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On the Required Number of Antennas in a Point-to-Point Large-but-Finite MIMO System

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***Abstract*—In this paper, we investigate the performance of the point-to-point multiple-input-multiple-output (MIMO) systems in the presence of a large but finite numbers of antennas at the transmitters and/or receivers[1]. Considering the cases with and without hybrid automatic repeat request (HARQ) feedback, we determine the minimum numbers of the transmit/receive antennas which are required to satisfy different outage probability constraints. We study the effect of the spatial correlation between the antennas on the system performance. Also, the required number of antennas are obtained for different fading conditions. Our results show that different outage requirements can be satisfied with relatively few transmit/receive antennas.**

I. Introduction

The next generation of wireless networks must provide high-rate data streams for everyone everywhere at any time. To address the demands, the main strategy persuaded in the last few years is the network *densification* [1]–[13]. One of the promising techniques to densify the network is to use many antennas at the transmit and/or receive terminals. This approach is referred to as massive or large multiple-input-multiple-output (MIMO) in the literature.

In general, the more antennas the transmitter and/or receiver are equipped with, the better the data rate/link reliability. Thus, the trend is towards asymptotically high number of antennas. This is specially because millimeter wave communication [14], which are indeed expected to be implemented in the next generation of wireless networks, makes it possible to assemble many antennas at the transmit/receive terminals. However, large MIMO implies challenges such as hardware impairments which may limit the number of antennas in practice. Also, one of the bottlenecks of large MIMO is channel state information (CSI) acquisition. Thus, it is interesting to use feedback schemes such as hybrid automatic repeat request (HARQ) whose overhead does not scale with the number of antennas.

The performance of HARQ protocols in MIMO systems is studied in, e.g., [15]–[17]. MIMO transmission with many antennas is advocated in [1], [2] where time-division duplex (TDD)-based training is utilized for CSI feedback. Also, [3]–[5] and [6]–[8] introduce TDD- and FDD-based (F: frequency) schemes for large systems, respectively. Considering imperfect CSI, [9] derives lower bounds for the uplink achievable rate of MIMO setups with large but finite number of antennas. Finally, [10] (resp. [11]) studies zero-forcing based TDD (resp. TDD/FDD) systems under the assumption that the number of transmit antennas and the single-antenna users are asymptotically large while their ratio remains bounded (For detailed review of the literature on massive MIMO, see [12], [13]).

To summarize, a large part of the literature on the point-to-point and multi-user large MIMO is based on the assumption of asymptotically many antennas. Then, a natural question is how many transmit/receive antennas do we require in practice to satisfy different quality-of-service requirements. The interesting answer this paper establishes is relatively few, for a large range of outage probabilities.

Here, we study the outage-limited performance of point-to-point MIMO systems in the cases with large but finite number of antennas. We derive closed-form expressions for the required number of transmit and/or receive antennas satisfying various outage probability requirements (Theorem 1). The results are obtained for different fading conditions and in the cases with or without HARQ. Furthermore, we analyze the effect of the antennas spatial correlation on the system performance (Section V.B).

As opposed to [15]–[17], we consider large MIMO setups and determine the required number of antennas in outage-limited conditions. Also, the paper is different from [1]–[13] because we study the outage-limited scenarios in point-to-point systems, implement HARQ and the number of antennas is considered to be finite. The differences in the problem formulation and the channel model makes the problem solved in this paper completely different from the ones in [1]–[13], [15]–[17], leading to different analytical/numerical results, as well as to different conclusions.

Our analytical and numerical results indicate that different quality-of-service requirements can be satisfied with relatively few transmit/receive antennas. Also, the implementation of HARQ reduces the required number of antennas significantly. Finally, the spatial correlation between the antennas increases the required number of antennas while, for a large range of correlation conditions, the same scaling rules hold for the uncorrelated and correlated fading scenarios.

