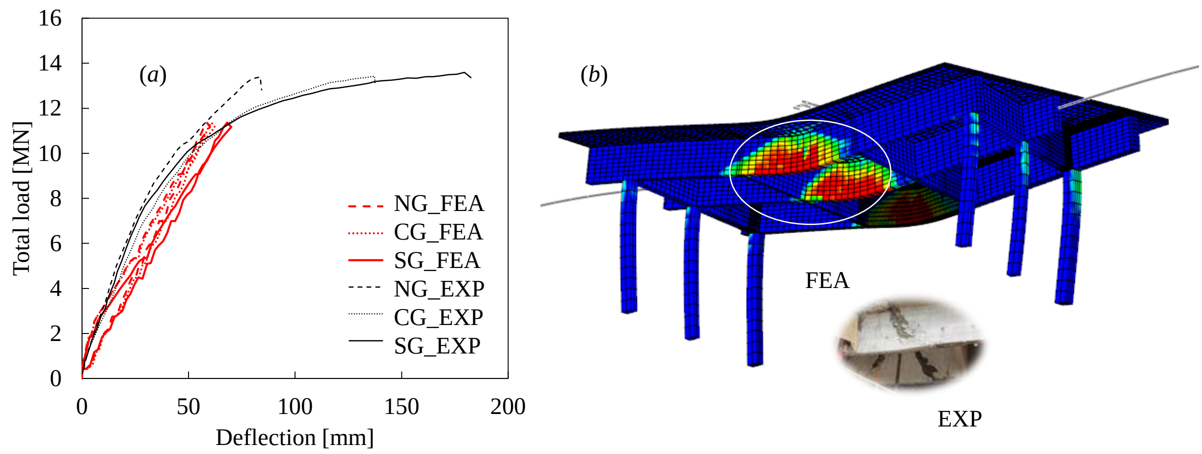


Figure 28. Comparison of (a) load-deflection curve and (b) crack pattern between field test and FE analysis; experimental results in (a) was obtained from Bagge et al.(2014).

5.1.3 Failure of the slab

After the calibration of the model has been finished, concentrated loads were added to the loading plates in the model to simulate the slab test until failure occurred on the slab at a load level of 3.27 kN, which was very close to the experimental result of 3.32 MN; see the load-deflection curve of the two loading plates in Figure 29 (*a*). However, it needs to be mentioned that the predicted shear capacity at this level may be influenced by the tolerance and size of loading steps during the non-linear FE analysis. The principal total strain based crack pattern before the failure load, at a cross-section through loading plate 1, from the FE analysis is displayed in Figure 29 (*b*). The strain threshold was set at 0.005, indicating crack widths larger than approximately 1 mm ($w = \varepsilon \times h_b$) displayed as black in the model. In the FE analysis, at approximately 60% of failure load, a large shear crack developed between the loading plates and the girder. Just before the punching failure occurred, another shear crack developed on the other side of the loading plates. Since the field-tested slab had not been cut from the bridge to examine the failure surface, the crack pattern from the experiment was not available. However, based on the sudden failure mode and the U-shaped failure crack surface from the bottom of the slab in experiment (*Paper VI*), an inclined shear crack in the cross-section could be expected to appear in the test as well.

