# Influence of hill-length on energy consumption for hybridized heavy transports in long-haul transports

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## Abstract

Goods transports are big producers of $CO_2$, i.e. consumers of energy. The conventional transport vehicles such as tractor-semitrailers can be replaced by long combination vehicles (LCVs). By doing so, fuel consumption will be reduced drastically, with up to 30%, mainly thanks to the reduced aerodynamic resistance per pay load mass and/or volume. Further reduction of $CO_2$ improvements can be made by hybridization, if the road topography demands variable propulsion power due to up- and downhills. This gain is emphasized for heavier vehicles. So, hybridized LCVs are of special interest.

When developing vehicles, or selecting vehicle for a certain transport, one needs to assume an operating cycle. To describe the operating cycle correctly is very important for this purpose. Traditionally, the magnitude of road grades is the only topography measure used to characterise the road. In this paper it is studied how an additional measure, hill length, influences these heavy hybridized LCVs. Together one can see these two measures as amplitude and wavelength.

It is shown how energy saving varies for different types of roads (combinations of grade magnitude and hill-length) and different energy buffer sizes. Road topography is statistically generated for a good coverage of road types, but also examples of real roads are marked within these synthetic roads. The result can be combined with estimates of hybridization costs and conclusions can be drawn when it is beneficial to hybridize and with how large buffer.

The main takeaways from the paper are that the potential energy savings for heavy LVCs due to hybridization are significant and that the hill-length is an important characteristic measure to include in operating cycle definitions.

## Keywords

Goods transport, Long Combination Vehicle, High Capacity Transport, Hybrid propulsion, Energy saving, Energy consumption, Road description, Road topography

## 1. Introduction

### 1.1 Background

Goods transports are big producers of $CO_2$ therefore potential ways to reduce the energy consumption are important to consider. One way of achieving a considerable reduction is to allow trucks with longer trailer combinations than the conventional tractor-semitrailer; for example the semitrailer-dolly-semitrailer (A-double) depicted in Figure 1. The reduction in energy consumption per pay load is mainly due to the reduction in aerodynamic resistance.

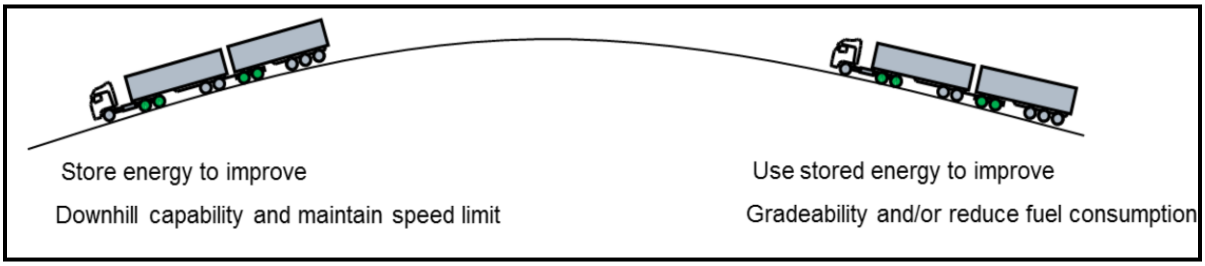


***Figure 1: Example of how LCVs can use hybridization to improve performance.***

A way to reduce the energy consumption further is to hybridize these kinds of combinations: since the slope resistance is mass dependent the fraction of fuel used to ascend slopes increase the more cargo a vehicle carries and, similarly, the more energy is available for regeneration in a downhill. In addition, there may also be a point in having extra propulsion power available for particularly steep slopes, to improve the gradeability.

Thus the topography (or hilliness) is an important factor to consider in the development process, something that is true not only for this specific application but for powertrain components in general. That means a good categorization of the parameter is necessary to guarantee that the components are allowed to work in the way that they were designed to do. For example, Volvo trucks have developed their Global Truck Application (GTA), see [1], for this type classification. It works as a vocabulary or guidebook and lists the parameters that are judged most important for operational environment (e.g. road condition), vehicle utilization (e.g. yearly usage) and transport mission (e.g. gross combination weight, GCW), and introduces requirements and restrictions based on these.

Topography is one of the parameters listed under operational environment. It is divided into four different classes: flat, predominantly flat, hilly and very hilly. The categorization is made based on the distribution of the inclination: how big a percentage of the road is below a certain grade. This is related to the variation of the grade but independent of the hill length: a road varying repeatedly between -2% and 5% would end up in the same class as a road with a steady decline of -2% for half the distance and 5% ascent the other half. The optimal choice of energy buffer size on these two roads is different, suggesting that an additional parameter – hill length – could be important.

In this paper a very simple and computationally efficient vehicle model is used to investigate how the hill length affects the energy consumption of a hybridized vehicle. A stochastic model of the topography, based on the grade variation (amplitude) and hill length, is used to quickly generate large amounts of realistic road profiles. These are used together with the vehicle model to simulate a conventional ICE vehicle and a HEV, and the results are compared. Note that the (target) speed is kept constant. We also show how design recommendations for buffer size can be made based on the two mentioned topography parameters and the expected GCW.

## 1.2 Objective

- Investigate the influence of hill length on energy saving for fixed buffer size.
- Find a method for how to make buffer size design recommendation based on topography parameters.

## 1.3 Limitations

- Simple vehicle model: e.g. inertia term is neglected (see appendix A.3).
- Simple driving patterns and transport mission, i.e. the speed is constant at 80 km/h (or maximum possible if less)
- Transport time gain or loss is not evaluated as both vehicles are forced to have same speed profile.
- Effects of downsizing the ICE is not studied since we do not investigate influence on gradeability.
- Effects of energy buffer weight is not included since the weight of the battery is deemed much smaller than the cargo weight (e.g. 500 kg << 40..80 ton).

