

Friction utilization for tyre-road friction estimation on snow: an experimental study

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ABSTRACT: Friction estimation using effect-based approaches are challenging during normal driving due to the large tyre excitation needed for an accurate estimate. The required excitation level varies for different tyres, road surfaces, road conditions and tyre models used in the estimator. Previous research has investigated the required friction utilization on different surfaces but due to the small sample sizes it is hard to draw any general conclusions. This paper investigates the tyre excitation required to estimate the tyre-road friction coefficient with a generic estimator for 76 different tyres on snow for five different tyre models and for different levels of measurement noise.

1 INTRODUCTION

The maximum tyre-road friction limits the horizontal forces that the tyres can generate. Knowledge about the tyre-road friction coefficient is thus useful for active safety and driver assistance systems where the appropriate intervention is dependent on the maximum available road grip. Vehicles with increasing levels of autonomous functions, such as autonomous emergency braking or, in the future, fully autonomous vehicle, can use this information to adapt the intervention thresholds or vehicle velocity based on the prevailing road conditions. The information can also be shared with other drivers or vehicles to warn them of an approaching area with low friction.

The tyre-road friction coefficient is difficult to estimate during normal driving, mainly due to the limited information about the maximum achievable tyre forces in the linear tyre region, see (Lex, 2015) and (Ray, 1997). Previous research have attempted to correlate the slip stiffness to the maximum road friction coefficient to remove the need for large tyre excitation (Gustafsson, 1997). However, the relation between the friction coefficient and the slip stiffness relies on empirical data. A-priori information about the tyres that are currently fitted to the vehicle is thus required. Another approach was evaluated experimentally in (Albinsson et al., 2015). Opposite wheel torques were added to the front and the rear axle to achieve large tyre excitation during normal driving, see also (Chen and Wang, 2011). This method, active tyre force excitation, makes it possible to achieve

large tyre forces during normal driving and thus to estimate the friction coefficient.

The required utilization, i.e. here defined as the ratio of the required and maximum normalized longitudinal force, to estimate the friction coefficient within an accuracy of $\pm 15\%$ for different tyres and road surfaces is investigated in (Svendenius, 2007). However, only a few different tyres were evaluated using the brush model and it is therefore difficult to draw any general conclusions.

This study investigates how well the tyre-road friction coefficient can be estimated by fitting a non-linear tyre model to measurements on a snow surface for 76 different tyres. The estimation error that can be expected without any a-priori information about the tyre can thus be evaluated with a large sample size. Furthermore, the estimation error for different tyre models are evaluated and compared, thus indicating which tyre models that are more suitable for friction estimation on snow. Noise was added to the measurements signals to investigate how different fitting error cost functions influence the estimation accuracy.

2 METHOD

The measurements were performed for another study with the BV12 measurement vehicle from VTI (The Swedish National Road and Transport Research Institute), (Hjort and Eriksson, 2015). Four to six slip ratio sweeps up to a fully locked wheel was done in each test run. The data was filtered and the offset at

zero slip was removed so that the tyre force is zero at zero longitudinal slip. The data was used as the reference tyre characteristics when evaluating the different tyre models. These tyre models assumes steady-state conditions and the slip ratio gradient should thus be kept small.

During the measurements, the maximum slip ratio gradient was around 0.2 $[(ms^{-1}) / (ms^{-1}) 1/s]$ at small tyre forces. With a first order relaxation model, Equation 1, the difference between the nominal $\sigma_{(x,nom)}$ and the actual slip ratio σ_x can be calculated as,

$$\dot{\sigma}_x = (\sigma_{x,nom} - \sigma_x) \frac{v_x}{r_{lx}} \quad (1)$$

$$(\sigma_{x,nom} - \sigma_x) = \dot{\sigma}_x \frac{r_{lx}}{v_x} \quad (2)$$

where r_{lx} is the relaxation length and v_x is the longitudinal velocity of the vehicle. Assuming that the relaxation length is 0.1 and with the longitudinal velocity of 8.33 m/s the difference between the nominal and the actual slip ratio is $0.0024[(ms^{-1}) / (ms^{-1})]$. At a slip level of 0.05 $[(ms^{-1}) / (ms^{-1})]$ this corresponds to a 5% error.

