Thesis for the degree of Licentiate of Engineering

Computational Methods for Deformable 1D Objects in Virtual Product Realization

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Abstract

In industry today, virtual design tools are used in the realization of a new product. As changes in the design and planning concepts are extremely costly in the later verification and production phases, much can be gained if a product design can be optimized and verified with respect to the assembly process with simulation tools as early as possible.

A topic of special interest is the virtual preparation of deformable 1D objects such as electrical cables and wiring harnesses, hoses, pipes and tubes. They are geometrically characterized as one-dimensional in the sense that one dimension is significantly larger than the other two and can deform when subject to external forces and moments. These types of flexible components are usually located where there is restricted design space and are often associated with quality problems and late on-line adjustments due to geometrical interference. Hence, there is a strong motivation to further strengthen the virtual product realization process in this area.

This thesis will present three computational methods for geometrical design and verification of deformable 1D objects. The main scientific challenge is to deal with the complexity of coupling a simulation model with iterative algorithms for optimization, path planning and variation simulation in an efficient way. The methods rely on a simulation model based on Cosserat rod theory that enables efficient and accurate computations of large spatial deformations of flexible 1D objects.

The first method solves the problem of routing a deformable 1D object with respect to geometrical design constraints. The method is segregated in a deterministic grid search step and a simulation-based local optimization step. The second method solves the assembly verification problem for a deformable 1D object with a given design. If the verification is true, the method produces a smooth manipulation of a set of grip points that installs the object in the target configuration. The third and final method is in fact a methodology for performing analysis and visualization of geometrical variation in a deformable 1D object. Here, the main innovation is the construction of a discrete envelope for given tolerances based on convex hull computations and silhouette generation.

Together, the three methods form a powerful tool set for geometrical design and verification. Quality problems and geometrical interference in the assembled product can now to a larger extent be addressed in the concept phase, thus saving significant development time and reducing the number of iterations between the design phase and the planning and verification phases.

The methods are implemented as an integral part in the commercial software IPS Cable Simulation as of version 3.1 (2016).

Keywords: assembly verification, automatic routing, deformable 1D objects, Cosserat rod theory, path planning, robust design, variation simulation.

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Tomas Hermansson
Göteborg, February 2017

List of publications

This thesis is based on the following appended papers:

Paper A. **T. Hermansson**, R. Bohlin, J. S. Carlson and R. Söderberg. *Automatic routing of flexible 1D components with functional and manufacturing constraints.* Journal of Computer-Aided Design Vol. 79. pp. 27-35, 2016.

Paper B. **T. Hermansson**, R. Bohlin, J. S. Carlson and R. Söderberg. *Automatic assembly path planning for wiring harness installations.* Journal of Manufacturing Systems Vol. 32. pp. 417-422, 2013.

Paper C. **T. Hermansson**, J. S. Carlson, S. Björkenstam and R. Söderberg. *Geometric variation simulation and robust design for flexible cables and hoses.* Journal of engineering manufacture Vol. 227, pp. 681-689, 2013.

Other relevant publications co-authored by Tomas Hermansson:

T. Hermansson, S. Vajedi, T. Forsberg, F. Ekstedt, J. Kressin, C. Toft and J. S. Carlson. *Identification of material parameters of complex cables from scanned 3D shapes.* Procedia CIRP Vol. 43. pp. 280-285, 2016, Gothenburg, Sweden.

C. Weischedel, A. Tuganov, **T. Hermansson**, J. Linn and M. Wardetzky. *Construction of discrete shell models by geometric finite differences.* The 2nd Joint Conference on Multibody System Dynamics, May 29–June 1, 2012, Stuttgart, Germany.

Contents

Part I

Introductory chapters

Chapter 1

Introduction

1.1 Background

The development of a new product from an existing idea to market introduction can be described by the *product realization process* (Figure 1.1). In the *design phase*, a new product design is developed based on different aesthetical and functional criteria. In the *planning phase*, a process for manufacturing and assembly of the suggested design is planned by generating a detailed set of tasks and instructions. Together, these two phases form the *concept phase*. Next, in the *verification phase*, the design and planning concepts are verified using physical prototypes and test series. Finally, in the *production phase*, the full production starts. The process is monitored in order to identify problems with the product and the process and to collect data and transfer knowledge to future product development.

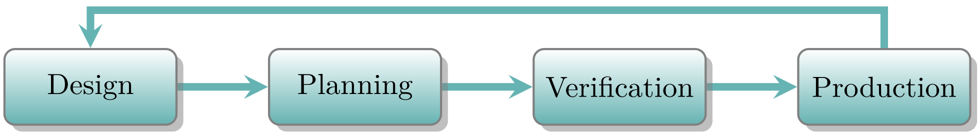


Figure 1.1: The product realization process

1.1.1 Virtual product realization

In industry today, virtual design tools are used throughout the concept phase to represent and form the geometry of the product. As changes in the design and planning concepts are extremely costly in the later verification and production phases, much can be gained if a product design can be optimized and verified with respect to the assembly process with simulation tools as early as possible.

Due to the ever-present geometrical variation in a process or a product part, the manufactured product will deviate from its intended nominal design. Variation simulation tools are therefore used in the geometry assurance process to predict and control how part variation propagates in an assembled product and to achieve a robust design. Structural simulation tools are used to compute the deformed shape of flexible components when subject to external forces and moments. Automatic path planning and sequencing tools are used to improve and verify the feasibility of the assembly process.

In this way, quality problems and geometrical interference between different parts and disciplines can be addressed early in the concept phase. The benefits are many from both a financial and sustainability perspective; shorter development times, a less demanding verification phase with a reduced number of physical prototypes, avoidance of unnecessary on-line adjustments and longer lasting products.

1.1.2 Deformable 1D objects (D1Os)

A topic of special interest is the virtual preparation of *deformable 1D objects* (D1Os) such as electrical cables and wiring harnesses, hoses, pipes and tubes. They are geometrically characterized as *one-dimensional* (or *slender*) in the sense that one dimension (the length) is significantly larger than the other two (the cross section). They are *deformable* in that they can exhibit large elastic deformations when subject to external forces and moments. In a complex product these types of components appear in a variety of applications, e.g. wired data communication, energy transfer (electrical power supply and hydraulics), heating and cooling systems, et cetera. As product failures and late on-line adjustments are often attributed to deformable objects(Ng et al. 2000; Falck et al. 2008), there is a high industrial impact if these issues can be addressed with simulation tools already in the concept phase.

