On the performance of amplifier-aware dense networks: Finite block-length analysis

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Behrooz Makki*, Chao Fang*, Tommy Svensson*, Masoumeh Nasiri-Kenari†
*Chalmers University of Technology, Gothenburg, Sweden, {behrooz.makki, fchao, tommy.svensson}@chalmers.se
† Sharif University of Technology, Tehran, Iran, Email: mnasiri@sharif.edu.

***Abstract*—In this paper, we investigate the performance of dense Poisson-point-process-based cellular networks using finite length codewords. Taking the properties of the power amplifiers (PAs) into account, we derive the outage probability, the per-user throughput and the area spectral efficiency in different conditions. Our analysis is based on some recent results on the achievable rates of finite-length codes and we investigate the effect of the codeword length/PAs properties on the system performance. Our numerical and analytical results indicate that the inefficiency of the PAs affects the performance of dense networks substantially. Also, for a given number of information nats per codeword, there is an optimal finite codeword length maximizing the throughput.**

I. Introduction

The next generation of wireless networks must provide high-rate reliable data streams for everyone everywhere at any time. To address the demands, the main strategy persuaded in the last few years is the network *densification* [1]. One of the key techniques to densify the network is to use many base stations (BSs) of different types to increase the capacity and service availability. In such networks, as the BSs vary in transmit power, coverage and spatial density, conventional methods such as the Wyner model [2] are not appropriate in capturing the characteristics of the network. For this reason, it is suggested in, e.g., [3], [4] to use stochastic geometry as a promising tool for modeling the randomness of the BSs' locations in dense networks. With stochastic geometry, the BSs are supposed to be arranged according to some, mostly Poisson, point process. Then, the users received signal-to-interference-plus-noise ratio (SINR) is characterized correspondingly which enables different performance analysis.

Following [3], [4], there are different works on the performance analysis of dense networks using stochastic geometry, e.g., [5]–[13]. Particularly, the SINR characteristics of M-tier networks [5]–[7], the analysis of optimal power allocation [8], [9], and the implementation of different techniques such as coordinated BS transmission [10], automatic repeat request (ARQ) [11], interference control strategy [12] and multi-antenna transmission [13] are considered.

In [3]–[13] (and many other papers) either the results are obtained under the assumption that the codewords are asymptotically long, such that the users' instantaneous achievable rates are given by $\log(1+x)$ with x standing for the user instantaneous received SINR, or it is only concentrated on the SINR characteristics. However, the effect of the codewords length is not considered in the analysis. On the other hand, in many applications, such as vehicle-to-vehicle and vehicle-to-infrastructure communications for traffic efficiency/safety or real-time video processing for augmented reality, the codewords are required to be short (in the order of ~ 100 channel uses) [14]–[17], and the system performance is significantly affected by the codewords length. Thus, as also highlighted in [18] which derives bounds for the error probability of decentralized networks, it is interesting to study the performance of PPP-based networks utilizing codewords of finite length.

From another perspective, to guarantee green communication, it is important to take the hardware impairments, specially the power amplifiers (PAs) efficiency, into account in the performance analysis of dense networks, e.g., [19]. The problem becomes more important when we remember that in life cycle studies of the state-of-the-art wireless communication systems the radio transmitters have been estimated to be responsible for $70\% - 90\%$ of the energy consumption during operation; most of the electrical energy is consumed by the final radio frequency (RF) PA stage [20], [21]. Therefore, designing amplifier-aware schemes is of fundamental importance [19]–[22]. However, to the best of the authors' knowledge, this problem has not been yet addressed in PPP-based networks.

In this paper, we study the data transmission efficiency of PPP-based dense networks using finite-length codewords and non-ideal (realistic) PAs. We use the recent results of [23], [24] on the achievable rates of finite block-length codes to investigate the system performance. Particularly, we study the outage probability, the per-user throughput and the area spectral efficiency (ASE) of PPP-based networks, and evaluate the effect of the codeword length/the PAs properties on the system performance. Our numerical and analytical results indicate that the outage probability and the throughput of dense networks are remarkably affected by the inefficiency of the PAs. Moreover, for a given number of information nats per codeword, there is an optimal finite codeword length maximizing the throughput/ASE. Finally, the per-user throughput increases with the path loss exponent/BSs density, and there is considerable potential for serving multiple users, if the code rate/length is properly designed.

II. System model

Let us consider a cellular network with BSs distributed randomly over a 2D plane. Each BS transmits the data to the corresponding user and is equipped with a PA which is modeled as follows.

