THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING IN SOLID AND STRUCTURAL MECHANICS

Prediction of structural dynamic behaviour under uncertainties

With applications to automotive structures

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Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY

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Abstract

The automotive industry is moving towards shorter development cycles for new car generations. This means that less expensive prototypes can be built and tested, and that, increasingly, computer models must be used for decision making. Further, the automotive industry is producing thousands of nominally identical cars which are known to exhibit noticeable spread in their vibration characteristics. A car's noise and vibration behaviour is therefore not the same between nominally identical cars. This implies a need for structural dynamic models considering uncertainties for robust decision making. Due to the final products complexity a substructuring approach is considered in this thesis, including experimental and computational methods, where predictive models of components are created, to be assembled for a predictive system response.

The first part of this thesis considers the reduction of uncertainties introduced from vibration experiments. A method for sensor placement in vibration experiments is developed, based on the method of effective independence, so that symmetric sensor positions are rejected using system gramians. Further, a measurement system is developed in MATLAB for fast and efficient stepped sine excitation.

The second part considers the spread between nominally identical components and the calibration, and an associated parameter uncertainty quantification, of industrial finite element models of said components. Results are reported here for three front and one rear subframe. For model calibration, a model updating procedure is employed that uses a frequency response function based deviation metric and equalised damping. A bootstrapping procedure is subsequently used to quantify parameter uncertainties with respect to the measurement noise. Calibrations are performed for an ensemble of front subframe components. Particular care is taken in the modelling of coupling elements and for the rear subframe the elastic modulus in rubber bushings is estimated using a mass loaded bushing boundary configuration.

In the automotive industry high fidelity models are common, with many interface degrees of freedom decreasing the efficiency of component mode synthesis methods. Therefore, a component mode synthesis interface reduction method is developed to speed up the process, using coarse meshes.

Keywords: Uncertainty quantification, structural dynamics, model updating, substructuring, experimental design, sensor localisation, noise and vibrations, automotive industry

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Preface

This thesis has been carried out from December 2014 to March 2017 in the scope of a Volvo Industrial PhD Project. During this time I have mostly been located at Chalmers University of Technology, with a day or two throughout a week at Volvo Car Corporation (VCC) Noise and Vibration Centre. Much of the time has gone into attending various postgraduate and generic skills courses, and participating as a teaching assistant in undergraduate solid mechanics and MATLAB programming courses. The rest of the time has been spent reading, writing, trying to wrap my head around the field of uncertainty quantification, using the MATLAB toolbox FEMcali and in the lab working on the project and developing the measurement software AbraDAQ as a MATLAB toolbox.

Acknowledgements

This thesis is the work of many people, whom I would like to thank. I am first of all very grateful to my supervisor Professor Thomas Abrahamsson for always being available, constantly encouraging me and coming with new ideas. Secondly I would like to thank my assistant supervisor Magnus Olsson, Technical Expert at VCC, for his support in all matters, technical, administrative and organisational. I am also very grateful towards VCC for providing funding for this project. Further, I would like to thank all of Professor Thomas Abrahamsson's previous and current PhD students that have been at Chalmers at some point during my stay here. Discussions with you have been very valuable in an environment otherwise dominated by other topics.

I have been lucky to have had two workplaces during this time, one at Chalmers and one at VCC. They have been very different, but complementary. This has also introduced me to how the two worlds of academia and industry work, and most importantly how they interact. I would like to thank all the co-workers at both locations for creating an interesting and enjoyable work environment. A special thanks to Mikaela Zetterberg for taking the time to read some of the work herein.

Lastly I want to thank my parents, without whom I wouldn't be here, and my brother and sister, without whom I wouldn't want to be here.

Göteborg, March 2017
Mladen Gibanica

Thesis

This thesis consists of an extended summary and the following appended papers:

Paper A M. Gibanica, T. J. S. Abrahamsson, and D. C. Kammer. "Redundant Information Rejection in Sensor Localisation Using System Gramians". *Topics in Modal Analysis & Testing, Volume 10*. Conference Proceedings of the Society for Experimental Mechanics Series. Springer International Publishing, 2016, pp. 325–333. ISBN: 978-3-319-30248-5. DOI: 10.1007/978-3-319-30249-2_29

Paper B M. Gibanica, T. J. S. Abrahamsson, and M. Olsson. "Calibration, Validation and Uncertainty Quantification of Nominally Identical Car Subframes". *Model Validation and Uncertainty Quantification, Volume 3*. Conference Proceedings of the Society for Experimental Mechanics Series. Springer International Publishing, 2016, pp. 315–326. ISBN: 978-3-319-29753-8. DOI: 10.1007/978-3-319-29754-5_31

Paper C M. Gibanica and T. J. S. Abrahamsson. "Parameter Estimation and Uncertainty Quantification of a Subframe with Mass Loaded Bushings". *Proceedings of the 35th IMAC*. International Modal Analysis Conference. Garden Grove, CA, 2017. In press

Paper D M. Gibanica, T. J. S. Abrahamsson, and D. J. Rixen. "A Reduced Interface Component Mode Synthesis Method Using Coarse Meshes". *X International Conference on Structural Dynamics*. EURODYN. Rome, 2017. Submitted

The thesis has also resulted in a MATLAB toolbox, AbraDAQ, for vibration measurements:

Software A M. Gibanica and T. J. S. Abrahamsson. *AbraDAQ*. URL: https://github.com/mgcth/abraDAQ (visited on 12/28/2016)

The appended papers were prepared in collaboration with co-authors. The author of this thesis was responsible for the major progress of the work, i.e. planning the papers, developing the theory, carrying out the numerical simulations, performing the experiments and writing the papers. A MATLAB toolbox, AbraDAQ, for vibration measurements has also been developed, in collaboration with Professor Thomas Abrahamsson.

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Part I

Extended Summary

1 Introduction

This chapter serves the purpose of creating a research context and further providing more details about the topic of study. It ends with a short description of this thesis aims within the topic. A more detailed description of many concepts will be given in subsequent chapters.

1.1 Research context

Some of the most important areas for customers purchasing a car are safety, fuel consumption and comfort. Safety is still a subject in which good insight in mechanics is important and with renewed interest now when composite structures are utilised to a greater extent [19, 22, 66]. Safety issues are increasingly being handled by autonomous systems with the goal to create crash free autonomous cars [15, 21]. Fuel consumption is another area in a state of change, which is the main motivator for the shift from steel and aluminium structures to composite structures in the design of lighter cars [19]. A slow shift is also made from combustion engines to electric engines and drivelines [18]. Comfort is commonly related to the area of noise, vibration and harshness (NVH). In NVH a car's sound and vibration characteristics are studied and designed. This is an area of great importance in the premium segment where customers have high expectations on the perceived sound and vibration characteristics. Harshness deals with human subjective perception involving the judgement of quality with respect to sound and vibration. This makes it a difficult quantity to measure. Furthermore, NVH is strongly influenced by the use of composite materials and electric drivelines. Both bring in new challenges in the form of computational predictability [72, 58] in general and in high frequency vibration and sound predictability in particular.

In designing a car the vibration and noise sources are many, e.g. forces propagating through the structure in form of vibrations from road contact, engine vibrations and forces acting on the car from wind. These sources create structural vibrations that are transformed into sound from the vibrating structure. Figure 1.1 illustrates some common vibration sources and their perceived output in form of vibrations and sound. Forces from road contact are propagated through the rear and front subframes, into the body of the car causing noise. The vibrations are also propagated into the seats possibly causing

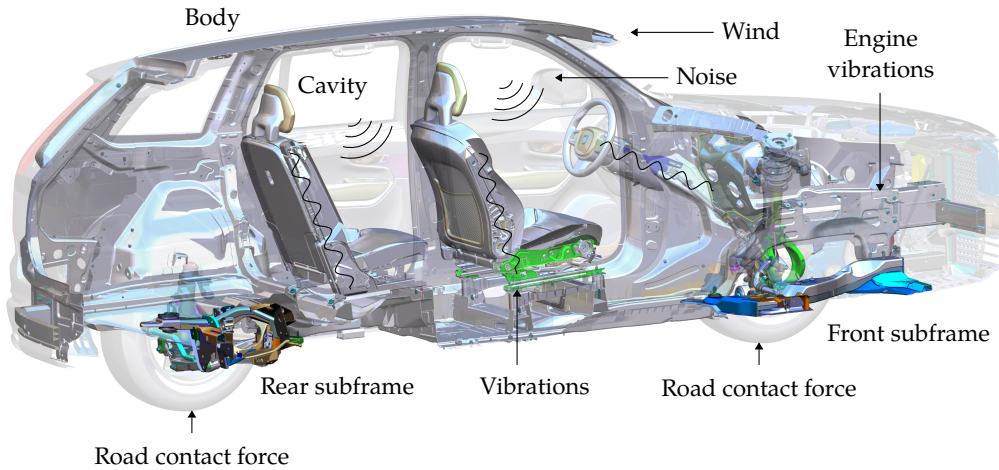


Figure 1.1: *Vibration sources and their perceived outputs as noise and vibrations in a car.*

discomfort. Engine forces are propagated through the front subframe and into the body, too. Furthermore, wind turbulence at high speeds cause the body to vibrate resulting in audible noise. All of these factors play an important role in the perceived premiumness and comfort of the car. It has also been observed that high noise and vibration levels can cause fatigue and health risks [88].