1.1.3 Geometrical design and verification

As the automotive industry today is focusing more specifically on electrified and hybrid solutions, both conventional combustion engines and battery supplied electrical engines need to fit in an already densely packed vehicle (Fig. 1.2). For D1O in particular, reduction of available design space makes it increasingly more difficult to make qualified geometrical design decisions. For example, the length of the object should be chosen so that the required over-length is minimized, but at the same time without violating functional constraints on the bending radius or making it impossible to assemble. Where should clips be introduced to control the deformation with respect to variation and/or a motion? Late design changes often result in solutions involving

Figure 1.2: Three cooling systems for a traditional engine, an electrical engine and a battery.

concealed routings, tricky assembly tasks and erratic clip placements in order to avoid quality problems. In conclusion, there is a strong motivation to develop methods for geometrical design and verification of D1Os in a restricted design space.

1.2 Scope

The main scope of this thesis is to investigate

> How can simulation of D1Os in a product and/or a production system be combined with practical and efficient methods to further strengthen the virtual product realization process?

The primary focus is to develop methods for geometrical design and verification. Based on needs and requests from the industry, three activities are of significant importance in this process.

Automatic Routing

Routing is the task of choosing a reference length and (if possible) a pre-formed shape of the D1O and choosing a position for the object to join given connection points. Usually geometrical design constraints (e.g. minimal bending radius) need to be satisfied for the manufacturing of the object or for the object to function properly. Routing is typically done in the concept phase based on concept drawing or spline curves that do not represent a true physical shape and often various forms of clips need to be applied to keep it in place.

Assembly Verification

Even if a D1O is routed in a good way there is still no guarantee that it can be installed in the routed position. It is then desired to verify that the object can be assembled into the routed position from for example, a stress-free 2D build-board layout, without stretching the object or colliding with the obstacles more than necessary. Typically, routings of cable harnesses are very concealed and consist of rigid components and several branches and break-outs for which this verification is not trivial.

Variation Analysis

All product realization processes are afflicted by variation. Geometrical variation from manufacturing and assembly propagates and accumulates during production, causing deviation from the reference design in the final product. It is important to strive towards a more robust design already in the design phase and to keep the variation under control using variation analysis tools. This is specifically true for D1Os, for which the deformation is hard to predict without simulation tools for given dimensional tolerances.

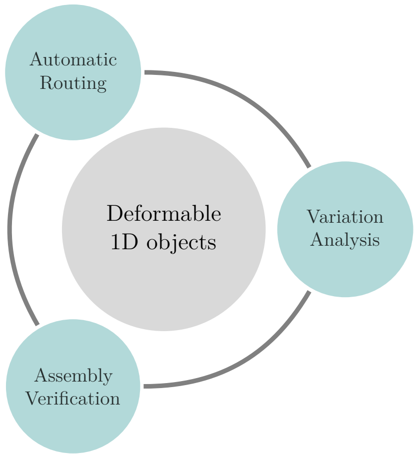


Figure 1.3: Geometrical design and verification of D1Os

A strong contribution to these activities will (i) help achieve an optimal geometrical design verified for robustness and assembly already in the design phase and (ii) shorten the development times as less time will be needed to deal with geometrical quality problems and verification in the following development phases.

1.3 Research questions

Three research questions have been formulated targeted at the activities in the geometrical design and verification process that were described in the previous section.

> **RQ1** (*Automatic Routing*): How shall a D1O be designed and routed when subject to manufacturing and functional constraints?
>
> **RQ2** (*Assembly Verification*): Can a D1O with a given design be installed in a target position?
>
> **RQ3** (*Variation Analysis*): How robust is the design of a D1O when it is subject to geometrical variation?

The main scientific challenge imposed by the research questions is to deal with the complexity of coupling an accurate simulation model with iterative algorithms for automatic path planning and variation simulation in an efficient way.

1.4 Research methodology

The research presented in this thesis is related to product design. *Design Research Methodology* (DRM) is a framework proposed in (Blessing and Chakrabarti 2009) that divides design research into four stages:

Research Clarification Formulate research goals and success criteria to evaluate the outcome of the research.

Descriptive Study I Understand and study the current situation and identify how to improve the situation.

Prescriptive Study Improve the current situation by developing methods and tools in order to reach the desired state.

Descriptive Study II Evaluate the methods and tools to verify that they satisfy the success criteria.

The research goals were set based on scientific challenges and needs and requests from the automotive industry. The success criteria were defined as the developed methods being able to accurately solve industrial size problems in reasonable time. A survey of available methods was performed and the relevant research areas were identified in order to formulate the research questions (**Research Clarification**). A literature review was then made in order to position the research questions within each identified research area and to identify any knowledge gaps (**Descriptive Study I**). Methods and tools were developed and the results were implemented in a demonstrator (**Prescriptive Study**). The results were consequently evaluated and refined based on both academic and industrial benchmark tests and feedback from industrial users (**Descriptive Study II**).

Implementation of results

The research results were implemented in the spirit of the Wingquist Laboratory implementation strategy in order to secure knowledge transfer in an integrated way. The results were presented and implemented in demonstrator versions of the virtual tool IPS Cable Simulation. The functionality is now an integral part in the commercial version of the software.

1.5 Contributions

The main contributions of the research presented in this thesis can be summarized as follows:

Scientific contributions

- A novel method for collision-free routing of 1D objects with geometrical design constraints combined with simulation-based optimization (Paper A).
- A novel method for collision-free assembly planning for (systems of) D1Os (Paper B).
- A scheme for generating envelopes for visualization of the worst case geometrical outcome of a D1O due to dimensional tolerances (Paper C).

Industrial contributions

- An efficient implementation of an accurate simulation model based on Cosserat rod theory well suited for coupling with iterative algorithms.
- A new set of tools for geometrical design and verification of D1Os in order to further strengthen the virtual product realization process.
  - The tools can help saving significant development time and reducing the number of iterations between the design phase and the planning and verification phases.
  - The tools have been implemented in the commercial version of the software IPS Cable Simulation.

1.6 Outline

In Chapter 2 a preliminary overview of simulation of D1Os, automatic path planning and variation simulation are given. In Chapter 3 the results are summarized and the methods presented in the appended papers are described. The findings are concluded in Chapter 4 and a discussion and outlook for future work is presented.

Chapter 2

Preliminaries

This chapter provides a preliminary overview of simulation of deformable 1D objects (D1Os), automatic path planning and variation simulation. These research areas are key ingredients in understanding and providing answers to the research questions.

2.1 Simulation of D1Os

The methods presented in this thesis rely on having a simulation model that enables *efficient* and *accurate* computations of large spatial deformations of flexible 1D objects. A limitation is applied to finding *static equilibria*, i.e. inertial effects are considered not important or of minor influence for the purpose of this research.