II. System Model

Consider a point-to-point MIMO setup with N_t transmit antennas and N_r receive antennas. In this way, the received signal is given by

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$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \mathbf{Z} \in \mathcal{C}^{N_\text{r}\times 1}, \tag{1}$$

where $\mathbf{H} \in \mathcal{C}^{N_\text{r}\times N_\text{t}}$ is the fading matrix, $\mathbf{X} \in \mathcal{C}^{N_\text{t}\times 1}$ is the transmitted signal and $\mathbf{Z} \in \mathcal{C}^{N_\text{r}\times 1}$ denotes the independent and identically distributed (IID) complex Gaussian noise matrix. The results are mainly given for IID Rayleigh-fading channels where each element of the channel matrix $\mathbf{H}$ follows a complex Gaussian distribution $\mathcal{CN}(0,1)$ (To analyze the effect of the antennas spatial correlation, see Fig. 4 and Section V.B). The channel coefficients are assumed to be known at the receiver which is an acceptable assumption in block-fading channels [15]–[17]. On the other hand, there is no CSI available at the transmitter except the HARQ feedback bits.

As the most promising HARQ approach [15]–[17], we consider the incremental redundancy (INR) HARQ with a maximum of M retransmissions. Note that setting $M = 1$ represents the cases without HARQ, i.e., open-loop communication. Also, a packet is defined as the transmission of a codeword along with all its possible retransmissions. We study the system performance for three different fading conditions:

- **Fast-fading**. Here, a finite number of channel realizations are experienced within each HARQ retransmission round.
- **Slow-fading**. In this model, the channel is supposed to change between two successive retransmission rounds, while it is fixed for the duration of each retransmission.
- **Quasi-static**. The channel is assumed to remain fixed within a packet period.

Fast-fading is an appropriate model for fast-moving users [17]. On the other hand, slow-fading can properly model the cases with users of moderate speeds or frequency-hopping schemes [16], [17]. Finally, the quasi-static represents the scenarios with slow-moving or stationary users [15], [17].

III. Problem Formulation

Considering INR HARQ with a maximum of M retransmissions, Q information nats are encoded into a codeword of length ML channel uses and the codeword is divided into M sub-codewords of length L. In each retransmission round, the transmitter sends a new sub-codeword and the receiver combines all signals received up to the end of that round. Thus, the equivalent rate at the end of round m is $\frac{Q}{mL} = \frac{R}{m}$ npcu where R denotes the initial transmission rate. The retransmissions continue until the message is correctly decoded by the receiver or the maximum permitted retransmission round is reached.

Let us denote the determinant and the Hermitian of the matrix $\mathbf{X}$ by $|\mathbf{X}|$ and $\mathbf{X}^\text{h}$, respectively. Assuming fast-fading conditions with T independent fading realizations $\mathbf{H}((m-1)T+1), \dots, \mathbf{H}(mT)$ in the mth round and an isotropic Gaussian input distribution over all transmit antennas, the results of, e.g., [18, Chapter 15], can be used to find the outage probability of the INR-based MIMO-HARQ scheme as

$$\Pr(\text{Outage})^{\text{Fast-fading}} = \Pr\left(\frac{1}{MT}\sum_{n=1}^{M}\sum_{t=(n-1)T+1}^{nT}\log\left|\mathbf{I}_{N_\text{r}} + \frac{\phi}{N_\text{t}}\mathbf{H}(t)\mathbf{H}(t)^h\right| \le \frac{R}{M}\right). \tag{2}$$

Here, ϕ is the total transmission power and $\frac{\phi}{N_\text{t}}$ is the transmission power per transmit antenna (in dB, we have $10\log_{10}\phi$ which, because the noise variance is set to 1, represents the signal-to-noise ratio (SNR) as well). Also, $\mathbf{I}_{N_\text{r}}$ represents the $N_\text{r}\times N_\text{r}$ identity matrix.

Considering $T=1$ in (2), the outage probability is

$$\Pr(\text{Outage})^{\text{Slow-fading}} = \Pr\left(\frac{1}{M}\sum_{n=1}^{M}\log\left|\mathbf{I}_{N_\text{r}} + \frac{\phi}{N_\text{t}}\mathbf{H}(n)\mathbf{H}(n)^h\right| \le \frac{R}{M}\right), \tag{3}$$

in a slow-fading channel. Also, setting $\mathbf{H}(t) = \mathbf{H}, \forall t = 1,\dots,MT$, outage probability in a quasi-static channel is

$$\Pr(\text{Outage})^{\text{Quasi-static}} = \Pr\left(\log\left|\mathbf{I}_{N_\text{r}} + \frac{\phi}{N_\text{t}}\mathbf{H}\mathbf{H}^h\right| \le \frac{R}{M}\right). \tag{4}$$