# 2. OCEAN

The Operating Cycle Energy mANagement project (OCEAN) is an ongoing project at Chalmers University of Technology with the purpose to improve the use of transport missions in full vehicle simulations. Initial studies [2] showed that vehicles driving on roads presently are not suited to the tasks they perform. Hence if the transport mission could be predicted and represented in a better way, the results from simulations would be more representative and allow for a more accurate computation of energy efficiency over extended periods of time. The application would be both in product development and the sales-to-order process.

The project is graphically represented in Figure 2 and, as can be seen, the work is twofold. The first part (blue box) deals with constructing a process such that relevant transport missions can be generated depending on purpose of the vehicle and its geographical operating area. This is done using data analysis of a large set of log data and the work is ongoing.

The second part (red box) concerns the actual format of the road for a vehicle simulation. The representation needs to consider all factors that are relevant for an accurate energy consumption evaluation. Topography is a good example, since the road inclination contributes with a direct resistive force it is essential to include.

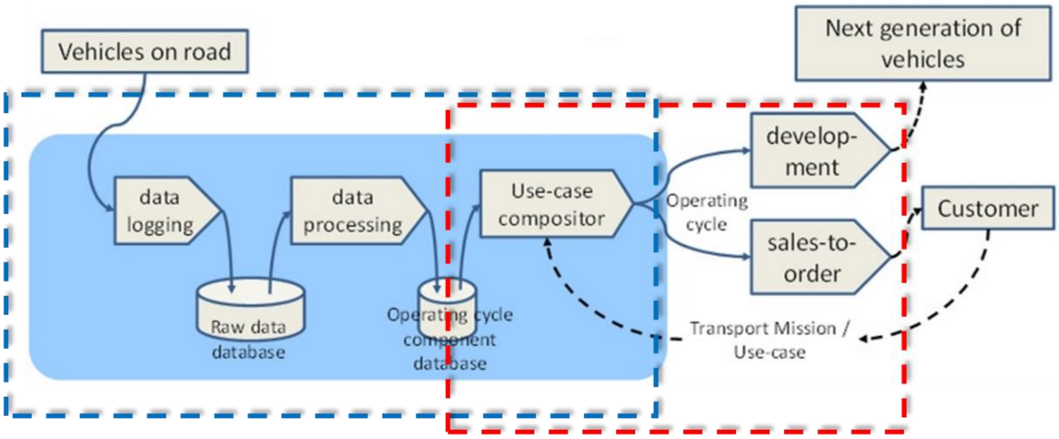


***Figure 2: Outline of the OCEAN project. Left dashed box marks activities in project, while right dashed box marks intended usage of the deliverables from project.***

# 3. Method

For the purpose of this article both the road model and the vehicle model are kept as simple as possible.

## 3.1 Road model

The operating cycle consists of distance, vehicle speed and a road model containing only topography. The distance is the governing variable and the target speed is kept constant at 80 km/h. The topography is simply given as road grade, but the origin is somewhat more sophisticated. It is modelled using a first order Markov chain due to Johannesson et al [3]. First we assume that the road inclination is piecewise constant over some sample distance $L_s$ = 50 m. The sample distance is related to the definition of the shortest wavelength ($\lambda = 2L_s$) included as road grade, as opposed to the characteristic lengths of road roughness or microstructure. For simplicity, we will also assume that the inclination is small enough to approximate the arc length with the horizontal distance, i.e.

$$\Delta s = \sqrt{\Delta x^2 + \Delta z^2} \approx \Delta x, \qquad \text{and} \qquad \frac{\Delta z_k}{\Delta s} \approx \frac{\Delta z_k}{\Delta x} \tag{1}$$

The road grade, in percentage, is defined as

$$y_\mathrm{k} = 100\frac{\Delta z_\mathrm{k}}{\Delta \mathrm{s}} \tag{2}$$

An auto-regressive model, AR(1), can then be written as in equation (3)

$$y_k = \ a y_{k-1} + e_k, \qquad e_k \sim N(0, \sigma_e^2) \tag{3}$$

The configurable parameters are $a$ and the standard deviation of the residual $\sigma_e$. Here it is assumed that the residual error in each step follows a normal distribution, which is also a simplification [3]. Usually when reasoning around this model it is more practical to use the road slope variance $\sigma_y$ and (mean) hill length $L_h$ than $a$ and $\sigma_e$, since the physical connection is much easier to grasp.

$$\sigma_y^2 = \frac{\sigma_e^2}{1-a^2}, \qquad L_h = \frac{2}{\frac{1}{2} - \arcsin a} L_s \tag{4}$$

The road slope variance is directly related to the probability of the road having a specific inclination and therefore, statistically, how big parts of the trip is driven on such inclination. Usually this is the measure that is used to categorize the road in different classes, as in [1]. The hill length is the mean length between two valleys, so in some sense this corresponds to the wavelength of the topography. This parameter does not affect the magnitude of inclination in any way and is typically not mentioned when considering different road grade classes.

There are some advantages with using a model as in equation (3) compared to e.g. using actual road profiles (from log data or road databases). First, it provides two (continuous) parameters that can be used to measure the severity of topography on different roads. Naturally these are independent of the trip length and thus the space needed for storage is a fraction of what would be needed for a real world trip topography with reasonable resolution. Second, it provides a very convenient and computationally effective way to generate new roads with a realistic slope profile. There are of course disadvantages too; it is a stochastic model and thus the statistical distribution (of the inclination) for a generated road can be guaranteed to coincide with the original one only if an infinite number of segments is generated. The accuracy decreases the fewer segments there are and therefore short trips are subject to large variations, especially if a long hill length is used. A good rule of thumb is that the trip length should be at least ten times as long as the hill length. A more subtle problem is that a model like this lacks interactions between different properties that may be present in reality. An example: supposing that road curvature was generated at the same time (see e.g. [4]) it may happen that a sharp curve shows up in the same spot as a steep incline. Generally, real roads are not constructed in this way [5].