The tyre relaxation will hence have an influence on the tyre characteristics and so will other measurement errors. However, the characteristics of specific road-tyre combinations are not interesting in themselves for the purpose of this study but rather the variation in the tyre-road characteristics represented by these different tyres.

The tested tyres were 9 new and 17 used studded tyres, 9 new and 16 used Nordic non-studded winter tyres, and 9 new and 16 used European non-studded winter tyres.

2.1 Tyre Models and Tyre Parameter Fitting

Four different tyre models are evaluated, the brush model with parabolic pressure distribution, the magic tyre formula with 4 parameters (Bakker et al., 1987), Burckhardt tyre model and Dugoff tyre model (Dugoff et al., 1969), see Equation 3-6. These tyre models are commonly used for online friction estimation, where the number of parameters is a concern due to the limited number of measurement points and since the estimation is normally done recursively.

$$f(\sigma_x) = \begin{cases} -C\sigma_x + \frac{C^2\sigma_x|\sigma_x|}{3\mu_{max}} - \frac{C^3\sigma_x^3}{27\mu_{max}^2} \text{ for } |\sigma_x| \leq \frac{3\mu_{max}}{C} \\ -sign(\sigma_x)\mu F_z \text{ else} \end{cases} \quad (3)$$

$$f(\sigma_x) = D \sin(C_m \tan^{-1}(B\sigma_x - E(B\sigma_x - \tan^{-1}(B\sigma_x)))) \quad (4)$$

$$f(\sigma_x) = c_1(1 - e^{c_2\sigma_x} - c_3\sigma_x) \quad (5)$$

$$f(\sigma_x) = \begin{cases} C\sigma_x(2 - \lambda)\lambda \text{ for } |\lambda| < 1 \\ C\sigma_x \text{ else} \end{cases} \quad (6)$$

$$\lambda = \frac{\mu_{max}}{2|C\sigma_x|}$$

The tyre model parameters are fitted to the measurements to minimize two different cost functions,

$$J_f = \sum_{i=1}^N (f_m[i] - f_d[i])^2 \quad (7)$$

$$J_s = \sum_{i=1}^N (f_m[i] - f_d[i])^2 + \left(\frac{\partial f}{\partial \sigma_d} |_{\sigma_d[i]} (\sigma_m[i] - \sigma_d[i]) \right)^2 \quad (8)$$

f_m is the normalized force from the tyre model at the measured slip ratio, f_d is the measured normalized force, σ_d is the measured slip ratio and σ_m is the slip ratio at the measured normalized force obtained from the inverse tyre model. The cost function J_f in Equation 7 does not penalize the slip ratio error and should thus reasonably make the tyre parameter estimates more sensitive to measurement noise in the slip measurements. Equation 8 is inspired by the total least square approach but has been simplified so that the terms in the cost function is available directly from the tyre model. Figure 1 illustrates the different components in the cost functions, J_f penalizes Δf , J_s penalizes Δf and $\Delta \sigma$.

By using the cost function in Equation 8 rather than the total least square method the evaluation of the cost function is made computationally more efficient. A total-least square approach use the perpendicular distance between the measurement point and the model. However, the expressions for finding the corresponding model point σ_m, f_m are nonlinear and not easily solved analytically which results in the high computational cost. The cost function J_s is hence a compromise to get an expression that penalizes both the slip and force error that is easily obtained from the tyre models and which is easy implement online.

The parameters of the tyre models were found using the gradient based optimization function `fmincon` with the interior-point algorithm in Matlab. The parameters of the tyre models were constrained to obtain reasonable tyre characteristics, Table 1. The gradient based optimization approach can in some circumstances converge to a local minimum. The cost function should hence preferably have one distinct minimum and not be too flat.

