Robust ambiguity estimation for automated analysis of Intensive sessions

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Abstract Very Long Baseline Interferometry (VLBI) is a unique space-geodetic technique that can directly access the Earth's phase of rotation, namely UT1. The daily estimates of the difference between UT1 and Coordinated Universal Time (UTC) are computed from 1-hour long VLBI Intensive sessions. These sessions are essential in providing timely UT1 estimates for satellite navigation systems. To produce timely UT1 estimates, efforts have been made to completely automate the analysis of VLBI Intensive sessions. This requires automated processing of X- and S-band group delays. These data often contain an unknown number of integer ambiguities in the observed group delays. In an automated analysis with the c5++ software the standard approach in resolving the ambiguities is to perform a simplified parameter estimation using a least-squares adjustment (L2-norm minimisation). We implement the robust L1-norm with an alternative estimation method in c5++. The implemented method is used to automatically estimate the ambiguities in VLBI Intensive sessions on the Kokee–Wettzell baseline. The results are compared to an analysis setup where the ambiguity estimation is computed using the L2-norm. Additionally, we investigate three alternative weighting strategies for the ambiguity estimation. The results show that in automated analysis the L1-norm resolves ambiguities better than the L2-norm. The use of the L1-norm leads to a significantly higher number of good quality UT1-UTC estimates with each of the three weighting strategies.

Keywords VLBI, UT1, robust estimation, ambiguities, Intensives

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1 Introduction

Very Long Baseline Interferometry (VLBI) is a unique space-geodetic technique capable of simultaneously determining all Earth Orientation Parameters (EOPs). These parameters include Universal Time (UT1), the Earth's phase of rotation. Space-geodetic techniques such as Global Satellite Navigation Systems (GNSS) depend on regular UT1 estimates from VLBI. For this purpose the International VLBI Service for Geodesy and Astrometry (IVS) [1] organises daily 1-hour Intensive (INT) VLBI sessions to provide timely UT1-UTC estimates. The currently observed INT sessions include INT1 (Kokee–Wettzell, Monday to Friday), INT2 (Tsukuba–Wettzell, Saturday and Sunday), and INT3 (Wettzell, Tsukuba, and Ny-Ålesund, Monday).

The relatively low number of observations and the baseline geometry of the INT sessions pose challenges for the data analysis and limit the UT1-UTC accuracy that can be achieved. To produce timely UT1 estimates the turnaround time of the INT sessions needs to be minimized. This requires a streamlined VLBI processing chain. A way to achieve this is to automatically process and analyse the data. Aspects and results related to automated near-real time analysis of INT sessions have been demonstrated in e.g. Hobiger et. al. (2010) [2] and Kareinen et. al. (2015) [3]. The operational INT sessions are observed on the legacy S/X frequency band. The two bands are formed by individual channels, which are in the post-correlation process combined on the respective bands using the bandwidth synthesis technique [4]. This leads to an unknown number of integer ambiguities in the observed group delays. These ambiguities need to be resolved before ionospheric calibration and parameter estimation. If unresolved, the group delay ambiguities will propagate into the UT1-UTC estimates.

C5++ [2], Vienna VLBI Software (VieVS) [5], CALC/SOLVE [6], GEOSAT [7], and OCCAM [8] are some of the currently used VLBI software packages. From these software packages only c5++ and CALC/SOLVE are capable to resolve the group delay ambiguities and to produce ambiguity- and ionosphere-free X-band databases.

The standard parameter estimation method in these VLBI software packages is the least-squares adjustment [9] (i.e. L2-norm minimisation). In this work we implement the robust L1-norm for the parameter estimation and apply it to automatically resolve the ambiguities in the INT sessions. Starting from Version-1 databases, we use a modified version of c5++ which includes the implemented L1-norm estimation to analyse and estimate UT1-UTC from 1885 INT1 sessions observed between 2001 and 2015.