## 3.2 Vehicle model

The idea is to keep the vehicle model as simple as possible to make it robust and computationally efficient. The longitudinal model of the chassis uses the basic forces for rolling, slope and air resistance

$$m\dot{v} = \frac{P_{prop}}{v} - mgf_R\cos\theta - mg\sin\theta - \frac{\rho_{air}AC_d v^2}{2} \tag{5}$$

where $P_{prop}$ is the power at the wheels. Since the target speed is constant we will neglect the acceleration-term here, turning the differential equation into an algebraic one. Furthermore, using the assumptions in equation (1), and attaching an index $k$ to separate between different road segments, it can be written

$$\frac{P_{prop,k}}{v_k} = mgf_R + mg\frac{y_k}{100} + \frac{\rho_{air}AC_d v_k^2}{2} \tag{6}$$

Though the target speed is 80 km/h, the actual speed is limited by the maximum propulsion power and the severity of the road inclination. Therefore, the assumption about the acceleration being negligible is only valid provided that the variation in speed in small in comparison to the other terms, see appendix A.3. Generally one can run into problems when both vehicle weight is heavy and the grade is large.

For the ICE-case we assume that the fuel efficiency $\eta_{ICE}$ is constant and that the engine can instantly deliver any power (or torque) below the maximum limit $P_{max,ICE}$. It is the second axle group that is driven and it is more convenient to use its power limit $P_{2,max} = \eta_{ICE}P_{max,ICE}$. Since the target speed is constant we will assume that no gearshifts are needed and refrain from modelling a gearbox. If the power is negative then the value instead denotes how much brake power is needed, supplied by the friction brakes.

The hybrid electric vehicle has the same properties as the ICE vehicle, and additionally it has a battery and an electrical motor capable of both propulsion and (regenerative) braking. We assume that the battery input power is the same as that of the electric motor, and that it has an energy storage upper limit $SoC_{max}$. The energy in the battery $E$ and the power in the electrical motor $P_{EM}$ is naturally related by

$$\dot{E} = P_{EM} \tag{7}$$

In accordance with the simplicity of the engine, we assume that the electrical motor can deliver or regenerate anything below its maximum power capability $P_{EM,max}$ and that it has an efficiency $\eta_{bat}$. The electrical motor is located on axle group four and as for the ICE we rather use its limit $P_{4,max} = \eta_{bat}P_{EM,max}$. The control strategy is a basic charge depletion charge sustaining variant: the electrical motor is used as long as it has some charge left and the combustion engine otherwise, whenever a braking force is needed as much energy as possible is regenerated and the friction brake employed for the excess.

Since the hybrid vehicle has all the capabilities that the ICE-vehicle has and more, we may use the ICE-vehicle's result from equation (6) to get an actual speed and a required propulsion power for the hybrid vehicle. With equation (7) this once again turns into a differential equation. In painful detail:

$P_{prop,k} \geq 0$: (8)

$$P_{4,k} = \min\left(P_{prop,k}, P_{4,max}, E_k/\Delta t_k\right)$$
$$P_{2,k} = P_{prop,k} - P_{4,k}$$
$$\dot{E}_k = -\frac{1}{\eta_{bat}} P_{4,k}$$

$P_{prop,k} < 0$: (9)

$$P_{4,k} = \max\left(P_{prop,k}, -P_{4,max}, (E_k - SoC_{max})/\Delta t_k\right)$$
$$P_{2,k} = P_{prop,k} - P_{4,k}$$
$$\dot{E}_k = -\eta_{bat}\, P_{4,k}$$

$$\Delta t_k = \frac{\Delta s_k}{v_k}, \qquad E_{k+1} = E_k + \dot{E}_\mathrm{k}\, \Delta t_k, \qquad E_0 = 0 \tag{10}$$

### 3.3 Comparison and metric

With these simple vehicle models it is straight forward to find a suitable energy comparison metric.

$$q = \frac{E_{ICE} - (E_{Hybrid} - \eta_{bat} E_{end})}{E_{ICE}} \tag{11}$$

Both $E_{ICE}$ and $E_{Hybrid}$ refer to the energy coming from the combustion engine, which means that in both cases it can easily be found as

$$E = \int_{t \in t_{trac}} P_{ICE}(t)\, dt = \int_{t\, \in t_{trac}} \frac{P_2(t)}{\eta_{ICE}} dt \tag{12}$$

where the traction region is defined as all-time intervals where the required propulsion force is positive: $\{i: t \in [t_i, t_{i+1}],\ F(t) > 0\}$, i.e. all points where energy is needed to keep the vehicle going.

In equation (11) the energy spent for the hybrid vehicle is compensated with whatever charge ($E_{end}$) is left in the battery at the end of the trip. It is debatable whether this is correct or not: having charge left does not decrease the amount of fuel consumed during the trip. In this case however, the objective is not an evaluation of the vehicle performance or control strategy but an investigation of whether a certain property of the road has an influence or not. To address that issue the energy spent during the trip is the important parameter and, in that regard, any charge remaining in the battery at the end represents an amount of unspent energy that must be compensated for.

The parameter $q$ in equation (11) is therefore a measure of the how much less energy the hybrid requires, in units of the ICE vehicle energy. Since $q \in [0,1]$, it can be seen as a percentage of the amount of saved energy.