2.1.1 Static equilibria

Assume that a deformable object is in static equilibrium and undergoes a small virtual deformation (or displacement). The principle of virtual work then states that the virtual external work $\delta W^{(ext)}$ done by forces acting on the object and the virtual internal work $\delta W^{(int)}$ done by elastic forces in the object add up to zero,

$$\delta W^{(int)} + \delta W^{(ext)} = 0. \tag{2.1}$$

The work accumulated by the internal forces is stored in the elastic strain energy E of the object. If the external forces are conservative and hence stem from a potential V, Eq. 2.1 is equivalent to

$$\delta \Pi = 0, \tag{2.2}$$

where $\Pi := E + V$ is the *total potential energy* for the object. Hence, a deformed state in static equilibrium is therefore characterized as a *stationary point* to the total potential energy functional. This special case is sometimes known as the *principle of minimum potential energy* (Reddy 2007). Although Eq. 2.2 only requires the point to be stationary, (local) minimization of the potential energy ensures that the static equilibrium is *stable*.

2.1.2 Cosserat rod theory

Special theories have been derived for the treatment of D1Os. In continuum mechanics this class of objects is commonly referred to as *rods*. The fiber-like structure of a rod suggests a mathematical model in terms of a space curve corresponding to its center line and a director frame defining the orientation of the local cross section plane. Geometrically exact *Cosserat rod theory* accounts for large deformations in the form of both shearing, stretching, bending and torsion. For a thorough discussion of the physical aspects of Cosserat rod theory, the reader is referred to (Antman 1974; Antman 2005) and the seminal work in (Reissner 1973; Simo 1985).

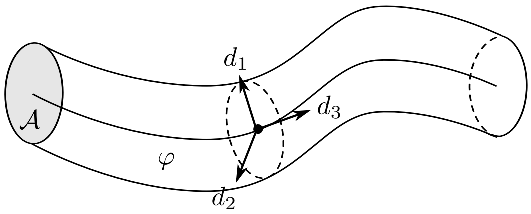


Figure 2.1: A configuration of a rod.

Formally, a configuration of a rod of reference length L can be described by a framed space curve q in $\mathbb{R}^3 \times \mathrm{SO}(3)$ parametrized by reference arc length s;

$$q : [0, L] \ni s \mapsto (\varphi(s), R(s)) \in \mathbb{R}^3 \times \mathrm{SO}(3). \tag{2.3}$$

Here, $R = (d_1, d_2, d_3) \in \mathrm{SO}(3)$ describes the evolution of the cross section orientation along the center curve φ. Furthermore, $\mathcal{S}(q) \subset \mathbb{R}^3$ denotes the volumetric shape of the rod.

In order to apply the principle of minimum potential energy, the stored strain energy relative to a stress free reference configuration $q_{(0)}$ needs to be computed for a given configuration. The elastic strain energy density of a rod is composed of quadratic forms in terms of frame invariant *strain measures*. For a configuration q, the shearing/stretching strain $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ and curvature/torsion strain $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ in material coordinates read

$$\Gamma(s) = R(s)^T \partial_s \varphi(s) - e_3, \tag{2.4a}$$

$$\hat{\Omega}(s) = R(s)^T \partial_s R(s). \tag{2.4b}$$

Here, $\Gamma_{1,2}$ are the shearing strain components in the d_1 and d_2 directions and Γ_3 is the tension strain, whereas $\Omega_{1,2}$ are the bending curvature strain components and Ω_3 is the torsion strain. With a hyper-elastic constitutive law, the corresponding stored shearing/stretching and bending/torsion energy densities with respect to the reference configuration read

$$w^{(\Gamma)}(s) = \frac{1}{2}(\Gamma(s) - \Gamma_{(0)}(s))^T K^{(\Gamma)}(\Gamma(s) - \Gamma_{(0)}(s)), \tag{2.5a}$$
$$w^{(\Omega)}(s) = \frac{1}{2}(\Omega(s) - \Omega_{(0)}(s))^T K^{(\Omega)}(\Omega(s) - \Omega_{(0)}(s)), \tag{2.5b}$$

for known effective stiffness tensors $K^{(\Gamma)}$ and $K^{(\Omega)}$ and reference configuration strains $\Gamma_{(0)}$ and $\Omega_{(0)}$. The total stored strain energy is then obtained from integration of the strain energy densities (Eqs. 2.5) along the rod,

$$E = \int_{s=0}^{L} \left\{ w^{(\Gamma)}(s) + w^{(\Omega)}(s) \right\} ds.$$

In a (conservative) gravitational field with field constant g, the external potential energy is evaluated as

$$V = -\int_{s=0}^{L} K^{(\rho)} g^T \varphi(s) ds,$$

where $K^{(\rho)}$ is the known length density.

2.1.3 Deformable 1D systems

The simulation model extends to the general case of deformable 1D systems, when the object is an interconnected system of n deformable 1D segments. A configuration q is then given by n configurations $q_1 \dots, q_n$. Furthermore, we assume that the object is constrained by m grip points such that grip point j places a boundary condition position Q_j on object i_j at reference arc length s_j according to $q_{i_j}(s_j) = Q_j$. Segments can then be connected to each other by constraints placed on the involved grip points.

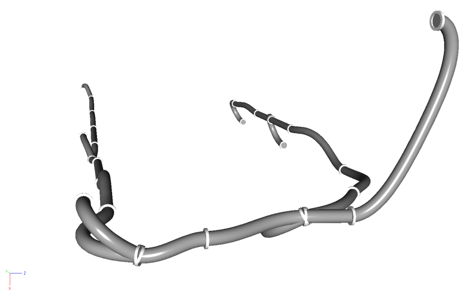


Figure 2.2: A wiring harness modelled as a deformable 1D system.

2.1.4 Numerical solutions

In order to find numerical solutions to the static equilibrium equation (Eq. 2.2), the representation of the rod as in Eq. 2.3 needs to be discretized by dividing it into a finite number of segments. Finite Element Methods approximate the rod with piecewise polynomial basis functions that satisfy the variational form in Eq. 2.3 for small linear deformations. Co-rotational finite elements can account for large deformations as well (Nour-Omid and Rankin 1991). An added benefit is that non-conservative forces, e.g. normal forces from internal hose pressure, can be prescribed in weak variational form.

For our purposes, we settle with a more straightforward approximation. We choose a quaternion-based parameterization for the geometrically exact rod model in Eq. 2.3. The strain measures in Eq. 2.4 are approximated by geometrical finite differences on the spatially discretized model as described in (Lang et al. 2011). The static equilibrium is then found by minimizing the potential energy Π with respect to the generalized variables of the discrete system. This can be done in a computationally efficient way with a Newton-based non-linear minimization solver. A modelling benefit of the energy minimization approach is that it easily extends to deformable 1D systems by summation of potential energy for the whole system and constraints on the system can be added as constraints to the minimization problem without the explicit administration of Lagrangian multipliers.

2.2 Automatic path planning

> Research questions **RQ1** and **RQ2** deal with finding collision-free configurations of D1Os. These problems can be stated as *path planning problems*.

