Estimating Scatterer Positions using Sparse Approximation

J. Winges, T. Rylander
Department of Signals and systems, Chalmers University of Technology, SE-41296 Göteborg
winges@chalmers.se

I. INTRODUCTION

We present a convex optimization method for a class of inverse scattering problems. The method is based on three steps: (i) compute a database of scattering data for the measurement situations of interest; (ii) find a sparse approximation of a measured response in terms of the database; and (iii) estimate a representative description from the sparse approximation as a weighted average.

Here, we intend to estimate the position of multiple scatters inside the microwave measurement system shown in Fig. 1, where we measure the 6-by-6 scattering matrix in the frequency band from 2.7 GHz to 4.2 GHz. The size and

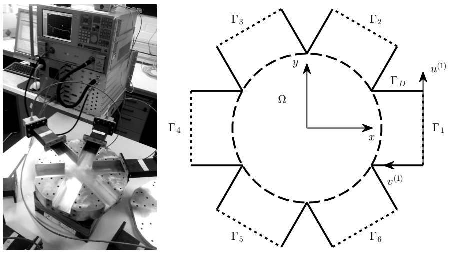


Fig. 1: To the left, partially disassembled measurement system. To the right, 2D model of the measurement region.

permittivity of the scatterers is a priori known, i.e. cylindrical acrylic-glass samples with $r = 0.52\,\text{cm}$ and $\epsilon_r \approx 2.54$ [1]. A 2D FEM model of the electromagnetic field problem is used to compute the scattering matrix.

First, we compute the databases $\mathbf{D}_1$ and $\mathbf{D}_2$ of the scattering data from (i) a single scatter and (ii) two closely spaced scatters. Here, we use 50 frequency points that are vectorized and stored in columns of $\mathbf{D}$. $\mathbf{D}_1$ and $\mathbf{D}_2$ are constructed for 2379 and 26171 different positions, which corresponds to a resolution of 2.1 mm and 5.3 mm, respectively. We also form $\mathbf{D}_{1\&2} = [\mathbf{D}_1, \mathbf{D}_2]$. The positions are described by the parameters $\mathbf{p}_n$ and the scattering data is found by subtracting an empty reference case from the scattering matrix.

Next, we find the sparse approximation $\mathbf{x}$ [2], with weights $x_n \in [0, 1]$, by solving the convex minimization problem

$$\min_{\widetilde{\mathbf{x}}} \tfrac{1}{2}||\widetilde{\mathbf{D}}\widetilde{\mathbf{x}} - \mathbf{m}||_2^2 + \gamma||\widetilde{\mathbf{x}}||_1, \tag{1a}$$

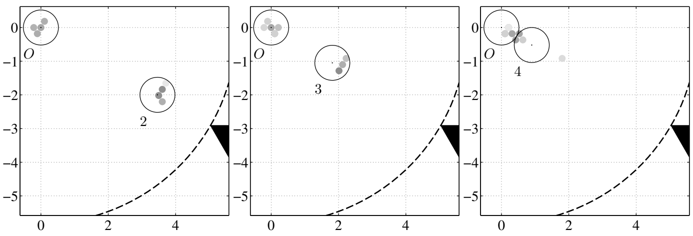
$$\text{s.t.} \quad \widetilde{x}_n \in [0, ||\mathbf{D}(:, n)||_2] \quad \forall n. \tag{1b}$$

Here, $\mathbf{m}$ is the measured scattering data and $\widetilde{\mathbf{D}}(:, n) = \mathbf{D}(:, n)/||\mathbf{D}(:, n)||_2$. Hence, $\mathbf{x}$ is given by $x_n = \widetilde{x}_n/||\mathbf{D}(:, n)||_2$ and it describes a sparse linear combination of situations in $\mathbf{D}$, which approximates $\mathbf{m}$.

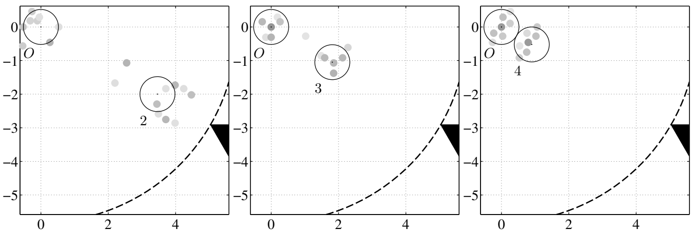
Finally, we estimate the scatterer positions of $\mathbf{m}$ by combining x_n and $\mathbf{p}_n$ as a weighted average $\hat{\mathbf{p}} \simeq \sum_n x_n \mathbf{p}_n / \sum_n x_n$, where the average is computed only for neighboring $\mathbf{p}_n$.

II. RESULTS

We solve (1) using either the database $\mathbf{D}_1$ or $\mathbf{D}_{1\&2}$ for three measurement cases with 2 samples. Here, we find that $\gamma = 0.1$ offers a good trade-off between residual and sparsity. The found non-zero x_n for each case is shown using dots at the positions $\mathbf{p}_n$ in Fig. 2, and the gray scale indicates the weight. For most cases, we find that there are a few non-zero elements



(a) Case 1 (b) Case 2 (c) Case 3



(d) Case 1 (e) Case 2 (f) Case 3

Fig. 2: Part of the measurement region with the two acrylic-glass cylinders shown by a solid circle for three different cases. In (a-c) we use $\mathbf{D}_1$, and in (d-f) we use $\mathbf{D}_{1\&2}$.

in $\mathbf{x}$ that can be associated with a single scatterer and, thereby, grouped by means of $\hat{\mathbf{p}}$. This gives the sample positions to an accuracy of 1 mm. Difficulties arise if we use (i) $\mathbf{D}_1$ and the two samples are close, or (ii) $\mathbf{D}_{1\&2}$ and the samples are further apart then the separation distance in $\mathbf{D}_2$. Both of these difficulties are addressed by using the appropriate database.

III. CONCLUSIONS

A convex optimization method for estimating multiple scatterer positions is presented and tested in a microwave measurement system.

ACKNOWLEDGEMENT

This work was supported by the Swedish Research Council (dnr 2010-4627), "Model-based Reconstruction and Classification Based on Near-Field Microwave Measurements". The computations were performed on resources at Chalmers Centre for Computational Science and Engineering (C3SE) provided by the Swedish National Infrastructure for Computing (SNIC).

REFERENCES

[1] L. Cerullo, J. Nohlert, J. Winges, and T. Rylander, "Microwave Measurements for Metal Vessels," in *7th Eur. Conf. Antennas Propagation, EuCAP*, pp. 3869–3873, 2013.

[2] A. Bruckstein, D. Donoho, and M. Elad, "From sparse solutions of systems of equations to sparse modeling of signals and images," *SIAM Rev.*, vol. 51, pp. 34–81, feb 2009.