## 4. Simulation and results

### 4.1 Reference hybridization

Using the road model in section 3.1, ten different parameter values for hill length and road variation were tested, resulting in 100 different combinations. 400 roads were generated for each combination, giving 40 000 trips in total. Each had the length 20 km but to always finish at the same altitude the vehicle were simulated driving both ways, making the actual transport mission length 40 km. The road parameters can be seen in Table 1 and the relevant vehicle parameters in Table 2.

***Table 1: Road topography parameter vectors.***

| $\boldsymbol{L_h}$ (km) | 0.3 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{\sigma_y}$ (%) | 0.50 | 0.90 | 1.29 | 1.60 | 2.00 | 2.35 | 2.60 | 2.90 | 3.21 | 3.50 |

***Table 2: Vehicle model parameters.***

| **Parameter** | **Value** | |
|---|---|---|
| $m$ | 40 ton | 80 ton |
| $f_r$ | 0.005 | |
| $A$ | 10 m$^2$ | |
| $C_d$ | 0.6 | |
| $P_{2,max}$ | 450 kW | |
| $P_{4,max}$ | 300 kW | |
| $\eta_{ICE}$ | 0.25 | |
| $\eta_{bat}$ | 0.90 | |
| $SoC_{max}$ | 12 kWh | |

Equation (11) was used to compute $q$ for each trip, which was in turn used to estimate a mean $\bar{q}$ and a standard deviation for each setting of $L_h$ and $\sigma_y$. The results can be found in Table 5, Table 6, Figure 3 and Figure 4.

Looking both at the 40 and 80 ton cases in the contour plot (Figure 3) there is a notable dependence on both amplitude and hill length, as well as a correlation between the two.

Consider first the hill length: for a fixed amplitude the potential energy saving decreases with an increasing hill length. This makes sense, when the hills are short the battery saturates less often. On the other hand, for very long slope lengths the battery may not be large enough to regenerate all energy even for rather small inclinations. Both the contour plot and Figure 4 show that the decrease is more rapid the larger the amplitude: shorter distance needed before full saturation.

For the amplitude in the 80 ton case, the energy saving increases up to a point and then starts to decrease. There are two reasons for this. One is that the battery starts to fully saturate in most hills and the friction brakes must be used to provide braking for the rest of the way. At this point the more severe the amplitude the more energy is lost in the process: the hybrid behaves more like a conventional vehicle and therefore the difference between them decreases. Naturally there is an effect from the length of the slope: the longer the hill the more energy lost.

The second reason is that for large amplitudes there is a limiting effect from the size of the electric motor. At some point the amplitude is severe enough that the electric motor more or less always operates at full power. Again the friction brakes must be used to provide braking to keep the vehicle at constant speed: at that point and onwards the regenerated power can never increase and always remains the same. This also explains the counter intuitive result that the 40 ton hybrid can save more energy than the 80 ton. The slope resistance is mass dependent, which means that the 80 ton vehicle saturates the electrical motor for a lower amplitude. According to the explanation above, beyond this point the heavier vehicle never improves but the 40 ton one still does. When both saturate fully the regenerated energy is the same in absolute numbers. Since the total energy is much greater for the heavy vehicle, the relative number is better for the 40 ton case.

Note that the **ideal** energy saving does increases with mass, see appendix A.2. This would correspond to the case with both a large enough battery (e.g. infinite) and an electrical motor with large enough maximum power (e.g. infinite).

The boundaries for $\sigma_y$ and $L_h$ have been selected based on roads that are commonly used in simulations and real log data from the OCEAN-project. It is likely that real roads have some correlation between grade and hill length: for example a road with very high amplitudes and very long hill lengths seem unreasonable, since the altitude gain (or decrease) would be very large. Further investigations could be done here, with the aim to reduce the “problem space” with one dimension or impose more realistic boundaries.