The basic path planning problem is to find a sequence of configurations that takes an object from a start configuration to a goal configuration without colliding with any obstacles. This problem arises in a wide range of industrial applications, e.g. off-line programming of industrial robots and assembly verification for parts in an assembly. Sometimes there is a desire to additionally optimize some integrated quantity along the path (e.g. robot cycle time) and in other cases it is the existence of a collision-free path that is of primary interest (e.g. assembly verification).

The computational complexity of automatically solving the path planning problem is in general believed to be very high. Determining the existence of a collision-free path has been proven to be NP-complete for the basic path planning problem and PSPACE-hard for polyhedral objects and obstacles in particular (Canny 1988). Since complete[1] algorithms are of little industrial relevance because they are too slow, different sampling based techniques trading completeness for speed and simplicity have gained much interest over the years. Probabilistic complete methods such as the Probabilistic Roadmap Method (L. Kavraki et al. 1996; Bohlin and L. Kavraki 2000) and Rapidly-Exploring Random Trees (Lavalle 1998) are capable of solving problems with many degrees of freedom. Deterministic and resolution complete methods solve the problem in finite time with a sufficiently fine resolution (Barraquand and Latombe 1993; Bohlin 2001). Common for these methods are the needs for efficient collision detection, graph representation, nearest neighbor searching, and graph

[1] An algorithm is complete if it will always find a solution or determine that none exist.

searching. For a comprehensive introduction to the theory of path planning, the reader is encouraged to read (Latombe 1991; Gupta and Plobil 1998; Laumond et al. 1998; Choset et al. 2005; LaValle 2006).

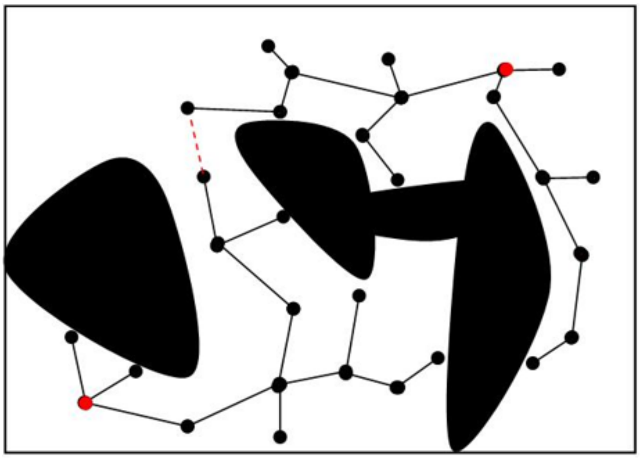


Figure 2.3: A probabilistic roadmap over a 2D configuration space.

2.2.1 Non-holonomic path planning

From a path planning perspective, routing a 1D object can be interpreted as the problem of finding a collision-free path for a 2D slice of the cross section of the object. The path will correspond to the centerline of the object and can be subject to e.g. curvature constraints due to the minimal allowed bending radius. In *non-holonomic path planning*, differential constraints usually reduce the controllable degrees of freedom to fewer than the configuration space dimension. Analytical methods have been derived for car-like vehicles in 2D in (Dubin 1957; Reeds and Shepp 1990). Probabalistic sampling methods have been adapted to different under actuated robots in the presence of obstacles; (Lamiraux and Laumond 2001; LaValle 2000; Karaman and Frazzoli 2013) and (Kuffner and LaValle 2011) implemented deterministic space-filling trees suited for this type of path planning. Specifically, methods for routing of steerable needles with curvature constraints have been presented in (Webster 2006; Patil and Alterovitz 2010).

> To answer research question **RQ1** (Automatic Routing), we need to extend these results and develop a method for D1Os in static equilibrium.

2.2.2 Path planning for deformable objects

Automatic path planning for deformable objects is acknowledged as a challenging research area (L.E. Kavraki et al. 1998); allowing the object to geometrically deform during manipulation adds complexity to the basic problem as the space of deformations is usually infinite dimensional. Also, evaluation of the simulation model and collision checking for the deformable object become huge computational bottlenecks. Another aggravating factor, as opposed to for rigid objects, is that many deformed states of the object in static equilibrium could correspond to the same set of boundary conditions. Nevertheless, there has been limited success in developing path planning algorithms for D1Os in special cases, most notably in (Bretl and McCarthy 2013; Moll and L. Kavraki 2006; Kabul et al. 2007). A comprehensive survey of path planning methods for deformable objects is given in (Jiménez 2012).

> To answer research question **RQ2** (Assembly Verification), we need to extend these results and develop a method that deals with the complexity of high-dimensional path planning for systems of D1Os.

2.3 Geometry assurance

> Research question **RQ3** deals with analysis of geometrical variation in D1Os.

All product realization processes are afflicted by variation. Geometrical variation from manufacturing and assembly propagates and accumulates during production, causing deviation from the reference design in the final product. The geometry assurance process is aimed at controlling geometrical variation in a number of activities. In the concept phase, the design is analyzed and optimized with respect to robustness and verified with respect to assumed dimensional tolerances by variation simulation (Gao et al. 1995; Cai et al. 1997; Glancy and Chase 1999; Söderberg and Lindkvist 1999). In the verification phase, physical prototypes and test series are used for verification. Non-nominal assembly planning techniques are used to reduce the need for physical test series (Berg et al. 2011; Carlson, Spensieri, et al. 2013). In the production phase, inspection data is used for diagnosis and root cause analysis (Johnson and Wichern 1998; Hu and Wu 1992; Ceglarek and Shi 1996; Jin and Shi 2001; Ding et al. 2000a; Ding et al. 2000b; Carlson and Söderberg 2003).

2.3.1 Virtual variation analysis

Virtual methods for robust design and variation simulation of rigid assemblies are well established in the industry. In the framework proposed in (Söderberg, Lindkvist, Wärmefjord, et al. 2016), a set of integrated tools are presented for the whole virtual geometry assurance process. The tools are available in the software RD&T. Here, a subset of the virtual tools for variation analysis is outlined.

Stability Analysis

In stability analysis (Söderberg and Lindkvist 1999), the robustness of the design and the locating scheme is evaluated. A stability matrix is generated by perturbing the locator point variables one at a time and computing the resulting displacements of key feature points distributed in the assembly. This matrix contains first-order information about how the locator point variables contribute to design sensitivity. The sensitivity in the reference points is visualized with color-coding (Fig. 2.5).

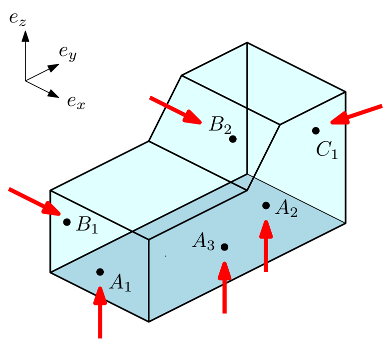


Figure 2.4: A 3-2-1 locating scheme.