Table 1, Tyre parameter constraints

C	[2,100]	B	[1,100]
μ_{max}	[0.05,1.5]	c_1	[0.05,1.5]
D	[0.05,1.5]	c_2	[0, 50]
E	[-1.5, 1]	c_3	[-0.01,0.01]
C_m	[1,1],[1,1.6]		

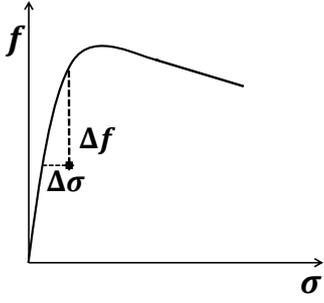


Figure 1. Cost function illustration

2.2 Evaluation Procedure

By using the reference tyre data from the measurements as a lookup table, an arbitrary excitation can be used to evaluate the different tyre models. However, it has to be ensured that the time derivative of the slip ratio is kept small enough to keep the tyre relaxation negligible. The excitation levels are described by the friction utilization μ_{util} , defined here as the ratio between the normalized force μ and the maximum friction coefficient μ_{max} .

The normalized estimated friction coefficient $\hat{\mu}_{\text{norm}}$ and the normalized friction estimation error $\tilde{\mu}_{\text{norm}}$ are defined as,

$$\hat{\mu}_{\text{norm}} = \frac{\hat{\mu}_{\text{max}}}{\mu_{\text{max}}} \quad (9)$$

$$\tilde{\mu}_{\text{norm}} = \frac{\hat{\mu}_{\text{max}} - \mu_{\text{max}}}{\mu_{\text{max}}} \quad (10)$$

These measures can be used to compare the results from different tyres since they are normalized with the maximum friction coefficient. The evaluations are done for different levels of white Gaussian noise added to the slip ratio and force signal. The noisy environment highlights the differences between the two different cost functions. The added noise signals are uncorrelated but has the same variance σ^2 . The lower noise level, $\sigma^2 = 0.003^2$, approximately corresponds to the noise levels found for the force and slip estimates during active tyre force excitation but with a lower variance for the force estimation (Albinsson et al., 2015). Other disturbances, due to for instance suspension compliance or uneven road surfaces, are not represented by the added noise. These disturbances will influence the measurements in reality but experience a behaviour that is hard to model and analyse.

3 RESULTS

The mean of the tyre-road friction coefficient estimate, normalized with the measured maximum friction coefficient, for the different tyre models as a function of the utilized friction are shown in Figure 2.

The standard deviation of the estimate is shown with an error bar.

The level of utilization required to obtain a friction estimation error of below 10% and 20% for two different noise level is presented in Figure 3-6. The noise level is the same on both the force and the slip ratio signal. Note that the limit of 10% and 20% is strict. The Dugoff model with noise std = 0.005 and using the force cost function J_f is close to the 10% error limit already from 75% utilization, see Figure 3.

The value of the cost functions for different tyre parameters has been plotted for one of the tyres with noise for the Dugoff model, see Figure 8. Different noise levels are used on the force and the slip ratio signals to more closely represent the noise found in previous work, see (Albinsson et al., 2015). Note that the cost function values has been limited to have the same scale in the different figures. The cost function J_f , which only penalizes the force error, does not have as distinct boundary at the lower friction values as the cost function J_s . The cost function J_s thus penalizes low friction estimates more than the cost function J_f . This means that the friction is more likely to be underestimated at low friction utilization when the cost function J_f is used compared to using the cost function J_s .

The reason for this can be understood by looking at Figure 1. If noise is present on both the slip signals and the force signal and only penalizes deviations in the force, measurement points as indicated in Figure 2 must be fitted by changing the friction coefficient. However, if the slip error is penalized as well the influence of these measurement points on the friction estimation is smaller. The tendency to underestimate the friction coefficient when only penalizing the force estimation can be seen in Figure 9 where the distribution of the normalized friction estimation error for low utilization is plotted.

