Cramér-Rao Lower Bounds for Battery Estimation

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EXTENDED ABSTRACT

To secure safety, reliability and performance of an electrified vehicle, it is important to monitor the State of Charge (SoC) of its battery. Today, there are no sensors that can measure SoC directly. Instead, it is usually estimated with an algorithmic filter. Since batteries are nonlinear, all feasible filters are only able to approximate the posterior densities which, in other words, means that their performances will be more or less suboptimal (Särkkä, 2013).

To be able to evaluate the performance of a filter, it is of great value to know how well a parameter or a state can be estimated. It can then be decided if it is worth spending time on tuning the filter, or implementing a more advanced filter. Furthermore, analyzing the achievable accuracy can be a way to better understand the application.

One suitable measure for benchmarking the performance is the Cramér-Rao Lower Bound (CRLB), which is a lower bound on the Mean Square Error (MSE) of any estimator.

In this paper we adopt a method to numerically determine the posterior CRLBs with a Monte Carlo-based algorithm. The posterior CRLBs are calculated for combined estimation of the states and the parameters of a commonly used equivalent circuit model. It is investigated how the posterior CRLBs depend on the amplitude and the frequency of the current. Furthermore, the posterior CRLBs are computed for a commercially available lithium- ion battery using data from laboratory experiments, and the results are compared to the MSEs of an Extended Kalman Filter (EKF). It is shown that the MSEs of the EKF are close to the posterior CRLBs, which means that the EKF seems to be a good observer for this application.

Battery model

In online applications it is common to model batteries as equivalent circuits (see e.g. (Seaman et al., 2014)). The drawback is that equivalent circuits have a limited relation to the nonlinear chemical phenomena inside the battery and, therefore, only describe the system in a neighborhood around the present state values. A remedy to this is to continuously adapt the parameters of the equivalent circuit in order to adapt the model to the present state of the battery. This means that, in addition to the states, also the parameters in the equivalent circuit have to be estimated.

The zero-order hold discrete time state space model of a commonly used equivalent circuit model is

$$\begin{bmatrix} u_1(k+1) \\ u_2(k+1) \\ z_{soc}(k+1) \end{bmatrix} = \begin{bmatrix} e^{-\frac{\Delta t}{\tau_1}} & 0 & 0 \\ 0 & e^{-\frac{\Delta t}{\tau_2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ z_{soc}(k) \end{bmatrix} + \tag{1a}$$

$$\begin{bmatrix} R_1(1-e^{-\frac{\Delta t}{\tau_1}}) \\ R_2(1-e^{-\frac{\Delta t}{\tau_2}}) \\ \dfrac{\eta_i \Delta t}{Q_n} \end{bmatrix} i_{cell}(k) \tag{1b}$$

$$y(k) = U_{ocv}(z_{soc}(k)) + u_1(k) + u_2(k) + R_0 i_{cell}(k), \tag{1c}$$

where k is the time step, z_{soc} is SoC, u_1 and u_2 are voltages, R_1 and R_2 are resistances in parallel with capacitors C_1 and C_2, R_0 is the ohmic resistance, $\tau_1 = R_1C_1$ and $\tau_2 = R_2C_2$ are time constants of the circuit, Δt is the sampling time, Q_n is the nominal capacity, and η_i is the Coulombic efficiency.

Cramér-Rao Lower Bound

The posterior CRLB was initially proposed in H. L. V. Trees (2001) as a lower bound for the MSE of any estimator, i.e.

$$MSE(\hat{x}_k) \geq J_k^{-1}, \tag{2}$$

where $\hat{x}$ is the estimator for an n-dimensional state vector and J represents the $n \times n$ Fischer matrix, defined as

$$J_k = E\{-\nabla_{x_k} \nabla_{x_k}^T \log\, p(x_k, y_k)\}, \tag{3}$$

where $p(x_k, y_k)$ denotes the joint probability density of the states and the measurement.

In general, analytical expressions of the posterior CRLBs are intractable for nonlinear systems and therefore, they are usually calculated numerically. In Taylor et al. (2003), a Monte Carlo-based algorithm, similar to a particle filter, is proposed. In Klintberg et al. (2016) this algorithm is adopted for calculating the posterior CRLBs for the battery model.

Combined state and resistance estimation

In combined estimation of the states and a parameter, it is expected that the accuracy is reduced compared to when the parameter is perfectly known. The question is, how much degradation of the accuracy can be expected and under what circumstances is it significant?

To investigate how the accuracy is affected when the resistance is estimated compared to when it is perfectly known, the CRLBs for the two cases are compared. As can be seen in Figure 1, the posterior CRLB for the SoC is higher in combined estimation whenever the current is nonzero.

