Ab initio Calculations of Charge Symmetry Breaking in the $A = 4$ Hypernuclei

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(Received 7 December 2015; published 22 March 2016)

We report on *ab initio* no-core shell model calculations of the mirror Λ hypernuclei ${}^4_\Lambda$H and ${}^4_\Lambda$He, using the Bonn-Jülich leading-order chiral effective field theory hyperon-nucleon potentials plus a charge symmetry breaking $\Lambda - \Sigma^0$ mixing vertex. In addition to reproducing rather well the $0^+_{\rm g.s.}$ and $1^+_{\rm exc}$ binding energies, these four-body calculations demonstrate for the first time that the observed charge symmetry breaking splitting of mirror levels, reaching hundreds of keV for $0^+_{\rm g.s.}$, can be reproduced using realistic theoretical interaction models, although with a non-negligible momentum cutoff dependence. Our results are discussed in relation to recent measurements of the ${}^4_\Lambda{\rm H}(0^+_{\rm g.s.})$ binding energy at the Mainz Microtron [A. Esser *et al.* (A1 Collaboration), Phys. Rev. Lett. 114, 232501 (2015)] and the ${}^4_\Lambda{\rm He}(1^+_{\rm exc})$ excitation energy [T.O. Yamamoto *et al.* (J-PARC E13 Collaboration), Phys. Rev. Lett. 115, 222501 (2015)].

DOI: 10.1103/PhysRevLett.116.122501

Introduction.—Charge symmetry in hadronic physics is broken in QCD by the up-down light quark mass difference and by the up and down quark QED interactions. Recent lattice QCD + QED simulations of SU(3) octet baryon mass differences within isospin multiplets, such as the neutron-proton mass difference Δ_{np}, which vanishes in the limit of charge symmetry, account nicely for the observed charge symmetry breaking (CSB) pattern in the lowest-mass nonstrange as well as strange baryon spectrum [1]. A comparable level of precision in reproducing theoretically CSB effects in the baryon-baryon interaction is lacking [2]. In practice, introducing two charge-dependent contact interaction terms in chiral effective field theory (EFT) applications, one is able at next-to-next-to-next-to-leading order (N3LO) to account quantitatively for the charge dependence of the low energy nucleon-nucleon (NN) scattering parameters [3]. For strangeness $S = -1$, however, given that low-energy Λp cross sections are poorly known and Λn scattering data do not exist, the available chiral EFT hyperon-nucleon (YN) interactions [4,5] do not include charge-dependent interaction terms. Potentially unique information on CSB in the ΛN interaction and in Λ hypernuclei is provided by the large Λ separation-energy difference $\Delta B_\Lambda^{J=0} = 350 \pm 60$ keV [6] in the $A = 4$ mirror hypernuclei 0^+ ground states (g.s.) and the apparently negligible difference $\Delta B_\Lambda^{J=1}$ in the 1^+ excited states [7]; see Fig. 1. Here, $\Delta B_\Lambda^J \equiv B_\Lambda^J({}^4_\Lambda{\rm He}) - B_\Lambda^J({}^4_\Lambda{\rm H})$. The recent precise measurement of the ${}^4_\Lambda{\rm H}_{\rm g.s.} \rightarrow {}^4{\rm He} + \pi^-$ decay at the Mainz Microtron (MAMI) [8] reaffirms a substantial CSB g.s. splitting $\Delta B_\Lambda^{J=0} = 270 \pm 95$ keV, which is consistent with the emulsion value cited above. Note that $\Delta B_\Lambda^{J=0}$ is considerably larger than the $\approx$70 keV assigned to CSB splitting in the mirror core nuclei ^{3}H and ^{3}He [9].