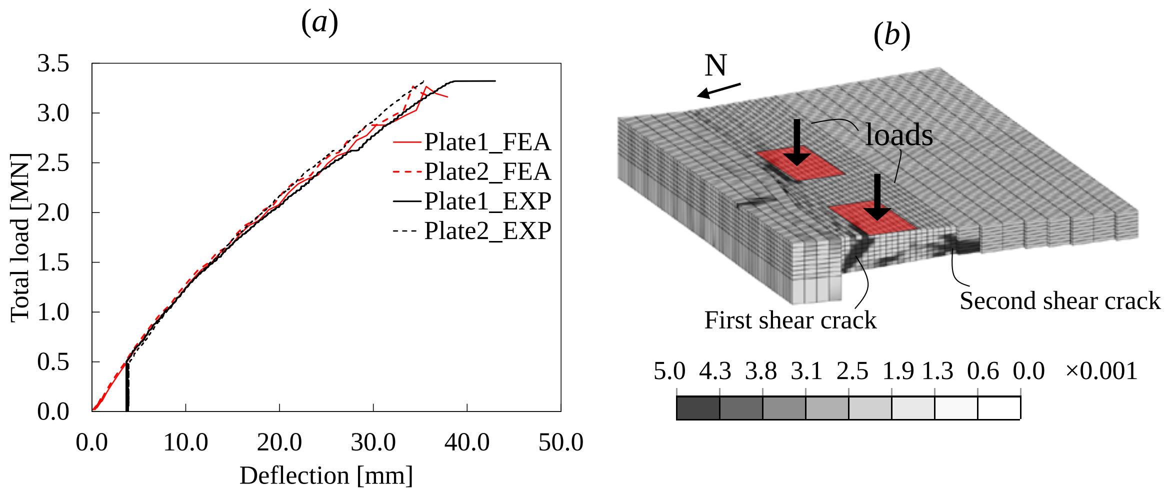


Figure 29. (a) Comparison of load-deflection curve between field test and FE analysis; (b) isometric view of crack pattern in cross-section.

The intention of this test was to approach punching failure of the slab, whereas, the results of the test showed that the appearance of the U-shaped semi-punching cone was the secondary effect after the one-way shear failure in the shorter span. When the one-way shear crack had been formed, all loads had to be carried by the remaining three sides of the loading plate, which resulted in a failure mode similar to punching. More evidence can be found in Figure 30, in which the shear force per unit length around two loading plates at a distance of $d/2$ to the edge of loading plates is presented at different load levels (V/V_R = 0.2, 0.4, 0.6, 0.8, 0.95). The divergence of shear force on different sides of the loading plates also confirms the assumption that the failure mode is a combination of one-way and punching shear, even though one-way shear is more predominant.

