

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN SOLID AND
STRUCTURAL MECHANICS

Stochastic model updating and model selection with
application to structural dynamics

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ABSTRACT

Uncertainty induced by our incomplete state of knowledge about engineering systems and their surrounding environment give rise to challenging problems in the process of building predictive models for the system behavior. One such challenge is the model selection problem, which arises due to the existence of invariably multiple candidate models with different mathematical forms to represent the system behavior, and so there is a need to assess their plausibility based on experimental data. However, model selection is a non-trivial problem since it involves a trade-off between predictive power and simplicity. Another challenge is the model updating problem, which refers to the process of inference of the unknown parameters of a specific model structure based on experimental data so that it makes more accurate predictions of the system behavior. However, the existence of modeling errors and uncertainties, e.g., the measurement noise and variability in material properties, along with sparsity of data regarding the parameters often make model updating an ill-conditioned problem. In this thesis, probabilistic tools and methodologies are established for model updating and selection of structural dynamic systems that can deal with the uncertainty arising from missing information, with special attention given to systems which can have high-dimensional uncertain parameter vector. The model updating problem is first formulated in the *Frequentist* school of statistical inference. A framework for stochastic updating of linear finite element models and the uncertainty associated to their parameters is developed. It uses the techniques of damping equalization to eliminate the need for mode matching and bootstrapping to construct uncertainty bounds on the parameters. A combination of ideas from bootstrapping and unsupervised machine learning algorithms lead to an automated modal updating algorithm suitable for identification of large-scale systems with many inputs and outputs. The model updating problem is then formulated in the *Bayesian* school of statistical inference. A recently appeared multi-level Markov chain Monte Carlo algorithm, ABC-SubSim, for approximate Bayesian computation is used to solve Bayesian model updating for dynamic systems. ABC-SubSim exploits the Subset Simulation method to efficiently draw samples from posterior distributions with high-dimensional parameter spaces. Formulating a dynamic system in form of a general hierarchical state-space model opens up the possibility of using ABC-SubSim for Bayesian model selection. Finally, to perform the exact Bayesian updating for dynamic models with high-dimensional uncertainties, a new multi-level Markov chain Monte Carlo algorithm called Sequential Gauss-Newton algorithm is proposed. The key to success for this algorithm is the construction of a proposal distribution which locally approximates the posterior distribution while it can be readily sampled.

Keywords: Uncertainty quantification, Bayesian model updating, Bayesian model selection, stochastic simulation, Bootstrapping, Subspace system identification, Finite element model

to my beloved family.

PREFACE

The Swedish Wind Power Technology Centre (SWPTC) is a research centre for design of wind turbines. The purpose of the centre is to support Swedish industry with knowledge of design techniques as well as maintenance in the field of wind power. The research in the centre is carried out within six thematic groups that represent design and operation of wind turbines. These are the Power and Control Systems, Turbine and Wind Loads, Mechanical Power Transmission and System Optimization, Structure and Foundation, Maintenance and Reliability and the Cold Climate theme groups. The work presented here is a part of the theme Structure and Foundation. SWPTC's work is funded by the Swedish Energy Agency, by three academic and thirteen industrial partners. Region Västra Götaland also contributes to the centre through several collaboration projects. The support of the centre to this work is gratefully acknowledged.

The work presented in this thesis was carried out at the Department of Applied Mechanics at Chalmers University of Technology during the years 2011 through 2016. I would like to express my sincere gratitude to my advisor Professor Thomas Abrahamsson for his invaluable mentoring throughout these years. I would also like to thank him for giving me the warm support and confidence to step into the professional world of science. I would like to thank my co-supervisor Dr. Anders T. Johansson for his guidance and enthusiastic encouragement. I enjoyed very much our numerous discussions at the coffee table.

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Gothenburg, September 2016
Majid Khorsand Vakilzadeh

THESIS

This thesis consists of an extended summary and the following appended papers:

- Paper A** M. K. Vakilzadeh, A. T. Johansson, C.-J. Lindholm, J. Hedlund, and T. J. Abrahamsson. “Development of simplified models for wind turbine blades with application to NREL 5 MW offshore research wind turbine”. *Dynamics of Coupled Structures, Volume 1*. Springer, 2014, pp. 389–402
- Paper B** M. K. Vakilzadeh, V. Yaghoubi, T. McKelvey, T. Abrahamsson, and L. Ljung. Experiment design for improved frequency domain subspace system identification of continuous-time systems. *IFAC-PapersOnLine* **48** (2015), 886–891
- Paper C** M. K. Vakilzadeh, V. Yaghoubi, A. T. Johansson, and T. Abrahamsson. Stochastic finite element model calibration based on frequency responses and bootstrap sampling. *Submitted for International Journal Publication* (2016)
- Paper D** V. Yaghoubi, M. K. Vakilzadeh, and T. Abrahamsson. Automated modal parameter estimation using correlation analysis and bootstrap sampling. *Submitted for International Journal Publication* (2016)
- Paper E** M. K. Vakilzadeh, Y. Huang, J. L. Beck, and T. Abrahamsson. Approximate Bayesian Computation by Subset Simulation using hierarchical state-space models. *Mechanical Systems and Signal Processing* (2016), in press
- Paper F** M. K. Vakilzadeh, J. L. Beck, and T. Abrahamsson. Using approximate Bayesian computation by Subset Simulation for efficient posterior assessment of dynamic state-space model classes. *Submitted for International Journal Publication* (2016)
- Paper G** M. K. Vakilzadeh, A. Sjögren, A. T. Johansson, and T. Abrahamsson. Sequential Gauss-Newton MCMC algorithm for high-dimensional Bayesian model updating. *In Prepration* (2016)

The appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of the work including taking part in planning the papers and developing theories, developing the numerical implementations, carrying out the numerical simulations and writing the papers.

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Part I

Extended Summary

1 Introduction

The problem of model updating has received much attention over the years because of its wide range of application in fields such as complex structural design, structural control, health monitoring, and reliability and risk assessment [8–12]. The usual goal of model updating is to use experimental data from a structural or mechanical system to reconstruct the unknown properties which appear as parameters in its numerical model such that it makes more precise predictions of the system response to a prescribed, or random, excitation [13]. The usual deterministic model updating takes a parameterized model of the system, whether it is based on physics or on a black-box model, and searches for the best parameter setting that minimizes the error between model predictions and experimental data. However, this approach may not be enough to yield credible models because there always exist uncertainties associated with the process of model construction. Such uncertainties include but are not restricted to measurement uncertainty, modeling uncertainty, and parameter uncertainty [14–17]. Besides assessing the fidelity of model predictions to test data, it is thus pivotal to account for such uncertainties in the process of model updating to quantify the level of confidence on the model predictions.