4 DISCUSSION

The brush model with parabolic pressure distribution does not perform as well as the other tyre models. The mean friction estimate is lower than the current utilization even at high utilization levels. This is due to the poor fit that the brush model has for tyre characteristics on snow, see Figure 7. The difference in shape between the data and the tyre model makes it difficult to find tyre parameters that provides a good fit.

The Dugoff model fits the snow measurements better than the brush model. The models are based on the same basic assumptions but with different vertical pressure distribution in the contact patch. Both these models have only two parameters and have therefore difficulty to fit the data when the curvature of the non-linear tyre region does not match the one described by the model.

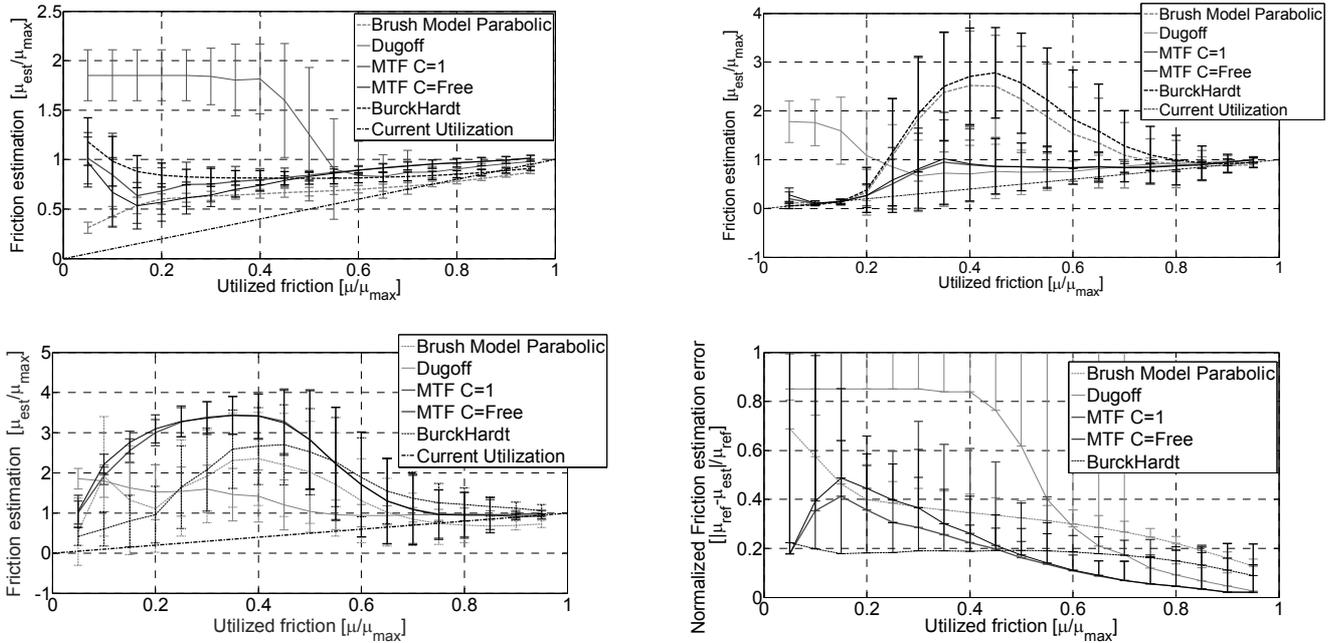


Figure 2. Mean normalized friction estimation of the measurements for the 76 tyres as a function of utilized friction with standard deviation shown as error bars. The current friction utilization is plotted as a dotted black line. Top left) Noise free input signals and minimizing force error. Top right) White Gaussian measurement noise ($\sigma^2 = 0.005^2$) added to the slip and force and minimizing force error. Bottom left) noise added to the slip and force and minimizing force & slip error (Equation 2). Bottom right) Mean and maximum friction estimation error, noise-free environment, force error minimization