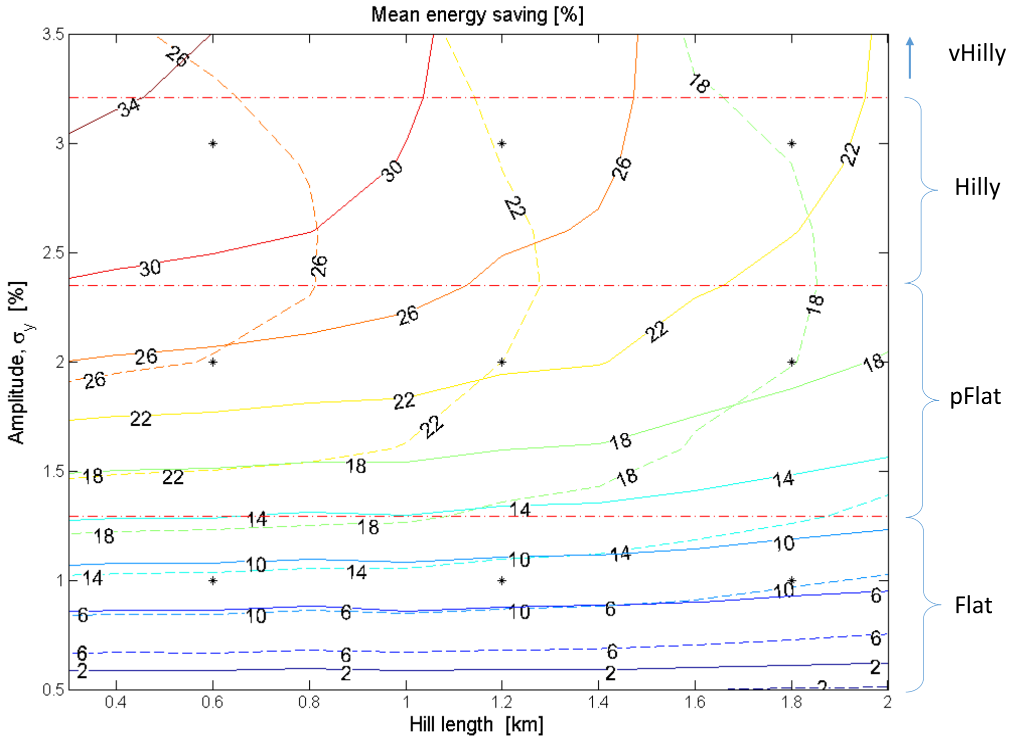


***Figure 3: Contour of the energy saving in percentage as a function of hill length and amplitude. The solid lines show the result of the 40 ton simulation and the dashed ones 80 ton. The red dash-dotted lines indicate the boundary between different GTA-classes, from bottom to top: flat, predominantly flat, hilly and very hilly. The black asterisks show the road parameters used in the buffer size simulation, in Figure 5.***

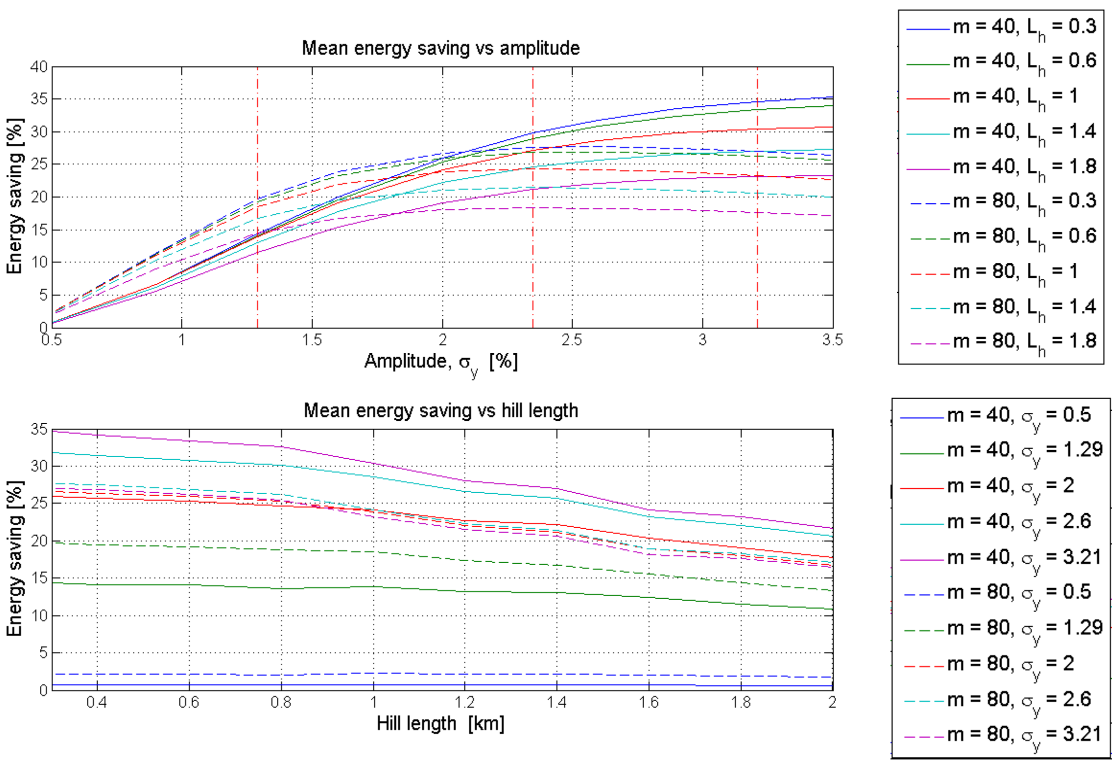


***Figure 4: Plots showing the energy saving with one parameter fixed. In the top graph the hill length is fixed and the amplitude is varied. In the bottom graph the amplitude is fixed and the hill length is varied. As before, the solid lines show results for the 40 ton vehicle and the dashed ones 80 ton. In the top graph the GTA limits are shown as dash dotted lines.***

*Table 3: Mean energy saving for GTA classes.*

| | Energy saving (%) | |
|---|---|---|
| **GTA class ↓ , GCW →** | **40 ton** | **80 ton** |
| FLAT | 0-14 | 2-20 |
| PFLAT | 11-30 | 13-28 |
| HILLY | 20-35 | 17-28 |
| VHILLY | 22-35 | 16-27 |

The result here could be used in an imagined sales-to-order process, using Table 3, to recommend what kind of truck to sell in a specific region or whether to recommend hybridization at all, based on the mean energy savings.

## 4.2 Influence of buffer size

Further simulations were conducted to investigate how the buffer size affected the energy saving. Nine different values of $L_h$ and $\sigma_y$ were chosen, marked with asterisks in Figure 3, and again 400 roads were generated for each setting. The battery size was varied up to twice the setting in Table 1 (i.e. from 0 to 24 kWh). The results are shown in Figure 5.

Note that here we are only focusing on the energy consumption. The investment grows the larger the battery, so considering these two together it may be that the most cost efficient point is much farther to the left than what is indicated here. Note especially that there is an approximate knee point in the figures below, meaning that whatever the battery (and electrical infrastructure) cost function would look like, there is never any reason to choose a larger buffer size than this point indicates. Furthermore, there is the question if the investment is favourable at all when considering that more cargo could be transported in the absence of the extra propulsion system.