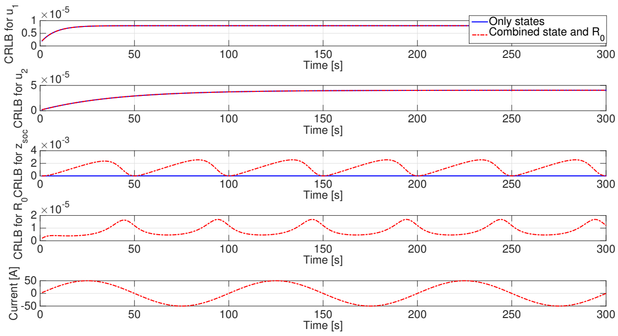

Fig. 1. Posterior CRLBs for standalone estimation of the states compared to combined estimation of the states and the resistance.

To investigate the dependency on current, the posterior CRLBs are calculated with sinusoidal current profiles with different amplitudes and with different frequencies. In Figure 2 it can be seen that the resulting posterior CRLB is lower for the resistance and higher for SoC for higher amplitudes of the current.

In Figure 3, it can be noticed that the posterior CRLBs decreases for both SoC and the resistance as the frequencies of the current increases.

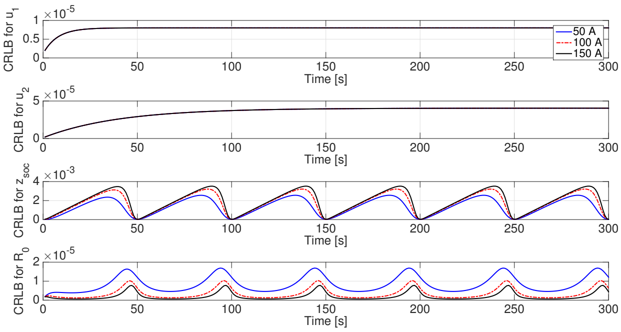

Fig. 2. Posterior CRLBs for different amplitudes of sinusoidally shaped currents. The amplitudes are 50 A, 100 A and 150 A.

Comparison with EKF on experimental data

In this section the EKF is evaluated against the posterior CRLBs in combined estimation of the states, the resistance and the capacity. The evaluation is performed on an equivalent circuit model tuned to fit experimental data, which means a nonlinear OCV-curve and parameters that depend nonlinearly on SoC and the current. The experimental data come from lab tests of a commercially available lithium-ion cell intended for use in a plug-in hybrid electric vehicle.

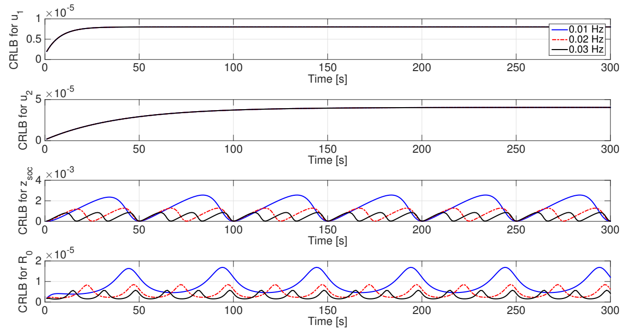

Fig. 3. Posterior CRLBs calculated with different frequencies of sinusoidally shaped currents.

The MSEs of the EKF were obtained from 100 Monte Carlo simulations where new trajectories were generated for every simulation. In Figure 4 it can be seen that the MSEs for the EKF are close to the posterior CRLBs, which means that an EKF seems to be a good observer for this application.

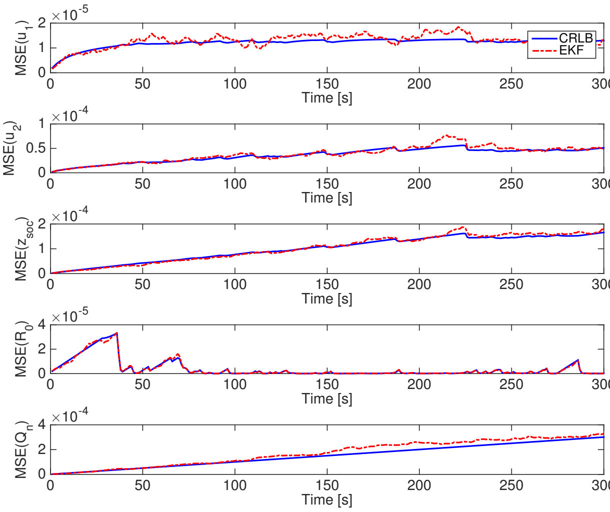

Fig. 4. Posterior CRLBs compared to MSEs from the EKF.

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