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TWO-SCALE MODELLING OF REINFORCED CONCRETE

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Summary. A two-scale model of reinforced concrete is presented. The subscale problem is derived, and the effective large-scale properties are obtained by computational homogenization. A reinforced deep beam in four-point bending has been analysed using two-scale formulation and numerical results are presented in this paper.

1 INTRODUCTION

Crack growth in reinforced concrete is of practical importance, since it directly influences the durability of the structure. In order to obtain reliable results, not only the cracking of concrete, but also the bond between the reinforcement bars and the concrete must be considered. If computational multi-scale modelling (CMM) is to be employed for the analysis of a reinforced concreted structure, it is of interest to develop the corresponding format of the large-scale and subscale problems. Furthermore, a suitable homogenization tool is needed in order to simulate material response at the large-scale level.

2 PROBLEM FORMULATION

2.1 Variational formulation

For a general two-dimensional reinforced concrete structure, the quasi-static problem can be characterised as follows: Find the concrete displacement and reinforcement slip $\boldsymbol{u}$, Δ that solves

$$\int_{\Omega} t_c \left[\boldsymbol{\sigma}_c \left(\boldsymbol{\nabla} \otimes \boldsymbol{u}\right)\right]^T : \left[\boldsymbol{\nabla} \otimes \delta \boldsymbol{u}\right] \mathrm{d}\Omega_c + \int_{\Gamma_{int}} A_s \sigma_s \left(\left[\boldsymbol{e}_l \otimes \boldsymbol{e}_l\right] : \left[\boldsymbol{\nabla} \otimes \boldsymbol{u}\right] + \frac{\partial \Delta}{\partial l}\right) \left[\boldsymbol{e}_l \otimes \boldsymbol{e}_l\right] : \left[\boldsymbol{\nabla} \otimes \delta \boldsymbol{u}\right] \ \mathrm{d}\Gamma_{int} = \int_{\Gamma_{ext}} t_c \boldsymbol{t}_p \cdot \delta \boldsymbol{u} \ \mathrm{d}\Gamma_{ext} \tag{1}$$

$$\int_{\Gamma_{int}} A_s \sigma_s \left(\left[\boldsymbol{e}_l \otimes \boldsymbol{e}_l\right] : \left[\boldsymbol{\nabla} \otimes \boldsymbol{u}\right] + \frac{\partial \Delta}{\partial l}\right) \frac{\partial \delta \Delta}{\partial l} \ \mathrm{d}\Gamma_{int} + \int_{\Gamma_{int}} S t_{\Gamma}\left(\Delta\right) \delta \Delta \ \mathrm{d}\Gamma_{int} = 0 \tag{2}$$

for suitable test function $\delta \boldsymbol{u}$ and $\delta\Delta$. Here, t_c is the thickness of the concrete, $\boldsymbol{\sigma_c}$ is the stress tensor present in the concrete. The reinforcement bars occupy the segments Γ_{int} with unit normal direction $\boldsymbol{e}_l$. A_s is the cross-sectional area of the reinforcement, σ_s is the stress in steel depending on both the steel strain and its slip, $\boldsymbol{t}_p$ are the prescribed tractions on the Neumann part of boundary, S is rebar's circumference and $t_\Gamma(\Delta)$ is the bond stress as a function of the slip.

2.2 Large-scale problem

A variational multi-scale ansatz[1] is employed here, and thus the global displacement field is split into the 'smooth' and 'fluctuation' parts, i.e. $u = u^M + u^s$. Furthermore, a prolongation operator $\mathcal{A}$ is defined, so that a macroscale variation $u^M(x)$ can be obtained over a region x from a smooth 'generating' macrofield $\bar{u}(x)$, i.e. $u^M(x) = \mathcal{A}\bar{u}(x)$. In this study, the slip of reinforcement is considered to vary only locally, i.e. $\Delta^s = \Delta$. The large-scale problem can be then expressed as: Find $\underline{\bar{u}}$, Δ that solves