Dalitz and von Hippel [10] suggested that the SU(3) octet $\Lambda_{I=0}$ and $\Sigma^0_{I=1}$ hyperons are admixed in the physical Λ hyperon, thereby generating a CSB direct ΛN potential $V_{\rm CSB}$ that consists of isovector meson exchanges, notably a long-range one-pion exchange (OPE) component. Although these exchanges are forbidden in the ΛN channel by the strong interactions (SI), they do contribute strongly to the $\Lambda N \leftrightarrow \Sigma N$ coupling potential. Quite generally, the matrix element of $V_{\rm CSB}$ arising from $\Lambda - \Sigma^0$ mixing is related to the $I_{NY} = 1/2$ SI matrix element $\langle N\Sigma|V_{\rm SI}|N\Lambda\rangle$ by [11]

$$\langle N\Lambda|V_{\rm CSB}|N\Lambda\rangle = -0.0297\tau_{Nz}\frac{1}{\sqrt{3}}\langle N\Sigma|V_{\rm SI}|N\Lambda\rangle, \quad (1)$$

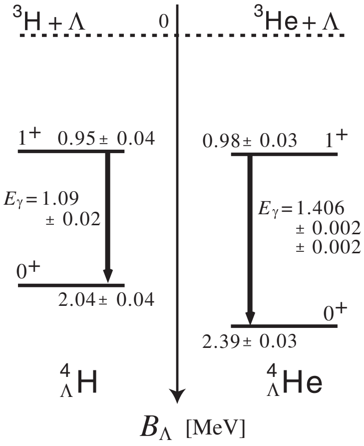

FIG. 1. (${}^4_\Lambda$H, ${}^4_\Lambda$He) mirror hypernuclei level diagram. The $0^+_{\rm g.s.}$ Λ separation energies B_Λ, loosely termed Λ binding energies, are from emulsion work [6], and the $1^+_{\rm exc}$ B_Λ values follow from γ-ray measurements of the excitation energies E_γ [7].

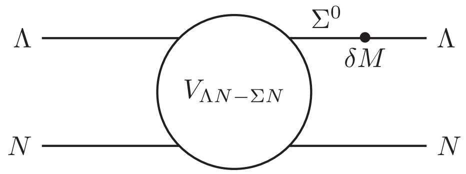

FIG. 2. CSB ΛN interaction diagram describing a SI $V_{\Lambda N-\Sigma N}$ interaction followed by a CSB $\Lambda - \Sigma^0$ mass-mixing vertex.

where $\tau_{Nz} = \pm 1$ for protons and neutrons, respectively, and the space-spin structure of this $N\Sigma$ state is taken identical with that of the $N\Lambda$ state embracing V_{CSB}. The CSB scale coefficient 0.0297 in Eq. (1) follows from the $\Lambda - \Sigma^0$ mass-mixing matrix element [12]

$$\langle \Sigma^0 | \delta M | \Lambda \rangle = \frac{1}{\sqrt{3}} (\Delta_{\Sigma^0\Sigma^+} - \Delta_{np}) = 1.14(5)\ \mathrm{MeV} \qquad (2)$$

and has been used in all previous CSB works listed below. A visualization of Eq. (1) is provided by the CSB ΛN interaction diagram of Fig. 2, where the $V_{\Lambda N-\Sigma N}$ blob represents *any* SI isovector meson exchange or contact term such as introduced in chiral EFT models [4].

Precise four-body calculations using the Nijmegen soft-core realistic meson exchange YN interaction models NSC97$_{e,f}$ [13], which include charge-dependent interactions induced by $\Lambda - \Sigma^0$ mixing and meson mixings, produced at most 30% of the observed CSB g.s. splitting $\Delta B_{\Lambda}^{J=0}$ [14–18]. Below, we comment on this insufficiency. More recent Nijmegen [19] or quark-cluster [20] models have not been used in four-body studies. With SI $\Lambda N \leftrightarrow \Sigma N$ potential energy contributions of order 10 MeV [14], and with a CSB scale of order 3%, Eq. (1) could yield CSB contributions of order 300 keV. Reproducing the observed CSB splitting poses a challenge for microscopic YN interaction models.

In this Letter we report on detailed *ab initio* no-core shell model (NCSM) calculations of the $A = 4$ Λ hypernuclei that employ the SI Bonn-Jülich LO chiral EFT YN interaction potentials [4], plus a CSB $\Lambda - \Sigma^0$ mixing interaction potential V_{CSB} generated by applying Eq. (1) to each one of the $\Lambda N \leftrightarrow \Sigma N$ V_{SI} components in this LO version. CSB meson mixings, with negligible contributions in the $A = 4$ hypernuclei [21], are disregarded here. In addition to reproducing reasonably well the $0^+_{\mathrm{g.s.}}$ and 1^+_{exc} binding energies, these four-body calculations establish for the first time as large CSB splittings $\Delta B_{\Lambda}^{J=0}$, as suggested by experiment, see Fig. 1, although with a non-negligible cutoff dependence. We also discuss possible implications to the recent Bonn-Jülich-Munich NLO chiral EFT YN interaction model [5].