1.1 L1-norm minimisation

Both the L1- and L2-norm minimisations can be derived from the general expression for a p-norm, which is given by

$$||x||_p = \left(\sum_{i=1}^{p} |x_i|^p \right)^{\frac{1}{p}}. \quad (1)$$

The objective functions to be minimised for the L1- and L2-norms are given respectively by

$$\mathrm{L1} : \mathbf{p}^\mathsf{T}|\mathbf{v}| \rightarrow min, \quad (2)$$

$$\mathrm{L2} : \mathbf{v}^\mathsf{T}\mathbf{P}\mathbf{v} \rightarrow min. \quad (3)$$

With the L2-norm the absolute value in the summand disappears. Thus it can be solved analytically, making it computationally straightforward. For the L1-norm, however, the absolute value of the residual vector $\mathbf{v}$ remains. Thus, the objective function is not differentiable at zero, and we are unable to directly derive the value for the vector of unknowns $\mathbf{x}$ that will minimise the sum of the weighted absolute values of the residuals.

The formulation for a L1-norm minimisation has been described in e.g. Amiri-Simkooei (2003) [10]. Following this general formulation, in order to deal with the absolute value function in the Equation 2, we re-write the vectors $\mathbf{v}$ and $\mathbf{x}$ with the help of slack variables. This will reduce the problem to that of linear programming. These vectors are now given by

$$\mathbf{v} = \mathbf{u} - \mathbf{w}, \quad \mathbf{u}, \mathbf{w} \geq 0, \quad (4)$$

$$\mathbf{x} = \boldsymbol{\alpha} - \boldsymbol{\beta}, \quad \boldsymbol{\alpha}, \boldsymbol{\beta} \geq 0, \quad (5)$$

where a condition u_i or $w_i = 0$ holds for the residual vector components. Now, given the conditions in Equation 4, Equation 2 can be written as

$$\mathbf{p}^\mathsf{T}|v| = \mathbf{p}^\mathsf{T}|\mathbf{u} - \mathbf{v}| = \mathbf{p}^\mathsf{T}(\mathbf{u} + \mathbf{w}), \quad (6)$$

subject to the conditions in Equation 5,

$$\mathbf{u} - \mathbf{w} = \mathbf{A}(\boldsymbol{\alpha} - \boldsymbol{\beta}) - \mathbf{y}. \quad (7)$$

The objective function can thus be written as

$$min \left(\begin{bmatrix} \mathbf{0}^\mathbf{T} & \mathbf{0}^\mathbf{T} & \mathbf{p}^\mathbf{T} & \mathbf{p}^\mathbf{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \mathbf{w} \\ \mathbf{u} \end{bmatrix} \right), \quad (8)$$

subject to

$$\begin{bmatrix} \mathbf{A} & -\mathbf{A} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \mathbf{w} \\ \mathbf{u} \end{bmatrix} = \mathbf{y}, \quad (9)$$

given the same conditions as earlier. Denoting the objective function with z this form is equivalent to

$$z = \mathbf{c}^\mathsf{T}\mathbf{x}, \quad (10)$$

subject to

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \quad \mathbf{x} \geq 0. \quad (11)$$

The L1-norm minimisation was implemented in c5++ with an external python script. The corresponding linear programming problem was solved using a Simplex-method [11] implemented in the *linprog* function of the optimisation module in SciPy [12].

The main advantage of the L1-norm over the L2-norm is its robustness against outliers. The L1-norm sums absolute deviations instead of squared values. This means that large residuals are not emphasized as they are with the L2-norm. Thus, the L1-norm is better at detecting the magnitude of large outliers, while the

L2-norm overcompensates the influence of large deviations. These errors will propagate to the unknown parameters in the adjustment.

2 Ambiguity estimation in c5++

The general ambiguity estimation process in c5++ is iterative. The X- and S-band group delays are processed as independent observations, which retains the integer-nature of the ambiguities. The estimated parameters are three clock polynomial terms for the non-reference station and local wet troposphere at both stations, giving a total of 5 parameters. When the UT1-UTC is estimated at a later stage, the number of estimated parameters increase to 6. In each iteration step the residuals are computed and if they are larger than 50 % of the ambiguity spacing in that band, the corresponding observations are shifted by one ambiguity spacing towards 0. This process of ambiguity shifting is iterated until the ratio of the Weighted Root Mean Square (WRMS) values from subsequent iterations reaches a specific level. In this analysis the ratio limit was set to 0.999. The maximum number of iterations was set to 60. During the estimation process, different weighting schemes can be applied. The effect of the choice of weighting was investigated using three different approaches, which are described in Table 1.

Table 1 The three different weighting approaches used in the ambiguity estimation.