All of these mentioned issues need to be considered in the development of a new car model, which relies heavily on computational models, in the use of computer aided engineering (CAE). In this thesis CAE will mostly refer to finite element (FE) models, but could, for example, also encompass computer aided design (CAD). In developing a new car model the previous generation is often used as a reference. A simplified schematic overview of the development of a new generation, from an NVH perspective, is shown in Figure 1.2. Provided that a model already exists, improvements are proposed after problems have been identified, e.g. when it has been found that road noise or idle powertrain vibration and noise levels might be to high. Customer satisfaction and benchmarking play an important role in the decision on which areas need to be improved. When improvements are needed the interest turn to the redesign of components, e.g. body, front and rear subframes. These areas are also influencing other attributes, e.g safety and durability. In other words, the modified parameters must not only fulfil the requirements on NVH, but also on levels of safety, durability and other attributes. Newly introduced features also often affect NVH, e.g. a new safety system might produce sound. This sound needs to fit into the overall sound design of the car. The next phase of the project is related to CAE. In here the computational models are created and analysed for various conditions, and decision are taken based on the results. If results from CAE are not satisfactory further redesign is needed. When CAE results are considered acceptable the project phase moves to the creation of a prototype, or prototypes, so that the simulation results can be verified. If the prototype is considered adequate with respect to the sought

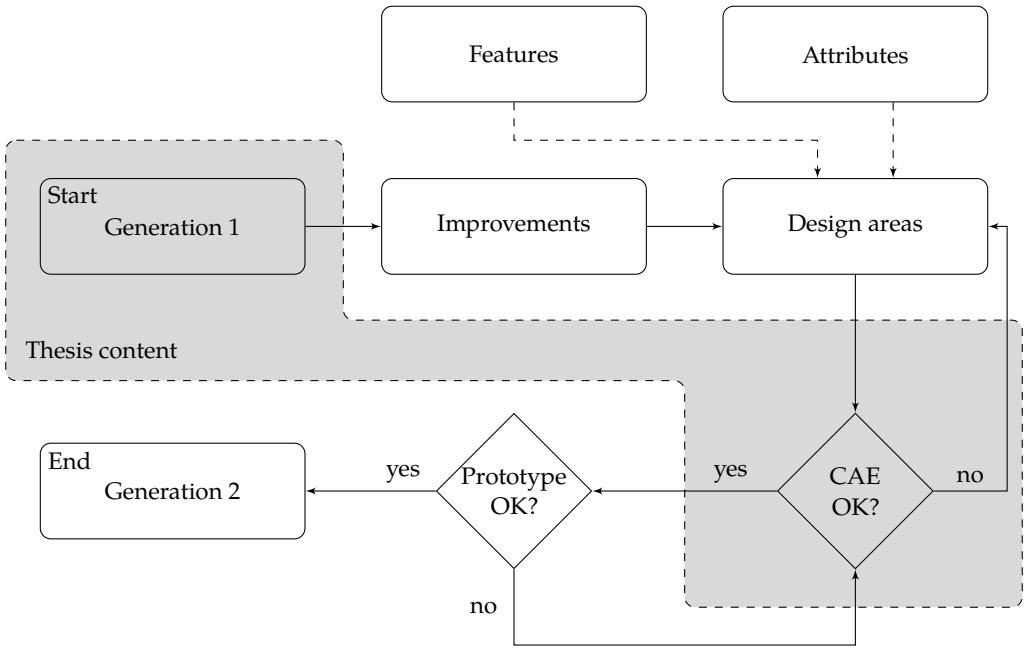


Figure 1.2: *Simplified project flow, utilising knowledge from previous generations.*

improvements, the new generation can be built. If it is decided that the requirements are not satisfied further work is needed in finding problems and correcting them, which often leads back to the CAE step. This loop is iterated until satisfactory results are reached. The shaded area of the project chart indicates where this thesis content fit in, which will be further explained below.

1.2 Predictive models

With the trend in decreased development time for each new car generation, meaning a decrease in expensive and time consuming prototypes, a CAE model's predictive accuracy is crucial to the automotive design's success. Today's computer models are generally very good in their predictive performance on component level. However, for complex structures, built up of many components the computational predictions are often much poorer. The joining parts between components are generally very hard to model and can influence the results considerably. Furthermore, in the automotive industry thousands of nominally identical but in reality slightly different products are produced. This will undoubtedly introduce variations in NVH behaviour. The product variations are caused by material variations, geometric variations and assembly tolerances. There have been many studies, on fully assembled cars, showing the level of spread in the measured output quantities such as frequency response functions, e.g [56, 57, 13, 73]. One single prototype

built for validation of CAE models will generally not provide knowledge about the spread in dynamical properties. This is problematic as there exist requirements and regulations on the noise and vibration levels. Since the levels can vary between nominally identical products it is many times not known if the requirements have been fulfilled before the start of production. Furthermore, CAE is of no help here due to its tendency to be simulated for a single, nominal, model producing a deterministic output. One option to solve this problem from a computational perspective is to perform stochastic simulations, later also termed uncertainty propagation, where the input parameters, e.g. material constants and geometry, have probability distributions associated with them, and in turn, the model outputs of interest will also be distributed. Information about such distributions can be used for more informed decision making. For this approach to work the input parameter uncertainties must be quantified, and at least the most important parameters must be identified and varied. Figure 1.2 indicates that the previous generation can be used as a reference in a new car project. Therefore, the existing experimental data gathered from the previous generation car models can be used, under the assumption that the previous generation and the new generation are not too different in design. This gives a statistical support in the quantification of model parameter variability.

Hence, for predictive simulation models the simulation results must be validated, commonly with respect to experiments. Uncertainties must be considered here, which can be, e.g. uncertainties associated with modelling or measurement errors. Variability between components should also be included, and due to scarce experimental data for new generation car models, measurements from previous generation car models can be used as a source for quantifying model parameter variability.

1.3 Uncertainty quantification and model validation

Uncertainty, or uncertainty quantification (UQ), has been used rather vaguely so far, and no formal definition has been given. One definition is given in [92] where it is described as a "quantitative characterisation and management of uncertainties ... in ... both computational models and observational data". Further, it mentions that UQ "encompasses many different tasks, including uncertainty propagation, sensitivity analysis, statistical inference and model calibration, decision making under uncertainty, experimental design, and model validation". Lastly, it is stated that "UQ has become an essential aspect of the development and use of predictive computational simulation tools". From this it can be realised that uncertainties are not only related to variability in model parameters between nominally identical components, but can be any type of uncertainty, e.g. modelling uncertainty and uncertainties associated with measurement noise. This is further exemplified in [11] where the concepts of verification and validation are defined. Two domains can be considered, a simulation and an experimental domain. On the simulation side, verification is the process of verifying that solutions from the implemented computer model from a mathematical description are substantiated, e.g. no errors exist in the software code and that FE mesh discretisation is sufficiently fine for the models

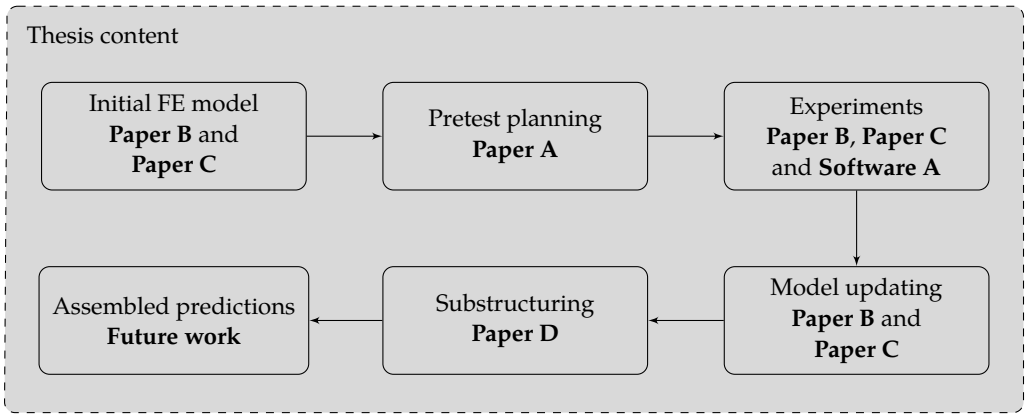


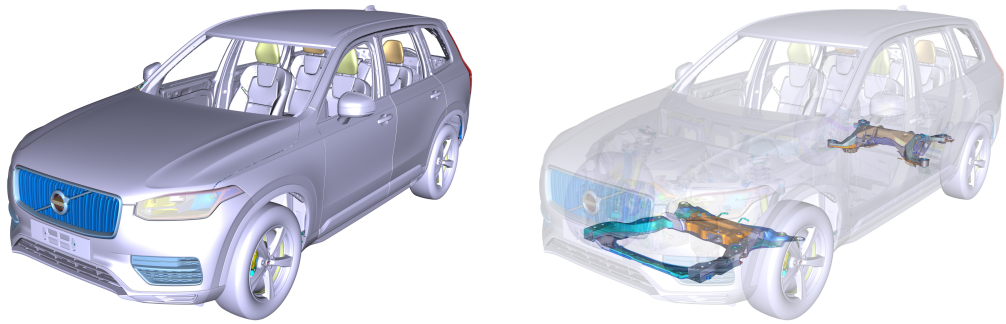
Figure 1.3: *Simplified thesis workflow with paper contributions.*

intended use. Sensitivity analysis can be used to investigate the model output sensitivities to changes in the model input parameters. Uncertainty propagation can be employed, as mentioned previously, to simulate a more realistic scenario where the input parameters can be considered stochastic so that variations in the resulting quantities are obtained. On the experimental side, UQ is the process of quantifying measurement noise and variability between components. For example, in experimental design, computer models are used in designing informative experiments such that experimental uncertainties can be minimised. Lastly, validation is the process of substantiating that the simulated model agrees with experimental results. If it does not, calibration (or model updating) can be used to update the computational model parameters so that the results agree to a higher degree. Most of these topics will be explained in subsequent chapters.

1.4 Aim and scope of research

This thesis is part of a project attempting to improve the predictability of complex industrial computational models built up of many components. The assumption that it is not possible to fully validate and understand the dynamics of an overly complex system (such as a car when fully assembled) is adopted here. Instead, a component approach is considered where the uncertainties and variability in the components are quantified, the models validated, and physical insight gained. The component models are then to be used in an assembly. It is the aim of this project to develop methods to improve the predictions of such assemblies, and in this thesis the first steps towards that goal are taken. This includes dealing with variability found between nominally identical components, and how uncertainties in general can be incorporated in a simulation environment for informative decision making, but also in quantifying and reducing experimental uncertainties.

The thesis content was outlined in Figure 1.2 and is shown in greater detail in Figure 1.3. The first aim of this thesis is to understand the cause of uncertainties and variations



(a) *Parts of a Volvo XC90 (2015).* **(b)** *Front and rear subframe.*

Figure 1.4: *The front and rear subframes of a Volvo XC90 (2015) considered in this thesis.*

found between nominally identical components. Therefore, much effort is initially put into obtaining good experimental data, so that uncertainties from experiments are reduced. A vibration measurement software, **Software A**, is developed enabling a stepped sine procedure with multiple simultaneous frequencies to be used so that experimental data can be acquired fast, and with low noise levels. Furthermore, a pretest planning methodology dealing with sensor placement, **Paper A**, is also developed. It is based on the widely used method of effective independence [45, 46], but extended so that sensor positions containing similar information are not selected. This is also related to the goal of obtaining good experimental data for use in validation and model updating.

The second goal of this thesis is to develop tools to handle the variations between components in a simulation environment, including identifying uncertain parameters. Therefore, two subcomponents of a Volvo XC90 (2015) are considered, seen in Figure 1.4. Three front subframe individuals and four rear subframe individuals have been tested. Models of the considered components are manually updated from basic CAD data to resemble to structures at hand, which **Paper B** and **Paper C** deals with. The FE models are then updated using a stochastic model updating method [95, 52, 98] towards experimental data in order to acquire better parameter estimates, quantify parameter uncertainties relative measurement noise, understand the parameter variability between components and identify sensitive and uncertain model parameters.