Methodology.—The nuclear NCSM technique used in the present four-body calculations employs realistic two-body and three-body model interactions and is formulated in translationally invariant Jacobi-coordinate harmonic-oscillator (HO) bases [22]. Antisymmetrization with respect to nucleons is exercised in order to satisfy the Pauli principle. The resulting Hamiltonian is diagonalized in finite four-body HO bases, admitting all HO excitation energies $N\hbar\omega$, $N \leq N_{\max}$, up to $N_{\max}$ HO quanta. Extrapolated energy values $E(\omega)$, $N_{\max} \to \infty$, are obtained by fitting an exponential function to $E(N_{\max}, \omega\,\mathrm{fixed})$ sequences in the vicinity of the variational minima with respect to the HO basis frequency ω. The reliability of such extrapolations is then reflected in the independence of $E(\omega)$ of the frequency ω.

TABLE I. Cutoff dependence of Λ separation energies B_{Λ}^{J} in ${}^{4}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$ (all in MeV) from *ab initio* NCSM calculations at $\hbar\omega = 30(32)$ MeV for $J = 0(1)$, using N3LO (LO) chiral NN (YN) interactions [3] (Ref. [4]) plus Coulomb interactions, and V_{CSB} generated by Eq. (1) from the LO SI YN potentials. Experimental values are from Fig. 1.

Cutoff	550	600	650	700	Experiment
$B_{\Lambda}^{J=0}({}^{4}_{\Lambda}\mathrm{H})$	2.556	2.308	2.154	2.196	2.04 ± 0.04
$B_{\Lambda}^{J=0}({}^{4}_{\Lambda}\mathrm{He})$	2.586	2.444	2.398	2.490	2.39 ± 0.03
$B_{\Lambda}^{J=1}({}^{4}_{\Lambda}\mathrm{H})$	1.744	1.359	1.067	0.877	0.95 ± 0.04
$B_{\Lambda}^{J=1}({}^{4}_{\Lambda}\mathrm{He})$	1.572	1.166	0.839	0.654	0.98 ± 0.03

This NCSM technique, extended recently to light hypernuclei [23,24], is applied here to the $A = 4$ mirror hypernuclei using chiral N3LO NN and N2LO NNN interactions [3,25], respectively, both with a momentum cutoff of 500 MeV. These together with the Coulomb interaction reproduce the binding energies of the $A = 3$ core nuclei. For the SI YN coupled-channel potentials V_{SI}, we use the Bonn-Jülich LO chiral EFT SU(3)-based model with cutoff momenta Λ from 550 to 700 MeV [4] plus V_{CSB} evaluated from V_{SI} by using Eq. (1). Baryon mass differences within isomultiplets are incorporated. The reported calculations consist of fully converged ${}^{3}\mathrm{H}$ and ${}^{3}\mathrm{He}$ binding energies and (${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$) $0^+_{\mathrm{g.s.}}$ and 1^+_{exc} binding energies extrapolated to infinite model spaces from $N_{\max} = 18(14)$ for $J = 0(1)$. The NNN interaction is excluded from the calculations reported here, in order to save computing time, after verifying that its inclusion makes a difference of only a few keV in the calculation of the CSB splittings ΔB_{Λ}^{J} for both $J = 0$, 1.