To quantify uncertainty, the model updating problem can be formulated probabilistically in two distinct ways based on either the Bayesian (e.g., [11, 18–26]) or the Frequentist paradigm (e.g., [3, 9, 10, 27, 28]). The ultimate goal of both approaches is to recover the statistical description of the model parameters based on the available experimental data and then propagate it through the forward simulation to obtain the uncertainty estimates of the model predictions. There are several characteristics of structural dynamic models making the stochastic model updating problem computationally very challenging: (*i*) the forward model which maps the model parameters to system outputs is a nonlinear function of the model parameters, (*ii*) the system output cannot be analytically formulated in terms of the model parameters, and (*iii*) the number of model parameters can be large.

Another problem which has attracted the interest of researchers from different areas of science is the model selection problem [29–33]. This problem arises from the fact that there always exists uncertainty in the selection of an appropriate model to represent a real system. One possibility is to select a set of competing model classes and perform the statistical inference at a model class level to rank models based on the available experimental data. It has long been recognized that the ranking of different model classes must comply the Principle of Model Parsimony or Ockham’s razor [34, 35], that is the simplest models that are consistent with data should be preferred to the more complex models which only slightly improve the fit to the data [32]. The challenge in the model selection problem is to compare multiple models without introducing any ad-hoc measures.

In this dissertation, appropriate statistical paradigms are adopted to provide rigorous solutions for the above challenges in updating and selection of structural dynamic models in the presence of uncertainty.

2 Background

This section begins with the formulation of the model updating problem in the deterministic setting. Afterwards, the model updating problem is defined in statistical frameworks to deal with existing uncertainties in the process of model development and ascertain their implications on the model predictions.

2.1 Deterministic model updating

In the setting of deterministic model updating (e.g., [36, 37]), one seeks to minimize an appropriate norm of discrepancy between the observed system output $\mathbf{D}^x \in \mathbb{R}^{n_o}$ and model outputs¹ $\mathbf{S}(\boldsymbol{\theta}) \in \mathbb{R}^{n_o}$, i.e.,

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \|\mathbf{D}^x - \mathbf{S}(\boldsymbol{\theta})\|_{\mathbf{W}}^2 \quad (2.1)$$

where $\mathbf{W} \in \mathbb{R}^{n_o \times n_o}$ is a weighting matrix. Unfortunately, the solution to the above minimization problem is ill-posed. In other words, there are many different sets of model parameters that are consistent with the observed data inasmuch as data are typically sparse and not informative about the full extent of the model. One way to alleviate the ill-posedness of model updating problem is to add a regularization term, $\mathbf{R}(\boldsymbol{\theta})$ to the discrepancy function. The regularization term penalizes the distance from a given parameter setting by minimizing:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \{ \|\mathbf{D}^x - \mathbf{S}(\boldsymbol{\theta})\|_{\mathbf{W}}^2 + \mathbf{R}(\boldsymbol{\theta}) \} \quad (2.2)$$

The solution to this inverse problem is the best parameter setting which simultaneously make the discrepancy function and the regularization term small [41].

Furthermore, the forward model can in general be a nonlinear mapping between the parameters $\boldsymbol{\theta}$ and the model output $\mathbf{S}(\boldsymbol{\theta})$. This may lead to convergence problems in the optimization routines employed due to existence of locally optimal parameter settings. This thus necessitates the initiation of the optimization routine from sufficiently high quality starting values [42].

However, the solution to the deterministic optimization problems stated in (2.1) or (2.2) provides only a point estimate of the model parameters for which no statistical spread in the test data is carried over to the estimator. Making predictions based on the point estimator model is not best practice since it does not represent our confidence in the predictions. Instead, we are interested to extract the statistical description of the model parameters from the available information in the observed experimental data. The next section sheds light on two paradigms to formulate a statistical model updating problem; the Frequentist and the Bayesian paradigms.

¹A challenge in the model updating problem, which has not been addressed in this thesis, is that the forward models are often large-scale models and their simulation leads to overwhelming demands on the computational resources. Thus, appropriate tools (e.g., [1, 38–40]) should be developed to speed up the model updating procedure.

2.2 Stochastic model updating

In the process of model development, there always exist errors because of imperfect modeling and uncertainties² because of lack of information about the real underlying system and its surrounding environment. This section presents two different statistical paradigms, namely the Bayesian and Frequentist paradigms, to account for such errors and uncertainties in the mathematical model in order to make credible predictions of system behavior.

2.2.1 Frequentist paradigm

The Frequentist method is one way to recovering the statistical description of the model parameters from the experimental data. In this approach, a measure of probability for each outcome of an experiment is the relative frequency of occurrence of that outcome in a long sequence of experiment repetitions [43]. Based on this definition of probability, the uncertainty in the parameter estimates is represented by the sampling distribution of the estimator which is the distribution that an estimator takes when it is applied to multiple experiment repetitions [44]. The notion of variation across the repeated experimental data sets is actually what makes the basis for modeling uncertainty in this approach.

The Maximum Likelihood Estimator (MLE) is one often used estimator in the Frequentist literature (e.g., [45, 46]). In general, MLE searches for the parameter setting which gives the highest likelihood to the experimental data. To formulate the likelihood function, one needs to form a bridge between the model output $\mathbf{S}(\boldsymbol{\theta}) \in \mathbb{R}^{n_o}$ and the experimental data (observed real system output) $\mathbf{D}^x \in \mathbb{R}^{n_o}$. This can be done using the output-additive error model [47] which represents the observed system output as:

$$\mathbf{D}^x = \mathbf{S}(\boldsymbol{\theta}) + \mathbf{e} \quad (2.3)$$

where $\mathbf{e} \in \mathbb{R}^{n_o}$ denotes the measurement\modeling error. Typically, \mathbf{e} is supposed to be a vector of independent Gaussian distributed random variables, with zero mean and covariance matrix $\boldsymbol{\Gamma}_e \in \mathbb{R}^{n_o \times n_o}$ [47]. Thus, the likelihood function $L(\mathbf{D}^x|\boldsymbol{\theta})$ can be written as [48]:

$$L(\mathbf{D}^x|\boldsymbol{\theta}) \propto \exp \left[-\frac{1}{2}(\mathbf{D}^x - \mathbf{S}(\boldsymbol{\theta}))^T \boldsymbol{\Gamma}_e^{-1} (\mathbf{D}^x - \mathbf{S}(\boldsymbol{\theta})) \right] \quad (2.4)$$