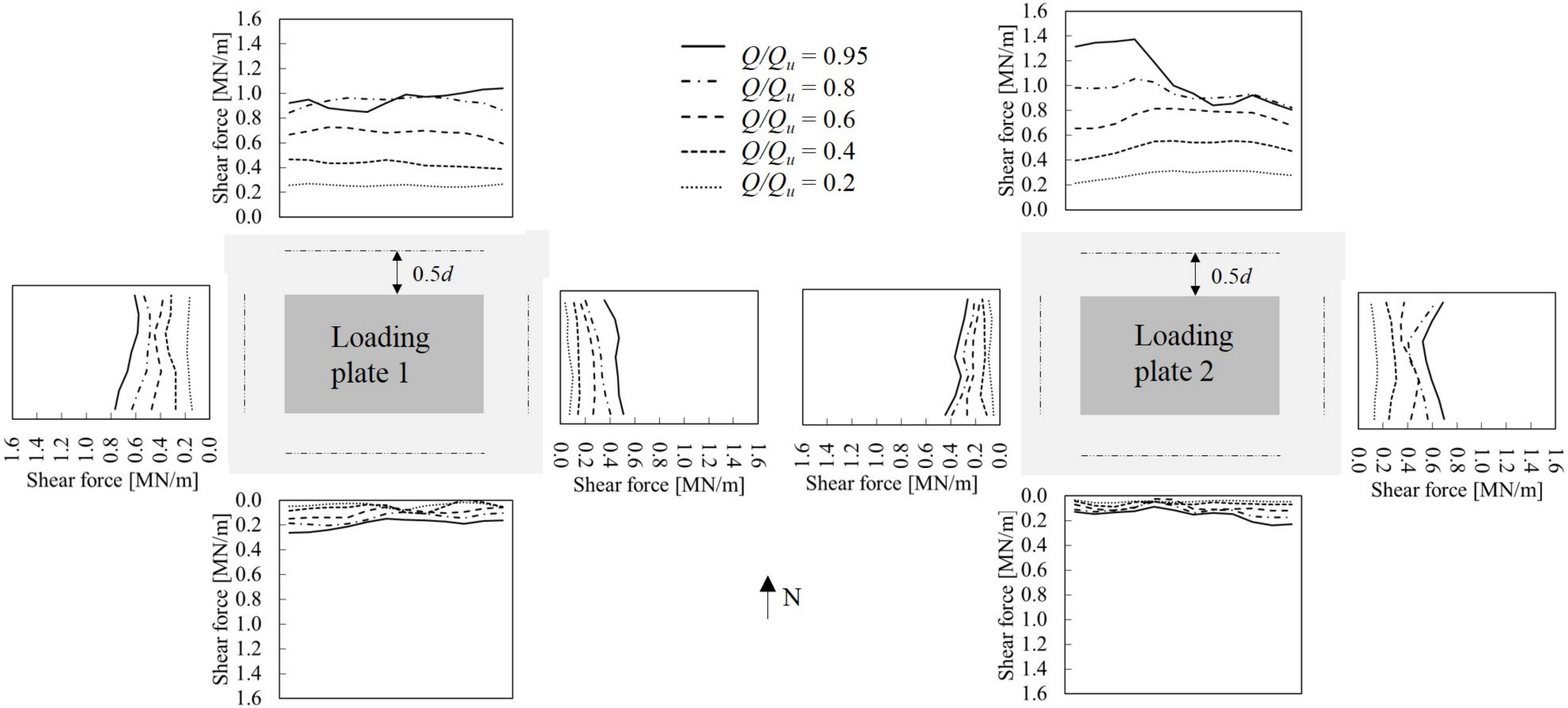


Figure 30. Shear force per unit length around two loading plates at a distance of 0.5d from the edge of loading plates at different loading stages (Paper VI).

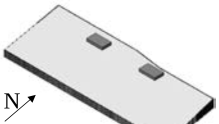
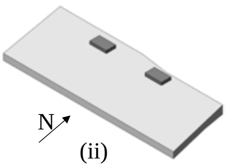
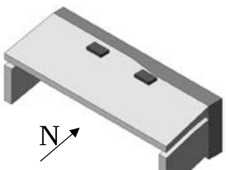
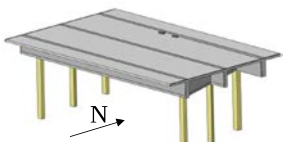
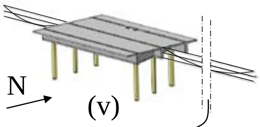
5.2 Parameter study

After the FE analysis of the tested bridge deck slab, three parameters were studied using continuum FE analysis at level IV to better understand the structural behaviour: (a) the influence of structural model simplifications and boundary conditions, (b) the influence of load positions and arching action and (c) shear force distribution.

5.2.1 Simplification of boundary conditions

The extent of the structural model and the boundary conditions assigned have often a decisive influence on the load effects determined in the structural analysis. In building codes, such as Eurocode 2 (CEN, 2004) and MC2010 (fib, 2013), a RC slab is usually analysed as an isolated component and boundary conditions are assumed as, for instance, fixed, pinned or simply supported at the edge of the component. However, in the FE analysis, it is possible to include extended parts of the structural system. The purpose is to study possible simplifications of the structural model and examine how closely the real response of the bridge can be predicted using the simplified models. In *Paper VI*, different boundary conditions were assumed and investigated, leading to the results listed in Table 11.

Table 11. Comparison of assumed boundary conditions and results.

Boundary conditions	Description of the model	Results and conclusions
 (i)	Including the loaded half-span of the slab, clamped on the north side at the connection to the main girder with symmetry boundary conditions on the other three edges.	Too stiff a response but the shear capacity was predicted reasonably accurately.
 (ii)	Including the loaded half-span of the slab, simply supported on the north side and symmetry boundary conditions on the other three sides.	Too soft a response and bending failure occurred instead of shear type failure at a low load level.
 (iii)	Including the loaded half-span of the slab, the closest girder and cross beams. The boundary conditions were assumed to be fixed at the end cross-sections of the girder but symmetric for the other edges of the slab and the end cross-sections of the cross-beams	Similar stiffness as in the experiment but the shear capacity was underestimated by 30%. Remaining part of the bridge and the effect of prestressing are important to include in the model to properly predict the shear capacity
 (iv)	Including the tested 2nd span of the bridge. The columns had fixed translations but free rotations in all three directions. The cross-sections at the end of the span were assumed to have symmetry boundary conditions;	Rather accurate estimation of the response, as well as load-carrying capacity.
 (v) Two spans in between	Including the entire bridge, where also the boundary conditions are specified.	Rather accurate estimation of the response as well as the load-carrying capacity, but higher computational cost compared to model (iv).