Results.—The cutoff dependence of Λ separation energies in both $A = 4$ mirror hypernuclei, obtained from NCSM calculations with LO chiral EFT coupled-channel YN potentials [4] and V_{CSB} from Eq. (1), is shown in Table I. We used $N_{\max} \to \infty$ extrapolated binding-energy values for the ${}^{4}_{\Lambda}\mathrm{He}$ and ${}^{4}_{\Lambda}\mathrm{H}$ $J = 0(1)$ levels at fixed $\hbar\omega = 30(32)$ MeV, which is where the absolute variational minima occur for $\Lambda = 550$ and 600 MeV. For higher values of Λ the four-body absolute variational minima occur at slightly higher $\hbar\omega$ values. Although the spread of $B_{\Lambda}^{J}(\hbar\omega)$ values for a given cutoff momentum is of the order of 100 keV, it is considerably smaller and in fact marginal for

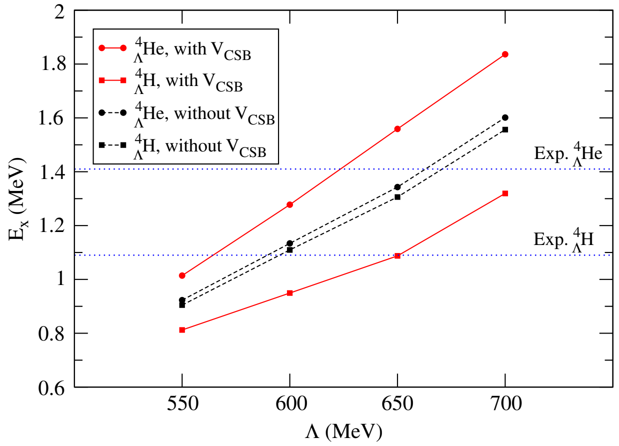

FIG. 3. Cutoff momentum dependence of excitation energies $E_x(0^+_{\rm g.s.} \to 1^+_{\rm exc})$ in ${}^4_\Lambda$H (squares) and ${}^4_\Lambda$He (circles) in *ab initio* NCSM calculations, at $\hbar\omega = 30(32)$ MeV for $J = 0(1)$, for LO chiral EFT coupled-channel YN potentials [4] with (solid lines) and without (dashed lines) $V_{\rm CSB}$ derived from these SI potentials using Eq. (1). The dotted horizontal lines denote E_x values from γ-ray measurements [7].

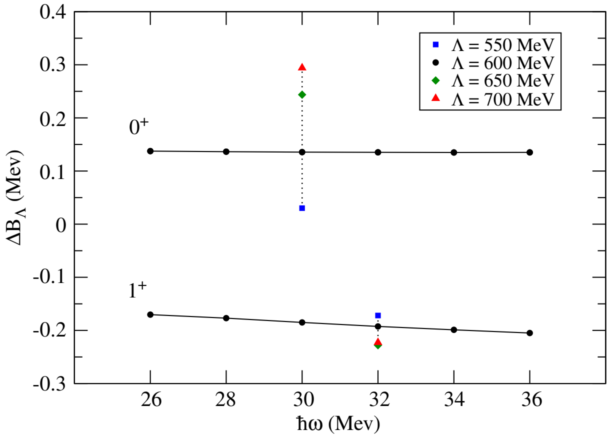

FIG. 4. Dependence of the separation-energy differences ΔB_Λ between ${}^4_\Lambda$He and ${}^4_\Lambda$H, for $0^+_{\rm g.s.}$ (upper curve) and for $1^+_{\rm exc}$ (lower curve) on the HO $\hbar\omega$ in *ab initio* NCSM calculations using LO chiral EFT coupled-channel YN potentials with cutoff momentum $\Lambda = 600$ MeV [4] plus $V_{\rm CSB}$ derived from these SI potentials using Eq. (1). Results for other values of Λ listed in the inset are shown at $\hbar\omega = 30(32)$ MeV for $J = 0(1)$.

the CSB splittings ΔB^J_Λ on which we focus here, as demonstrated by Fig. 4 below.

The Λ separation energies listed in Table I show a moderate cutoff dependence for the $0^+_{\rm g.s.}$ mirror levels and a stronger dependence for the $1^+_{\rm exc}$ mirror levels, with mean values for their charge-symmetric (CS) averages given by $\overline{B}{}^{\rm CS}_\Lambda(0^+_{\rm g.s.}) = 2.39^{+0.18}_{-0.12}$ MeV and $\overline{B}{}^{\rm CS}_\Lambda(1^+_{\rm exc.}) = 1.16^{+0.50}_{-0.39}$ MeV, which compare well with the CS-averaged experimental values derived from the last column in Table I. Furthermore, considering NCSM $N_{\rm max} \to \infty$ extrapolation uncertainties, our CS-averaged B_Λ values are in fair agreement with those reported in other four-body calculations using CS LO YN chiral EFT interactions [16–18,23,24]. A detailed analysis of calculational uncertainties will be given elsewhere.