Using (2)-(4) for given initial transmission rate and SNR, the problem formulation of the paper can be expressed as

$$\{\hat{N}_\text{t}, \hat{N}_\text{r}\} = \arg\min_{N_\text{t},N_\text{r}}\{\Pr(\text{Outage}) \le \theta\}. \tag{5}$$

Here, θ denotes an outage probability constraint and $\hat{N}_\text{t}, \hat{N}_\text{r}$ are the minimum numbers of transmit/receive antennas that are required to satisfy the outage probability constraint. In the following, we study (5) in four distinct cases:

- Case 1: N_r is large but N_t is given.
- Case 2: N_r is given but N_t is large.
- Case 3: Both N_t and N_r are large and the transmission SNR is low.
- Case 4: Both N_t and N_r are large and the transmission SNR is high.

Note that Cases 1-3 are commonly of interest in large MIMO systems. However, for the completeness of discussions, we consider Case 4 as well. Moreover, in harmony with the literature [10], [11][2], we analyze Cases 3-4 under the assumption

$$\frac{N_\text{t}}{N_\text{r}} = K, \tag{6}$$

with K being a constant. However, it is straightforward to extend the results of the paper to the cases with other relations between the numbers of antennas.

IV. Performance Analysis

To solve (5), let us first introduce Lemma 1. The lemma is of interest because it represents the outage probability as a function of the number of antennas, and simplifies the performance analysis remarkably.

Lemma 1: Considering Cases 1-4, the outage probability of the INR-based MIMO-HARQ system is given by

$$\begin{cases} \Pr(\text{Outage})^{\text{Fast-fading}} = Q\left(\frac{\sqrt{MT}(\mu - \frac{R}{M})}{\sigma}\right), & \text{(i)} \\ \Pr(\text{Outage})^{\text{Slow-fading}} = Q\left(\frac{\sqrt{M}(\mu - \frac{R}{M})}{\sigma}\right), & \text{(ii)} \\ \Pr(\text{Outage})^{\text{Quasi-static}} = Q\left(\frac{\mu - \frac{R}{M}}{\sigma}\right), & \text{(iii)} \end{cases} \tag{7}$$

[2]In [10], [11], which study multi-user MIMO setups, N_t and N_r are supposed to follow (6) while, as opposed to our work, they are considered to be asymptotically large.

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}\mathrm{d}u$ is the Gaussian Q-function and for different cases μ and σ are given in (8).

Proof. The proof is based on (2)-(4) and [19, Theorems 1-3], where considering Cases 1-4 the random variable $Z(t)=\log|\mathbf{I}_{N_\text{r}}+\frac{\phi}{N_\text{t}}\mathbf{H}(t)\mathbf{H}(t)^h|$ converges in distribution to a Gaussian random variable $Y\sim\mathcal{N}(\mu,\sigma^2)$ which, depending on the numbers of antennas, has the following characteristics

$$(\mu,\sigma^2)=\begin{cases}\left(N_\text{t}\log\left(1+\frac{N_\text{r}\phi}{N_\text{t}}\right),\frac{N_\text{t}}{N_\text{r}}\right), & \text{if Case 1}\\ \left(N_\text{r}\log\left(1+\phi\right),\frac{N_\text{r}\phi^2}{N_\text{t}(1+\phi)^2}\right), & \text{if Case 2}\\ \left(N_\text{r}\phi,\frac{N_\text{r}}{N_\text{t}}\phi^2\right), & \text{if Case 3}\\ \left(\tilde{\mu},\tilde{\sigma}^2\right), & \text{if Case 4}\\ \tilde{\mu}=N_\text{min}\log\left(\frac{\phi}{N_\text{t}}\right)+N_\text{min}\left(\sum_{i=1}^{N_\text{max}-N_\text{min}}\frac{1}{i}-\gamma\right) & \\ \quad+\sum_{i=1}^{N_\text{min}-1}\frac{i}{N_\text{max}-i},\ \gamma=0.5772\dots & \\ \tilde{\sigma}^2=\sum_{i=1}^{N_\text{min}-1}\frac{i}{(N_\text{max}-N_\text{min}+i)^2} & \\ \quad+N_\text{min}\left(\frac{\pi^2}{6}-\sum_{i=1}^{N_\text{max}-1}\frac{1}{i^2}\right), & \\ N_\text{max}\doteq\max(N_\text{t},N_\text{r}),\ N_\text{min}\doteq\min(N_\text{t},N_\text{r}). & \end{cases}\tag{8}$$