5.2.2 Location of loads: arching action

Considering the loads on the field-tested slab were rather close to the supportive girder and one-way shear was the dominant failure mode, the arching action should be taken into account in assessing the load-carrying capacity. The arching action is accounted for in EC2 (CEN, 2004) and MC2010 (fib, 2013) for beams and one-way slabs by a reduction factor of $\beta = a_v/2d \leq 1$. Moreover, Natario *et al*. (2014) have shown that the loading action of shear force can be even further reduced compared to that stated in Eurocode 2 (CEN, 2004), suggesting that the reduction factor should be modified to $\beta = a_v/2.75d \leq 1$. However, compared to the arching action of beams (Kani, 1966; Muttoni & Ruiz, 2008; Campana *et al*., 2013), this increase in capacity is not as remarkable for slabs, because the direct load transfer is contracted by the decrease of the effective width when the loads moved closer to the support (Lantsoght *et al.*, 2014). Similar tests were also carried out by researchers such as Graf (1933), Regan (1982), Cullington *et al*. (1996) as well as Furuuchi *et al*. (1998). In the current field test, the distance from the edge of the loads to the edge to the girder for load plate 1 and load plate 2 was only 1.09d and 0.6d, respectively (*Paper VI*). To study the influence of arching action, the position of loads in FE analyses were gradually loaded further away from the girder (see Figure 31 (*a*)) to test the capacity. Position 1 was the same as that used in the field test. The nominal shear capacities were compared to previous laboratory tests with respect to nominal shear capacity; see Figure 31 (*b*). It can be observed that the shear capacity also decreases when the loads moved further away from the support. In particular, when the loading plates are placed on position 4, the failure mode changed from shear to bending.

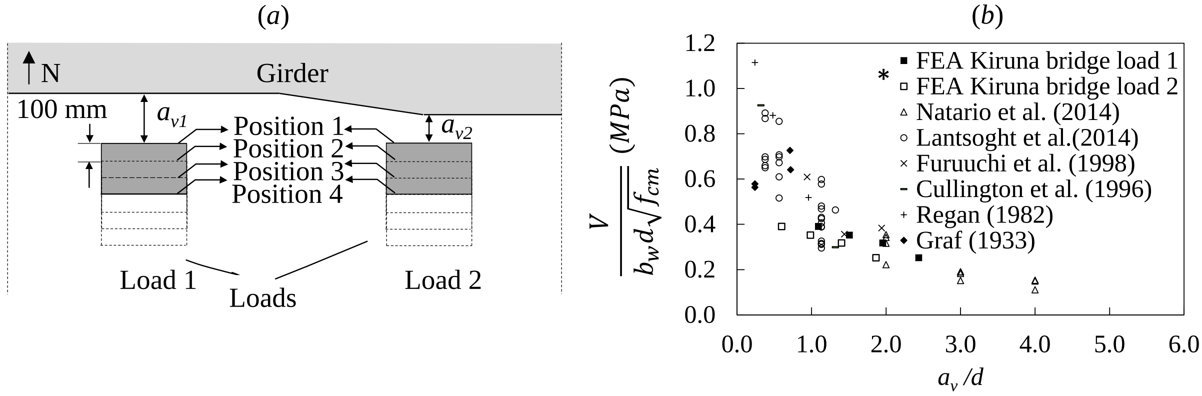


*Figure 31. (a) Variation of load position and (b) nominal shear capacity of the slab subjected to loads at different positions and comparison to previous research;*shear failure in FE analysis only occurred to load 1, but the loads were evenly distributed in load 1 and load 2; adapted from Paper VI.*

5.2.3 Shear force distribution

In order to better understand how the loads distributed in the area close to the supportive girder in the field-tested slab, the load distribution should be studied for RC slabs in one-way shear behaviour. Figure 32 (*a*) shows the force distribution of a laboratory tested cantilever slab subjected to a concentrated load (Natário, 2015). It can be observed that the shear force, in the direction perpendicular to the line support, is highly concentrated close to the load, and decreases parallel to the direction of the line support. Similar observations have been also obtained from FE analyses by Hakimi (2012) with modelling of shear test of cantilever slabs by Vaz Rodrigues (2008). However, redistribution of such load effects may occur due to cracking and non-linear behaviour of a RC slabs. Figure 32 (*b*) illustrates the reaction force measured along the supports with close concentrated load, plotted for different stages of concentrated load (0.3*P*, 0.6*P*, 0.9*P* and *P*) on clamped slabs in Natário (2015). It can be observed that the shear forces close to the loading area increase fast as the applied load increases at low load levels (< 0.9*P*), but these shear forces increase more slowly at higher load levels (≥ 0.9*P*). Instead, the shear forces in the adjacent region continue to increase rapidly.