The MLE estimator $\hat{\boldsymbol{\theta}}_{MLE}$ is therefore the solution to the following optimization problem:

$$\hat{\boldsymbol{\theta}}_{MLE} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \{L(\mathbf{D}^x|\boldsymbol{\theta})\} \quad (2.5)$$

² In general uncertainty can be classified into two categories, epistemic uncertainty and aleatoric uncertainty. Epistemic uncertainty, also known as systematic uncertainty, basically refers to the type of uncertainty that could be made known but they are unknown in practice. These uncertainties are associated to lack of knowledge. In contrast, aleatory uncertainty, also known as statistical uncertainty, is due to inherent randomness in the real system. This type of uncertainty cannot be reduced other than by putting in unreasonable resources. The Brownian motion of molecules or momentary distribution of gust wind velocity at a wind turbine site are examples of processes subjected to aleatoric uncertainty. In this thesis we only address methods and tools for accounting for the epistemic uncertainty in the mathematical models of structural systems.

The sampling distribution for MLE can be computed in two distinct ways:

Asymptotic distribution

If the model is globally identifiable (see Remark 1), the modeling\measurement error can be modeled as identically and independently distributed random variables, and certain regularity conditions are met, it can be shown that the distribution of the MLE estimator $\hat{\boldsymbol{\theta}}_{MLE}$ asymptotically tends to Gaussian for large number n_o of observed data points [47]. The center of the Gaussian distribution is $\hat{\boldsymbol{\theta}}_{MLE}$ and its variance is approximated by the inverse of Fisher information matrix computed at $\hat{\boldsymbol{\theta}}_{MLE}$, where the Fisher information matrix $\mathbf{I}(\boldsymbol{\theta})$ is the negative Hessian of the logarithm of the likelihood function [48].

Since most often the ultimate goal of model development is to make predictions, it is of particular interest to propagate the parameter uncertainty to the model predictions. When the parameter uncertainty can be approximated by a Gaussian distribution, the most straightforward approach to approximate the covariance matrix of model predictions of interest $\mathbf{S}^p(\boldsymbol{\theta})$ is to use its first order derivatives as [49]:

$$\text{var}(\mathbf{S}^p) = (\nabla \mathbf{S}^p(\boldsymbol{\theta}))^T \mathbf{I}(\boldsymbol{\theta})^{-1} (\nabla \mathbf{S}^p(\boldsymbol{\theta})) \Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_{MLE}} \quad (2.6)$$

where $\nabla = [\frac{\partial}{\partial \theta_1}, \dots, \frac{\partial}{\partial \theta_{n_p}}]^T$ denotes the gradient.

Bootstrapping

A different approach to approximate the sampling distribution for MLE is bootstrapping. The idea behind bootstrapping is to repeatedly draw random datasets with replacement from the experimental data \mathbf{D}^x . This would be repeated a number of times to generate n_B bootstrap datasets $\{\mathbf{D}_{(1)}^x, \dots, \mathbf{D}_{(n_B)}^x\}$ of the same size as the experimental data \mathbf{D}^x . Then, applying the MLE to bootstrap datasets gives a set of estimates $\{\hat{\boldsymbol{\theta}}_{MLE}^{(1)}, \dots, \hat{\boldsymbol{\theta}}_{MLE}^{(n_B)}\}$ which approximates the sampling distribution for MLE. To adequately reflect the uncertainty in the model parameters, bootstrapping requires a sufficient number of bootstrap datasets (e.g., $n_B = 100$ [50]), and, additionally requires to start optimizations (2.5) towards bootstrap datasets from widely dispersed starting values [49].

Bootstrapping is also a well-developed approach to estimate the prediction uncertainty. In particular, this method is of interest when the goal is to quantify the prediction uncertainty for model responses at which there exists no measured data [51]. To this end, several bootstrap rules such as the 0.632 bootstrap rule have been developed which basically keep track of how well a model evaluated at an individual sample $\hat{\boldsymbol{\theta}}_{MLE}^{(i)}$ from the MLE distribution predicts the response of interest at data points that are not included in the associated bootstrap dataset $\mathbf{D}_{(i)}^x$ [51]. This will be discussed in detail in **paper C**.

Remark 1 *One concept that plays an important role in model updating is identifiability [18]. Based on the topology of the likelihood function, a model can be categorized into one the following classes; (i) globally identifiable: if there is a unique $\hat{\boldsymbol{\theta}}_{MLE}$, (ii) locally identifiable: if there are a finite number of $\hat{\boldsymbol{\theta}}_{MLE}$'s, and (iii) unidentifiable: if there are infinitely many $\hat{\boldsymbol{\theta}}_{MLE}$'s.*

2.2.2 Bayesian paradigm

Bayesian model updating provides a systematic framework to take into account various sources of uncertainty to characterize the uncertainty in the model parameters through updates of their joint probability density function after testing. In contrast to the Frequentist interpretation of a probability measure, the Bayesian approach is based on the probability logic axioms [52]. In probability logic, a probability measure demonstrates the degree of plausibility of an event n given the information covered by a proposition m , with probability of n given m to be $p(n|m)$. If the information in m gives a complete knowledge about the event n , and thus makes it deterministic, the probability logic collapses to the Boolean logic which means that information in m implies that n is either true or false. Presence of uncertainty, due to lack of knowledge, reduces information in the proposition m about n , and then $p(n|m)$ is not binary but instead $p(n|m) \in [0, 1]$. Thus, uncertainty extends the Boolean logic to a multi-valued logic. The remaining part of this section demonstrates that the exclusive foundation of Bayesian paradigm on the probability logic axioms provides a rigorous framework to treat uncertainty in updating and selection of models in order to make credible predictions.