In future work the propagation of uncertainties in assembled components will be considered. Therefore, in **Paper D** an approach to an interface reduction for the component mode synthesis method is given so that efficient coupling of high fidelity FE models, as dealt with in this thesis, is possible.

2 Preliminary theory

In this chapter the theory that will serve as a basis throughout this thesis is presented. The notations used in this chapter might be changed in subsequent chapters, but changes should be obvious from the text.

2.1 Time domain models

In structural dynamics the linear dynamical behaviour of structures is commonly modelled as a discrete linear system through the use of second order ordinary differential equations (ODEs). For complex systems, with complex geometry, the FE method is normally used [7], which yields such a system. Models of this type are refereed to as white box or first principle models as the models are built up based on laws of physics, such as Newton's laws of motions and Hooke's law [75, 27]. The equations of motion (EOMs) of an m degrees of freedom (DOFs) system, such as those acquired from the FE method, can be written as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{V}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{2.1}$$

where the dot notation is used for time differentiation and $\mathbf{M}$, $\mathbf{V}$ and $\mathbf{K} \in \mathbb{R}^{m\times m}$ represent the mass, damping and stiffness matrices, respectively. The general displacement vector is denoted by $\mathbf{q}(t) \in \mathbb{R}^{m\times 1}$ and the external force vector by $\mathbf{f}(t) \in \mathbb{R}^{m\times 1}$. Displacement outputs can be obtained as $\mathbf{y}(t) = \mathbf{I}\mathbf{q}(t)$, with $\mathbf{I}$ of appropriate dimension.

Systems on second order form, such as in Equation (2.1), can be cast into first order form by forming a state vector [64]

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix} \tag{2.2}$$

which gives the state-space system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{2.3a}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{2.3b}$$

with $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times n_u}$, $\mathbf{C} \in \mathbb{R}^{n_y\times n}$ and $\mathbf{D} \in \mathbb{R}^{n_y\times n_u}$ representing the system, input, output and direct throughput matrices, respectively. The load vector $\mathbf{f}(t)$ in Equation

(2.1) can be obtained from the input vector $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ with the transformation matrix $\mathbf{U}$ as $\mathbf{f}(t) = \mathbf{U}\mathbf{u}(t)$. The relationship between the state dimension n and degrees of freedom m is $n = 2m$. The system matrices can be formed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{V} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{U} \end{bmatrix} \tag{2.4}$$

where $\mathbf{C}$ and $\mathbf{D}$ are formed appropriately so that linear combinations of the system states $\mathbf{x}(t)$ and inputs $\mathbf{u}(t)$ form the system outputs $\mathbf{y}(t)$. The state-space system matrices are commonly refereed to as the state-space matrix quadruple $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$.

2.2 Frequency domain models

The Laplace transform of the displacement vector $\mathbf{Q}(j\omega) = \mathcal{L}(\mathbf{q}(t))$, with $j^2 = -1$, and force vector $\mathbf{F}(j\omega) = \mathcal{L}(\mathbf{f}(t))$ yields the frequency domain formulation of the system in Equation (2.1) [64]

$$\mathbf{Z}(\omega)\mathbf{Q}(\omega) = \left(-\omega^2\mathbf{M} + j\omega\mathbf{V} + \mathbf{K}\right)\mathbf{Q}(\omega) = \mathbf{F}(\omega) \tag{2.5}$$

in which ω is the angular frequency. Here $\mathbf{Z}(\omega) \in \mathbb{C}^{m\times m}$ is identified as the dynamic stiffness matrix, from which the frequency response function (FRF) matrix can be computed as $\mathbf{H}(\omega) = \mathbf{Z}(\omega)^{-1}$.

Similarly, the Laplace transformation can be applied to the state-space system in Equation (2.3) to yield the FRF matrix $\mathbf{H}(\omega)$ as

$$\mathbf{H}(\omega) = \mathbf{C}(j\omega\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}. \tag{2.6}$$

It is noted that the frequency domain formulation is easily computed from first principle models. The reverse, however, is not true, which will be discussed in a following chapter.

2.3 Eigenvalue problem

Let $\mathbf{q}(t) = e^{j\omega t}\boldsymbol{\phi}$ be the solution to the homogeneous undamped EOMs from Equation (2.1). Inserting the solution, and eliminating the common time factor $e^{j\omega t}$ yields the algebraic eigenvalue problem.

$$\left(\mathbf{K} - \omega_i^2\mathbf{M}\right)\boldsymbol{\phi}_i = \mathbf{0}, \quad i = 1, \dots, m \tag{2.7}$$

Here ω_i denote the i:th eigenfrequency and $\boldsymbol{\phi}_i \in \mathbb{R}^{m\times 1}$ the i:th eigenvector. The eigenvalues and eigenvectors can be put in matrix form $\boldsymbol{\Omega} = \text{diag}(\omega_1, \dots, \omega_m)$ and $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_m]$, respectively. Note that

$$\text{diag}(\omega_1, \dots, \omega_m) = \begin{bmatrix} \omega_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \omega_m \end{bmatrix}. \tag{2.8}$$

If the solution $\mathbf{x}(t) = e^{\lambda t}\boldsymbol{\rho}$ is used in the first order state-space system in Equation (2.3), the following eigenvalue problem is obtained.

$$\mathbf{A}\boldsymbol{\rho}_i = \boldsymbol{\rho}_i\lambda_i, \quad i = 1, \dots, n \tag{2.9}$$

Now the eigenvalues λ_i and eigenvectors $\boldsymbol{\rho}_i \in \mathbb{C}^{n\times 1}$ are in general complex. In matrix form the eigenvalues are $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_n)$ and the eigenvectors $\mathrm{P} = [\boldsymbol{\rho}_1, \dots, \boldsymbol{\rho}_n]$.

2.4 Controllability and observability

State controllability for a linear system on first order form, as in Equation (2.3), can answer whether all the system states can be controlled to be forced from one to another arbitrary state by a stimuli $\mathbf{u}(t)$. State observability, on the other hand, answers the question whether all system states are observable from a known output $\mathbf{y}(t)$ time history [2]. These concepts are important in pretest planning of experimental work, which is the focus of **Paper A** and will be further discussed in Chapter 3.

The controllability matrix $C \in \mathbb{R}^{n\times nn_b}$ can be formed from a system's state and input matrices as [2]

$$C = \left[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \dots, \mathbf{A}^{n-1}\mathbf{B}\right] \tag{2.10}$$

and the system is controllable if $\text{rank}(C) = n$. The observability matrix $O \in \mathbb{R}^{n_y n\times n}$ can be formed from a system's state and output matrices as [2]

$$O^T = \left[\mathbf{C}^T, (\mathbf{CA})^T, (\mathbf{CA}^2)^T, \dots, (\mathbf{CA}^{n-1})^T\right]^T \tag{2.11}$$

and is observable if $\text{rank}(O) = n$. Here superscript T denote the matrix transpose.

2.5 Black box models

In comparison to first principle models, it is possible to form models solely from the system's input and output relation, e.g. from measured FRFs. These models are commonly denoted black box models where nothing is known beforehand about the model's internal structure. In structural dynamics this modelling technique is denoted experimental modal analysis (EMA) in which the eigenstructure of the system under consideration is determined from experiments. EMA is based on system identification in which many methods exist, many of which are iterative in nature, see [60]. The most useful methods in structural dynamics are non-iterative such as the eigensystem realisation algorithm (ERA) [44] and numerical algorithms for subspace state space system identification (N4SID) [96, 63]. Other methods exist, and one very commonly used in industry is the polyreference least squares complex frequency domain method (PolyMAX) [37]. In this thesis the N4SID method, as implemented in MATLAB's System Identification Toolbox, is used in **Paper B** and **Paper C**.

2.6 Reduced order models

High fidelity FE model solutions can take a very long time to solve. Therefore, it is common to reduce the full FE model to a reduced order system. This is the basis of most substructuring techniques, which is used in **Paper D**, and is further explained in Chapter 4. In optimisation or uncertainty propagation procedures a parametric reduction is necessary so that model parameters can be varied in the reduced model and results obtained in reasonable time. In **Paper B** and **Paper C** a parametric model reduction is used in the model updating procedure which is based on first order Taylor expansion, see [24, 1]. The parametric model reduction is explained in **Paper B** and **Paper C**. For a thorough review of model reduction see [4].

In structural dynamics the most commonly found methods for model reduction are based on projection methods, such as modal truncation. From the EOMs in Equation (2.1) the eigenvalue problem is solved as in Equation (2.7) so that the system's modal matrix $\mathbf{\Phi}$ can be obtained. A subset of all modes can be selected in forming a reduction basis $\mathbf{T} = [\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_{n_r}]$ with $n_r < m$. The EOMs can now be reduced by defining $\mathbf{q}(t) \triangleq \mathbf{T}\boldsymbol{\eta}(t)$ and premultiplying with $\mathbf{T}^T$, yielding

$$\mathbf{M}_r\ddot{\boldsymbol{\eta}}(t) + \mathbf{V}_r\dot{\boldsymbol{\eta}}(t) + \mathbf{K}_r\boldsymbol{\eta}(t) = \mathbf{T}^T\mathbf{f}(t) \tag{2.12}$$

with system matrices

$$\mathbf{M}_r = \mathbf{T}^T\mathbf{M}\mathbf{T}, \quad \mathbf{V}_r = \mathbf{T}^T\mathbf{V}\mathbf{T} \quad \text{and} \quad \mathbf{K}_r = \mathbf{T}^T\mathbf{K}\mathbf{T}. \tag{2.13}$$

A similar reduction can be performed for the state-space system in Equation (2.3), or the state-space quadruple can directly be formed from the reduced matrices in Equation (2.13).