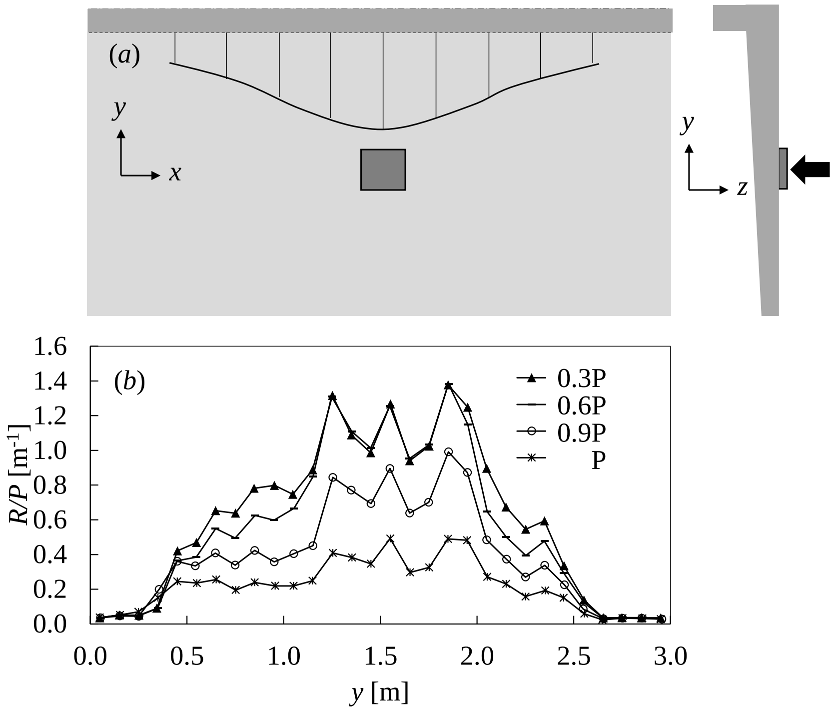


Figure 32. (a) Shear force distribution of a cantilever slab subjected to a concentrated load from linear FE analysis, (b) Nominal reaction force distribution from experiment; adapted from Natário (2015).

Similar observations can be found in the results of FE analysis of the field-tested slab in Figure 33. It can be observed that the location of the load also plays a significant role in the shear redistribution in the slab. The phenomenon of shear force redistribution mentioned above is remarkable for loading plate 1 but less so for plate 2. A possible explanation is that there is not enough space for shear force redistribution since loading plate 2 is much closer to the girder.

According to French practice (Chauvel *et al*., 2007), the effective width should be limited in the area within a 45º angle. Lantsoght (2013) indicated that the French load spreading method agrees best with these experimental data. For the case of field-tested slabs in Figure 33, when Q/Q_u = 0.95, the integrated shear force within this region (grey shadow) is calculated to be 83.4% of the integrated total shear force along the support, indicating that the effective width according to French Annex (Chauvel *et al*., 2007) is reasonable yet conservative.