Stochastic model class

To solve the problem of model updating through the Bayesian approach, a key idea is to describe the uncertain behavior of a system by constructing a *stochastic model class* \mathcal{M} which consists of two fundamental probability distributions [13]. First, a set of parameterized probability models $p(\mathbf{D}|\boldsymbol{\theta}, \mathcal{M})$ is established to partially quantify the relative plausibility of the possible values of the system output $\mathbf{D} \in \mathbb{R}^{n_o}$ given the vector of uncertain parameters $\boldsymbol{\theta} \in \mathbb{R}^{n_p}$. This can be constructed by *stochastic embedding* of any deterministic model, e.g., a state-space model or a finite element model, of the system that gives the relationship between the parameter vector $\boldsymbol{\theta}$ and the model output $\mathbf{S}(\boldsymbol{\theta}) \in \mathbb{R}^{n_o}$. To this end, an uncertain prediction error can be introduced:

$$\mathbf{D} = \mathbf{S}(\boldsymbol{\theta}) + \mathbf{e} \quad (2.7)$$

to connect the output of the deterministic model to the observable system output. The probability model of the prediction error \mathbf{e} is chosen to be a Gaussian white noise $\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma}_e)$ based on the Principle of Maximum (Information) Entropy [52] under the first and second moment constraints. Then, the predictive PDF for the observable system output is given by:

$$p(\mathbf{D}|\boldsymbol{\theta}, \mathcal{M}) = \frac{1}{(2\pi)^{n_o} |\boldsymbol{\Gamma}_e|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{D} - \mathbf{S}(\boldsymbol{\theta}))^T \boldsymbol{\Gamma}_e^{-1} (\mathbf{D} - \mathbf{S}(\boldsymbol{\theta})) \right] \quad (2.8)$$

where $|\cdot|$ denotes the determinant of a matrix. For the second part of the stochastic model class, a *prior* distribution $p(\boldsymbol{\theta}|\mathcal{M})$ over the parameter space $\Theta \in \mathbb{R}^{n_p}$ is selected that encodes the initial relative degree of plausibility of each probability model $p(\mathbf{D}|\boldsymbol{\theta}, \mathcal{M})$. Here, we assume that the prior distribution has a probability density function (PDF) and also that the negative log-prior has a positive semidefinite Hessian over Θ .

Bayesian model updating and robust posterior predictive analysis

When the measurement data \mathbf{D}^x is available from the dynamic system, the predictive PDF in (2.8) gives the likelihood function $p(\mathbf{D}^x|\boldsymbol{\theta}, \mathcal{M})$ which can be used to update the prior probability for the uncertain parameters $\boldsymbol{\theta}$ through Bayes' Theorem to obtain the *posterior* PDF as:

$$p(\boldsymbol{\theta}|\mathbf{D}^x, \mathcal{M}) = \frac{p(\mathbf{D}^x|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})}{p(\mathbf{D}^x|\mathcal{M})} \quad (2.9)$$

where $p(\mathbf{D}^x|\mathcal{M}) = \int_{\Theta} p(\mathbf{D}^x|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M}) d\boldsymbol{\theta}$ denotes the *evidence*, or marginal likelihood. A central challenge in Bayesian model updating is to devise an efficient algorithm to characterize the posterior PDF, especially when it is defined over a high-dimensional parameter space or its support has a complex geometry.

When a large amount of data is available and the model class \mathcal{M} is globally identifiable based on data \mathbf{D}^x , the posterior PDF can be asymptotically approximated by a Gaussian PDF centered at the Maximum A Posteriori (MAP) point, i.e., the parameter setting maximizing the posterior PDF:

$$\hat{\boldsymbol{\theta}}_{MAP} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} p(\boldsymbol{\theta}|\mathbf{D}^x, \mathcal{M}) \quad (2.10)$$

and characterized by covariance matrix equal to the inverse of the Hessian of the negative log posterior PDF evaluated at the MAP point. However, application of the asymptotic approximation faces difficulties in dealing with locally identifiable and unidentifiable model classes. In recent years, the focus has shifted from asymptotic approximations to using Markov Chain Monte Carlo methods which generate samples from the posterior PDF [5, 11, 19–21, 24, 25, 53–56]. MCMC methods used to solve the problem of Bayesian model updating in the field of structural dynamics include, but are not limited to: the multilevel Metropolis-Hastings [19, 20, 55], the Gibbs [24], the Hybrid Monte Carlo [11], the Manifold-based Metropolis adjusted Langevin algorithms [53], and the Approximate Bayesian Computation by Subset Simulation (ABC-SubSim) [5, 57, 58].

A useful application of Bayesian model updating within a particular model class \mathcal{M} is to make robust posterior predictions about uncertain events based on past observations. Given the Model class \mathcal{M} , the Total Probability Theorem can be used to obtain the robust predictive PDF for a future system response of interest \mathbf{D}^p as [13, 59]:

$$p(\mathbf{D}^p|\mathbf{D}^x, \mathcal{M}) = \int p(\mathbf{D}^p|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathbf{D}^x, \mathcal{M}) d\boldsymbol{\theta} \quad (2.11)$$

This expression can be interpreted as a weighted average of the predictive PDFs $p(\mathbf{D}^p|\boldsymbol{\theta}, \mathbf{D}^x, \mathcal{M})$ for each specific model $\boldsymbol{\theta} \in \Theta$ within the model class \mathcal{M} , where the weight is the posterior probability $p(\boldsymbol{\theta}|\mathbf{D}^x, \mathcal{M}) d\boldsymbol{\theta}$. As pointed out by Beck and Taflanidis [59], one interesting application of the robust posterior analysis is to improve the predictive modeling of already operating systems. This can be done by defining a system performance measure as expectation of some performance function with respect to the posterior robust predictive PDF as follows:

$$\mathbb{E}[f(\mathbf{D}^p)|\mathbf{D}^x, \mathcal{M}] = \int f(\mathbf{D}^p)p(\mathbf{D}^p|\mathbf{D}^x, \mathcal{M}) d\mathbf{D}^p \quad (2.12)$$

A common example of a performance function used in reliability analysis is the indicator function, $f(\mathbf{D}^p) = I_F(\mathbf{D}^p)$ in which $I_F(\mathbf{D}^p) = 1$ if \mathbf{D}^p meets a given failure criterion and zero otherwise. Using this performance function, (2.12) gives the *posterior robust failure probability*.