2.7 Cramér-Rao lower bound

For an unbiased estimator $\hat{\boldsymbol{\theta}} \in \mathbb{R}^{p\times 1}$ of the parameters of a model deduced from noisy experimental data the Cramér-Rao lower bound (CRLB) defines a lower bound on the estimator variance. Assuming that the probability density function (PDF) of the random variable $\mathbf{z} \in \mathbb{R}^{n_z\times 1}$ conditioned on the parameters $\boldsymbol{\theta} \in \mathbb{R}^{p\times 1}$ is $p(\mathbf{z};\boldsymbol{\theta})$ and that it satisfies the regularity conditions

$$\mathrm{E}_{\mathbf{z}}\left[\frac{\partial \ln p(\mathbf{z};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right] = \mathbf{0} \quad \forall \boldsymbol{\theta} \tag{2.14}$$

the covariance matrix $\mathbf{\Sigma}_{\hat{\boldsymbol{\theta}}} \in \mathbb{R}^{p\times p}$, defined as

$$\mathbf{\Sigma}_{\hat{\boldsymbol{\theta}}} = \mathrm{E}_{\mathbf{z}}\left[\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right)^T\right] \tag{2.15}$$

for any unbiased estimator $\hat{\boldsymbol{\theta}}$, satisfies

$$\mathbf{\Sigma}_{\hat{\boldsymbol{\theta}}} - \mathcal{F}^{-1}(\boldsymbol{\theta}) \succeq \mathbf{0} \tag{2.16}$$

where $\succeq \mathbf{0}$ denotes a positive semidefinite matrix. Here $\mathrm{E}_{\mathbf{z}}$ is the expected value over $p(\mathbf{z};\boldsymbol{\theta})$. The Fisher information matrix (FIM) $\mathcal{F} \in \mathbb{R}^{p\times p}$ is defined as follows.

$$\mathcal{F}_{i,j}^{-1} = -\mathrm{E}_{\mathbf{z}}\left[\frac{\partial^2 \ln p(\mathbf{z};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_i \partial\boldsymbol{\theta}_j}\right] \tag{2.17}$$

The FIM can be interpreted as the average curvature sharpness of the PDF. The PDF with higher curvature is said to be more informative, and at the same time less uncertain in that it has a lower variance. An unbiased estimator $\hat{\boldsymbol{\theta}}$ that attains the CRLB, i.e. $\boldsymbol{\Sigma}_{\hat{\boldsymbol{\theta}}} = \mathcal{F}^{-1}(\boldsymbol{\theta})$, is said to be efficient, and hence is the minimum variance unbiased (MVU) estimator [50].

The concept of Fisher information is used in **Paper B** and **Paper C** in the parameter selection, which was proposed in [1]. The CRLB also forms the basis of the widely used method of effective independence (EfI) [45] which **Paper A** is based on. The CRLB will further be discussed in Chapters 3 and 5.

2.8 Linear models

Linear models are an important class of models in estimation theory. The MVU estimator can easily be found if the problem at hand can be structured as a linear in the parameters problem such as

$$\mathbf{z} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{2.18}$$

with white Gaussian noise (WGN) model $\mathbf{w} \sim \mathcal{N}(0, \sigma^2\mathbf{I})$. Here $\mathbf{H} \in \mathbb{R}^{n_z\times p}$ is the observation matrix with $n_z > p$ and $\mathrm{rank}(\mathbf{H}) = p$, $\boldsymbol{\theta} \in \mathbb{R}^{p\times 1}$ is the parameter vector and $\mathbf{z} \in \mathbb{R}^{n_z\times 1}$ the experimentally obtained observations. The MVU estimator, which is efficient, is then simply [50]

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{z}. \tag{2.19}$$

Linear models form the basis of the EfI method [45] for sensor placement in vibration measurements, which is used in **Paper A**. Linear models will also briefly be mentioned in Chapter 5.

2.9 Least squares

The least squares estimator does not assume any probabilistic assumption about the data, and therefore in general have no parameter variance optimality properties. The squares objective can be written, for a parameter vector $\boldsymbol{\theta} \in \mathbb{R}^{p\times 1}$, as

$$J(\boldsymbol{\theta}) = (\mathbf{z} - \mathbf{s}(\boldsymbol{\theta}))^T(\mathbf{z} - \mathbf{s}(\boldsymbol{\theta})) \tag{2.20}$$

where $J(\boldsymbol{\theta})$ is the error criterion whos value depend on the parameters. Here again $\mathbf{z} \in \mathbb{R}^{n_z\times 1}$ is the observed data vector and $\mathbf{s}(\boldsymbol{\theta}) \in \mathbb{R}^{n_z\times 1}$ is the model output from

simulation of the same data. For a linear model, $\mathbf{s}(\boldsymbol{\theta}) = \mathbf{H}\boldsymbol{\theta}$, the solution is given by Equation (2.19) [50]. In the case when $\mathbf{s}(\boldsymbol{\theta})$ is a nonlinear function of $\boldsymbol{\theta}$ iterative based nonlinear regression methods have to be used, as in general no closed form solution can be found [50]. Nonlinear regression will be considered further in Chapter 5 and is used throughout **Paper B** and **Paper C** in the model updating procedure.

3 Experiments

Vibration experiments have been an integral part of this thesis, and has resulted in the MATLAB toolbox AbraDAQ, **Software A**, for vibration measurements. For a theoretical review of vibration measurements see Ewins [20]. In this section a short overview of the important subject of pretest planning is given, with focus on the method of effective independence (EfI) [45]. **Paper A** develops a method, based on the EfI method, where sensors giving redundant information are rejected. This redundancy was also noted and investigated by Stephan [90]. The developed method is later used in **Paper B** and **Paper C**. There exist many other methods for sensor positioning, e.g. gramian based methods [26], and methods that account for uncertainties in sensor placement [70]. Further, it should be noted that sensor placement is important in many fields which deal with control systems, and there exist a vast literature related to it. This thesis is concerned with structural dynamics, and in particular using experimental data with parametric models, to which the discussion is limited. Also, no actuator placement method has been studied in this thesis.

3.1 Sensor localisation

If there exist some discrepancy in system dynamics between FE models and experiments, e.g. missing modes, it calls for further investigation which possibly will add new physical insight. There might be a modelling error present in the FE model, or the experimental data might lack some information. For FE model errors, model updating can be used to update model parameters so that they represent the experimental data better, which is further discussed in Chapter 5. If the experimental data lack some important dynamical behaviour model updating cannot be used. Then more information rich experimental data is necessary. Because experiments are time consuming and expensive, compared to FE model analyses, it is of great importance to obtain good experimental data from the beginning. Hence, pretest planning is an important step in the experimental work. In particular, it is important to place the limited number of sensors and actuators correctly so that the dynamical behaviour, in a frequency region of interest, can be observed and controlled, in some optimal sense. This is especially important for complex structures, as those found in the automotive industry, in which it can be hard to predict the dynamic

behaviour from experience or simplified models.

One of the earliest records of sensor placement for structural parametric identification is the paper by Shah and Udwadia [79]. In [93] Udwadia and Garba developed a methodology for best identification of parameters, in the sense of minimising the covariance matrix of the parameter estimate. This was further extended by Kammer [45] in which the widely used method of EfI was developed. The method is derived from a linear model, as proposed in [93], which is based on an FE model of the structure to be tested. Therefore, the method assumes that a sufficiently good model already exists. The method also assumes that a candidate set of nodes for sensor placement have been preselected by the user, from which it will find an sub-optimal set for well selected sensor locations. In [49] Kammer and Tinker extended the method for triaxial accelerometers. Further, in [46] Kammer improved the method's efficiency, and ease of use, by letting the user select the initial set and build up the final set from a candidate set. This method will briefly be described below, and how it has been used in this thesis. Only the case of a linearly independent initial set will be treated. The case when the initial set is linearly dependent is more complicated, and described in [46].

It is assumed that a good FE model is available, governed by the EOMs in Equation (2.1). Solving the associated undamped eigenvalue problem yields the modal matrix $\mathbf{\Phi}$. In industrial FE models it is infeasible to select all DOFs as candidates for ranking, and many times most DOFs are not available for sensor placement in reality. That might be due to sharp curvature in geometry, or positions not being reachable. Therefore, a candidate set of positions are selected manually, such that n_e DOFs are selected. Thus only a subset of the full modal matrix $\mathbf{\Phi}$ is used, $\mathbf{\Phi}_c \in \mathbb{R}^{n_e \times n_s}$. Here n_s stands for the number of modes selected, i.e. how many modes are of interest to observe as good as possible in the EfI sense. It should be noted that it is important that the initial candidate set, i.e. the modal matrix $\mathbf{\Phi}_c$, be linearly independent and thus is full rank. It is now possible to state the sensor output equation

$$\mathbf{q}_s = \mathbf{\Phi}_c \boldsymbol{\eta} + \mathbf{w} \tag{3.1}$$

where $\mathbf{q}_s$ are the outputs and $\boldsymbol{\eta}$ the generalised DOFs. For simplicity the noise is assumed to be uncorrelated white Gaussian noise (WGN) $\mathbf{w} \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$. The linear model in Equation (3.1) has an efficient estimator

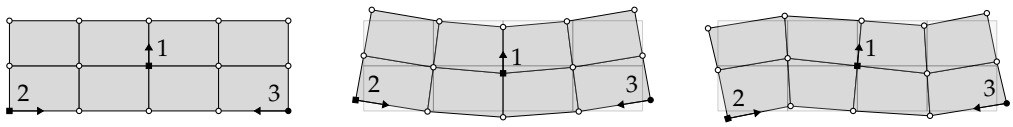
$$\hat{\boldsymbol{\eta}} = (\mathbf{\Phi}_c^T \mathbf{\Phi}_c)^{-1} \mathbf{\Phi}_s^c \mathbf{q}_s. \tag{3.2}$$

The covariance matrix of the estimator $\hat{\boldsymbol{\eta}}$ is

$$\mathbf{\Sigma} = \mathrm{E}\left[(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}) (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})^T \right] = \sigma^2 \left[\mathbf{\Phi}_c^T \mathbf{\Phi}_c \right]^{-1} = \mathcal{F}^{-1}. \tag{3.3}$$

The last inequality comes from that $\hat{\boldsymbol{\eta}}$ is efficient. Therefore the Fisher information matrix (FIM) $\mathcal{F}$ is

$$\mathcal{F} = \frac{1}{\sigma^2} \mathbf{\Phi}_c^T \mathbf{\Phi}_c = \frac{1}{\sigma^2} \mathbf{Q}_c. \tag{3.4}$$



(a) *Model.* **(b)** *First flexible mode.* **(c)** *Second flexible mode.*

Figure 3.1: *Initial sensor position denoted with filled squares ▪, candidate set denoted with circles ∘ (all positions), and final set denoted with filled markings, i.e • and ▪.*

The method of EfI seeks to maximising the FIM $\mathbf{Q}_c$, so that the covariance matrix is minimised. This is achieved with increasing the matrix determinant as candidates are added to the final set. It will be assumed that n_d uni-axial sensors are to be placed. Therefore, the candidate modal matrix $\mathbf{\Phi}_c$ is partitioned accordingly, from which $\mathbf{Q}_c$ is formed. The initial sensor set's modal matrix is denoted $\mathbf{\Phi}_0 \in \mathbb{R}^{n_0 \times n_s}$, where n_0 is the number of initial sensors, with corresponding FIM

$$\mathbf{Q}_0 = \mathbf{\Phi}_0^T \mathbf{\Phi}_0 \tag{3.5}$$

The candidate set now consist of $n_s - n_0$ positions, and the EfI method seeks to select the best positions from that set for the remaining $n_d - n_0$ positions. The EfI measure can then be formed as

$$EfI = \frac{|\mathbf{Q}^+| - |\mathbf{Q}_0|}{|\mathbf{Q}_0|} \tag{3.6}$$

where

$$\mathbf{Q}^+ = \mathbf{Q}_0 + \mathbf{\Phi}_{ci}^T \mathbf{\Phi}_{ci}. \tag{3.7}$$

Here $|\cdot|$ denote the matrix determinant and $\mathbf{\Phi}_{ci}$ denote the i:th row of the candidate sensor set. The EfI ranking represent the fractional increase of the FIM determinant if the i:th sensor is added. The procedure is iterative where a ranking of all the positions available in the candidate set are performed, and the position providing the maximum information increase is selected.