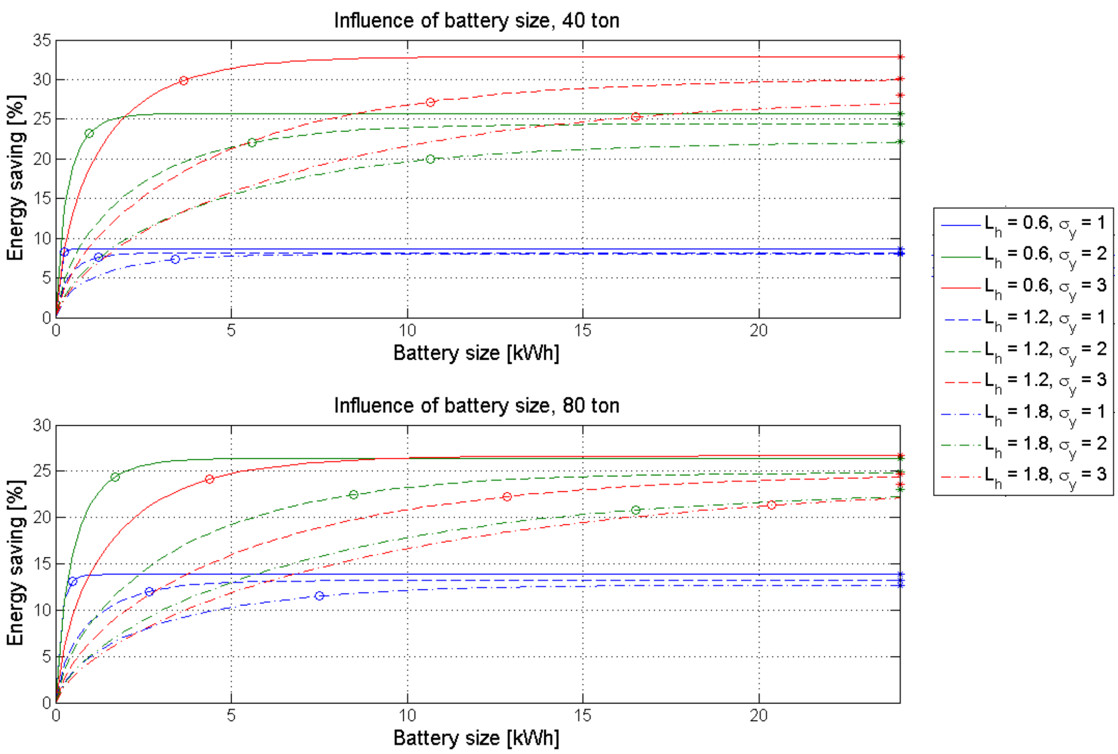


***Figure 5: Energy saving as a function of battery size for the nine different topography settings. The asterisks at the right edge of the picture show the best possible energy saving, corresponding to a buffer of unlimited size. The circles indicate the minimum recommended battery size when using the design criterion in the example.***

With the comments above in mind, the results in Figure 5 could be used directly in a hypothetical design process. First, we would need to know the general purpose of the truck to find an approximate gross combination weight. Second, we would need to know in which region it is supposed to operate. From that information suitable values for the amplitude and hill length could be found. With these three pieces of information, the correct diagram could be selected, the appropriate line found and the battery size determined by saying, for example, that the design criterion is 90% of the optimal energy saving. Example: if the new truck is to transport, on average, 40 ton and it will be operated in a hilly region with moderate hill lengths ($L_h \approx 1.2$

km, $\sigma_y \approx 3.0$), the red dashed line in the first diagram shows that the maximum saving is about 30%, meaning that the smallest battery size choice would be 10.7 kWh (resulting in an energy saving of 27%). Table 4 below shows the recommended battery size based on this design criteria.

***Table 4: Battery sizes (in kWh) for the exemplified design criteria.***

| | M = 40 ton | | |
|---|---|---|---|
| $\sigma_y \downarrow , L_h \rightarrow$ | **0.6** | **1.2** | **1.8** |
| **1.00** | 0.2 | 1.0 | 3.6 |
| **2.00** | 1.2 | 5.6 | 10.7 |
| **3.00** | 3.4 | 10.7 | 16.5 |

| | M = 80 ton | | |
|---|---|---|---|
| $\sigma_y \downarrow , L_h \rightarrow$ | **0.6** | **1.2** | **1.8** |
| **1.00** | 0.5 | 2.7 | 7.5 |
| **2.00** | 1.7 | 8.5 | 16.5 |
| **3.00** | 4.4 | 12.9 | 20.4 |

# 5. Conclusions

- A road topography generator and a simple vehicle model has been used to investigate grade and hill-length influence on energy consumption for heavy vehicles with regenerative braking and limited energy buffer and power.
- Hybridization of heavy vehicles can save a significant amount of energy, typically 10-25%.
- Hill length is an important road parameter besides grade.
- It is shown how to use the results in a sales-to-order process to evaluate the gain of hybridizing a customer vehicle depending on geographic region.
- It is shown how to derive a quantitative design recommendation for buffer size, given vehicle weight and road parameters.

# 6. Future work

It is likely that hill length also is important for energy storage in vehicle kinetic energy, i.e. intelligent speed variation (e.g. the Volvo iSee). Investigating this would require a more detailed study.

In this work the maximum power of the extra propulsion system was kept fixed, but it too could be turned into a design parameter and varied in a similar way to find a design recommendation. It seems likely that the hill length has a much lesser influence on its design choice, but the vehicle weight greater.

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# 8. Appendices

## A.1. Tabulated data of energy saving

Mean energy saving in percentage of the 400 roads for each setting of $L_h$ and $\sigma_y$. The last row shows the pooled standard deviation for each hill length.