Shown in Fig. 3 by solid lines is the cutoff momentum dependence of the $0^+_{\rm g.s.} \to 1^+_{\rm exc}$ excitation energies E_x formed from the B_Λ values listed in Table I for both $A = 4$ mirror hypernuclei. As observed in several few-body calculations of s-shell hypernuclei [26–29], E_x is strongly correlated with the $\Lambda N \leftrightarrow \Sigma N$ coupling potential which in the present context, through $\Lambda - \Sigma^0$ mixing, gives rise to CSB splittings of the $A = 4$ mirror levels.

Figure 3 demonstrates a steady rise of both $E_x({}^4_\Lambda{\rm He})$ and $E_x({}^4_\Lambda{\rm H})$ as a function of the cutoff momentum Λ, with a CS-averaged value $\overline{E}{}^{\rm CS}_x = 1.23^{+0.35}_{-0.32}$ MeV compared to 1.25 ± 0.02 MeV deduced from the two γ-ray energies shown in Fig. 1. A steady rise is also observed in the difference $\Delta E^{\rm CSB}_x$ with a mean value 380^{+140}_{-180} keV compared to 320 ± 20 keV, again from Fig. 1. In agreement with previous calculations [14–18], residual CSB contributions of up to 30 keV from electromagnetic mass differences, mostly of Σ hyperons, and from the increased Coulomb repulsion in the ^{3}He core of ${}^4_\Lambda$He, survive upon switching off $V_{\rm CSB}$, as demonstrated by the slight difference between the two middle dashed lines in the figure.

In Fig. 4 we show the $\hbar\omega$ dependence of separation-energy differences ΔB^J_Λ between ${}^4_\Lambda$He and ${}^4_\Lambda$H levels of a given spin J, for $0^+_{\rm g.s.}$ and $1^+_{\rm exc}$, using $N_{\rm max} \to \infty$ extrapolated values for the four possible binding energies which are calculated for a cutoff $\Lambda = 600$ MeV and including $V_{\rm CSB}$ from Eq. (1). Extrapolation uncertainties for ΔB^J_Λ are about 20 keV. The variation of $\Delta B^{J=0}_\Lambda$ in the spanned $\hbar\omega$ range amounts to a few keV, whereas that of $\Delta B^{J=1}_\Lambda$ is larger, amounting to ~30 keV. It is worth noting that the difference $\Delta B^{J=0}_\Lambda - \Delta B^{J=1}_\Lambda$ between the upper and lower curves assumes at $\Lambda = 600$ MeV the value 0.33 ± 0.04 MeV, in perfect agreement with the difference $E_\gamma({}^4_\Lambda{\rm He}) - E_\gamma({}^4_\Lambda{\rm H}) = 0.32 \pm 0.02$ MeV between the two γ-ray energies shown in Fig. 1. The figure also demonstrates a strong cutoff dependence of $\Delta B^{J=0}_\Lambda$, varying between 30 to 300 keV upon increasing Λ, together with a considerably weaker cutoff dependence of $\Delta B^{J=1}_\Lambda$, varying between −170 and −230 keV. Note that $\Delta B^{J=0}_\Lambda$ comes out invariably positive, whereas $\Delta B^{J=1}_\Lambda$ is robustly negative. With mean values $\overline{\Delta B}{}^{J=0}_\Lambda = 176^{+118}_{-146}$ keV and $\overline{\Delta B}{}^{J=1}_\Lambda = -204^{+32}_{-24}$ keV, the mean values $\overline{\Delta B}{}^J_\Lambda$ satisfy

$$\overline{\Delta B}{}^{J=1}_\Lambda \approx -\overline{\Delta B}{}^{J=0}_\Lambda < 0. \qquad (3)$$