Multi-dimensional integrals arising in the above formulations, (2.11) and (2.12), are most often high-dimensional and cannot be evaluated analytically, nor numerically. This renders the use of the posterior PDF for the purpose of robust posterior analysis challenging. However, Laplace's method of asymptotic approximation and stochastic simulations are useful methods to approximate these integrals. The Laplace's method for asymptotic approximation can be used when the posterior distribution of the model parameters can be approximated as a Gaussian distribution and a large amount of experimental data is available. An interested reader is referred to Beck and Katafygiotis [18] for a detailed description of the asymptotic approximation method. Recently, the use of stochastic simulation methods [11, 53, 55] for evaluation of the integrals involved in the Bayesian model updating problem has become widespread. In these methods, the integral in (2.11) can be approximated by:

$$p(\mathbf{D}^p|\mathbf{D}^x, \mathcal{M}) = \frac{1}{N} \sum_{n=1}^N p(\mathbf{D}^p|\boldsymbol{\theta}_n, \mathbf{D}^x, \mathcal{M}) \quad (2.13)$$

where $\boldsymbol{\theta}_n$, $n = 1, \dots, N$ are samples drawn from the posterior distribution $p(\boldsymbol{\theta}|\mathbf{D}^x, \mathcal{M})$ using a stochastic simulation method.

Bayesian model selection and model averaging

Bayesian model class selection provides a rigorous framework to compare the performance of a set of candidate model classes in describing the experimental data [13, 29, 32, 60]. Consider a set $\mathbf{M} \equiv \{\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_{N_M}\}$ of N_M model classes for representing a system. In Bayesian model selection, models in \mathbf{M} are ranked based on their probabilities conditioned on the data \mathbf{D}^x that is given by Bayes' Theorem:

$$P(\mathcal{M}_j|\mathbf{D}^x, \mathbf{M}) = \frac{p(\mathbf{D}^x|\mathcal{M}_j)P(\mathcal{M}_j|\mathbf{M})}{\sum_{l=1}^{N_M} p(\mathbf{D}^x|\mathcal{M}_l)P(\mathcal{M}_l|\mathbf{M})} \quad (2.14)$$

where $P(\mathcal{M}_j|\mathbf{M})$ denotes the prior probability of \mathcal{M}_j that indicates the modeler's belief about initial relative plausibility of \mathcal{M}_j within the set \mathbf{M} . The factor $p(\mathbf{D}^x|\mathcal{M}_j)$, which is called the evidence for \mathcal{M}_j , indicates the probability of data \mathbf{D}^x according to \mathcal{M}_j . Evidence can be calculated by use of the Total Probability Theorem as:

$$p(\mathbf{D}^x|\mathcal{M}_j) = \int_{\boldsymbol{\Theta}} p(\mathbf{D}^x|\boldsymbol{\theta}, \mathcal{M}_j) p(\boldsymbol{\theta}|\mathcal{M}_j) d\boldsymbol{\theta} \quad (2.15)$$

The interpretation of the evidence is similar to the one given for (2.11) except now the likelihood function $p(\mathbf{D}^x|\boldsymbol{\theta}, \mathcal{M}_j)$ is weighted by the prior probability $p(\boldsymbol{\theta}|\mathcal{M}_j)d\boldsymbol{\theta}$. However, the calculation of evidence requires the evaluation of a multi-dimensional integral which is the computationally challenging step in Bayesian model selection, specially when the

number of parameters n_p is large (see [60] for a detailed discussion on model evidence calculation).

It is worthy to mention that the log-evidence, which most often dominates the posterior probability of a model in (2.14), can be written as the difference of two terms [22, 32]:

$$\ln p(\mathbf{D}^x | \mathcal{M}_j) = \mathbb{E}[\ln p(\mathbf{D}^x | \boldsymbol{\theta}, \mathcal{M}_j)] - \mathbb{E}\left[\ln \frac{p(\boldsymbol{\theta} | \mathbf{D}^x, \mathcal{M}_j)}{p(\boldsymbol{\theta} | \mathcal{M}_j)}\right] \quad (2.16)$$

in which expectation $\mathbb{E}[\cdot]$ is taken with respect to the posterior PDF $p(\boldsymbol{\theta} | \mathbf{D}^x, \mathcal{M}_j)$. In this expression, the first term is the posterior mean of the log-likelihood function, which encodes the degree to which the model class \mathcal{M}_j fits to the data, and the second term is the Kullback-Leibler divergence, which reflects the amount of information extracted from data and is always non-negative [13, 32]. The above expression gives insight into how rigorously the log-evidence for a model class, without introducing ad-hoc penalty terms, trades off between the data-fit and complexity of a model class.

A useful application of the Bayesian model class selection is the hyper-robust posterior predictive analysis [13] (posterior model averaging) which combines the predictions from all model classes in the set \mathbf{M} . By use of the Total Probability Theorem, the posterior hyper-robust predictive PDF for the set of model classes \mathbf{M} is given by:

$$p(\mathbf{D}^p | \mathbf{D}^x, \mathbf{M}) = \frac{1}{N_M} \sum_{l=1}^{N_M} p(\mathbf{D}^p | \mathbf{D}^x, \mathcal{M}_j) p(\mathcal{M}_j | \mathbf{D}^x, \mathbf{M}) \quad (2.17)$$

which is in fact a weighted average of the posterior robust predictive PDF of each model class in the set \mathbf{M} , where the weight is its posterior probability $p(\mathcal{M}_j | \mathbf{D}^x, \mathbf{M})$.

3 Research challenges

The present dissertation has been carried out with a focus on the quantification of the uncertainties, in two different levels of the parameter uncertainty within a particular model class and the model uncertainty within a set of alternative model classes, and the assessment of their effects on the model predictions. This is performed using both the Frequentist and the Bayesian paradigms each giving rise to its own specific challenges as summarized below:

- **Frequentist paradigm**

The aim of the model updating problem is to reconstruct the unknown parameters of a mathematical description of a real system based on the observed data from the system. The characteristics of the system and the intended use of the model strongly influence the choice of the model structure and, as a consequence, the challenges arising from the building of these models. Two popular model structures in structural dynamics are modal models, in which the parameters are natural frequencies, damping ratios and mode shapes, and FE models, in which the parameters are physically motivated parameters such as Young's moduli or mass density. The present work has attempted to address the following challenges pertinent to the application of Frequentist paradigm to identification of the parameters of these model structures.