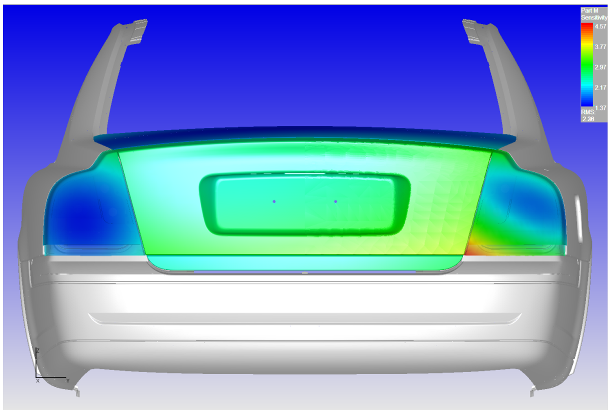


Figure 2.5: Color-coding of sensitivity in a rigid assembly.

Variation Simulation

In variation simulation, the variation in critical measures is evaluated for given statistical distributions and tolerances for the design dimensions. Here, the Monte Carlo simulation method is used. The method randomly generates numbers for the design dimensions according to their distributions and evaluates the critical measures to estimate their statistical properties. The method captures non-linearities and allows any kind of statistical distribution as input variation.

Tolerance Envelope Analysis

As part of an extensive visualization tool set, tolerance envelope analysis provides a visualization of the geometrical effect of variation by computing the smallest volume enclosing the object for given tolerances. In (Lööf et al. 2006), a tolerance envelope is generated for a rigid assembly using convex partitioning and convex hull computations for a large set of variation simulation outcomes.

2.3.2 Analysis of non-rigid assemblies

For non-rigid assemblies (i.e. assemblies consisting of deformable parts), over-constrained locating schemes may be used. There are many significant results related to variation analysis of 2D sheet metal assemblies, e.g. in (Liu et al. 1996; Cai et al. 1997; Ceglarek and Shi 1997; Camelio et al. 2004; Lindau et al. 2015). The framework for variation analysis described in the previous section has been extended to sheet metal assemblies in (Söderberg, Lindkvist, and Dahlström 2006), however not to complicated deformable 1D systems such as wiring harnesses with multiple branches and break-outs.

> To answer research question **RQ3** (Variation Analysis), we need to extend the framework for virtual variation analysis to deformable 1D systems.

Chapter 3

Results

In this chapter we give a brief summary of the results and methods presented in the appended papers. Each paper and method addresses one corresponding research question.

As an illustrative example, the methods are applied to a scenario from the automotive industry as shown in Fig. 3.1. The scenario consists of three cooler hoses joined in a T-section connecting the radiator with the oil cooler and the engine cooling system. The geometries are courtesy of NEVS.

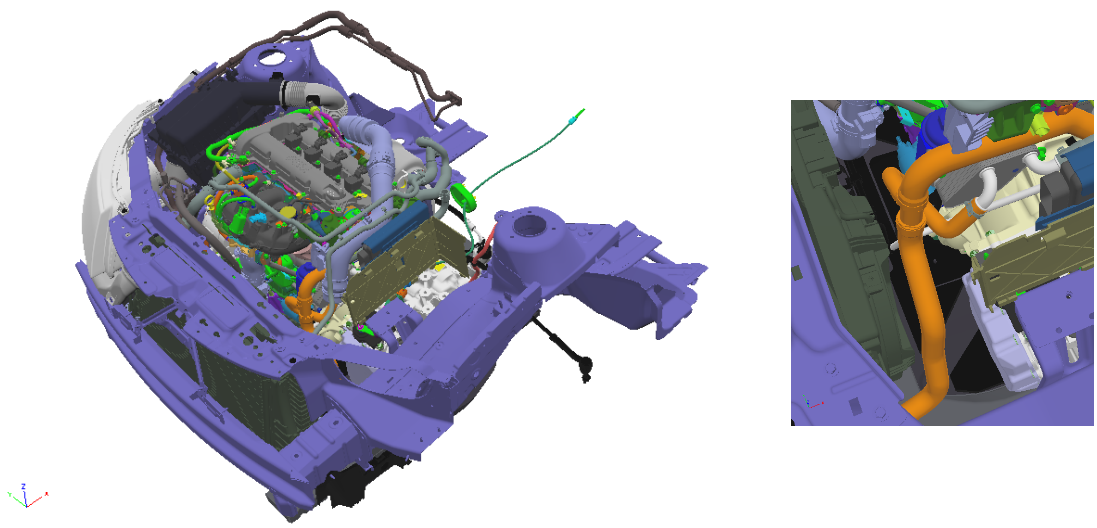


Figure 3.1: Cooler hoses in the engine compartment of a car.

3.1 Paper A: Automatic Routing

Paper A presents a method tailored to answer the first research question restated as follows:

> **RQ1** How shall a D1O be designed and routed when subject to manufacturing and functional constraints?

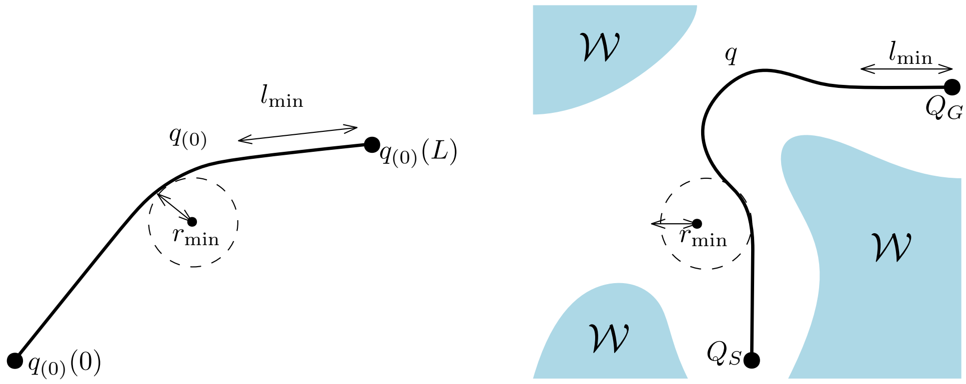
The research question is formalized in a problem description.

3.1.1 Problem description

Routing a D1O is the task of finding

- a reference configuration $q_{(0)}$ and a reference length L (the manufactured shape of the object at rest) that satisfies given manufacturing constraints on the form $h_m(q_{(0)}) \geq 0$,
- a routed configuration q of reference length L that connects given start and end points ($q(0) = Q_S$, $q(L) = Q_G$), is collision-free with respect to the obstacles $\mathcal{W}$, ($\mathcal{S}(q) \cap \mathcal{W} = \emptyset$), satisfies given functional constraints on the form $h_f(q) \geq 0$. The routed configuration should also be in static equilibrium with respect to $q_{(0)}$, $\delta\Pi(q) = 0$.