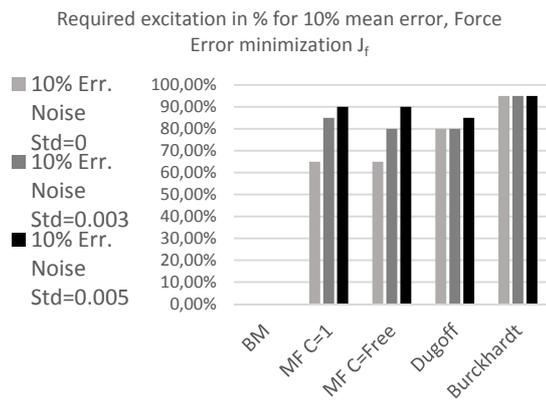


Figure 3. Required excitation in % for 10% mean error, Force Error minimization J_f

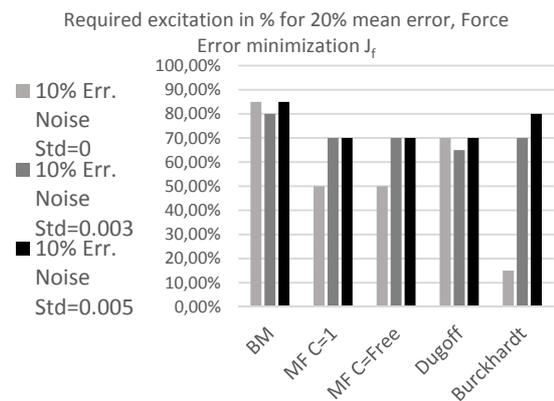


Figure 5. Required excitation in % for 20% mean error, Force Error minimization J_f

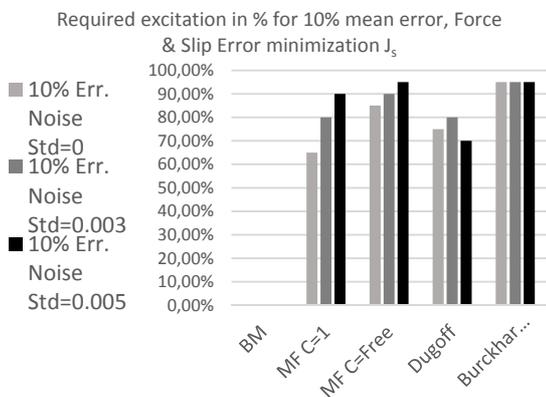


Figure 4. Required excitation in % for 10% mean error, Force and Slip Error minimization J_s

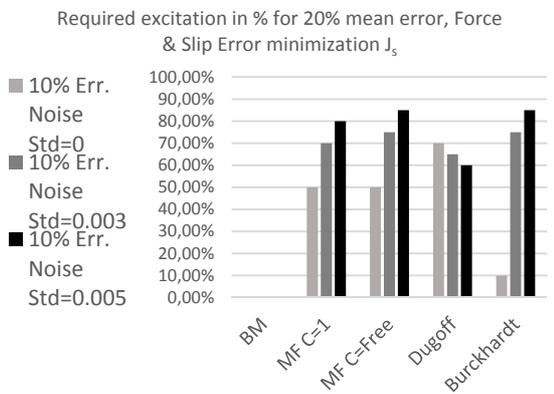


Figure 6. Required excitation in % for 20% mean error, Force and Slip Error minimization J_s

The Magic formula and the Burckhardt model have more than two parameters and can fit the snow measurements better. As seen from Figure 3-6, the magic tyre formula has the lowest required utilization for a 10% error in the friction estimate without any measurement noise.