Discussion.—To understand the CSB pattern Eq. (3) for the $A = 4$ hypernuclei, we note that the SI $\Lambda N \leftrightarrow \Sigma N$ coupling potential in the LO chiral EFT YN model of Ref. [4] consists of a pseudoscalar meson exchange, dominated by OPE, plus two s-wave interaction contact terms (CT) of which the ${}^3S_1$ CT is negligible and the ${}^1S_0$ CT is large. In a zeroth-order single-particle description of the $A = 4$ hypernuclei, and using Eq. (1), these $\Lambda N \leftrightarrow \Sigma N$ coupling-potential components contribute to the CSB separation-energy differences as follows:

$$\Delta B_\Lambda^{J=0} = \frac{3}{2}C_1 - \frac{1}{2}C_0, \qquad \Delta B_\Lambda^{J=1} = \frac{1}{2}C_1 + \frac{1}{2}C_0, \tag{4}$$

with $C_S = C_S^{\mathrm{CT}} + C_S^{\pi}$ the sum of contributions to the triplet ($S = 1$) and singlet ($S = 0$) matrix elements from CT and from OPE. The $\vec{\sigma}_Y \cdot \vec{\sigma}_N$ spin dependence of C_S^{π} leads in this approximation to $C_1^{\pi} = -\frac{1}{3}C_0^{\pi}$. Recalling that these matrix elements already incorporate isospin, both C_0^{π} and C_0^{CT} are negative. Hence,

$$\Delta B_\Lambda^{J=0} \approx -\left(C_0^{\pi} + \frac{1}{2}C_0^{\mathrm{CT}}\right) > 0, \tag{5}$$

$$\Delta B_\Lambda^{J=1} \approx +\left(\frac{1}{3}C_0^{\pi} + \frac{1}{2}C_0^{\mathrm{CT}}\right) < 0, \tag{6}$$

in agreement with the signs of the calculated CSB splittings. In the limit that C_0^{π} is negligible with respect to C_0^{CT}, Eq. (3) is recovered. We conclude that it is the sizable ${}^1S_0$ $\Lambda N \leftrightarrow \Sigma N$ coupling-potential CT in the LO chiral EFT YN interaction model [4] that makes it possible to generate sufficiently large values of $\Delta B_\Lambda^{J=0}$ to explain the observed CSB splitting of the $0^+_{\mathrm{g.s.}}$ mirror levels. However, the opposite-sign values of $\Delta B_\Lambda^{J=1}$ appear too large with respect to the near degeneracy observed for the 1^+_{exc} mirror levels, even when preliminary results of the latest MAMI measurement are considered [30].

In contrast to the ability of the LO chiral EFT YN interaction model to generate sizable CSB g.s. splittings $\Delta B_\Lambda^{J=0}$ owing to a dominant ${}^1S_0$ $\Lambda N \leftrightarrow \Sigma N$ coupling-potential CT, the $\Lambda N \leftrightarrow \Sigma N$ coupling potential in NSC97 models is dominated by a ${}^3S_1 - {}^3D_1$ tensor component which is ineffective in generating a large CSB contribution when used in the right-hand side of Eq. (1). The reason is that the SI ΛN states on the left-hand side, in the case of NSC97, are dominated by purely s-wave channels [14]. The NSC97 ${}^1S_0$ $\Lambda N \leftrightarrow \Sigma N$ coupling-potential contribution that replaces C_0^{π} in Eq. (5) is too weak to generate on its own a sizable $\Delta B_\Lambda^{J=0}$. A detailed account of this item will be given elsewhere.