$$\int_{\Omega} \bar{\boldsymbol{\sigma}}^T : \left[\boldsymbol{\nabla} \otimes \delta \bar{\boldsymbol{u}}\right] \mathrm{d}\Omega = \int_{\Gamma_{ext}} t_c \boldsymbol{t}_p \cdot \delta \bar{\boldsymbol{u}} \ \mathrm{d}\Gamma_{ext} \tag{3}$$

where $\bar{\boldsymbol{\sigma}}$ is an implicit functional (average stress) of $\bar{\boldsymbol{u}}$, Δ and $(\boldsymbol{\nabla} \otimes \bar{\boldsymbol{u}})$ that can be computed from the subscale problem.

2.3 Subscale problem

The definition of the subscale problem follows directly from (1) and (2) upon restriction to a so-called Representative Volume Element (RVE) $\Omega_\square$ and choosing approprate boundary conditions. In this study, Dirichlet boundary conditions on both concrete and steel were used. Hence, the fluctuation u^s on the boudary of studied subscale region is zero, i.e. $u^s = 0$ and $\Delta = 0$ on $\partial\Omega_\square$. The corresponding subscale problem gives expressions for the average stress functional defined in Section 2.2, i.e.

$$\bar{\boldsymbol{\sigma}} = \frac{1}{|\Omega_\square|} \left[\int_{\Omega_\square} t_c \boldsymbol{\sigma}_c \mathrm{d}\Omega + \int_{\Gamma_\square^{int}} A_s \sigma_s \boldsymbol{e}_l \otimes \boldsymbol{e}_l \mathrm{d}\Gamma\right] \tag{4}$$

3 NUMERICAL EXAMPLE

For the numerical example, a 4 m high, 10.5 m long and 0.2 m thick reinforced deep beam in four-point bending was chosen. The structure was analysed as a 2D solid in plane stress using the symmetry line at mid span. For simplicity, a uniform reinforcement layout across the structure was used. The reinforcement grid comprised rebars in horizontal ($\phi_1 = 20$ mm) and vertical ($\phi_2 = 8$ mm) direction placed every 200 mm. The subscale unit cell was modelled as a concrete square with a 400 mm side and two rebars in each direction, see Figure 1.

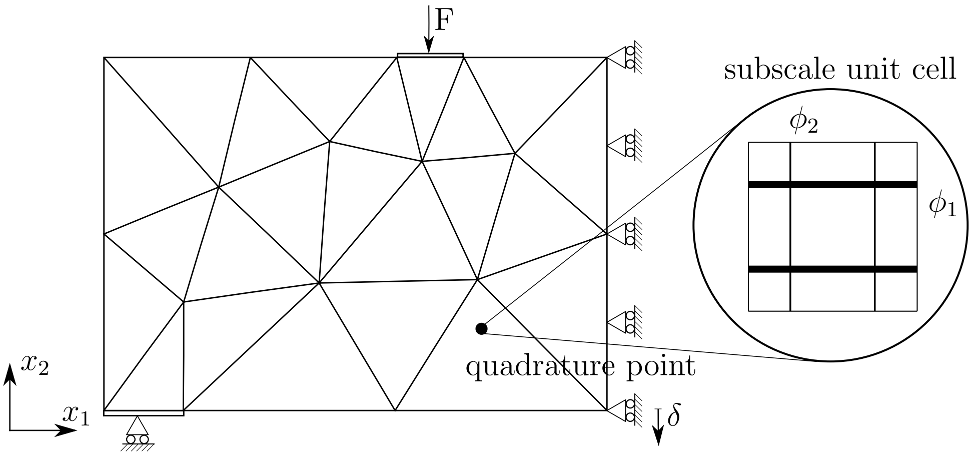

Figure 1: Two-scale model of the reinforced deep beam.