Both the Magic tyre formula together with the Dugoff tyre model shows good promise to separate between low and high friction at low excitation and to achieve accurate friction estimation at larger excitation. The tyre model of choice should thus depend on the expected noise levels. However, it is important to note that these results are only valid for one road surface. On other road surfaces the tyre models may not perform as well. Similar studies should thus be performed on other road surfaces to investigate the change in friction estimation error for the different road surfaces. The Burckhardt model maintains the same performance in terms of required utilization (for ten percent error) when adding noise, however the Burckhardt model seems to require a larger excitation level compared to the magic formula and the Dugoff model.

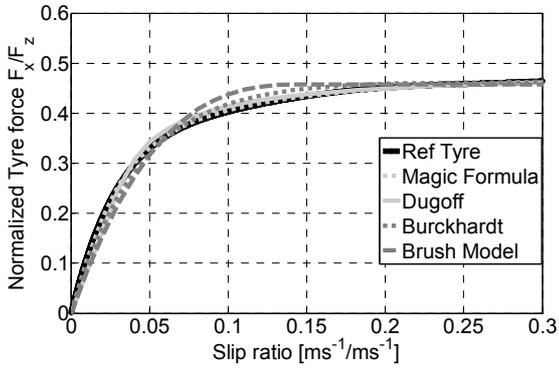


Figure 7. Example of brush model modelling error.

When adding noise, the performance of the Dugoff model does not decrease much in terms of required utilization. This makes it one of the models which require the lowest excitation levels in a noisy environment. With measurement noise, the Dugoff tyre model has lower required utilization than the magic

4.1 Cost function and measurement noise

When noise is present on both the force and the wheel speed signal, only minimizing the force error can provide poor estimates since the friction coefficient estimate tends to follow the current utilization at low tyre utilizations.

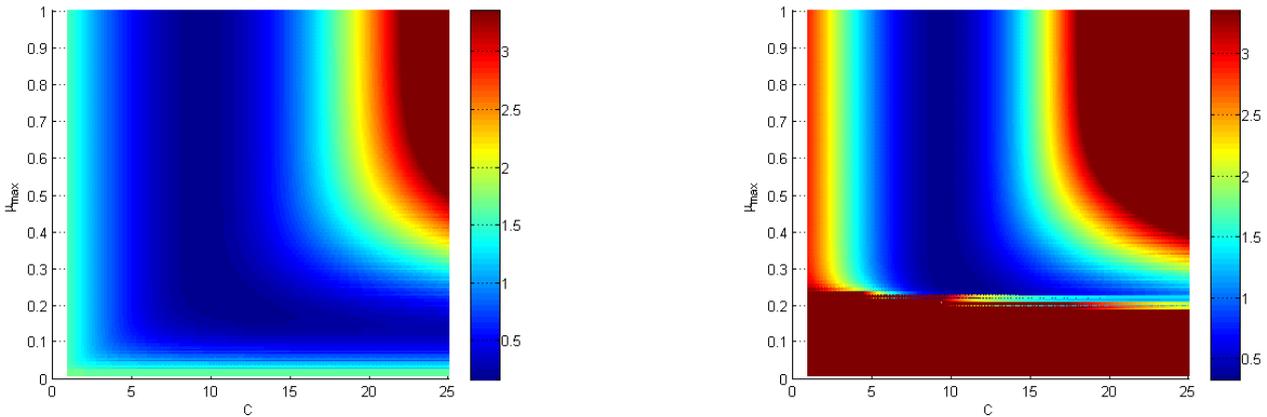


Figure 8. Cost function value for Dugoff tyre model for one of the tyres, with added noise on the force signal $\sigma_f^2 = 0.03^2$ and the slip signal $\sigma_x^2 = 0.003^2$. Right plot for J_s , left plot for J_f . All plots at 30% friction utilization with $\mu_{ref} = 0.54$.