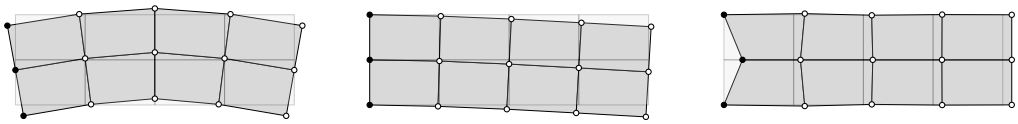
As an illustrating example in Figure 3.1 a simple model and its first two eigenmodes are shown. Three uni-axial sensors are available for positioning of which two are placed initially and the method of EfI used for the placement of the third. Every DOF in the two dimensional model is available in the candidate set. It is noted first that sensor 1 is placed such that mode 2 is not observable. This is the reason for a pretest planning procedure. In this case sensor 2 can observe both modes and no such problem exist. However, it is noted that the placement of sensor 3 by the EfI method is symmetric with respect to position 2. This behaviour was noted and addressed by Stephan [90]. In **Paper A** another approach to the rejection of such redundant positions is proposed. The example was illustrated for a linearly independent initial set, but equal results are obtained for linearly dependent initial sets.

4 Substructuring

Dividing a problem into smaller portions is a frequently used strategy in many fields. Within structural dynamics it is denoted substructuring and is a common technique used in analysis of complex systems. It works by splitting a system into smaller systems, called substructures, which are individually analysed and the solutions coupled into the global solution. This approach brings very many benefits, such as easy parallelisation as each substructure can be individually analysed, analysis of complex systems otherwise not solvable in reasonable time and identification of local dynamic behaviour [97]. The last argument is very important in the automotive industry where the full system is very complex, consisting of thousands of components. A full system level analysis will seldom bring much physical insight into local dynamical behaviour, which can be the cause of too high vibration or noise levels resulting in ride discomfort. Using substructuring, local systems can be analysed individually and even updated towards experimental data, providing physical insight, and later assembled into the global solution. The flexibility offered with this methodolgy allows for experimental substructures, i.e. models identified from experimental data [55]. This is especially useful for hard-to-model components and has recently gathered much research focus, e.g. see [62], and resulted in a large body of literature, e.g. [82, 81, 76, 34, 59].

One of the most well-known and most used substructuring methods is the component mode synthesis (CMS) method, which will be outlined in this chapter. It was first proposed by Hurty [43, 42] and Gladwell [36]. This quickly resulted in the well-known Guyan reduction method [38], and later in the Craig-Bampton method [6]. A common problem encountered with high fidelity models, e.g. most models used within the automotive industry, is associated with interfaces between subtructures consisting of many DOFs. For such cases the CMS method will decrease in efficiency. Methods have been proposed for reducing the interfaces which was initiated by Craig and Chang [16], and later further developed by Balmés [5]. In **Paper D** an approach for efficient interface reduction is proposed.

In deriving the CMS method the EOMs from Equation (2.1) of some component (s) can be generalised for the external excitation vector which is split into an external excitation vector $\mathbf{f}(t)$ and an interface vector $\mathbf{g}(t)$ containing counteracting forces from neighbouring



(a) *Free interface mode.* **(b)** *Fixed interface mode.* **(c)** *Static constraint mode.*

Figure 4.1: *Three common mode types. Here • denote interface boundary nodes and ◦ denote internal nodes.*

structures as follows.

$$\mathbf{M}^{(s)}\ddot{\mathbf{q}}^{(s)}(t) + \mathbf{V}^{(s)}\dot{\mathbf{q}}^{(s)}(t) + \mathbf{K}^{(s)}\mathbf{q}^{(s)}(t) = \mathbf{f}^{(s)}(t) + \mathbf{g}^{(s)}(t) \tag{4.1}$$

Explicit time dependence (t), and substructure (s) notation is here on dropped for brevity. It is now possible to partition the vector of DOFs in Equation (4.1) into a boundary (or interface) set b and an internal set i, with n_b and n_i DOFs each. The substructures can then be split into boundary and internal representations.

$$\begin{bmatrix} \mathbf{M}_{bb} & \mathbf{M}_{bi} \\ \mathbf{M}_{ib} & \mathbf{M}_{ii} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_b \\ \ddot{\mathbf{q}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{bb} & \mathbf{V}_{bi} \\ \mathbf{V}_{ib} & \mathbf{V}_{ii} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_b \\ \dot{\mathbf{q}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{q}_b \\ \mathbf{q}_i \end{bmatrix} = \begin{bmatrix} \mathbf{g}_b \\ \mathbf{g}_i \end{bmatrix} + \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_i \end{bmatrix} \tag{4.2}$$

What follows in this chapter are some commonly used modes for model reduction and the formulation of the CMS reduction basis, followed by coupling in the physical domain and the CMS assembly procedure.

4.1 Component modes

From the undamped eigenvalue problem of the EOMs for a substructure (s) in Equation (4.1) the system modes can be obtained. These modes can be categorised according to their behaviour, and system boundary conditions. A very common type of mode is the free interface mode $\boldsymbol{\phi}_i^{free}$ in which the model interfaces are free, and in fact the whole boundary is free. It can be seen in Figure 4.1a. This type of mode is also commonly obtained from vibration measurements, and is therefore often used in correlation analyses to FE data, such as in the modal assurance criterion (MAC) [3]. The free interface mode is included here only because of its commonness.

The fixed interface modes are similarly obtained from the undamped EOMs, but here the interfaces DOFs $\mathbf{q}_b$ are fixed, and assuming that $\mathbf{g} = \mathbf{0}$ and $\mathbf{f}_i = 0$ in Equation (4.2), the EOMs reduces to the internal DOFs

$$\mathbf{M}_{ii}\mathbf{q}_i + \mathbf{K}_{ii}\mathbf{q}_i = \mathbf{0}. \tag{4.3}$$

Solving the associated eigenvalue problem gives the fixed interface modes $\boldsymbol{\phi}_i^{fixed}$, in matrix form $\boldsymbol{\Phi}^{fixed} \in \mathbb{R}^{n_i \times n_i}$. A fixed interface mode can be seen in Figure 4.1b. The free and fixed

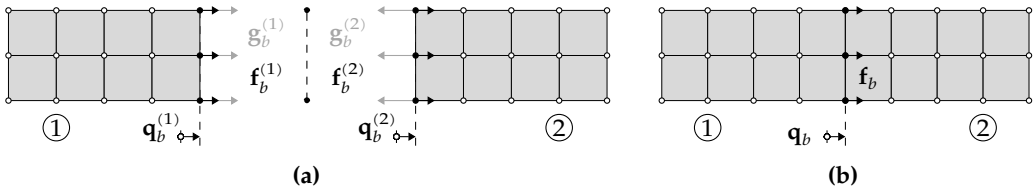


Figure 4.2: *In (a) two substructures to be assembled are shown and in (b) their assembly. Here ∘ denote internal nodes associated with $\mathbf{q}_i^{(s)}$ and • denote boundary nodes associated with $\mathbf{q}_b^{(s)}$. The black arrows → denote external forces $\mathbf{f}_b^{(s)}$ acting on the boundary while the grey arrows → denote boundary forces $\mathbf{g}_b^{(s)}$ felt from neighbouring structures.*

interface modes are vibration modes, i.e. the systems dynamical behaviour is accounted for.

The last mode type discussed here is the static constraint modes, which describe the interaction with neighbouring substructures. It is derived by decomposing the internal DOFs vector into a static and dynamic contribution $\mathbf{q}_i = \mathbf{q}_i^{static} + \mathbf{q}_i^{dynamic}$. Further, for the undamped EOMs in Equation (4.2) the accelerations are set to zero and assuming that $\mathbf{g} = \mathbf{0}$ and $\mathbf{f}_i = \mathbf{0}$ the EOMs are reduced to the boundary DOFs

$$\mathbf{q}_i^{static} = \mathbf{K}_{ii}^{-1}\mathbf{K}_{ib}\mathbf{q}_b = \mathbf{\Psi}\mathbf{q}_b. \tag{4.4}$$

This is in fact a Guyan reduction [38] and $\mathbf{\Psi} \in \mathbb{R}^{n_i \times n_b}$ is the Guyan reduction basis. The static constraint modes describe the static deformation for the DOFs in $\mathbf{q}_i$ as a result of displacement at one boundary DOF at a time. A static constraint mode can be seen in Figure 4.1c.

Now the reduction basis for the CMS method can be formed from the static constraint and fixed interface modes, as introduced by Craig and Bampton [6]. Let the internal DOFs be defined as

$$\mathbf{q}_i \triangleq \mathbf{\Psi}\mathbf{q}_b + \mathbf{\Phi}_{n_r}^{fixed}\boldsymbol{\eta}_i \tag{4.5}$$

where $\mathbf{\Phi}_{n_r}^{fixed} \in \mathbb{R}^{n_i \times n_r}$ consist of the n_r lowest fixed interface modes selected from $\mathbf{\Phi}^{fixed}$. Here $\boldsymbol{\eta}_i$ are the generalised internal DOFs. The CMS reduction basis for substructure (s), $\mathbf{R}^{(s)} \in \mathbb{R}^{m \times n_b + n_r}$, can then be formed as

$$\mathbf{q}^{(s)} \triangleq \mathbf{R}^{(s)}\boldsymbol{\eta}^{(s)} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{\Psi} & \mathbf{\Phi}_{n_r}^{fixed} \end{bmatrix} \begin{bmatrix} \mathbf{q}_b \\ \boldsymbol{\eta}_i \end{bmatrix} \tag{4.6}$$

where $\boldsymbol{\eta}$ are the generalised DOFs.