In this way, from (2) and for different cases, the outage probability in fast-fading condition is given by

$$\Pr(\text{Outage})^\text{Fast-fading}=\Pr\left(Z\leq\frac{R}{M}\right),Z\doteq\frac{1}{MT}\sum_{t=1}^{MT}Z(t),\tag{9}$$

where, because Z is the average of MT independent Gaussian random variables $Y\sim\mathcal{N}(\mu,\sigma^2)$, we have $Z\sim\mathcal{N}(\mu,\frac{1}{MT}\sigma^2)$. Consequently, using the cumulative distribution function (CDF) of the Gaussian random variables, the outage probability of the fast-fading condition is given by (7.i). The same arguments can be applied to derive (7.ii-iii) in the slow-fading and quasi-static conditions. □

Using Lemma 1, the minimum numbers of antennas satisfying different outage probability constraints are determined as stated in Theorem 1.

Theorem 1: The minimum numbers of the transmit/receive antennas in an INR-based MIMO-HARQ system that satisfy the outage probability constraint $\Pr(\text{Outage})\leq\theta$ are given by

$$\begin{cases}\hat{N}_\text{r}=\frac{\left(Q^{-1}(\theta)\right)^2}{4MTN_\text{t}W^2\left(\frac{Q^{-1}(\theta)\sqrt{\phi}}{2\sqrt{MT}N_\text{t}}e^{-\frac{R}{2MN_\text{t}}}\right)}, & \text{if Case 1}\\ \hat{N}_\text{t}=\left(\frac{\phi\sqrt{N_\text{r}}Q^{-1}(\theta)}{\sqrt{MT}(1+\phi)\left(N_\text{r}\log(1+\phi)-\frac{R}{M}\right)}\right)^2, & \text{if Case 2}\\ \hat{N}_\text{r}=\hat{N},\hat{N}_\text{t}=K\hat{N},\hat{N}=\frac{R}{M\phi}+\frac{Q^{-1}(\theta)}{\sqrt{MTK}}, & \text{if Case 3}\\ \hat{N}_\text{r}=\hat{N},\hat{N}_\text{t}=K\hat{N} & \text{if Case 4}\\ \begin{cases}\hat{N}\simeq\frac{\frac{R}{M}+\frac{Q^{-1}(\theta)}{\sqrt{MT}}\sqrt{\log\left(\frac{K}{K-1}\right)}}{\log(\phi)-\gamma-1+(K-1)\log\left(\frac{K}{K-1}\right)}, & K>1\\ \hat{N}\simeq\frac{\frac{R}{M}+\frac{Q^{-1}(\theta)}{\sqrt{MT}}\sqrt{-\log(1-K)}}{K\left(\log(\phi)-\gamma-1-\log(K)+\left(\frac{K-1}{K}\right)\log(1-K)\right)} & K<1,\end{cases}\end{cases}\tag{10}$$

if the channel is fast-fading. Here, $Q^{-1}(x)$ and $W(x)$ are the inverse Q-function and the Lambert W function, respectively. For the slow-fading and quasi-static conditions, the minimum numbers of the antennas are obtained by (10) where the term $\frac{Q^{-1}(\theta)}{\sqrt{MT}}$ is replaced by $\frac{Q^{-1}(\theta)}{\sqrt{M}}$ and $Q^{-1}(\theta)$, respectively.

Proof. The proof is based on the fact that, considering Lemma 1 and a fast-fading condition, (5) is rephrased as

$$\{\hat{N}_\text{t},\hat{N}_\text{r}\}^\text{Fast-fading}=\underset{N_\text{t},N_\text{r}}{\arg}\left\{\frac{\mu-\frac{R}{M}}{\sigma}=\frac{Q^{-1}(\theta)}{\sqrt{MT}}\right\}.\tag{11}$$

Then, implementing (8) into (11) for different cases leads to (10). More detailed proof of the theorem is presented in the extended version of the paper [20]. □

From Theorem 1, the following conclusions can be drawn:

1) Using the tight approximation $W(e^{a+x})\simeq x+a-\log(a+x)$ in (10), the required number of receive antennas in Case 1 is rephrased as

$$\hat{N}_\text{r}\simeq\frac{\left(Q^{-1}(\theta)\right)^2}{4MTN_\text{t}\mathcal{Y}}\simeq\frac{\left(Q^{-1}(\theta)\right)^2}{MTN_\text{t}\left(\log(\phi)\right)^2},$$
$$\mathcal{Y}\doteq\left(\log\left(\frac{Q^{-1}(\theta)\sqrt{\phi}}{2\sqrt{MT}N_\text{t}}\right)-\frac{R}{2MN_\text{t}}-\log\left(\log\left(\frac{Q^{-1}(\theta)\sqrt{\phi}}{2\sqrt{MT}N_\text{t}}\right)-\frac{R}{2MN_\text{t}}\right)\right)^2,\tag{12}$$

where the last approximation holds for moderate/high SNRs. Thus, at moderate/high SNRs, the required number of receive antennas increases with $(Q^{-1}(\theta))^2$ linearly. On the other hand, the required number of receive antennas is inversely proportional to the number of experienced fading realizations MT, the number of transmit antennas N_t, and $(\log(\phi))^2$. Interestingly, we can use (10.Case 2) to show that at high SNRs the same scaling laws hold for Cases 1 and 2. That is, in Case 2, the required number of transmit antennas decreases (resp. increases) with MT, N_t, and $(\log(\phi))^2$ (resp. $(Q^{-1}(\theta))^2$) linearly.
2) The same scaling laws are valid in Cases 3-4, i.e., when the numbers of transmit and receive antennas increase simultaneously. For instance, the required number of antennas increases with $Q^{-1}(\theta)$ and code rate R semi-linearly[3] (see (10.Cases 3-4)). At hard outage constraints, i.e., small values of θ, the required number of antennas decreases with the number of retransmissions according to $\frac{1}{\sqrt{M}}$. On the other hand, the number of antennas decreases with M linearly when the outage constraint is relaxed, i.e., θ increases. The only difference between Cases 3 and 4 is that in Case 3 (resp. Case 4) the number of antennas decreases with ϕ (resp. $\log(\phi)$) linearly.

V. Simulation Results and Discussions

In this section, we verify the accuracy of the derived results, and present the simulation results in spatially independent and correlated fading conditions as follows.

[3]The variable y is semilinear with x if $y=a+bx$ for constants a and b.

A. Performance Analysis in Spatially-independent Conditions

In Figs. 1-3, we verify the accuracy of the results in Theorem 1 and derive the required number of transmit/receive antennas in outage-limited conditions. Setting $N_\mathrm{r} = 1$ or 2, Fig. 1 shows the required number of transmit antennas in Case 2 with large N_t and given N_r. Here, we consider quasi-static, slow- and fast-fading conditions with $\theta = 10^{-4}, T = 2, M = 2, \phi = 15$ dB. In Fig. 2, we verify the effect of HARQ on the system performance. Here, assuming Case 1 (large N_r and $N_\mathrm{t} = 1, 5$), the required number of antennas is derived in the scenarios with ($M = 2$) and without ($M = 1$) HARQ. The results of the figure are given for $\phi = 5$ dB and $\theta = 10^{-4}$. Figure 3 studies the required number of antennas in Cases 3 and 4 with low and high SNRs, respectively, large number of transmit and receive antennas, and $\frac{N_\mathrm{t}}{N_\mathrm{r}} = K$. Here, we consider the quasi-static conditions, $M = 1$, and $\theta = 10^{-3}$. Note that, to have the simulation results of Case 4 in reasonable running time, we have stopped the simulations at moderate initial transmission rates. For this reason, the simulation results of Case 4, i.e., the red solid-line curves of Case 4 in Fig. 3, are plotted for the moderate initial rates. According to the results, the following conclusions can be drawn:

- For Cases 1-3 and different fading conditions, the analytical results of Theorem 1 are very tight for a broad range of initial transmission rates, outage probability constraints and SNRs (Figs. 1-3). Also, in Case 1 (resp. Case 2) the tightness of the approximations increases with the number of receive (resp. transmit) antennas (Figs. 1-2). For Case 4 (which is not of practical interest in large MIMO setups), although the approximation is not tight, the curves still follow the same trend as in the simulation results. For instance, with the approximation approaches of Case 4, the required number of antennas increases with the initial rate linearly, in harmony with the simulation results (Fig. 3). Also, as shown in [20], we can improve the analytical results of Theorem 1 in Case 4, such that the analytical results match the simulation results with high accuracy (Fig. 3). Finally, the scaling laws of Theorem 1 are valid because, as demonstrated in Figs. 1-3, in all cases the analytical and the simulation results follow the same trends (see Theorem 1 and its following discussions).
- Fewer antennas are required when the number of fading realizations experienced during the HARQ packet transmission increases. Intuitively, this is because more diversity is exploited by HARQ in fast-fading (resp. slow-fading) condition compared to slow-fading (resp. quasi-static) conditions and, consequently, different outage probability constraints are satisfied with fewer antennas in the fast-fading (resp. slow-fading) conditions (Fig. 1).
- The HARQ reduces the required number of antennas significantly (Fig. 2). For instance, consider the quasi-static conditions, the outage probability constraint $\Pr(\text{Outage}) \leq 10^{-4}$, $N_\mathrm{t} = 5$, $\phi = 5$ dB and the code rate 20 npcu. Then, the implementation of HARQ with a maximum of $M = 2$ retransmissions reduces the required number of receive antennas from 95 without HARQ to 15 (Fig. 2). Moreover, the effect of HARQ increases with the number of transmit/receive antennas (Fig. 2).

B. On the Effect of Spatial Correlation

Throughout the paper, we considered IID fading conditions motivated by the fact that the millimeter-wave communication, which will definitely be a part in the next generation of wireless networks, makes it possible to assemble many antennas close together with negligible spatial correlations [14]. However, it is still interesting to analyze the effect of the antennas spatial correlation on the system performance. For this reason, considering Case 2 with $N_\mathrm{r} = 1$, Fig. 4 demonstrates the required number of antennas in spatially-correlated conditions where, denoting the transpose operator by $()^\mathrm{T}$, the successive elements of the channel vector $\mathbf{H} = [h_1, \dots, h_{N_\mathrm{t}}]^\mathrm{T}$ follow

$$h_i = \beta h_{i-1} + \sqrt{1-\beta^2}\varpi, \varpi \sim \mathcal{CN}(0,1), h_0 \sim \mathcal{CN}(0,1). \tag{13}$$

Here, β is a correlation coefficient where $\beta = 0$ and $\beta = 1$ corresponds to the uncorrelated and fully correlated conditions.

As shown in the figure, the effect of the antennas spatial correlation on the required number of antennas is negligible for correlation coefficients of, say, $\beta \lesssim 0.4$. Then, the sensitivity to the spatial correlation increases for large values of the correlation coefficients, and the required number of antennas increases with β. However, the important point is that the curves follow the same trend, for a large range of correlation coefficients (Fig. 4). Thus, with high accuracy, the same scaling laws as in the IID scenario also hold for the correlated conditions, as long as the correlation coefficient is not impractically high. Also, we observe the same conclusions in the other cases, although not demonstrated in the figure.

VI. Conclusion

This paper studied the required number of antennas satisfying different outage probability constraints in large but finite MIMO setups. We showed that different quality-of-service requirements can be satisfied with relatively few transmit/receiver antennas. As demonstrated, the required number of antennas decreases by the HARQ remarkably. The effect of the antennas spatial correlation on the required number of antennas is negligible for small/moderate correlation coefficients, while its effect increases in highly correlated conditions. In the extended paper [20], we analyze the effect of power amplifiers imperfection, adaptive power allocation as well as the asymptotic performance analysis with large antennas.

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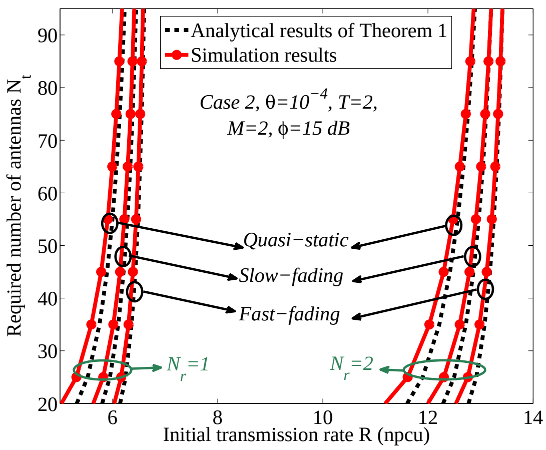

Figure 1. The required number of transmit antennas vs the initial transmission rate R for the quasi-static, slow- and fast-fading conditions (Case 2: large N_t, given N_r). Outage probability constraint $\Pr(\text{Outage}) < \theta$, ($\theta = 10^{-4}$), $\phi = 15$ dB, $T = 2$, $M = 2$, and $N_\mathrm{r} = 1$ or 2.