- *FE model updating*: As mentioned in Section 2.1, the challenge for the model updating problem is to find a starting value for the parameters that is reasonably close to the global minimum of the deviation between model predictions and measurement data. Recently, Abrahamsson and Kammer [36] proposed a model updating algorithm, called “FE model calibration with damping equalization”, which formulates the distance metric in (2.1) as the deviation between the logarithm of the frequency responses of FE model and a test data model found from measurement where the same level of modal damping is imposed on all modes. This formulation gives a smooth metric with a large radius of convergence to the global minimum. Unfortunately, it was found that the estimates provided by this model updating algorithm become biased in the presence of measurement noise. Besides, analogous to other deterministic model updating algorithms, it does not provide the uncertainty in the model parameters. Therefore, the challenge here is to enhance the performance of the technique of “FE model calibration with damping equalization” in dealing with noisy experimental data and to recover the uncertainty bounds on the model parameters.
- *Modal updating*: A central problem in the identification of the modal parameters using experimental data is to determine the true model order to capture the physical modes of the structure under study [61]. The common practice is to identify a model with an order that is much higher than motivated by physics to ensure that all physical eigenmodes within the frequency band of interest are safely captured [62]. However, this inevitably results in the appearance of noise modes in the identified model, i.e., modes which are present in the model due to measurement noise or computational imprecision but have no relevance to the physics of the tested system. The detection and elimination of such noise modes usually demand considerable interaction from an experienced user which hinders the use of developed modal analysis techniques for the applications which require a periodic estimation of the modal parameters like continuous health monitoring of structures. Therefore, the challenge is to develop a fully automated modal parameter estimation algorithm such that it provides uncertainty bounds on the estimated modal parameters and also avoids high-dimensional optimization.

- **Bayesian paradigm**

The MCMC methods are currently the most popular simulation techniques for solving Bayesian model updating problems. Their popularity stems from the facts that they can handle model classes with various degrees of identifiability based on available experimental data, and also can deal with applications with a small amount of experimental data. However, MCMC methods often encounter difficulties when: (i) the parameter space is high-dimensional, this is a problem which is known as so-called *curse of dimensionality*, a situation in which an algorithm works efficiently in low dimensions but fails in high dimensions, and (ii) the posterior distribution has a complex topology, e.g., multi-modal, very peaked, or nonlinear correlation among the uncertain parameters. It is highly desirable to devise MCMC methods that can efficiently draw samples from posterior distributions exhibiting any or all of such characteristics.

4 Thesis contribution

The contribution of the present work is to develop novel tools and methodologies to respond to the research challenges exposed in the previous section which can be summarized as follows:

- **Frequentist paradigm**

- *FE model updating*: The development of a novel stochastic FE model updating framework for estimation of the uncertainty in model parameters and predictions from the measured frequency responses, see **Paper C**. This framework combines the technique of “FE model calibration with damping equalization” ([36] and **Paper A**) with the principles of bootstrapping. The performance of the former is improved in dealing with noisy measurements by: *(i)* use of a new dedicated frequency sampling strategy that gives the frequencies at which the experimental FRF of the structure needs to be measured, see **Paper B**, and *(ii)* use of a weighted log-least-squares objective function. The bootstrapping technique is used to take into account the uncertainties in the measurements and forward simulations in order to quantify the uncertainty in the parameters and to assess their effects on the predictions made by the FE model.
- *Modal updating*: The development and validation of a modal parameter estimation algorithm, see **Paper D**, that satisfies the following criteria: *(i)* it allows for fast and robust identification of MIMO systems of a given order, *(ii)* it avoids high-dimensional optimization, *(iii)* it provides uncertainty bounds on the estimated modal parameters, and *(iv)* it needs no user-specified parameters or thresholds. This algorithm combines the principles of bootstrapping for uncertainty quantification with the technique of subspace based system identification [63] and also with unsupervised learning algorithms. The key to success of the engaged unsupervised learning algorithm is a novel correlation metric [64] that is able to treat the problems of spatial eigenvector aliasing and nonunique eigenvectors of coalescent modes simultaneously.

- **Bayesian paradigm**

The contribution of the present dissertation to Bayesian model updating is around two stochastic simulation algorithms which avoid the problem of drawing samples from difficult posterior distributions (i.e., high-dimensional, multimodal, or very peaked distributions) by sequentially constructing a series of intermediate distributions that interpolates between the prior and posterior PDFs. These contributions can be summarized as follows:

- *Approximate Bayesian computation by subset simulation*, ABC-SubSim, is a recently appeared algorithm for Approximation Bayesian Computation (ABC) which exploits the subset simulation for efficient rare-event simulation. The merits of ABC-SubSim stems from the facts that *(i)* it does not suffer from the curse of dimensionality, and *(ii)* it bypasses the explicit evaluation of the likelihood function and, as a result, is applicable to any model for which forward simulation

is available. One example of such models is a nonlinear state-space model with state and output uncertain prediction errors for which an analytical formula for the likelihood function is difficult to establish. In this thesis, a dynamic problem is formulated in form of a general hierarchical state-space model to show that the ABC-SubSim algorithm is performing an exact Bayesian updating for a new model in which its output is corrupted by a uniform additive error term, see **Paper E**. This formulation not only allows understanding the quality of the ABC approximations of the posterior distribution and model evidence, but also makes it possible to independently approximate the model evidence for each of the competing models as a simple by-product of the ABC-SubSim algorithm, see **Paper F**.

- A new multi-level MCMC algorithm called *Sequential Gauss-Newton* algorithm is developed, see **Paper G**, that enables sampling from difficult posterior distributions appearing in Bayesian updating of structural models. This algorithm has two novel facets: (i) the systematic resampling algorithm is utilized to avoid the loss of diversity among the samples, and (ii) a new MCMC algorithm, called Gauss-Newton MCMC algorithm, is proposed which is essentially a Metropolis-Hastings algorithm with a Gaussian proposal PDF tailored to the posterior PDF using the gradient and Hessian information of the negative log posterior. The statistical efficiency of the Sequential Gauss-Newton algorithm is improved by adopting a self-regulating technique to automatically tune the Gauss-Newton MCMC algorithm at each level such that the average acceptance rate is coerced to a target value.