4.2 Physical system synthesis

An assembly consisting of two equal, unreduced, substructures with EOMs as in Equation (4.2) can be seen in Figure 4.2. For these two substructures the compatibility condition

states that $\mathbf{q}_b^{(1)} = \mathbf{q}_b^{(2)}$ and the equilibrium condition states that $\mathbf{f}_b^{(1)} + \mathbf{f}_b^{(2)} + \mathbf{g}_b^{(1)} + \mathbf{g}_b^{(2)} = \mathbf{f}_b^{(1)} + \mathbf{f}_b^{(2)} = \mathbf{f}_b$ where $\mathbf{g}_b^{(1)} + \mathbf{g}_b^{(2)} = \mathbf{0}$. Moreover, $\mathbf{g}_i^{(s)} = \mathbf{0}$, as neighbouring substructures do not affect the internal nodes directly. Generalising the coupling procedure to any number of substructures the primal physical assembly technique presented in [55, 97] is adopted here. Hence, the EOMs of the $s = 1, \dots, n_m$ substructure systems from Equation (4.1) to be coupled can be written on block diagonal form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{V}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} + \mathbf{g} \tag{4.7}$$

with

$$\begin{aligned}
&\mathbf{K} = \operatorname{diag}\left(\mathbf{K}^{(1)}, \dots, \mathbf{K}^{(n_m)}\right) = \begin{bmatrix} \mathbf{K}^{(1)} & \cdot & \cdot \\ \cdot & \ddots & \cdot \\ \cdot & \cdot & \mathbf{K}^{(n_m)} \end{bmatrix}, \\
&\mathbf{V} = \operatorname{diag}\left(\mathbf{V}^{(1)}, \dots, \mathbf{V}^{(n_m)}\right), \quad \mathbf{M} = \operatorname{diag}\left(\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(n_m)}\right), \\
&\mathbf{q} = \begin{bmatrix} \mathbf{q}^{(1)} \\ \vdots \\ \mathbf{q}^{(n_m)} \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \mathbf{g}^{(1)} \\ \vdots \\ \mathbf{g}^{(n_m)} \end{bmatrix} \quad \text{and} \quad \mathbf{f} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(n_m)} \end{bmatrix}.
\end{aligned} \tag{4.8}$$

The compatibility and equilibrium conditions can be generalised for the (s) substructures in matrix form as

$$\mathbf{E}\mathbf{q} = \mathbf{0} \quad \text{and} \quad \mathbf{L}^T\mathbf{g} = \mathbf{0} \tag{4.9}$$

where $\mathbf{E} \in \{-1, 0, 1\}^{n_b \times m n_m}$ is a signed Boolean matrix and $\mathbf{L} \in \{0, 1\}^{m n_m \times n_u}$ is a Boolean matrix, with n_u unique DOFs for the coupled substructures. Here Boolean matrix means a matrix possibly consisting of only ones and zeros, and signed Boolean matrix is an extension containing negative ones, too. The matrix $\mathbf{L}$ can be viewed as a localisation matrix transforming the coupled substructures unique DOFs $\mathbf{q}_u \in \mathbb{R}^{n_u \times 1}$ to the coupled substructures total DOFs $\mathbf{q}$ as

$$\mathbf{q} = \mathbf{L}\mathbf{q}_u. \tag{4.10}$$

The compatibility condition can now be express in terms of the unique set of DOFs $\mathbf{q}_u$ as

$$\mathbf{E}\mathbf{q} = \mathbf{E}\mathbf{L}\mathbf{q}_u = \mathbf{0} \tag{4.11}$$

where $\mathbf{L} = null(\mathbf{E})$ as $\mathbf{q}_u$ is in general not zero. Now the systems in Equation (4.7) can be coupled by using $\mathbf{q} = \mathbf{L}\mathbf{q}_u$ and premultiplying with $\mathbf{L}^T$ giving

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}_u + \tilde{\mathbf{V}}\dot{\mathbf{q}}_u + \tilde{\mathbf{K}}\mathbf{q}_u = \tilde{\mathbf{f}} \tag{4.12}$$

with

$$\tilde{\mathbf{M}} = \mathbf{L}^T\mathbf{M}\mathbf{L}, \quad \tilde{\mathbf{V}} = \mathbf{L}^T\mathbf{V}\mathbf{L}, \quad \tilde{\mathbf{K}} = \mathbf{L}^T\mathbf{K}\mathbf{L} \quad \text{and} \quad \tilde{\mathbf{f}} = \mathbf{L}^T\mathbf{f} \tag{4.13}$$

where $\mathbf{L}^T\mathbf{g} = \mathbf{0}$ has been utilised.

4.3 Component mode synthesis

In the CMS method reduced systems are being coupled. The reduction basis for a substructure (s) defined in Equation (4.6) can be substituted into the EOMs for substructure (s) as

$$\mathbf{M}^{(s)}\mathbf{R}^{(s)}\ddot{\boldsymbol{\eta}}^{(s)} + \mathbf{V}^{(s)}\mathbf{R}^{(s)}\dot{\boldsymbol{\eta}}^{(s)} + \mathbf{K}^{(s)}\mathbf{R}^{(s)}\boldsymbol{\eta}^{(s)} = \mathbf{f}^{(s)} + \mathbf{g}^{(s)}. \tag{4.14}$$

The reduction basis can be cast in block diagonal form as

$$\mathbf{q} \triangleq \mathbf{R}\boldsymbol{\eta}, \quad \text{with} \quad \mathbf{R} = \operatorname{diag}\left(\mathbf{R}^{(1)}, \dots, \mathbf{R}^{(n_m)}\right). \tag{4.15}$$

The EOMs for all uncoupled substructures in Equation (4.7) can be reduced with the introduced reduction basis and a premultiplication of $\mathbf{R}^T$ so that

$$\mathbf{M}_m\ddot{\boldsymbol{\eta}} + \mathbf{V}_m\dot{\boldsymbol{\eta}} + \mathbf{K}_m\boldsymbol{\eta} = \mathbf{f}_m + \mathbf{g}_m \tag{4.16}$$

with

$$\begin{aligned} &\mathbf{M}_m = \mathbf{R}^T\mathbf{M}\mathbf{R}, \quad \mathbf{R}_m = \mathbf{R}^T\mathbf{V}\mathbf{R}, \quad \mathbf{K}_m = \mathbf{R}^T\mathbf{K}\mathbf{R}, \\ &\mathbf{f}_m = \mathbf{R}^T\mathbf{f} \quad \text{and} \quad \mathbf{g}_m = \mathbf{R}^T\mathbf{g}. \end{aligned} \tag{4.17}$$

Coupling of the reduced systems requires the compatibility and equilibrium conditions to be enforced in the generalised DOFs. The compatibility condition can then be expressed as

$$\mathbf{E}_m\boldsymbol{\eta} = \mathbf{0}, \quad \text{with} \quad \mathbf{E}_m \triangleq \mathbf{E}\mathbf{R}. \tag{4.18}$$

The generalised DOFs $\boldsymbol{\eta}$ can be expressed in terms of a set of unique generalised DOFs $\boldsymbol{\xi}$ for the assembled systems through a localisation matrix $\mathbf{L}_m$ as

$$\boldsymbol{\eta} = \mathbf{L}_m\boldsymbol{\xi}. \tag{4.19}$$

Substituting Equation (4.19) into Equation (4.18) yields

$$\mathbf{E}_m\mathbf{L}_m\boldsymbol{\xi} = \mathbf{0} \tag{4.20}$$

with $\mathbf{L}_m = \mathit{null}(\mathbf{E}_m)$. The coupling is now performed as in the physical domain by substituting $\boldsymbol{\eta} = \mathbf{L}_m\boldsymbol{\xi}$ into the EOMs in Equation (4.16) and premultiplying with $\mathbf{L}^T$ so that the coupled system is obtained as

$$\tilde{\mathbf{M}}_m\ddot{\boldsymbol{\xi}} + \tilde{\mathbf{V}}_m\dot{\boldsymbol{\xi}} + \tilde{\mathbf{K}}_m\boldsymbol{\xi} = \tilde{\mathbf{f}}_m \tag{4.21}$$

with

$$\begin{aligned} &\tilde{\mathbf{M}}_m = \mathbf{L}_m^T\mathbf{M}_m\mathbf{L}_m, \quad \tilde{\mathbf{V}}_m = \mathbf{L}_m^T\mathbf{V}_m\mathbf{L}_m, \quad \tilde{\mathbf{K}}_m = \mathbf{L}_m^T\mathbf{K}_m\mathbf{L}_m \quad \text{and} \\ &\tilde{\mathbf{f}}_m = \mathbf{L}_m^T\mathbf{f}_m \end{aligned} \tag{4.22}$$

where the fact that $\mathbf{L}_m^T\mathbf{g}_m = \mathbf{0}$ has been used.

5 Uncertainty quantification

Uncertainty quantification is an essential tool in the process of developing predictive models in CAE driven fields, such as the automotive industry. It is a vast area with active research that spans most scientific fields. Therefore, it is not the intent of this chapter to provide an overview of the whole field, but rather point towards some recent developments within the field of structural dynamics. First a short summary is made of the different types of possible-to-encounter uncertainties. Thereafter, a brief overview of model updating is presented with focus on the particular method used in this thesis, followed by a short summary of inverse uncertainty quantification, which can be seen as a generalisation of model updating. Then a section is devoted to a short summary of some possible techniques related to uncertainty propagation for forward predictions. The last two sections are included mainly as a short review of the subject area.

5.1 Types of uncertainties

Many types of uncertainties can be encountered in simulation and experimental results. It is commonly said that two categories of model uncertainties exist. One is reducible uncertainty, also called lack-of-knowledge or epistemic uncertainty. The other being irreducible uncertainty, also called random or aleatory uncertainty [53]. Essentially the division into the two categories is left to the user for the problem at hand, if at all needed. Take the example of measurement noise. If no other measurement method is available the uncertainty can probably not be reduced and should be categorised as irreducible. If other means of measurement exists the uncertainty can be categorised as reducible. Another example is variability between components. If no control of the tolerances in production exist the uncertainty should be treated as irreducible, and reducible if there is. As can be noted the problem of categorising the uncertainty is not an easy task, and not something this thesis will consider in great detail. What follows are definitions of some types of model uncertainties, as defined by Kennedy and O'Hagan [51].

Parametric uncertainty describes the statistical knowledge associated with model parameters when considering test data from a single individual from a population. In an experiment the results obtained contain noise, and multiple experiments will never produce exactly the same results. The estimated parameters from a model updating

procedure will therefore be uncertain to some extent.