At large-scale the model had a rather coarse mesh consisting of linear strain triangles. At the subscale, solid quadrilateral elements were used for concrete, while the reinforcement was modelled with beam elements, which resulted in them having a bending stiffness, but the ends were free to rotate. The interaction between concrete and steel was modelled with interface elements describing the bond-slip relation. The subscale representative volume element was analysed with help of the software DIANA using the total strain rotating crack model (which follows a smeared approach for the fracture energy[2]) with a Hordijk tension softening curve[3]. In compression, the concrete was modelled as elastic-ideally plastic. Strain hardening elastoplasticity was used to model the constitutive response of steel. All material parameters for the concrete, steel and the bond-slip relation were taken from literature[4]. This two-scale analysis, here denoted FE^2, has a nested character as the constitutive response at the large-scale was obtained with help of computational homogenization of the subscale response. Since in practice the numerical integration is carried out at the quadrature points, this is where the subscale unit cells were located. For comparison, a reference analysis featuring the full-detail model at the large-scale was also carried out in DIANA. The force-deflection results for both analyses is presented in Figure 2, as can be seen, they agree well. In Figure 3, an example of results from a

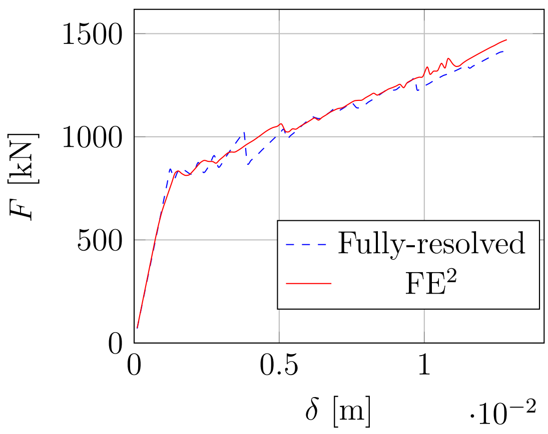

Figure 2: Applied force versus midspan deflection for fully-resolved and FE^2 analyses.

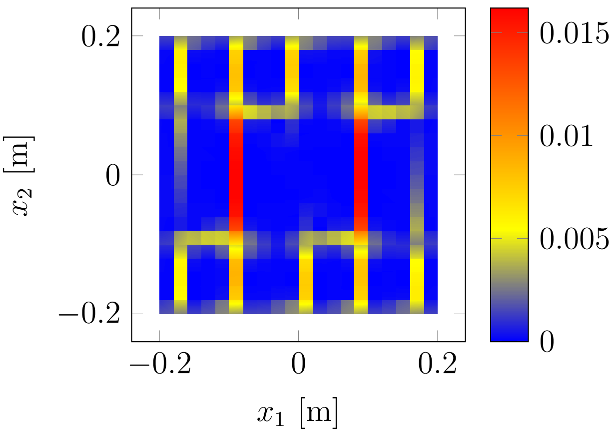

Figure 3: Principal strain ε_1 in the subscale unit cell marked in Figure 1 at $\delta = 12.85$ mm.

subscale analysis is presented, namely the principal strain (ε_1) pattern for the last load step (for δ =12.85 mm). This particular unit cell corresponds to the location indicated by the point in Figure 1.

4 CONCLUSIONS

By employing the variationally consistent multi-scale modelling approach (treated extensively in[1]) it was possible to create a two-scale model of reinforced concrete. In this way, it is possible to study how the subscale composition of the material affects the overall large-scale response. The results show a good agreement between the fully-resolved analysis and the FE^2 procedure.

REFERENCES

[1] Larsson, F., Runesson, K. & Su, F. Variationally consistent computational homogenization of transient heat flow. *International Journal for Numerical Methods in Engineering* 1659–1686 (2009).

[2] Bažant, Z. & Oh, B. Crack band theory of concrete. *Materials and Structures* **16**, 155–177 (1983).

[3] Hordijk, D. A. *Local approach to fatigue of concrete*. Ph.D. thesis, Delft University of Technology (1991).

[4] fib. *Model Code for Concrete Structures 2010* (Ernst & Sohn, 2013).