5 Summary of appended papers

Paper A: *Development of simplified models for wind turbine blades with application to NREL 5 MW offshore research wind turbine*

Integration of complex models of wind turbine blades in aeroelastic simulations places an untenable demand on computational resources and, hence, means of speed-up become necessary. This paper considers the process of producing simplified rotor blade models which accurately approximate the dynamics of interest. The novelty, besides applying an efficient model updating procedure to the wind turbine blade, is to challenge the conventional beam element formulation utilized in the majority of aeroelastic codes. First, a 61.5 m blade, previously reported by the National Renewable Energy Laboratory, is selected as a case study and a verified industry-standard three dimensional shell model is developed based on its actual geometry. Next, given the reported spanwise cross sectional properties of the blade, a calibrated beam model is developed, using an efficient model updating process, that shows an excellent agreement to the low frequency dynamics of the baseline model in terms of mode shapes, resonance frequency and frequency response function. The simulation study provides evidence that a beam model cannot capture all the important features found in a large-scale 3D blade. This motivates a departure from conventional beam element formulation and suggests addressing the problem of

producing simplified models in the framework of model reduction techniques. A modified modal truncation algorithm is applied to the baseline model to produce a simpler model which accurately approximates its input–output behavior in a given frequency range. It is concluded that besides the computational efficiency of the reduction algorithm, the resulting approximation error is guaranteed to be bounded and the yielded low-order model can, in turn, be served in wind turbine design codes.

Paper B: *Experiment design for improved frequency domain subspace system identification of continuous-time systems*

A widely used approach for identification of linear, time-invariant, MIMO (multi-input/multi output) systems from continuous-time frequency response data is to solve it in discrete-time domain using subspace based identification algorithm incorporated with a bilinear transformation. However, the bilinear transformation maps the distribution of the frequency lines from continuous-time domain to discrete-time domain in a non-linear fashion which may make identification algorithm to be ill-conditioned. In this paper we propose a solution to get around this problem by designing a dedicated frequency sampling strategy. Promising results are obtained when the algorithm is applied to synthetic data from a 6DOF mass-spring model.

Paper C: *Stochastic finite element model calibration based on frequency responses and bootstrap sampling*

A new stochastic finite element model calibration framework for estimation of the uncertainty in model parameters and predictions from the measured frequency responses is proposed in this paper. It combines the principles of bootstrapping with the technique of FE model calibration with damping equalization. The challenge for the calibration problem is to find an initial estimate of the parameters that is reasonably close to the global minimum of the deviation between model predictions and measurement data. The idea of model calibration with damping equalization is to formulate the calibration metric as the deviation between the logarithm of the frequency responses of FE model and a test data model found from measurement where the same level of modal damping is imposed on all modes. This formulation gives a smooth metric with a large radius of convergence to the global minimum. In this study, practical suggestions are made to improve the performance of this calibration procedure in dealing with noisy measurements. A dedicated frequency sampling strategy is suggested for measurement of frequency responses in order to improve the estimate of a test data model. The deviation metric at each frequency line is weighted using the signal-to-noise ratio of the measured frequency responses. The solution to the improved calibration procedure with damping equalization is viewed as a starting value for the optimization procedure used for uncertainty quantification. The experimental data is then resampled using the bootstrapping approach and the FE model calibration problem, initiating from the estimated starting value, is solved on each individual resampled dataset to produce uncertainty bounds on the model parameters and predictions. The proposed stochastic model calibration framework is demonstrated on a six degree-of-freedom spring-mass system prior to being applied to a general purpose satellite structure.

Paper D: *Automated modal parameter estimation using correlation analysis and bootstrap sampling*

The estimation of modal parameters from a set of noisy measured data is a highly judgmental task, with user expertise playing a significant role in distinguishing between estimated physical and noise modes of a test-piece. Various methods have been developed to automate this procedure. The common approach is to identify models with different orders and cluster similar modes together. However, most proposed methods based on this approach suffer from high-dimensional optimization problems in either the estimation or clustering step. To overcome this problem, this study presents an algorithm for autonomous modal parameter estimation in which the only required optimization is performed in a three-dimensional space. To this end, a subspace-based identification method is employed for the estimation and a non-iterative correlation-based method is used for the clustering. This clustering is at the heart of the paper. The keys to success are correlation metrics that are able to treat the problems of spatial eigenvector aliasing and nonunique eigenvectors of coalescent modes simultaneously. The algorithm commences by the identification of an excessively high-order model from frequency response function test data. The high number of modes of this model provide bases for two subspaces: one for likely physical modes of the tested system and one for its complement, dubbed the subspace of noise modes. By employing the bootstrap resampling technique, several subsets are generated from the same basic dataset and for each of them a model is identified to form a set of models. Then, by correlation analysis with the two aforementioned subspaces, highly correlated modes of these models which appear repeatedly are clustered together and the noise modes are collected in a so-called Trashbox cluster. Stray noise modes attracted to the mode clusters are trimmed away in a second step by correlation analysis. The final step of the algorithm is a fuzzy c-means clustering procedure applied to a three-dimensional feature space to assign a degree of physicalness to each cluster. The proposed algorithm is applied to two case studies: one with synthetic data and one with real test data obtained from a hammer impact test. The results indicate that the algorithm successfully clusters similar modes and gives a reasonable quantification of the extent to which each cluster is physical.

Paper E: *Approximate Bayesian Computation by Subset Simulation using hierarchical state-space models*

A new multi-level Markov Chain Monte Carlo algorithm for Approximate Bayesian Computation, ABC-SubSim, has recently appeared that exploits the Subset Simulation method for efficient rare-event simulation. ABC-SubSim adaptively creates a nested decreasing sequence of data-approximating regions in the output space that correspond to increasingly closer approximations of the observed output vector in this output space. At each level, multiple samples of the model parameter vector are generated by a component-wise Metropolis algorithm so that the predicted output corresponding to each parameter value falls in the current data-approximating region. Theoretically, if continued to the limit, the sequence of data-approximating regions would converge on to the observed output vector and the approximate posterior distributions, which are conditional on the data-approximation region, would become exact, but this is not practically feasible. This paper studies the performance of the ABC-SubSim algorithm for Bayesian updating

of the parameters of dynamical systems using a general hierarchical state-space model. We note that the ABC methodology gives an approximate posterior distribution that actually corresponds to an exact posterior where a uniformly distributed combined measurement and modeling error is added. We also note that ABC algorithms have a problem with learning the uncertain error variances in a stochastic state-space model and so we treat them as nuisance parameters and analytically integrate them out of the posterior distribution. In addition, the statistical efficiency of the original ABC-SubSim algorithm is improved by developing a novel strategy to regulate the proposal variance for the component-wise Metropolis algorithm at each level. We demonstrate that Self-regulated ABC-SubSim is well suited for Bayesian system identification by first applying it successfully to model updating of a two degree-of-freedom linear structure for three cases: globally, locally and un-identifiable model classes, and then to model updating of a two degree-of-freedom nonlinear structure with Duffing nonlinearities in its interstory force-deflection relationship.