Model adequacy refers to how well the model at hand is capturing reality. Many times simplifications are introduced, which might be overly crude and make the model inadequate. For example, modelling a car's body in white with beam elements might be adequate in some situations where simple analyses are of interest. It is not adequate in most situations, such as when the vibration predictability is of interest. In situations of model inadequacy it is important not to update model parameters to account for modelling simplifications. The obtained model parameters would act as surrogates for other errors, and the obtained model would in most cases not be usable for predicting other results.

Residual variability means that for a real process with specified inputs the outputs are not necessarily always equal. Therefore, a model will not in general be able to predict the output of a real process for some specified input condition. This is a consequence of either a form of model inadequacy, i.e. some conditions are not modelled, or the process might be inherently random. For example, between equal experiments the input conditions will always vary to some degree, which are not captured by the model.

Observation errors are caused due to measurement noise, e.g. electronic noise in the equipment or fluctuations in temperature. This type of uncertainty might be very hard to separate from residual variability.

Parametric variability represent the actual variability of the physical properties. For a mass produced component manufacturing tolerances will cause the nominally identical components produced to always show some property variation. Such variation can be captured as parameter variation after an update of model parameters that represent such physical properties, e.g by updated geometry properties or the elastic modulus deviates from nominal values.

Code uncertainty is related to the uncertainty associated with complex computer codes. For most problems of interest complex models are used where no analytical solution exist. Therefore, computer codes are used to obtain a numerical solution. The computer code implementation is usually not free from errors, which can cause erroneous results related to uncertainties.

5.2 Model updating

The forward problem, or forward propagation, was presented in Chapter 2 as a second and first order system of ODEs. This problem deals with obtaining outputs from a model given some inputs, and has in most cases a unique solution and is therefore well-posed. The inverse problem however, which is that of obtaining model inputs from the model outputs, is in general ill-posed, i.e. there may exist multiple solutions or no solution at all [25]. In Figure 5.1 the two problem types are shown. Model inputs are commonly parameters $\boldsymbol{\theta}$ and system inputs $\mathbf{u}$, while outputs $\mathbf{y}$ can be any quantity of interest, but will here most often be FRFs. In both cases it has been assumed that the model is known. FE model updating is an inverse problem and strives to update an FE model, or the input parameters

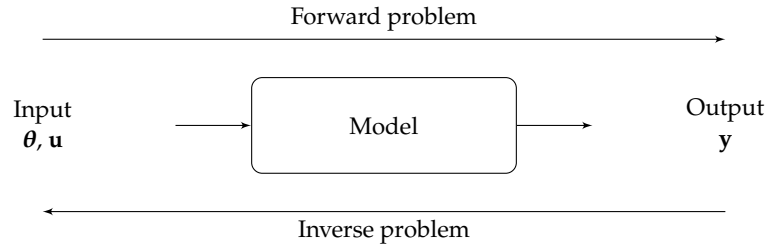


Figure 5.1: *Forward and inverse problems.*

$\boldsymbol{\theta}$, so that the model outputs represent experimental data better. Early attempts to FE model updating were made by direct updating where the elements of mass and stiffness matrices were altered. While these methods could represent the measurement data very precisely, the physical meaning of the obtained models was lost. Later approaches considered updating of physically meaningful parameters where deviation functions are minimised, e.g. deviations in modal data between FE and experimental models. However, using modal data introduces the problem of identifying modal parameters from experimental data. Using a deviation based on FRFs alleviates this problem, but introduces problems associated with mode pairing. For a thorough background on FE model updating see [68, 25]. In this thesis an FRF based model updating procedure with equalised damping is used [1], to circumvent the mode pairing problem, and will be the basis for this section presentation. Note that explicit frequency dependency is dropped from the FRF matrices below, and the used frequencies are assumed to take discrete values.

The discrepancy between the a model $\mathbf{H}_R$ and the experimentally measured model $\mathbf{H}_X$ can be denoted by $\mathbf{N}_o$ [51, 80, 52, 98]. Further, the true model $\mathbf{H}_R$ can be thought to be composed of an FE model $\mathbf{H}_{FE}(\boldsymbol{\theta})$ and the model prediction error $\mathbf{N}_m$.

$$\mathbf{H}_X = \mathbf{H}_R + \mathbf{N}_o = (\mathbf{H}_{FE}(\boldsymbol{\theta}) + \mathbf{N}_m) + \mathbf{N}_o = \mathbf{H}_{FE}(\boldsymbol{\theta}) + \mathbf{N}_G \tag{5.1}$$

Under the assumption that the bias introduced by the FE model is small, $\mathbf{N}_G$ can be modelled as an independent, zero mean, multivariate normally distributed random variable with a known covariance matrix $\boldsymbol{\Sigma}$.

The goal of the calibration is to estimate a calibration parameter setting $\hat{\boldsymbol{\theta}}$ that minimises the deviation between the experimentally obtained FRFs $\mathbf{H}_X$ and the FE model FRFs $\mathbf{H}_{FE}(\boldsymbol{\theta})$ under some norm. In the method used in this thesis [1] the discrepancy between a finite element model $\mathbf{H}_{FE}$ and an experimentally identified model $\mathbf{H}_\Sigma$ is minimised instead. The experimental model can be obtained by various system identification algorithms, e.g. N4SID [63] or PolyMAX [37]. The damping can therefore be equalised in both models, and the mode pairing issue is circumvented [1]. The deviation metric can be formed as a nonlinear least squares problem, as in Equation (2.20), that is smooth and weights high

and low structural responses equally.

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg\min_{\boldsymbol{\theta}} \frac{\boldsymbol{\epsilon}^H(\boldsymbol{\theta})\boldsymbol{\epsilon}(\boldsymbol{\theta})}{N} \\ \boldsymbol{\epsilon}(\boldsymbol{\theta}) &= \log_{10}\text{vect}(\mathbf{H}_{FE}(\boldsymbol{\theta})) - \log_{10}\text{vect}(\mathbf{H}_{\Sigma}) \end{aligned} \tag{5.2}$$

The superscript H denote the conjugate transpose and $\text{vect}(\cdot)$ stands for the vectorisation operation in which a matrix is transformed into a column vector by stacking the columns of the matrix.

Model updating is used in **Paper B** to update model parameters towards three front subframes, individually. In **Paper C** bushing rubber stiffness is estimated using model updating of one rear subframe, together with other model parameters. The procedure is deterministic and does not consider any types of uncertainty discussed above, other than parametric variability in **Paper A** for the three components. Therefore, other methods need to be used so that influences of noise and parameter variability can be accounted for. This is discussed in the next sections.

The parameter selection is a non-trivial subject [25, 80]. Selecting too many parameters easily renders the model updating problem ill-posed as all of the parameters might not be identifiable from the observed data. Selecting too few might not yield satisfactory results. Which parameters to select might be the hardest problem in a calibration procedure. Ideally parameters that are not well known, or uncertain, should be selected. Geometric parameters are recommended in [25]. Many times this is not possible, and parameters considered uncertain might not be identifiable. Therefore, often a selection of surrogate parameters has to be done. An identifiability study can be performed to exclude parameters that would render the model updating problem ill-posed. The CRLB can be used where the FIM is computed from the output sensitivities relative the parameters [80, 50].

$$\mathcal{F} = \mathbf{J}^T\mathbf{J} \quad \text{with} \quad \mathbf{J}_{ij} = \frac{\partial\left[\boldsymbol{\epsilon}(\boldsymbol{\theta})^H\boldsymbol{\epsilon}(\boldsymbol{\theta})\right]_i}{N\partial\boldsymbol{\theta}_j} \tag{5.3}$$

Here $i = 1, \ldots, N$ where N is the number of points in the deviation metric, and $j = 1, \ldots, p$ with p representing the number of parameters. A parameter with very high variance compared to other parameters should therefore be discarded, as model outputs are not very sensitive to small changes to it.

5.3 Inverse uncertainty quantification

A deterministic calibration yields a point estimate in the parameter space, meaning that no statistical description about the estimator exist. Performing predictions with such a parameter setting is not best practice as no statistical confidence bounds in the results are obtained. Therefore, methods exist to account for the uncertainties in the observed data. Generally two viewpoints for the statistical inference problem exist, the frequentist and Bayesian viewpoints [74].

The frequentist view of probability is many times considered the traditional viewpoint. Here the probability for en experimental outcome is interpreted as the relative frequency of occurrence of that outcome in a long sequence of experiment repetitions [50, 74]. Thus, the estimator uncertainty is represented by the probability distribution obtained by applying an estimator to the experiment repetitions. Parameters of interest are viewed as deterministic but unknown. Structural dynamics methods taking the frequentist approach is common, see [61, 95]. A common estimator used is the maximum likelihood estimator (MLE) which gives a parameter setting that maximises the likelihood function [50]. From Equation (5.1), assuming that $\mathbf{h}_X = vect(\mathbf{H}_X)$ and $\mathbf{h}_{FE} = vect(\mathbf{H}_{FE})$, and with the assumption that $\mathbf{N}_G \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$ the likelihood function can be formed as

$$l(\mathbf{h}_x|\boldsymbol{\theta}) \propto exp\left[-\frac{1}{2}\left(\mathbf{h}_X - \mathbf{h}_{FE}\right)^T \boldsymbol{\Sigma}^{-1}\left(\mathbf{h}_X - \mathbf{h}_{FE}\right)\right]. \tag{5.4}$$

Here $\propto$ denote proportionality between two expressions. The MLE estimator $\hat{\boldsymbol{\theta}}_{MLE}$ is then found as

$$\hat{\boldsymbol{\theta}}_{MLE} = \arg\max_{\boldsymbol{\theta}} l(\mathbf{h}_x|\boldsymbol{\theta}). \tag{5.5}$$

Under certain conditions the MLE estimator $\hat{\boldsymbol{\theta}}_{MLE}$ distribution can be shown to be [50]

$$\hat{\boldsymbol{\theta}}_{MLE} \overset{a}{\sim} \mathcal{N}\left(\mathbf{0}, \mathcal{F}^{-1}(\boldsymbol{\theta})\right). \tag{5.6}$$

Here $\overset{a}{\sim}$ denote the asymptotically distributed symbol.

Another approach to find the sampling distribution of the MLE estimator is based on using bootstrapping [40, 52, 98, 95]. This approach is used in this thesis in **Paper B** and **Paper C**. The procedure works by repeatedly drawing random datasets $\mathbf{h}_X^b$ with replacement from the original data set $\mathbf{h}_X$. The calibration procedure is then performed towards this new data set $\mathbf{h}_X^b$ in Equation (5.2) so that a new calibration parameter setting $\hat{\boldsymbol{\theta}}^b$ is obtained. This procedure is repeated n_b times. Thus n_b vectors of calibrated parameters $\hat{\boldsymbol{\theta}}^b$ will be obtained, from which statistics can be computed [52, 98].