The manufacturing and functional constraints are typically a minimally allowed bending radius or straight length. If a feasible routing can be found, there are usually infinitely many solutions to choose from. It is then desired that the solution is *optimal* in some sense. Usually the notion of optimality is related to shortest



(a) Reference configuration $q_{(0)}$ of length L satisfying manufacturing constraints.

(b) Optimal routed configuration q satisfying functional constraints.

Figure 3.2: Illustration of the routing problem.

possible reference length L and the object staying in preferred regions of space (e.g. due to heat). We assume that these preferences can be collected in a weighted cost functional $q \mapsto J_w(q) \in \mathbb{R}^+$ for given weights $w \geq 0$.

3.1.2 Method

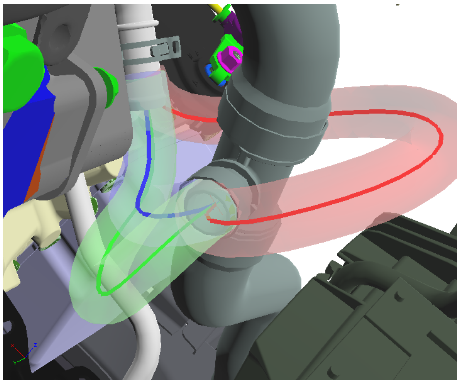
Routing a 1D object can (as noted in Section 2.2.1) be stated as a non-holonomic path planning problem. When the object is deformable, it is not clear from literature how to incorporate the static equilibrium condition in the problem in an efficient way. Therefore, we propose a segregated approach in two steps:

First, we find an optimal configuration of the object that connects the start and end points, is collision-free, satisfies the manufacturing constraints (**Nominal Routing**). Then, using the simulation model to ensure that static equilibrium with respect to the boundary conditions holds, we locally refine the solution so that it satisfies the functional constraints (**Local Refinement**). If an acceptable solution is not found, we restart the procedure with a revised set of weights.

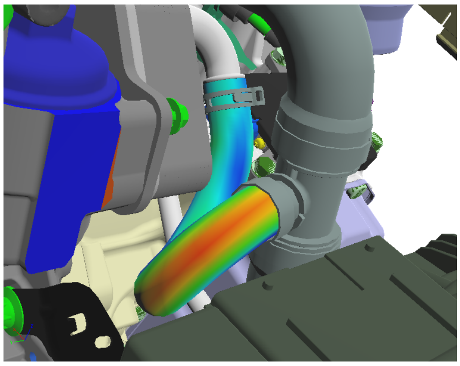
Nominal Routing Nominal routing of a 1D object is the task of finding an optimal configuration $q^{(i)}$ that connects the start and end points, is collision-free and satisfies the manufacturing constraints. We assume that the cross section profile is constant. Furthermore, we are at this point only interested in torsion-free configurations. The nominal routing problem can then be stated as a non-holonomic path planning problem for a slice of the cross section in the configuration space $\mathbb{R}^3 \times S^2$. To solve this problem numerically, we construct a discrete roadmap over a uniformly sampled grid in the configuration space that encode the manufacturing constraints in the feasible edge set. We then compute the path of minimal cost between the start and end points on the roadmap. If a solution $q^{(i)}$ is found, it determines the initial reference length $L^{(i)}$ and is used as the initial solution to the local refinement problem. Optionally, it can serve as a reference configuration $q_{(0)}$ for objects for which a non-straight pre-formed design can be realized.

Local Refinement Local refinement is the task of starting from an initial configuration $q^{(i)}$ and trying to retain both optimality and feasibility when the object is in static equilibrium. The adjustable design variables x are typically the reference length L or rotation of the start and end points. For a given set of variables x and an initial configuration $q^{(i)}$, the simulation model gives back a configuration q that satisfies the boundary conditions and is in static equilibrium, $x, q^{(i)} \mapsto q(x, q^{(i)})$. We minimize the cost functional J_w with respect to x using a gradient-free solver for unconstrained minimization problems (Nelder and Mead 1965) and enforce the functional constraints with an added cost penalty term. Collisions between the object and the obstacles $\mathcal{W}$ are resolved by adding normal contact forces in the simulation model.

If an acceptable solution is not found, we either (1) restart the nominal routing procedure with a revised set of parameters to the problem (e.g. the cost functional weights w) or (2) control the deformation by introducing clips along $q^{(i)}$.



(a) Red, green and blue nominal routing alternatives corresponding to different weights.



(b) The green routing alternative in static equilibrium after local refinement.

Figure 3.3: Routing of a cooler hose from the T-section to the oil cooler.

Finally, we note that the local refinement step can be done as a stand-alone procedure (independently of the nominal routing step), to optimize the design of a D1O that already has a specified configuration.

3.1.3 Summary of Paper A

Paper A presents a method for automatic routing of D1Os with geometrical design constraints. In order to deal with the complexity of path planning for D1Os, the method is segregated into a resolution complete grid search step and a simulation-based local optimization step. The method was verified for an academic benchmark case and was successfully applied to an industrial scenario.

3.2 Paper B: Assembly Verification

Paper B presents a method tailored to answer the second research question restated as follows:

> **RQ2** Can a D1O with a given design be installed in a target position?

The research question is formalized in a problem description.

3.2.1 Problem description

We treat the general case when the D1O is a system of deformable 1D segments and is manipulated by moving a set of grip points (see Section 2.1.3).

Let $q_{(T)}$ be a configuration of the D1O in its assembled (target) position. The assembly verification problem is the task of verifying whether or not the D1O can be manipulated into $q_{(T)}$ by moving the grip points. We require that the manipulation is quasi-static, i.e. the configuration q of the D1O at each time instant is in static equilibrium. Furthermore, if a solution exists, we are interested in establishing a manipulation that does not strain the D1O more than necessary and stays away from the obstacles $\mathcal{W}$.

3.2.2 Method

There are several difficulties associated with path planning for deformable objects (see Section 2.2.2). When there are multiple grip points to manipulate we venture into the area of high-dimensional path planning. Also, many equilibrium configurations can satisfy the same set of boundary conditions, meaning that after manipulation of the object its configuration could be topologically inconsistent with $q_{(T)}$ even when the grip point positions are aligned. To overcome these difficulties, we propose a low-dimensional path planning approach in four steps:

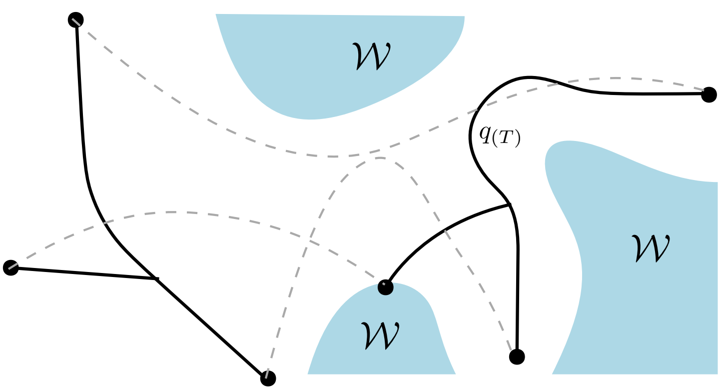


Figure 3.4: Assembly manipulation of a D1O into the assembled position.