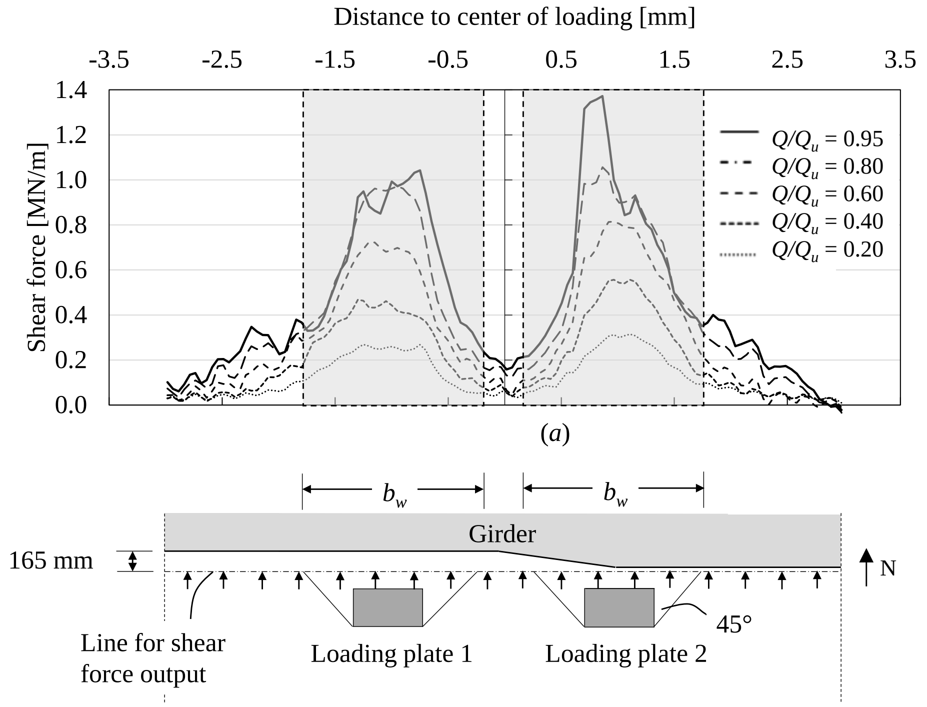


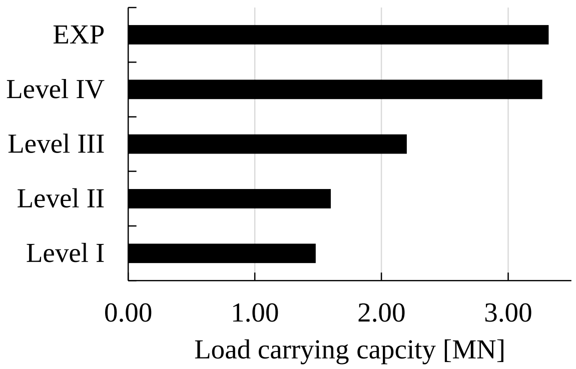
Figure 33. Shear force per unit length along longitudinal direction close to girder (Paper VI).

5.3 Results of Multi-level Assessment

The shear capacity of the tested slab has been calculated at different assessment levels according to the Multi-level Assessment Strategy in *Paper II*; see Table 12. Only one-way shear and punching resistance were calculated since bending and anchorage failures have already been checked in the design phase of the experiment. The calculated shear and punching capacity $Q_{u.cal}$ of the deck slab is compared to the failure load $Q_{u.exp}$ from the experiment in Figure 34. Level II yields similar results to level I, indicating that improved representation of the geometry when determining the load effect is not sufficient. The level III analysis provides a notably higher, still considerably underestimated, load-carrying capacity only by representing the non-linear bending response more correctly. Finally, the continuum non-linear FE analysis at level IV provides a load-carrying capacity which is close to that obtained in the experiment. The shear resistance calculated based on EC2 (CEN, 2004) at level I largely underestimated the real capacity. By upgrading the level of approximation, the accuracy of the calculated capacity increases.

Table 12. Analysis methods at different assessment levels (Paper VI).

	One-way shear	Punching shear	Procedure
Field test			
Level IV	Non-linear continuum FE analysis		One-step
Level III	Non-linear shell FE analysis + Eurocode 2	Non-linear shell FE analysis + MC2010	Two-step
Level II	Linear shell FE analysis + Eurocode 2	Linear shell FE analysis + MC2010	Two-step
Level I	Eurocode 2	Eurocode 2	Two-step



Capacity	$Q_{u,cal}/Q_{u,exp}$	Control failure mode
3.32 MN		One-way shear & Punching
3.27 MN	0.98	One-way shear & Punching
2.20 MN	0.66	One-way shear
1.60 MN	0.48	One-way shear
1.48 MN	0.45	One-way shear

Figure 34. Load-carrying capacity calculated based on Multi-level Assessment and comparison with the experiment (Paper VI).

6 CONCLUSIONS

6.1 Summary and general conclusions

The objective of the study reported in this thesis was to develop and calibrate improved methods for the assessment of load-carrying capacity and response of reinforced concrete (RC) slabs. This study proposes an enhanced assessment through an improved structural analysis and resistance evaluation in order to achieve a higher detectable load-carrying capacity. To achieve this objective, the scientific approaches adopted included literature study, laboratory tests, analytical analyses and finite element (FE) analyses. Major studies and conclusions can be summarized as follows:

Existing standards are not capable of accurately reflecting the behaviour of RC slabs. The ultimate load-carrying capacity of existing RC slabs is largely underestimated by the traditional assessment approach because factors such as membrane action and strain hardening of reinforcement are neglected. To better reflect structural behaviour and make use of the inherent capacity of existing structures, enhanced assessment methods, such as the non-linear FE method, should be utilized. A Multi-level Assessment Strategy, based on the principle of successively improved evaluation in structural assessment, is proposed, providing a structured approach to the use of simplified as well as enhanced non-linear analysis methods.