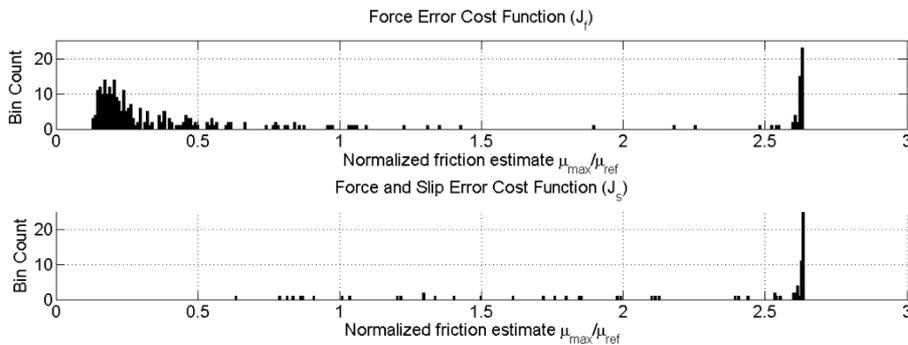


Figure 9. Normalized Friction estimation distribution μ_{max}/μ_{ref} for the brush model at 20% friction utilization and for 300 noise realizations. Force noise variance $\sigma_f^2 = 0.03^2$ and slip noise variance $\sigma_x^2 = 0.003^2$.

The additional slip term in the alternative cost function, Equation 2, creates a more distinguished separation between low and high friction at low tyre utilizations, see Figure 2. It penalizes low friction estimates more compared to only minimizing the force error, see Fig 10. Although the actual friction coefficient is unknown, the separation makes it possible to distinguish between high- and low-friction surfaces at low excitation levels. The Dugoff model is not as sensitive to measurement noise as the other models at low excitation. This is likely due to the fact that the Dugoff model is exactly linear at low tyre excitations. The cost function should thus be chosen depending on the noise level and the chosen tyre model.

4.2 Required Tyre Excitation

The required tyre excitation to estimate the friction coefficient with an average error of less than 10% and 20% is presented in Figure 3-6. As seen from the figures, the lowest required excitation can be found for the magic tyre formula (both with locked and estimated C_m parameter) that has a required excitation of 65% (for 10% error) in a noise free environment. As expected the required excitation increases for increasing noise levels and with a white Gaussian noise with a standard deviation of 0.005 the required excitation has increased to 90% for the magic formula. This trend is similar for both cost functions.

Naturally, noise and disturbances cannot be avoided in the real-world application where compliances in the suspension, uneven road surfaces, varying pitch angle all affect the measurement signals. These disturbance are most likely not zero mean white Gaussian noise as used in this study and may further increase the need for large excitations. Nevertheless, the results illustrates the importance of actively working towards minimizing any noise and disturbances.

It should also be noted that this is the excitation required to reach a mean error of below 10%. The error distribution is not shown here. However, it was found that at the excitation level required for 10% mean error in a noise free environment the maximum error was within 15% for all tyre models.

In this study, 200 samples were used for friction estimation. This corresponds to 2s in a system with 100Hz sampling time. With the vehicle used in previous studies, see (Albinsson et al., 2015), which had a maximum torque gradient of 1200 Nm/s the maximum front axle torque that can be reached is 2400 Nm or approximately 7500 N. For the vehicle in question it is enough to reach a normalized force of 0.92 [N/N] and thus enough to achieve 100% friction utilization on snow. Naturally the required torque may not be reached at higher velocities for vehicles with less powerful engines.

The brush model is not suitable for friction estimation on snow. The friction estimation error never goes below 10% and it has large modelling errors, see Figure 3-6 and Figure 7. The Dugoff model and the Magic formula on the other hand performs better with a lower required excitation. The Magic Formula requires lower excitation levels in a noise free environment. However, when adding noise to the slip and force signal the difference between the Dugoff Model and the Magic Formula in terms of required excitation is smaller. With noise added to the slip and force signals, the required excitation generally increase. In order to increase the performance of the estimator the noise and disturbance should hence be reduced as much as possible.

Similar measurements have been performed on wet asphalt and a similar study should be performed for these measurements as well. It is expected that the relative performance of some tyre models will change when the surface is changed from snow to wet asphalt due to modelling errors which favours one of the surfaces.

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