Paper F: *Using approximate Bayesian computation by Subset Simulation for efficient posterior assessment of dynamic state-space model classes*

Approximate Bayesian Computation (ABC) methods have gained in their popularity over the last decade because they expand the horizon of Bayesian parameter inference methods to the range of models for which only forward simulation is available. The majority of the ABC methods rely on the choice of a set of summary statistics to reduce the dimension of the data. However, as has been noted in the ABC literature, the lack of convergence guarantees that is induced by the absence of a vector of sufficient summary statistics that assures inter-model sufficiency over the set of competing models, hinders the use of the usual ABC methods when applied to Bayesian model selection or assessment. In this paper, we present a novel ABC model selection procedure for dynamical systems based on a newly appeared multi-level Markov chain Monte Carlo method, self-regulating ABC-SubSim, and a hierarchical state-space formulation of dynamic models. We show that this formulation makes it possible to independently approximate the model evidence required for assessing the posterior probability of each of the competing models. We also show that ABC-SubSim not only provides an estimate of the model evidence as a simple by-product but also it gives the posterior probability of each model as a function of the tolerance level, which allows the ABC model choices made in previous studies to be understood. We illustrate the performance of the proposed framework for ABC model updating and model class selection by applying it to two problems in Bayesian system identification: a single degree-of-freedom bilinear hysteretic oscillator and a three-story shear building with Masing hysteresis, both of which are subject to a seismic excitation.

Paper G: *Sequential Gauss-Newton MCMC algorithm for high-dimensional Bayesian model updating*

Bayesian model updating provides a rigorous framework to account for uncertainty induced by lack of knowledge about engineering systems in their respective mathematical models through updates of the joint probability density function (PDF), the so-called posterior PDF, of the unknown model parameters. The Markov chain Monte Carlo (MCMC) methods are currently the most popular approaches for generating samples from

the posterior PDF. However, these methods often found wanting when sampling from difficult distributions (e.g., high-dimensional PDFs, PDFs with flat manifolds, multimodal PDFs, and very peaked PDFs). This paper introduces a new multi-level sampling approach for Bayesian model updating, called Sequential Gauss-Newton algorithm, which is inspired by the Transitional Markov chain Monte Carlo (TMCMC) algorithm. The Sequential Gauss-Newton algorithm improves two aspects of TMCMC to make an efficient and effective MCMC algorithm for drawing samples from difficult posterior PDFs. First, the statistical efficiency of the algorithm is enhanced by use of the systematic resampling scheme. Second, a new MCMC algorithm, called Gauss-Newton MCMC algorithm, is proposed which is essentially an M-H algorithm with a Gaussian proposal PDF tailored to the posterior PDF using the gradient and Hessian information of the negative log posterior. To further improve the statistical efficiency of the Sequential Gauss-Newton algorithm, a self-regulating technique is adopted to tune the Gauss-Newton MCMC algorithm at each level. The effectiveness of the proposed algorithm for solving the Bayesian model updating problem is illustrated using three examples with irregularly shaped posterior PDFs.

6 Concluding remarks and future works

The overall goal of the research presented in this thesis is to establish statistical tools and methodologies for model updating and selection of structural dynamic models that account for uncertainty arising from our incomplete knowledge of the underlying system and its environment, with special attention given to systems which can have high-dimensional uncertainties. One specific goal is in wind turbine engineering in which fatigue predictions are made early in the design process using loading assumptions and computational models. However, after time passes by and data are collected from the operating wind turbine, more information about the actual loading situation is gained. In a stochastic framework, like presented here, such information can be used for updated remaining lifespan predictions based on a rational procedure.

Based on the ideas from the *Frequentist* school of statistical inference, two model updating algorithms are presented. The first one is a novel framework for stochastic FE model updating based on noisy FRFs which enjoys the merits of the techniques of bootstrapping and “FE model calibration with damping equalization”. The former is employed to quantify the uncertainty in the model parameters and predictions. The latter is used to provide high-quality starting value for the optimizations towards the bootstrap data sets. The accuracy of the starting values are increased by proposing a dedicated frequency sampling strategy that gives the frequencies at which the experimental FRF of the structure needs to be measured. The results show that the proposed stochastic FE model calibration framework provides an unbiased estimator which is almost statistically efficient. The second one is an automated modal parameter estimation algorithm which uses bootstrap sampling in conjunction with a correlation-based unsupervised machine learning algorithm. The former is employed to quantify the uncertainty in the modal parameters. A new mode correlation measure is proposed which is able to deal with the spatial aliasing phenomenon and the non-unique eigenvector of the modes of coalescent

eigenvalues. The proposed algorithm has been successfully applied to two cases, a case with synthetic test data and a case with real test data. Both indicate the method's adequacy to distinguish between physical and noise modes.

Based on the ideas from the *Bayesian* school of statistical inference, two stochastic simulation algorithms are presented for solving high-dimensional updating problems. The first stochastic simulation algorithm is ABC-SubSim which solves the Bayesian model updating problem in the framework of approximate Bayesian computation methods to adopt the Subset Simulation algorithm, which is an efficient rare-event simulation algorithm, for drawing samples from high-dimensional distributions. Formulating a dynamic problem in form of a general hierarchical state-space problem opens up the possibility to use the ABC-SubSim algorithm to estimate the model evidence for each model class in a set of competing model classes. The examples show the successful application of the ABC-SubSim algorithm for Bayesian model updating and model selection of linear and nonlinear structures. The second stochastic simulation algorithm is the Sequential Gauss-Newton algorithm which is a new multi-level MCMC algorithm that avoids the problem of generating samples from difficult distributions by drawing samples from a sequence of intermediate distributions that can be readily sampled. The combination of a systematic resampling scheme, which updates samples between intermediate distributions and the Gauss-Newton MCMC algorithm, which performs exploration within one intermediate distribution, boosted the statistical efficiency of the algorithm.

It is noteworthy to mention that the stochastic model updating algorithms that rely on the asymptotic approximations have difficulties to deal with the ill-conditioned problems. However, the model updating algorithms proposed in this thesis provide potential solutions for such challenging problems.

The Bayesian model updating frameworks developed in this thesis are applied to illustrative examples with synthetic data. One possible extension is to apply these frameworks to more complex structures and to cases with real experimental data.

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