The proposed assessment strategy focuses on an enhanced assessment through improved structural analyses and resistance evaluations. In the scope of the Multi-level Assessment Strategy, it is already known that the 3D non-linear continuum FE analysis has the highest potential for discovering any additional sources of load-carrying capacity in RC slabs. However, a 3D non-linear continuum FE analysis, at the highest level of the proposed strategy, is demanding and requires skilled and experienced structural engineers. Furthermore, such an analysis involves many modelling choices that are decisive for how well the analysis results reflect the response of the real structure. For the purpose of mapping important factors regarding modelling choices for the structural behaviour of the FE model of RC slabs, sensitivity analyses have been conducted. The selected modelling choices included five major categories: geometry non-linearity, element properties, modelling of concrete and reinforcement, as well as modelling of support.

Through a sensitivity study of RC slabs subjected to bending, it can be concluded that geometric non-linearity, crack bandwidth and Poisson's ratio have significant impacts on load-carrying capacity. The crack pattern is influenced by element properties and how the reinforcement is modelled. The stiffness of support can considerably affect load distribution in a slab. Through a sensitivity study of RC slabs subjected to shear and punching, it can be concluded that for the Total Strain rotating crack model, fracture energy is an important influencing factor. For the Total Strain fixed crack model, punching capacity is considerably affected by the shear retention factor β. Thus, the rotating crack model is easier to use when the shear retention factor cannot be determined accurately for the specific case. Based on the statements above and the fact that the assumed reference models are capable of reflecting RC slabs with reasonable accuracy, recommendations can be derived for using 3D continuum finite elements to model RC slabs:

a. A Total Strain rotating crack model can be used for concrete and the fully bonded reinforcement model can be used for steel reinforcement when a detailed simulation of the crack pattern is not required.
b. Geometric non-linearity should be included to capture the increase of load-carrying capacity due to membrane actions when deflections larger than half the slab thickness may occur.
c. First-order eight-node brick elements with at least seven element layers over the cross-section height are sufficient; first-order four-node tetrahedral elements and second-order brick elements are also applicable alternatives.
d. The crack bandwidth should be estimated as the mean crack distance if fully bonded reinforcement is used.
e. When estimating fracture energy based on concrete strength, MC1990 can be used. With MC2010, aggregate size is not taken into account and load-carrying capacity may be overestimated.
f. The effect of lateral confinement on the compressive strength of concrete is also recommended, since it reflects real behaviour.
g. The stiffness of supports needs to be modelled properly to correctly describe support reaction distribution, e.g. by using interface elements with calibrated normal stiffness and friction. One appropriate approach to model column support is to use non-tension springs in both the shell element model and continuum element model. This approach realistically reflects the reaction force distribution of the column support.

A parameter study shows that the FE model using reference modelling choices, according to the recommendations given, does not only provide a good estimation of load-carrying capacity, but also accurately reflects the size effect and influence of parameters such as the flexural reinforcement ratio. The predicted punching capacity using non-linear FE analysis demonstrates higher scatter when the flexural reinforcement ratio increases. Through further investigation of the punching behaviour of RC slabs, it has been observed that the shear force distribution and redistribution can be reflected in non-linear FE analyses. The shear distribution at the non-linear stage shows a significant difference compared to that at the linear elastic stage (corresponding to low applied load values). The shear force redistribution of RC slabs is influenced by factors such as the layout of reinforcement; when yielding of reinforcement occurs, the shear force redistributes to regions where the reinforcement ratio is higher. The shear force redistribution is also affected by the formation of the critical shear crack; the shear force increases faster within the area of the critical section, but more slowly outside the critical section. The shear force distribution of RC slabs is also influenced by the geometry of a column. A square-shaped column causes stress concentrations near the corners but they do not extend as far from the column as to the basic control perimeter. A rectangular shaped column has a significant impact on shear distribution. Shear stress is more concentrated to the shorter ends of the column support, which becomes even more pronounced after the redistribution due to non-linear behaviour.

In analysing a field-tested bridge deck slab, it is concluded that the tested deck slab has failed in a combination of one-way shear and punching shear. This kind of shear type failure, as well as the structural response of the slab, can be reflected using continuum non-linear FE analyses of a model including the entire bridge. The extension of the FE model and the assumption of boundary conditions when limiting the extension of the model to include only part of the bridge have a significant influence on the analysis results. If the model is limited to the loaded half-span of the slab alone, neither fixed nor simply supportive boundary conditions can accurately describe the real response. The position of the applied load plays an important role for both load-carrying capacity and failure mode; when the load is close to a support, the arching action exerts a large influence. When the load is moved further away from the girder supporting the tested slab, the shear capacity of the slab decreases until the failure mode changes to bending failure. However, the rate of decrease is not as large as in beam tests or pure one-way shear tests due to the two-way load-carrying mechanism. The analysis of shear force distribution shows that the method to define effective width assuming 45º limit lines for the force distribution is reasonable yet conservative.