***Table 5: Numerical results for m = 40 ton.***

| | **Hill length (km)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Amplitude (%)** | **0.30** | **0.40** | **0.60** | **0.80** | **1.00** | **1.20** | **1.40** | **1.60** | **1.80** | **2.00** |
| **0.50** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **0.90** | 7 | 7 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 5 |
| **1.29** | 14 | 14 | 14 | 14 | 14 | 13 | 13 | 12 | 12 | 11 |
| **1.60** | 20 | 20 | 20 | 19 | 19 | 18 | 18 | 17 | 16 | 15 |
| **2.00** | 26 | 26 | 25 | 25 | 24 | 23 | 22 | 20 | 19 | 18 |
| **2.35** | 30 | 29 | 29 | 28 | 27 | 25 | 25 | 22 | 21 | 20 |
| **2.60** | 32 | 31 | 31 | 30 | 29 | 27 | 26 | 23 | 22 | 21 |
| **2.90** | 33 | 33 | 32 | 32 | 30 | 28 | 27 | 24 | 23 | 21 |
| **3.21** | 35 | 34 | 33 | 33 | 31 | 28 | 27 | 24 | 23 | 22 |
| **3.50** | 35 | 35 | 34 | 33 | 31 | 28 | 27 | 24 | 23 | 22 |
| **SE** | 0,05 | 0,07 | 0,1 | 0,13 | 0,19 | 0,23 | 0,25 | 0,27 | 0,28 | 0,29 |

***Table 6: Numerical results for m = 80 ton.***

| | **Hill length (km)** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Amplitude (%)** | **0.30** | **0.40** | **0.60** | **0.80** | **1.00** | **1.20** | **1.40** | **1.60** | **1.80** | **2.00** |
| **0.50** | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| **0.90** | 11 | 11 | 11 | 11 | 11 | 11 | 10 | 10 | 9 | 8 |
| **1.29** | 20 | 19 | 19 | 19 | 19 | 17 | 17 | 16 | 14 | 13 |
| **1.60** | 24 | 24 | 23 | 23 | 22 | 20 | 20 | 18 | 17 | 16 |
| **2.00** | 27 | 26 | 26 | 25 | 24 | 22 | 21 | 19 | 18 | 17 |
| **2.35** | 28 | 27 | 27 | 26 | 24 | 22 | 21 | 19 | 18 | 17 |
| **2.60** | 28 | 27 | 27 | 26 | 24 | 22 | 21 | 19 | 18 | 17 |
| **2.90** | 27 | 27 | 27 | 26 | 24 | 22 | 21 | 19 | 18 | 17 |
| **3.21** | 27 | 27 | 26 | 25 | 23 | 22 | 21 | 18 | 18 | 17 |
| **3.50** | 26 | 26 | 26 | 25 | 23 | 21 | 20 | 18 | 17 | 16 |
| **SE** | 0,04 | 0,05 | 0,08 | 0,11 | 0,17 | 0,21 | 0,23 | 0,24 | 0,26 | 0,27 |

## A.2. Ideal energy saving

Assuming that all the energy could be recovered in the downhills, i.e. axle capacity and battery size are both unlimited, the ideal, i.e. best possible, energy saving can be evaluated. In this case it is also assumed that the electrical motor efficiency is 100%. The results are shown in Figure 6.

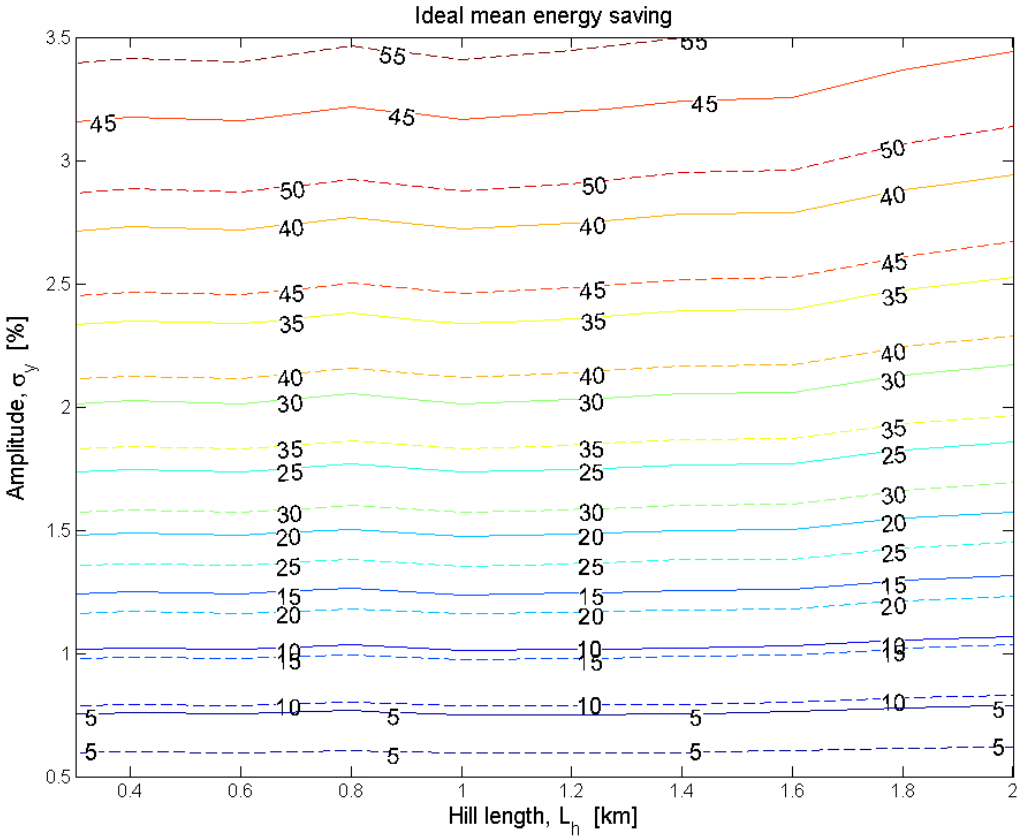


***Figure 6: The ideal energy saving if all the energy could be recovered in the downhills. The solid lines show the 40 ton case and the dashed lines 80 ton.***