It is tempting to speculate on the $A = 4$ CSB separation-energy differences ΔB_Λ^{J} anticipated from applying Eq. (1) to the recently published NLO chiral EFT YN interaction [5]. The $\Lambda N \leftrightarrow \Sigma N$ coupling-potential contact terms differ considerably in NLO from those in LO, with a very large C_1^{CT} that dominates in NLO over C_0^{CT}, and with a new ${}^3S_1 - {}^3D_1$ CT. It is fair to assume that pseudoscalar one- and two-meson exchange contributions in NLO are still dominated by OPE. Dominance of C_1^{CT} over all other allowed contributions would result in *negative* values of ΔB_Λ^{J}, with $\Delta B_\Lambda^{J=0}$ three times as large as $\Delta B_\Lambda^{J=1}$; this would disagree with the observed positive value for $\Delta B_\Lambda^{J=0}$, see Fig. 1, confirmed also by the new MAMI measurement [8]. We note, furthermore, that the NLO version underestimates the $A = 4$ hypernuclear g.s. separation energy, with $B_\Lambda^{\mathrm{CS}} \approx$ 1.5–1.6 MeV [17], compared to ≈2.2 MeV from Fig. 1. Three-body YNN interaction terms introduced in higher-order versions in order to recover the missing g.s. attraction might provide an additional source of CSB in Λ hypernuclei. However, expecting that the dominant YNN terms correspond to $\Sigma^*(1385)NN$ intermediate states [31] and realizing that, unlike Σ^0, $\Sigma^{*0}(\frac{3}{2}^+)$ cannot mix with $\Lambda^0(\frac{1}{2}^+)$ to generate CSB, these YNN interaction terms will not produce as strong CSB as evaluated here using Eq. (1), which is based on the Dalitz–von Hippel $\Lambda^0 - \Sigma^0$ mixing mechanism [10]. It is therefore questionable whether the NLO version [5] offers an advantage over the LO version [4] for Λ hypernuclei, given also that both provide comparably reasonable fits to the low-energy YN scattering data.

Summary and outlook.—In conclusion, we have presented the first CSB *ab initio* calculation in hypernuclei with chiral EFT coupled-channel YN interactions, showing that the LO version [4] is capable of producing a *large* CSB $0^+_{\mathrm{g.s.}}$ splitting $\overline{\Delta B_\Lambda^{J=0}} \sim 180 \pm 130$ keV. This is consistent with a g.s. splitting of 270 ± 95 keV reported by the MAMI experiment [8]. Our NCSM calculation reproduces quantitatively and with weak cutoff dependence the $0^+_{\mathrm{g.s.}}$ binding energies of the $A = 4$ mirror hypernuclei, whereas the 1^+_{exc} binding-energy calculation, which is known to be numerically more challenging [14], displays a strong cutoff dependence. The calculated CSB 1^+_{exc} splitting is of opposite sign to that of the $0^+_{\mathrm{g.s.}}$ splitting and fairly large: $\overline{\Delta B_\Lambda^{J=1}} \approx -200 \pm 30$ keV, with a weak cutoff dependence. While preliminary results from MAMI suggest a relatively small and negative CSB splitting of -84 ± 85 keV for the 1^+_{exc} mirror levels [30], the measurement systematic uncertainty is still too large to rule out the prediction of the LO version.

In future work it would be of great interest to apply the CSB generating equation (1) in *ab initio* calculations of the $A = 4$ mirror hypernuclei using the recent NLO EFT version [5], and also to readjust the $\Lambda N \leftrightarrow \Sigma N$ contact terms in NLO by imposing the accurate CSB datum $E_\gamma({}^4_\Lambda\mathrm{He}) - E_\gamma({}^4_\Lambda\mathrm{H}) = 0.32 \pm 0.02$ MeV, so it is reproduced in four-body calculations with as weak cutoff dependence as possible. Another natural follow-up would be to extend these CSB calculations in LO and NLO to p-shell hypernuclei. Recent shell model calculations [11], using a schematic $\Lambda N \leftrightarrow \Sigma N$ coupling-potential model, suggest that CSB splittings of g.s. mirror levels in p-shell hypernuclei decrease in size with respect to $A = 4$, and perhaps even reverse sign, in rough agreement with old emulsion data [6]. Such extensions of the present work pose a valuable theoretical challenge to the microscopic understanding of strange nuclear systems.

We are grateful to Petr Navrátil for providing valuable advice and help on extensions of nuclear-physics NCSM codes, to Johann Haidenbauer and Andreas Nogga for providing us with the input LO EFT YN potentials used in the present work, to Nir Barnea for stimulating discussions on EFT few-body applications, and to Jiří Mareš for a critical reading of this manuscript. The research of D. G. was supported by the Grant Agency of the Czech Republic (GACR) Grant No. P203/15/04301S.

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