Let the D1O be positioned in its target configuration $q_{(T)}$. Imagine that we first relax all boundary conditions (**Grip Point Relaxation**). We then pull the object away from the obstacles by moving a handle grip point and let the object unfold (**Handle Path Planning**). During this process we trace the trajectory of the free grip points and reverse and smooth the resulting manipulation (**Smoothing**). In a final step, contacts during the manipulation (if any) are resolved by adding supplementary grip points (**Grip Point Supplementation**).

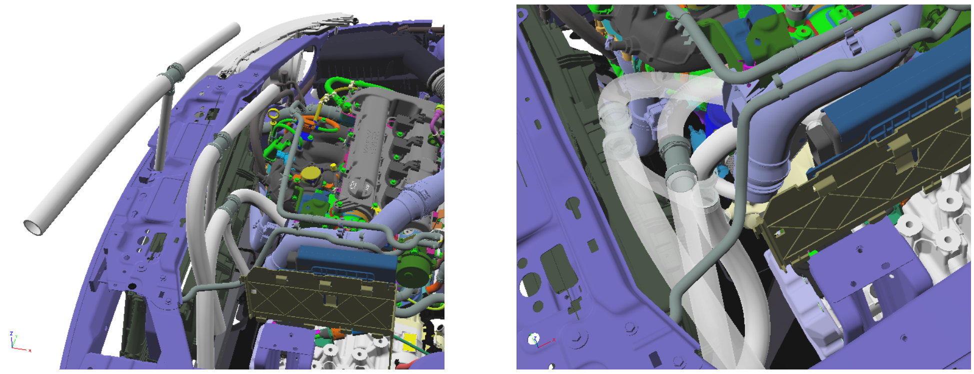
Constraint Relaxation First, an additional handle grip point is added to the object in its target configuration $q_{(T)}$. Then, all other grip points are relaxed, i.e. the boundary conditions are no longer imposed. The object will attain an equilibrium configuration held in place by an applied force at the handle grip point and external contact forces from the obstacles. When solving for equilibrium by minimizing the object's potential energy, the trajectory of each (free-hanging) grip point is traced and added to the solution manipulation.

Handle Path Planning A collision-free path is now sought for a ball $B_r \in \mathbb{R}^3$ of radius $r > 0$ starting from the handle grip point position to a goal position far away from the obstacles. This path planning problem in $\mathbb{R}^3$ is solved on a uniform grid using standard graph search techniques. If a collision-free path is found, for a large enough radius r, the object will passively follow due to internal forces when the handle grip point is moved along the path and unfold completely when there is free space. Again, the trajectory of each grip point is traced and appended to the solution manipulation.

Smoothing The solution manipulation is reversed so that we now have a manipulation where the grip points end up at their target positions. Also, the handle grip point is removed. For the trajectory of each grip point, we employ a simple smoothing procedure with respect to curvature and clearance.

Grip Point Supplementation Finally, when applying the quasi-static solution manipulation to the object, unwanted contact points between the object and the obstacles could still occur. They can be resolved by attaching one or several supplementary grip points to the object.

If an acceptable assembly manipulation is not found, it might be necessary to increase or decrease the length of the object depending on the situation.



(a) The first part of the assembly manipulation.

(b) The final part of the assembly manipulation.

Figure 3.5: Assembly manipulation of three cooler hoses.

3.2.3 Summary of Paper B

Paper B presents a method for verifying if a D1O with a given design can be installed in a target position by manipulating a set of grip points. If the verification is true, the method outputs a smooth synchronized manipulation. In contrast to existing path planning methods for deformable objects the method is based on low-dimensional path planning for a handle grip point and generalizes to when the D1O is a complex system of deformable 1D segments. The method was verified for an academic benchmark case and was successfully applied to an industrial scenario.

3.3 Paper C: Variation Analysis

Paper C presents a method tailored to answer the third and final research question restated as follows:

> **RQ3** How robust is the design of a D1O when it is subject to geometrical variation?

The research question is formalized in a problem description.

3.3.1 Problem description

We treat the general case when the D1O is a system of deformable 1D segments and is positioned by fixation of a set of grip points (see Section 2.1.3).

Let the deformable 1D system be affected by variation in a set of design dimensions. The design dimensions can e.g. be the length dimensions $L_1, \dots, L_n$, the fixed grip point locations $s_1, \dots s_m$ or the fixed grip point positions $Q_1, \dots, Q_m$. The configuration of the object then formally depends on a set of N stochastic design variables collected in a vector $x = (x_1, \dots, x_N)^T$, each x_i belonging to a statistical distribution and having a symmetrical tolerance $[t_i^{(-)}, t_i^{(+)}]$ about the reference value $\bar{x}_i$. One or more critical measures may be defined in a vector $y = (y_1, \dots, y_M)^T$.

Variation analysis is then the task of evaluating the robustness of a design and performing a variation simulation in order to estimate the statistical distributions of y and to visualize the geometrical variation.

3.3.2 Methodology

In order to perform virtual variation analysis for D1Os, we extend the framework described for rigid assemblies in Section 2.3.1.

The generalization to D1Os is conceptually straightforward. Each perturbation of the design variables $\tilde{x}$ now yields a responding equilibrium configuration, for which

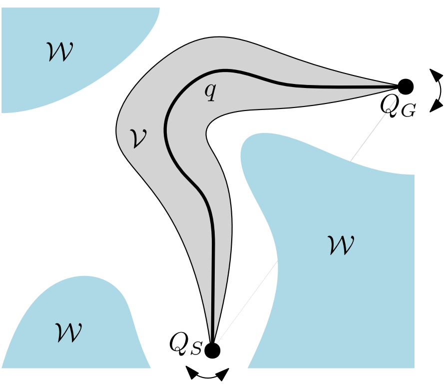


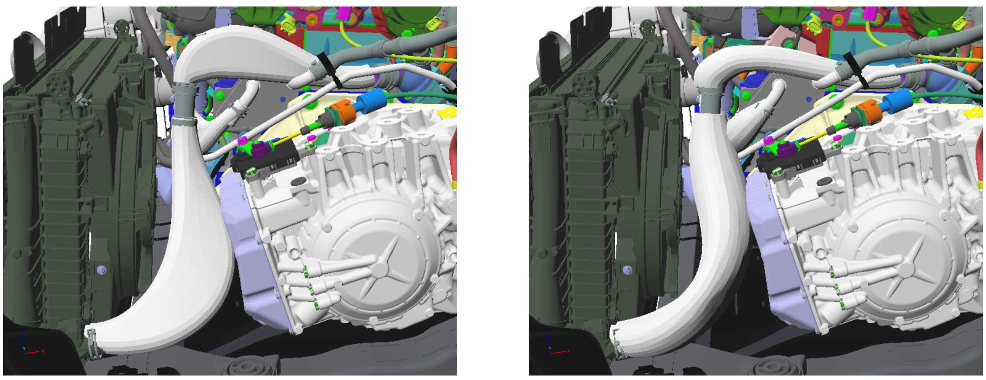
Figure 3.6: A tolerance envelope of a D1O with respect to rotational fixture tolerances at grip points Q_S and Q_G.