## A.3 Assess influence of neglected acceleration term

As was mentioned in section 3.2, the assumption about the acceleration term in equation (5) being negligible is only valid as long as its influence is small in comparison with the resistive terms. A smaller set of simulations were performed to investigate how well the approximation worked. The simulation model was basically equation (5) together with appropriate limitations for the available propulsion power, compare to equations (8) and (9). The evaluation measure is the ratio of the acceleration inertia term and the resistive force term

$$\rho(\sigma_y, L_h) = \left( \sum_{i=1, \dot{v}_i > 0}^{i:\, s_i = s_{end}} m\dot{v}_i \, \Delta s_i \right) \Bigg/ \left( \sum_{i=1, \dot{v}_i > 0}^{i:\, s_i = s_{end}} F_{res,i} \, \Delta s_i \right) \tag{13}$$

Figure 7 and Figure 8 show the mean value for 100 generated roads for each setting of $\sigma_y$ (10 different) and $L_h$ (7 different). The results show that in the 40 ton case the approximation is sound: in the worst case (short slopes and large amplitude) the acceleration term is on average ten times smaller than the resistance.

The scenario is somewhat different for the 80 ton vehicle where the acceleration term can be as big as 35% of the resistive term on average, in worst case. Thus the simplified model has questionable validity in this regime, meaning that for the largest inclination amplitudes the conclusions about the 80 ton vehicle are not reliable.

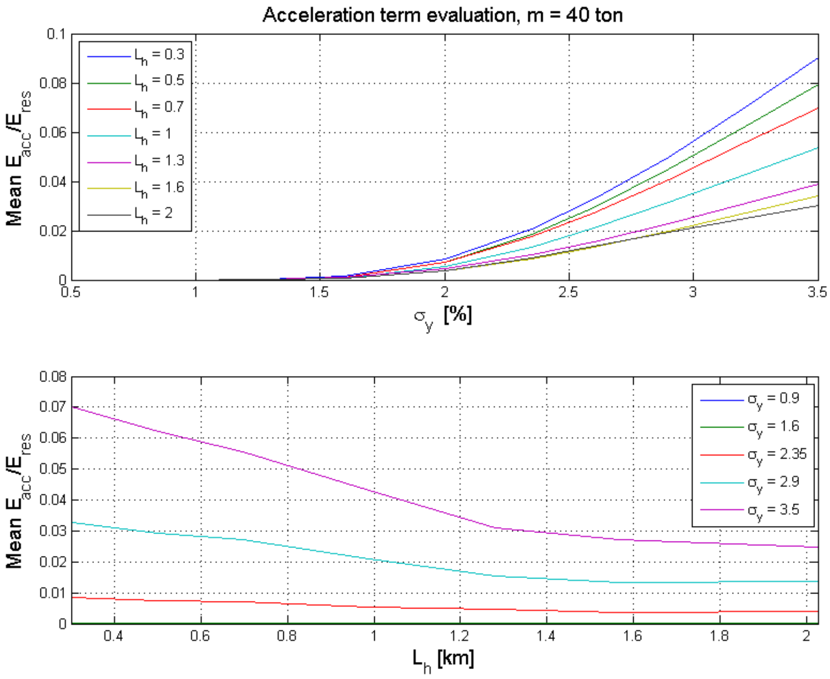


***Figure 7: Acceleration influence for the 40 ton case.***

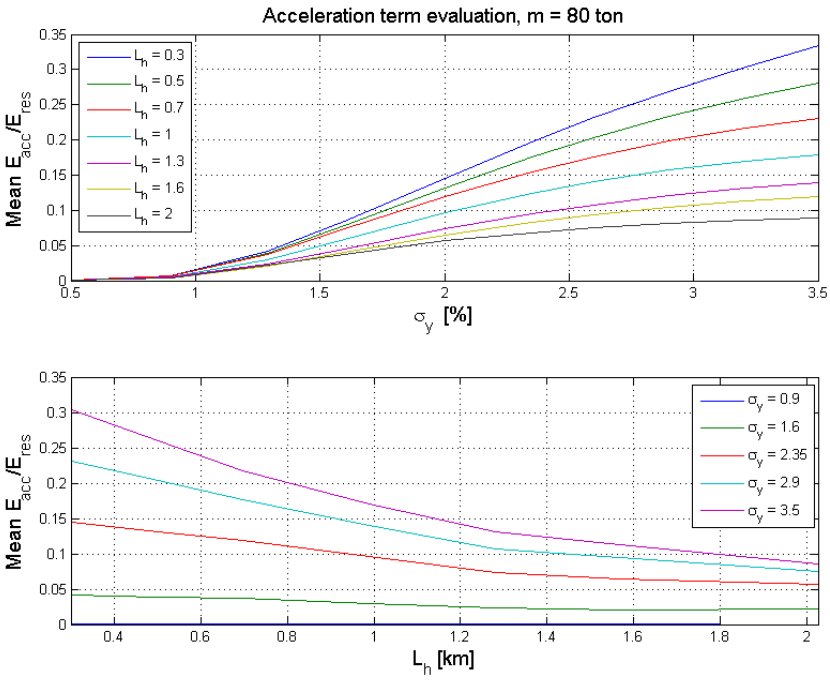


***Figure 8: Acceleration term influence for the 80 ton case.***