	Description	Weighting
W1	Unit weighting	1
W2	Formal errors	$\frac{1}{\sigma_\tau}$
W3	Formal errors × wmf(e)	$\frac{1}{\sigma_\tau\sqrt{\mathrm{mf}(e)^2_{\mathrm{wet},1}+\mathrm{mf}(e)^2_{\mathrm{wet},2}}}$

Once the ambiguities are resolved, the X- and S-band data are combined to produce an ionosphere free X-band database. This database is then subsequently used as an input in the UT1-UTC estimation step. At this stage the observations were weighted according to the elevation dependent approach, W3. The schematics in Figure 1 illustrate the ambiguity and the UT1-UTC estimation process in c5++.

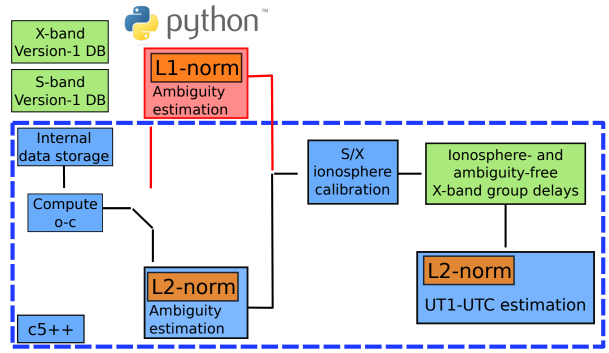

Fig. 1 Schematics of the automated ambiguity and UT1-UTC estimation in c5++.

3 Results

The impact of using the L1-norm was evaluated by investigating the post-fit residuals from the ambiguity and UT1-UTC estimation stages as well as the UT1-UTC results computed using the ambiguity resolved databases.

All EOP except UT1-UTC were fixed to their EOP C04 08 [13] values. The UT1-UTC were estimated with respect to the a priori C04 08 values. From now on these values will be referenced simply as the UT1-UTC estimates. In order to focus on the sessions which produced meaningful UT1-UTC estimates and to eliminate gross errors that would distort the derived statistics of the UT1-UTC, the estimates were filtered with a condition where the absolute values of the estimates are larger than 1000 μs and/or the formal errors are larger that 50 μs. After this initial outlier elimination was applied, we obtained a set of sessions for each ambiguity estimation method–weighting mode pair, for which RMS and WRMS of the UT1-UTC values were computed. The RMS and WRMS values for the post-fit residuals from the ambiguity estimation for both norms and all weighting strategies are listed in Table 2. Table 3 lists statistics for the post-fit residuals from the UT1-UTC estimation.

The Venn-diagrams in Figure 2 illustrate the overlap between the good sessions from the L1- and L2-norm approaches. These diagrams show the number of good sessions which are found in both L1 and L2, only L1, or only L2 results. To investigate the differences in the sessions, where either the L1- or L2-norm approaches fail or succeed, we consider subsets of the di-

Table 2 Mean RMS and WRMS of the post-fit residuals from the ambiguity estimation for L1- and L2-norms for all weighting strategies.

	L1		L2	
	$\overline{\text{RMS}}$ [ns]	$\overline{\text{WRMS}}$ [ns]	$\overline{\text{RMS}}$ [ns]	$\overline{\text{WRMS}}$ [ns]
W1	3.61	3.61	4.17	4.17
W2	6.23	1.43	6.20	1.71
W3	6.10	1.22	6.23	1.46

Table 3 Statistics for the post-fit residuals from the UT1-UTC estimation with the L1- and L2-norm approaches and weighting strategies W1, W2, and W3. Included are RMS, mean of absolute values, and median of absolute values.

	L1			L2		
	$\overline{\text{RMS}}$ [ns]	$\overline{\lvert\text{res}\rvert}$ [ns]	M(|res|) [ps]	$\overline{\text{RMS}}$ [ns]	$\overline{\lvert\text{res}\rvert}$ [ns]	M(|res|) [ps]
W1	0.39	0.26	29.68	1.07	0.75	32.24
W2	1.51	1.10	35.69	1.98	1.47	38.69
W3	1.38	1.00	34.69	1.81	1.34	37.69