critical measures $y(\tilde{x})$ and displacements can be computed. Hence, the simulation model will be evaluated for each perturbed evaluation. For the first-order **Stability Analysis**, this overhead is not a major problem. However, for a full-scale **Variation Simulation**, the number of random samples needed in the Monte Carlo method can become potentially huge. This stresses the fact that it is important that the simulation model is computationally efficient to evaluate. Also, even for small tolerances, it is important that the simulation model accounts for large spatial deformations when treating non-robust configurations.

Tolerance Envelope Analysis

The main innovation in the proposed methodology is the generation of tolerance envelopes for deformable 1D systems. Formally, a tolerance envelope for a D1O is defined as the smallest possible volume $\mathcal{V} \subset \mathbb{R}^3$ enclosing the object satisfying the given set of tolerances: $S(\tilde{q}) \subseteq \mathcal{V}$ for all x such that $x_i \in [\bar{x}_i - t_i^{(-)}, \bar{x}_i + t_i^{(+)}]$. Hence, a tolerance envelope is a visualization of the worst case geometrical outcome.

First, we use variation simulation to generate a spanning set of configurations of the object. The statistical distributions of the design variables are not important, as the tolerance envelope is a worst case visualization. We therefore use the Monte Carlo simulation method and sample the design variables uniformly within their tolerances. We now assume that the volumetric shape $\mathcal{S}$ of a D1O is approximated by a sequence of triangulated cylinders (representing its boundary surface). A discrete approximation of $\partial\mathcal{V}$ is then achieved by generating the convex hull for each such cylinder when the object assumes all the configurations in the spanning set, and then taking the union of all convex hulls. Besides the obvious computational bottleneck in evaluating the simulation model, there is a potential build-up in required memory to hold the discrete envelope. To overcome this, we remove all the interior triangulated patches from the envelope after the union operation by employing a marching cube algorithm, thus keeping only its silhouette.



(a) Tolerance envelope with respect to a length tolerance of $+/-50mm$ on each hose.

(b) Tolerance envelope with respect to a rotational cone tolerance of $+/-15°$ on each hose connection.

Figure 3.7: Tolerance envelope analysis of three cooler hoses.

Finally, we note that tolerance envelopes need not necessarily be generated with respect to just dimensional design variables. They can also be used to visualize the effect of variation in for example, the material parameters of the D1O.

3.3.3 Summary of Paper C

Paper C presents a methodology for analysis and visualization of geometrical variation in D1Os. The methodology naturally extends on the existing framework for virtual geometry assurance for rigid and sheet metal assemblies described in Section 2.3.1. The main innovation in the paper is the construction of tolerance envelopes of D1Os based on convex hull computations and silhouette generation. The method can be applied to complex systems of deformable 1D segments and accounts for physically correct deformations. The methodology was successfully applied to two industrial scenarios.

Chapter 4

Conclusions and future work

This thesis has presented three computational methods for geometrical design and verification of deformable 1D objects (D1Os). The first method solves the problem of routing a D1O with respect to geometrical design constraints. The second method solves the assembly verification problem for a D1O with a given design. The third and final method is in fact a methodology for performing variation analysis and visualization for D1Os. Together with existing virtual tools, the three methods form a powerful tool set for design and process engineers. Quality problems and geometrical interference in the assembled product can now to a larger extent be addressed in the concept phase, thus saving significant development time and reducing the number of iterations between the design phase and the planning and verification phases.

This research work has been following the Wingquist Laboratory philosophy, where the starting point is an industrial need and an industrial challenge to be investigated and solved. Part of the focus has been to meet a need from the manufacturing industry by developing new practical methods that are implemented and ready to use. As a result, the functionality is now an integral part in the commercial software IPS Cable Simulation as of version 3.1 (2016).

The main scientific challenge has been to deal with the complexity of coupling a simulation model with iterative algorithms for optimization, automatic path planning and variation simulation in an efficient way. The path planning problems are solved in two steps separating the path planning algorithm from the simulation model, which can sometimes result in suboptimal solutions. In terms of simulation accuracy, all methods rely on having a simulation model that enables efficient and accurate computations of large special deformations of flexible 1D objects. The model of choice is a finite-difference based Cosserat rod model that extends to systems of D1Os. Of course, the accuracy of the results also heavily depends on that the material parameters supplied to the model are authentic.

Some important steps have been taken towards strengthening the virtual product realization process. Considered topics for future work are the following:

Geometry Assurance Apart from variation analysis tools, the framework in (Söderberg, Lindkvist, and Carlson 2006) also consists of optimization tools for the locating scheme. It should be considered to also extend these tools to D1Os.

Process Planning Problems with robot dress packs are one of the major reasons for on-line adjustments of robot motions and for down time in robot stations (Eriksson 2005b; Eriksson 2005a). The dress packs consist of attached cables and hoses which typically have significant impact on allowed robot configurations and motions in the station. It would be interesting to take the dresspack into account in off-line programming and optimization of robot stations and continue the work presented in (Carlson, Kressin, et al. 2016).

Human Assembly Most often the installation of D1Os in an assembly is a manual process. It would be interesting to incorporate D1Os in a virtual human simulation to analyse different ergonomic aspects of human assembly and continue the work presented in (Delfs et al. 2014).

Automatic Routing The routing problem can be stated as an optimal control problem. Although solving a path planning problem is probably still needed in order to obtain a collision-free initial solution, the physics and the functional constraints could be incorporated as differential constraints in the optimal control problem.

Assembly Verification If a feasible assembly operation exists, then the Assembly Verification method produces a manipulation of a set of given grip points that takes the object to its target position. As the number of grip points that can be manipulated at once is usually limited, questions of a combinatorial nature arise: How many grip points (hands) are required to perform an assembly operation? Is there a preferred connecting sequence order?

On a final note, a common task in electrical CAD software is to flatten (i.e. transform to the unstressed configuration) a wiring harness geometry modelled in its connected configuration. By converting the geometrical representation of the harness to a rod configuration and using the Grip Point Relaxation procedure described in Section 3.2.2, this would become a useful tool for optimization of harness manufacturing.

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