It has been previously shown that the PA efficiency can be written as [19], [22]

$$\frac{P}{P^{\text{cons}}} = \epsilon \left(\frac{P}{P^{\text{max}}}\right)^{\vartheta} \Rightarrow P = \sqrt[1-\vartheta]{\frac{\epsilon P^{\text{cons}}}{(P^{\text{max}})^{\vartheta}}}. \qquad (1)$$

Here, P, P^{max} and P^{cons} are the output, the maximum output and the consumed power of the PA, respectively, $\epsilon \in [0,1]$ denotes the maximum power efficiency achieved at $P = P^{\text{max}}$ and $\vartheta \in [0,1]$ is a parameter depending on the PA classes.

With ALOHA medium access protocol [3], in each time slot all BSs transmit independently. We consider a single frequency band when the locations of the transmitting BSs form a homogeneous PPP Φ with density λ. According to Slivnyaks's theorem [25], a user can be conditioned on an arbitrary point in the plane without changing the distribution of the rest of the process. Therefore, for simplicity, a typical user is assumed to be placed in the origin of the plane.

Each user is served by only one BS. The user selects the BS providing the maximum received signal power averaged over fading as its serving BS. Because the average received power only depends on the distance between the user and the BS, the cell association strategy in this paper is equivalent to the nearest BS association. This is the best choice in the cases with no channel state information (CSI) at the BSs and with long-term objective functions, e.g., ASE/throughput/outage probability, on which we concentrate [3]. The serving BS and the distance between the user and its serving BS are denoted as b_0 and r, respectively. Also, all other BSs $x_i \in \Phi$ are assumed to cause interference to the considered user.

Quasi-static Rayleigh fading conditions are considered where the channel coefficients remain constant during a codeword transmission and then change to other values based on their corresponding probability distribution functions (PDFs)[1]. Perfect CSI is assumed to be available at the receiver which is an acceptable assumption in quasi-static conditions. Due to the randomness of the positions of the BSs in Poisson networks, the distance from the user to the connecting BS r follows a PDF $f_r(r) = e^{-\lambda\pi r^2} 2\pi\lambda r$ [3]. The useful signal power is subject to a path loss $r^{-\alpha}$ and fading coefficient h with unit-mean exponential distribution, where $\alpha > 2$ is the path loss exponent. The interference power comprises powers from all BSs except for b_0 and is given by

$$I = \sqrt[1-\vartheta]{\frac{\epsilon P^{\text{cons}}}{(P^{\text{max}})^{\vartheta}}} \sum_{x_i \in \Phi / b_0} g_{x_i} d_{x_i}^{-\alpha}, \qquad (2)$$

where g_{x_i} is the independent and identically distributed exponential fading power coefficient of the interfering channels, and d_{x_i} is the distance from x_i to the user. We further assume that additive noise with power σ^2 is present at the receiver.

III. Performance Analysis

With finite-length codewords, in each slot the serving BS of a user encodes K information nats into a codeword of length L channel uses and rate $R = \frac{K}{L}$ nats-per-channel-use (npcu). The codeword is sent to the user and is dropped if the receiver can not decode it correctly. Thus, the per-user throughput and the ASE [4], averaged over many codeword transmissions, are given by

$$\eta = R(1-\chi), \qquad (3)$$

and

$$\mathcal{A} = \lambda R(1-\chi), \qquad (4)$$

respectively, where χ denotes the message outage probability. Note that $\mathcal{A} = \lambda\eta$, so analyzing the throughput provides the same information for the ASE.

To analyze (3) and (4), let us first review the recent results of [23], [24] for the cases with codewords of finite length. Define an (L, N, P, δ) code as the collection of

- An encoder $\Gamma : \{1, \dots, N\} \mapsto \mathcal{C}^L$ which maps the message $n \in \{1, \dots, N\}$ into a length-L codeword $c_n \in \{c_1, \dots, c_N\}$ satisfying the transmit power constraint
$$\frac{1}{L} \|c_j\|^2 \leq P, \forall j. \qquad (5)$$
- A decoder $\Omega : \mathcal{C}^L \mapsto \{1, \dots, N\}$ that satisfies the maximum error probability constraint
$$\max_{\forall j} \Pr(\Omega(y) \neq J | J = j) \leq \delta \qquad (6)$$
with y representing the channel output induced by the transmitted codeword.

The maximum achievable rate of the code is defined as

$$R_{\text{max}}(L, P, \delta) = \sup\left\{\frac{\log N}{L} : \exists (L, N, P, \delta)\text{code}\right\} \text{ (npcu)}. \qquad (7)$$

Considering quasi-static conditions, [24] has recently presented a very tight approximation for the maximum achievable rate (7) as

$$R_{\text{max}}(L, P, \delta) = \sup\{R : \Pr(\log(1+GP) < R) < \delta\} - \mathcal{O}\left(\frac{\log L}{L}\right) \text{ (npcu)}, \qquad (8)$$

which, for codes of rate R npcu, leads to the following error probability [24]

$$\delta(L, R, P) \approx E\left[Q\left(\frac{\sqrt{L}\,(\log(1+GP) - R)}{\sqrt{1 - \frac{1}{(1+GP)^2}}}\right)\right]. \qquad (9)$$

Here, G represents the simultaneous channel gain, $E\{\cdot\}$ is the expectation with respect to G and $B(x) = \mathcal{O}(C(x)), x \to \infty$ is defined as $\lim_{x\to\infty} \sup |\frac{B(x)}{C(x)}| < \infty$. Also, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} \mathrm{d}u$ denotes the Gaussian Q-function. Note that, according to [24], the approximations in (8) and (9) are very tight for sufficiently large values of L[2].