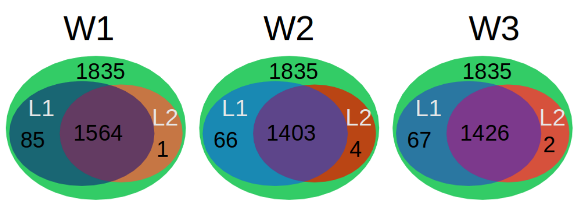

Fig. 2 Venn-diagrams for the weighting strategies W1, W2, and W3 illustrating the overlap between the sets of sessions obtained with the L1- and L2-norm ambiguity estimation, that pass the $|\text{UT1-UTC}| < 1000\ \mu$s and $\sigma_{\text{UT1-UTC}} < 50\ \mu$s criteria.

agrams illustrated in Figure 2. The following subsets are considered for all weighting strategies:

- Subset-1: select all sessions that are good with the L1-norm approach, select the same sessions from the L2-norm solutions.
- Subset-2: select all sessions that are exclusively good with the L1-norm approach, select the same sessions from the L2-norm solutions.
- Subset-3: select all sessions that are exclusively good in the L2-norm approach, select the same sessions from the L1-norm solutions.

The number of sessions in each of the Subsets and all three weighting strategies are given in Table 4.

The L1-norm leads to approximately 5 % more sessions compared to the L2-norm. In general, the big advantage in using the L1-norm for the ambiguity estimation is the increased number of succesfully processed sessions. The overall accuracy in WRMS remains at the level of approximately 18 μs.

Table 4 The number of sessions in for Subset-1, -2, and -3 for all weighting strategies W1, W2, and W3.

	Number of sessions		
	Subset-1	Subset-2	Subset-3
W1	1564 + 85 = 1649	85	1
W2	1403 + 66 = 1469	66	4
W3	1426 + 67 = 1493	56	2

Table 5 Number of sessions and corresponding RMS/WRMS of UT1-UTC values for the sessions included in Subset-1.

		L1		L2	
Units: [μs]	#Sessions	RMS	WRMS	RMS	WRMS
W1	1649	22.58	18.39	938.09	18.70
W2	1469	22.32	18.43	805.21	18.82
W3	1493	22.25	18.43	1096.07	18.74

Table 6 Number of sessions and corresponding RMS/WRMS of UT1-UTC values for the sessions included in Subset-2.

		L1		L2	
Units: [μs]	#Sessions	RMS	WRMS	RMS	WRMS
W1	85	18.83	22.54	3934.66	4130.73
W2	66	17.11	19.38	3280.11	3797.42
W3	67	19.48	19.55	2646.68	5173.04

In Subset-1 (see Table 5) the inclusion of sessions which fail with the L2-norm shows up as high RMS values compared to that of the L1-norm. This is not seen in the WRMS values, which are on a normal level for both norms, with only sligthly (sub-microsecond) higher values for the L2-norm.

In Subset-1 (see Table 5) the inclusion of the sessions that are filtered in the L2-norm solution is seen as high L2-norm WRMS values compared to the respective L1-norm WRMS values. This indicates that the large values for the UT1-UTC corrections, causing the large RMS in the L2-norm, have correspondingly large formal errors. Thus, these sessions get downweighted in the WRMS computation.

In Subset-2 (see Table 6) we see a clear difference both in RMS and WRMS values between the two norms. The large WRMS values for the L2-norm indicate that the formal errors for the filtered UT1-UTC estimates have similar magnitude compared to one another.

4 Conclusions

The use of the L1-norm shows a clear improvement for the automated ambiguity estimation for the INT sessions in terms of the increased number of sessions that produce a good quality UT1-UTC estimate. Smaller RMS and WRMS values of the post-fit residuals from the ambiguity estimation also indicate that the ambiguity estimation benefits from the L1-norm. The sessions where the L1-norm performs better in ambiguity estimation are almost identical in terms of average number of observations compared to the whole data set of the analysed INT1 sessions. Thus, the benefit gained with the L1-norm is not correlated with particularly high or low number of observations in these sessions. The number of sessions that are improved by the L1-norm approach greatly outnumber the ones where the issues of stability result in a failed ambiguity estimation. The computational complexity of solving the linear programming problem compared to inverting the normal equations does not generally cause significant overhead in the processing time of an individual session. The L1-norm using the W1 weighting (i.e. equally weighted) produced the biggest increase in good quality UT1-UTC estimates. Further information can be found in [14].

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