Through case studies, the Multi-level Assessment Strategy was applied to tested slabs, both laboratory tests and a field test. It is illustrated that the proposed strategy provides a straight forward approach to evaluate the load-carrying capacity of existing RC slabs. The results show that in general, advanced models are more capable of demonstrating a load-carrying capacity that better reflects reality. Level II analysis usually yields similar results to level I, indicating that improved representation of the geometry when determining load effects is not sufficient. Level III analysis provides a notably higher, yet considerably underestimated, load-carrying capacity only by representing the non-linear bending response more correctly. However, coupled with a mechanical resistance model such as the Critical Shear Crack Theory (CSCT), the shell FE analysis yields robust results in predicting punching strength, with lower computational effort than continuum FE analysis. For slabs subjected to shear and punching, the continuum non-linear FE analysis at level IV is capable of reflecting the structural behaviour very close to the experiment. In general, the non-linear FE analysis (levels III, IV and V) does not only have the advantage of predicting bending and shear (punching) capacity, but also contributing to an improved understanding of the structural response of RC slabs.

6.2 Suggestions for the future research

To form general recommendations for assessments based on the Multi-level Assessment Strategy, additional case studies with a variation of parameters are needed for different kinds of structures. Such recommendations also need to be more practically oriented. Three possible alternatives to expand and consolidate the proposed assessment strategy have been identified:

The first suggestion for future research concerns shear strength of corroded RC slabs without shear reinforcement. The assessment strategy proposed in this study has been developed for existing RC structures, which are likely to be subjected to deterioration due to environmental impact. Therefore, the assessment strategy should be supplemented by clear directions on how to model structures with potential damage, e.g. due to corrosion of reinforcement and frost damage of concrete. RC slab structures are among the structures directly exposed to harsh environmental conditions, especially in cold climates such as Sweden's. They are usually exposed to water, snow, ice, and de-icing salts which increase the probability of corrosion of reinforcement. Shear failure of structures is a dangerous brittle failure mode, which can cause huge losses of human life and property to the society. Internal cracking due to corrosion and other degradation mechanisms may largely affect shear strength, resulting in a reduction of remaining service life, or even the sudden collapse of existing structures. Currently, due to the lack of methods to assess the shear capacity of corroded RC structures, it is on the one hand difficult to predict the sudden collapse of such structures; on the other hand, many buildings and infrastructures are most likely being strengthened and even replaced unnecessarily. Combining the considerable amount of research on the shear capacity of RC slab structures with current research on the structural effects of reinforcement corrosion would make it possible to properly assess also the shear capacity of corroded RC slab structures.

The second suggestion for future research is to develop a safety format for the non-linear FE analysis of RC structures. In the non-linear FE analyses carried out in this study, it has been found that large model uncertainty exists due to the increased complexity of models. Such uncertainty varies according to different modelling methods and failure modes. There are two major sources of uncertainty, i.e. an uncertainty due to a lack of knowledge of how well the model performs and an uncertainty due to random properties of input such as material properties. The safety format used in current building codes are based on reliability methods and calculated using partial factors. However, a global safety format may be needed for the non-linear FE analysis due to the global nature of this method. A probabilistic description of the modelling uncertainty may be used to facilitate the use of a global safety format.

The third suggestion for future research is to develop the lower level assessment methods with the help of a higher level assessment within the scope of the Multi-level Assessment Strategy. The enhanced FE analysis methods for RC slabs have been proven capable of predicting load-carrying capacity and structural behaviour. However, the required computational effort makes them impractical for current real assessment situations. The currently used simplified calculation method at lower assessment levels should be improved with the help of the enhanced FE analysis methods at higher assessment levels. For example, the effective width of the one-way shear capacity of RC slabs could be optimized by looking into shear force distribution. In addition, shell FE analysis has been extensively used in the analysis of RC slabs for bending. In the present study regarding shear and punching, shell elements have been used coupled with the Critical Shear Crack Theory (CSCT) to predict the punching capacity of RC slabs. In order to achieve the one-step procedure in this analysis, a shell FE analysis, with appropriate element and material modelling formulations, that make them applicable for out-of-plane shear and punching analysis, can be developed.

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