In this way, e.g., the per-user throughput is rephrased as

[1] The cumulative distribution functions (CDF) and PDF of a random variable X are denoted by $F_X(.)$ and $f_X(.)$, respectively.

[2] As shown in [24], the error probability (9) is reached by uniform distribution over the surface of L-dimensional sphere. Then, as the codeword length L increases the difference between this code and Gaussian code becomes negligible. Thus, we can assume the codes to be Gaussian.

$$\eta = R\left(1 - E\left[Q\left(\frac{\sqrt{L}\,(\log(1+\gamma_P)-R)}{\sqrt{1-\frac{1}{(1+\gamma_P)^2}}}\right)\right]\right)$$
$$= R\left(1-\int_0^\infty f_{\gamma_P}(x)Q\left(\frac{\sqrt{L}\,(\log(1+x)-R)}{\sqrt{1-\frac{1}{(1+x)^2}}}\right)\mathrm{d}x\right), \tag{10}$$

where γ_P represents the simultaneous SINR received by the data transmission of the BSs with transmit power P. Also, the expectation is over the SINR with interference terms following (2). For Rayleigh fading conditions, we can combine the results of [3, Eq. (11)-(12)] and (1) to derive the PDF of the received SINR as $f_{\gamma_P}(x)=\frac{\mathrm{d}F_{\gamma_P}(x)}{\mathrm{d}x}$ with

$$F_{\gamma_P}(x) = \Pr(\text{SINR} \leq x)$$
$$= 1-\pi\lambda\int_0^\infty e^{-\pi\lambda v(1+\rho(x,\alpha))-\sqrt[1-\vartheta]{\frac{\epsilon P^{\text{cons}}}{(P^{\text{max}})^\vartheta}}x\sigma^2 v^{\frac{\alpha}{2}}}\mathrm{d}v, \tag{11}$$

$$\rho(x,\alpha) = x^{\frac{2}{\alpha}}\int_{x^{-\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}}\mathrm{d}u. \tag{12}$$

Taking (10)-(12) into account, it is difficult to derive (10) even numerically because there is no closed-form expression for the PDF of the SINR. Therefore, we need to implement different approximation/bounding schemes.

Using the first order linear approximation of the Q function in (10), we write

$$Q\left(\frac{\sqrt{L}\,(\log(1+x)-R)}{\sqrt{1-\frac{1}{(1+x)^2}}}\right) \simeq Z(x),$$
$$Z(x) = \begin{cases} 1 & x<\frac{1}{2m}+n,\\ \frac{1}{2}+m(x-n) & x\in\left[\frac{1}{2m}+n,\frac{-1}{2m}+n\right],\\ 0 & x>\frac{-1}{2m}+n,\end{cases} \tag{13}$$

with $n \doteq e^R-1$ and

$$m \doteq \left.\frac{\partial Q\left(\frac{\sqrt{L}(\log(1+x)-R)}{\sqrt{1-\frac{1}{(1+x)^2}}}\right)}{\partial x}\right|_{x=n} = -\sqrt{\frac{L}{2\pi(e^{2R}-1)}}, \tag{14}$$

which is found by the derivative of $Q\left(\frac{\sqrt{L}(\log(1+x)-R)}{\sqrt{1-\frac{1}{(1+x)^2}}}\right)$ at point $x=n$. From (10)-(14), we have

$$\eta \overset{(a)}{\simeq} R\bigg(1-\int_0^{\frac{1}{2m}+n} f_{\gamma_P}(x)\mathrm{d}x$$
$$+\left(mn-\frac{1}{2}\right)\int_{\frac{1}{2m}+n}^{\frac{-1}{2m}+n} f_{\gamma_P}(x)\mathrm{d}x - m\int_{\frac{1}{2m}+n}^{\frac{-1}{2m}+n} x f_{\gamma_P}(x)\mathrm{d}x\bigg)$$
$$\overset{(b)}{\simeq} R\bigg(1-F_{\gamma_P}\left(\frac{1}{2m}+n\right)$$
$$+\left(mn-\frac{1}{2}\right)\left(F_{\gamma_P}\left(\frac{-1}{2m}+n\right)-F_{\gamma_P}\left(\frac{1}{2m}+n\right)\right)$$
$$+\left(\frac{1}{2}+mn\right)F_{\gamma_P}\left(\frac{1}{2m}+n\right)-\left(mn-\frac{1}{2}\right)F_{\gamma_P}\left(\frac{-1}{2m}+n\right)$$
$$-F_{\gamma_P}(n)\bigg). \tag{15}$$

Here, (a) comes from using (13) in (10) and (b) follows from partial integration and the first order Riemann integral approximation $\int_{c_0}^{c_1} f(x)\mathrm{d}x \simeq (c_1-c_0)f(\frac{c_1+c_0}{2})$ with $F_{\gamma_P}(x)$ given in (11).