The Bayesian viewpoint, on the other hand, interprets probability as a degree of plausibility. Therefore, the parameters are considered random variables and a prior belief about their distribution can be used, if it exist. In structural dynamics much research is presently focusing on Bayesian techniques and especially their computational challenge using Markov chain Monte Carlo methods, see [9, 8, 14, 94]. The methods work by using Bayes' theorem [12]

$$p(\boldsymbol{\theta}|\mathbf{h}_X) = \frac{p(\mathbf{h}_X|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{h}_X)} \tag{5.7}$$

where $p(\mathbf{h}_X|\boldsymbol{\theta})$ is the likelihood function, $p(\boldsymbol{\theta})$ the prior distribution and $p(\mathbf{h}_X)$ a normalisation constant so that the posterior distribution $p(\boldsymbol{\theta}|\mathbf{h}_X)$ becomes a valid probability density function. The posterior distribution is therefore associated with the prior distribution, but is updated with respect to the data, and so new information is incorporated

in this framework. The posterior distribution can be used in an estimator. It is common to use the maximum a posteriori (MAP) estimator [50].

$$\hat{\boldsymbol{\theta}}_{MAP} = \arg\max_{\boldsymbol{\theta}} p(\boldsymbol{\theta}|\mathbf{h}_x) \tag{5.8}$$

This estimator selects a parameter setting $\hat{\boldsymbol{\theta}}_{MAP}$ that will maximise the posterior distribution, i.e. find the posterior distributions mode. Another common estimator is the minimum mean squared error (MMSE) estimator [50].

$$\hat{\boldsymbol{\theta}}_{MMSE} = \int \boldsymbol{\theta} p(\boldsymbol{\theta}|\mathbf{h}_x) d\boldsymbol{\theta} = \mathrm{E}\left[\boldsymbol{\theta}|\mathbf{h}_x\right] \tag{5.9}$$

Here the parameter setting $\hat{\boldsymbol{\theta}}_{MMSE}$ is chosen so that the estimator minimises the mean square error (MSE). For other estimators see [50]. The computational burden associated with Bayesian techniques is very high for high dimensional parameter spaces (curse of dimensionality) which is one of the drawbacks of the method. The method has also been criticised as biased due to the prior distribution, which can sometimes be chosen arbitrarily. Furthermore, the implementation is fairly complex compared to the simpler frequentist methods.

Other approaches exist for the treatment of uncertainties and variability in model parameters for the inverse problem. Non-probabilistic methods have surfaced recently, such as the fuzzy set method [39, 80]. They have been introduced due to the inability of probabilistic methods to account for epistemic uncertainty [67], i.e. uncertainty that very little is known about or non-random uncertainty. In other words many authors propose that probabilistic methods should be avoided when the uncertainties are epistemic, i.e. lack-of-knowledge type uncertainties are present [10].

5.4 Forward uncertainty quantification

When uncertainties are associated with the simulation model one single deterministic simulation will generally not be sufficient for predictive purposes. That is because no confidence bounds for the results are present. In reality uncertainties about simulation models are always present. For instance when no experimental data about the model exist, or in cases when it is known that a particular parameter is representing a property created by a production process with some randomness with a known probability distribution. In such situations uncertainty propagation, or forward uncertainty quantification methods can be used. The most well known method is the Monte Carlo method, which is also often used as a reference for other methods. It can be seen as a brute force method in which the deterministic simulation is run very many times with different realisations of input parameters and controlled by the parameter distribution. Statistics about the output quantities are then obtained which in turn provide more confidence in the simulated results. This method is usually very computationally expensive as a single deterministic simulation can take many hours, and many simulations are needed for just one varying parameter. It is common to have hundreds of uncertain parameters, and hence a very

large parameter space. Therefore, other methods have been developed to circumvent this problem. Good overviews of methods are given in [91, 77, 78, 78, 89]. It should be noted here that the inverse quantification methods in the previous section generally solve the forward problems many times, and so the output uncertainties are implicitly obtained.

For probabilistic finite element analysis uncertainty propagation methods using perturbation techniques exist [54]. These are normally based on Taylor series expansion, which are generally well suited when the first and second moments, i.e. mean and variance, of the outputs are sought [91]. From reliability engineering the first and second order reliability methods exist which are well suited when the tail probabilities are sought [91], e.g. to assess a structures probability of failure. Further, spectral finite element methods exist [28] which strive to be suited for any type of problem. Although, these methods are not well suited for nonlinear problems and can be computationally intensive [89]. Sensitivity analysis is another important part of stochastic mechanics, which was briefly explained above where a local sensitivity analysis method was outlined in terms of the CRLB. It can be used to asses the importance of parameters and hence reduce the cost of simulations. Global sensitivity analysis on the other hand strives to quantify the output uncertainty due to uncertainty in all input parameters combined [91].

Most of the focus so far has been on parametric methods, in which specific model parameters are varied. Non-parametric approaches have surfaced recently based on random matrix theory [84, 87, 85, 86, 83]. The main idea is that the EOMs system matrices can be treated as random, i.e. the nominal model can be considered as the mean and very few parameters are used to represent the uncertainties in the matrices. The obvious disadvantage of the method is that no physical insight is usually retained, but on the other hand all types of uncertainties might be considered. This has resulted in an application for structural-acoustic modelling in automotive vehicles [17].

Possible ways of circumventing the computational burden of Monte Carlo methods is to use reduced order models, or surrogate models, e.g. through a multi-fidelity approach [35, 71, 69]. Another approach relies on substructuring, or component mode synthesis, where reduced order models are assembled to obtain system responses [41]. In comparison to the non-parametric methods, substructuring methods will generally allow for a greater physical insight as simpler components are coupled to form the complex system. Further, surrogate techniques exist, such as Gaussian processes [23]. Many of these techniques are computationally expensive in the creation of the surrogate models, but cheap in the evaluation of the surrogate models. Therefore, the sometimes millions of runs required by the Monte Carlo method can be performed cheaply. The drawback is that the surrogate models need to be recreated for other parameter configurations, and that surrogate models introduce errors in the system outputs.

Other approaches exist for the treatment of uncertainties and variability in model parameters, e.g. non-probabilistic methods. Common methods are the convex interval method [10] and the fuzzy set method [65]. These methods are especially useful in the treatment of epistemic uncertainty, i.e. lack-of-knowledge uncertainty rather than random uncertainty. The convex interval method is based on the assumption that many times the parameter range is constrained and as such it tends to form a convex set which simplifies

the analysis. The fuzzy set approach is based on fuzzy sets [99], which are sets where elements have a degree of membership associated with them, i.e. the degree of belonging to the set is defined by the membership function. The methods have surfaced due to the many times arbitrary assumption of the probability density function governing the uncertainties, e.g. when not enough data about the uncertain parameters exist which is many times the case. Experiments tend to be time consuming and expensive and only a limited number can be performed. These methods could thus be classified as a middle ground between deterministic simulations and stochastic methods.

6 Summary of appended papers

Paper A: Redundant information rejection in sensor localisation using system gramians

In this paper the method of effective independence (EfI) is shown to select redundant sensor positions in symmetric models and very detailed finite element models. An extension of the EfI method is proposed where sensors are ranked by the EfI metric and sensor positions containing redundant information are identified and rejected from the final sensor set. The redundant sensor position criteria is based on system gramians and works for sensors measuring in any direction. The method is verified on a simple symmetric clamped plate.

Paper B: Calibration, validation and uncertainty quantification of nominally identical car subframes

Vibration experiments of three nominal Volvo XC 90 (2015) front subframes are performed. Experimental models are acquired through system identification. Sensor placement is based on the method proposed in **Paper A**. The spread in dynamical and static properties is reported. It is found that within certain frequency regions the spread in dynamical properties is considerable. The damping equalisation, frequency based, calibration method FEMcali is used to calibrate 9 physical parameters for the three subframes in independent calibrations up to 400 Hz towards the experimentally identified models. Considerable improvement in model predictability is achieved. A bootstrapping procedure, towards raw experimental data, is utilised in assessing the parameter uncertainty with respect to the measurement noise. It is found that one parameter is sensitive to the measurement noise for two components.

Paper C: Parameter estimation and uncertainty quantification of a subframe with mass loaded bushings

One Volvo XC 90 (2015) rear subframe is considered. Vibration experiments, with sensor placement based on the method proposed in **Paper A**, are performed with and without

mass loaded bushings. Experimental models are identified using system identification. Model calibrations of configurations with and without mass loaded bushings are performed with the FEMcali method towards the experimentally identified models. High fidelity bushings models are created and the rubber stiffness updated towards the mass loaded experimental models. The updated subframe model is then further updated towards the configuration without mass loaded bushings for various physical parameters. Very good model predictability is achieved.

Paper D: A reduced interface component mode synthesis method using coarse meshes

A component mode synthesis (CMS) method with efficient interface reduction is proposed. CMS methods provide an efficient modelling methodology for large scale structural models. When a high density mesh is used, with detailed interfaces, the method's efficiency is reduced. It is proposed that coarse mesh models are used in forming a reduction basis of the interface degrees of freedom. The method is verified on a simple plate model consisting of two substructures.

7 Conclusion and future work

This thesis deals with the development of methods for creation of predictive models under uncertainties for use in structural dynamics. The goal is twofold. Minimising measurement uncertainties and enabling computational models to account for uncertainties. The focus lies on complex industrial structures from the automotive industry. Therefore a component approach is considered so that physical insight is retained. The more complex system of interest can be obtained from an assembly of the individual components.

In reducing measurement uncertainties an existing procedure for sensor position selection, the method of effective independence, is extended for rejection of sensor positions carrying similar information using system gramians. Further, a measurement software, implemented in MATLAB, for faster stepped sine vibration experiments is developed such that experimental data with low noise levels can be obtained relatively fast.

To quantify model parameter uncertainties a model updating method using frequency response functions and equalised damping with a bootstrapping uncertainty quantification procedure is used on two automotive structures. A front and a rear subframe are considered. Very good models are obtained, that can be used in a future synthesis. Physical parameters are updated, but it is noted that geometric parameters are highly desirable for a better parametrisation. It is found that the identification of an experimental system is very difficult, and tools for ease of use should be further developed. Spread in dynamical properties between nominally identical components of the two structures is also noted.

A component mode synthesis interface reduction method is also developed which enables fast computations of high fidelity finite element models. The method is based on the usage of coarse meshes to reduce the dimension of the linear system solved in a static condensation.

For future work the assembly of components will be considered, and how the variability between nominally identical components propagate through such assemblies, in both experimental and computational models. Some possible methods to consider for propagation of uncertainties in built up structures are component mode based techniques [41, 48, 47].

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Part II

Appended Papers A–D