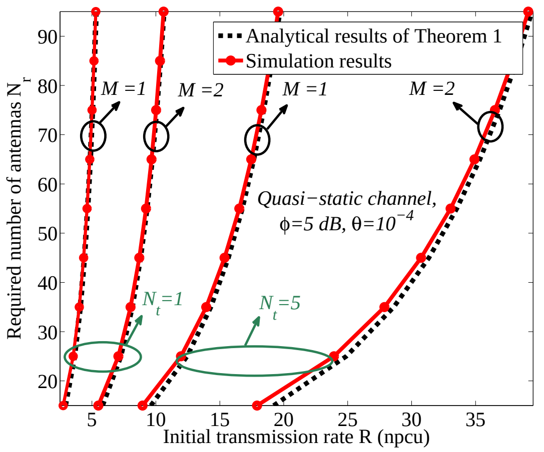

Figure 2. The required number of transmit antennas in the scenarios with HARQ ($M = 2$) and without HARQ ($M = 1$), Case 1: (large N_r, given N_t). Outage probability constraint $\Pr(\text{Outage}) < \theta$ with $\theta = 10^{-4}$, $\phi = 5$ dB, , $N_\mathrm{t} = 1$ or 5, and quasi-static conditions.

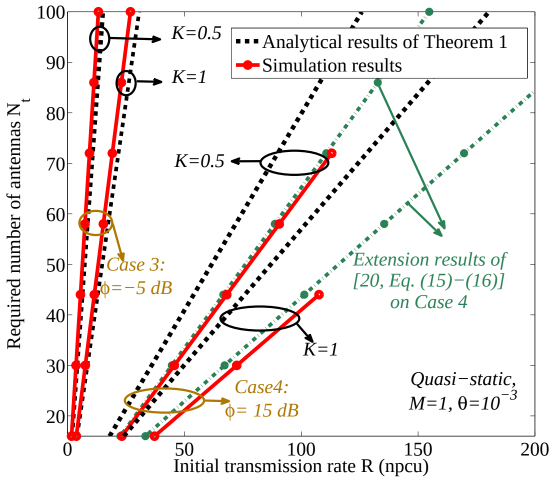

Figure 3. The required number of transmit antennas vs the initial transmission rate, Cases 3 and 4: (large N_t, N_r, $\frac{N_\mathrm{t}}{N_\mathrm{r}} = K$). Outage probability constraint $\Pr(\text{Outage}) < \theta$ with $\theta = 10^{-3}$, $\phi = -5$ or 15 dB, , $M = 1$, and quasi-static conditions.

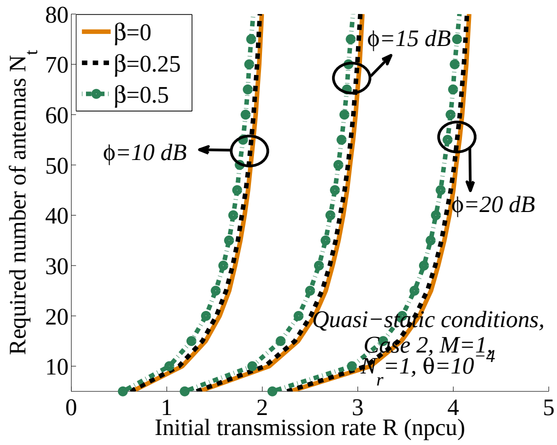

Figure 4. The required number of antennas in spatially-correlated conditions. Case 2: (large N_t, given N_r), outage probability constraint $\Pr(\text{Outage}) < \theta$ with $\theta = 10^{-4}$, $M = 1$, quasi-static conditions, and $N_\mathrm{r} = 1$.

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