On the other hand, in the cases with $\alpha=4$, which is of interest in PPP-based networks [3], [11], we can use some manipulations and (11)-(12) to derive the CDF and the PDF of the received SINR as

$$F_{\gamma_P}(x) = 1-\frac{\sqrt{P}\pi^{\frac{3}{2}}\lambda}{\sqrt{x}}e^{\frac{P\lambda^2\pi^2\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)^2}{4x}}\times$$
$$Q\left(\frac{\sqrt{P}\lambda\pi\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)}{\sqrt{2x}}\right),$$
$$P = \sqrt[1-\vartheta]{\frac{\epsilon P^{\text{cons}}}{(P^{\text{max}})^\vartheta}}, \tag{16}$$

and

$$f_{\gamma_P}(x) = -Q\left(\frac{\sqrt{P}\lambda\pi\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)}{\sqrt{2x}}\right)\times$$
$$\Bigg(\Bigg(\frac{P\sqrt{P}\lambda^3\pi^3\sqrt{\pi}}{4x\sqrt{x}}\left(\frac{\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}}+\frac{1}{x+1}\right)\times$$
$$\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)-$$
$$\frac{P\sqrt{P}\lambda^3\pi^3\sqrt{\pi}}{4x^2\sqrt{x}}\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)\Bigg)$$
$$\times e^{\frac{\frac{P\lambda^2\pi^2}{4}\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)^2}{x}}$$
$$-\frac{\sqrt{P}\lambda\pi\sqrt{\pi}e^{\frac{P\lambda^2\pi^2\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)^2}{4x}}}{2x\sqrt{x}}\Bigg)$$
$$+\frac{\sqrt{P}\pi\lambda}{\sqrt{2x}}e^{\frac{P\lambda^2\pi^2\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)^2}{4x}}\times$$
$$\frac{\sqrt{P}\lambda\pi}{2\sqrt{2}x\sqrt{x}(x+1)}e^{-\frac{1}{2}\left(\frac{\sqrt{P}\lambda\pi\left(1+\sqrt{x}\left(\frac{\pi}{2}-\arctan\left(\frac{1}{\sqrt{x}}\right)\right)\right)}{\sqrt{2x}}\right)^2},$$
$$P = \sqrt[1-\vartheta]{\frac{\epsilon P^{\text{cons}}}{(P^{\text{max}})^\vartheta}}, \tag{17}$$

respectively. Thus, from (1), (3), (4), (10) and (17) the per-user throughput, the outage probability and the ASE are derived based on a one-dimensional integration which can be calculated numerically. In Section IV, we use the derived results to analyze the performance of dense networks in the presence of finite-length codewords and non-ideal PAs.

IV. Simulation Results

According to [23], [24], the approximations in (8) and (9) are very tight for sufficiently long codewords, and the tightness increases with the codewords' length. For the numerical results, we consider the cases with $L \geq 100$ channel uses, for which the approximation is tight enough, and we do not consider shorter codewords. Our choice of $L \geq 100$ as the minimum possible length is motivated by [24, Fig. 2] where the relative difference of the exact and the approximate achievable rates is less than 2% for the cases with codewords of length ≥ 100. We are further motivated for our choice of the codewords length by reports such as [14], that suggest the practical codes of interest for, e.g., vehicle-to-vehicle communication to be in the range of $100-300$ channel uses.

In all figures, we consider $\sigma^2 = 1$ and set the PAs parameters to $\epsilon = 0.65$, $P^{\max} = 10$ dB, and $\vartheta = 0.5$, unless otherwise stated. Considering $\alpha = 4$, Figs. 1-3 study the outage probability, the per-user throughput and the ASE of PPP-based networks with different codeword lengths, respectively. Also, the figures investigate the accuracy of the analytical results (15)-(17) (Note that with $\alpha = 4$ the numerical evaluation of (10) matches the result achieved by (16)-(17) exactly). For numerical evaluation of (10), we first find the SINR CDF (11) numerically and then use the result to calculate the expectation in (10). To analyze the effect of the PAs efficiency, Fig. 4 shows $\tilde{P}^{\text{cons}}(x) = \min\{P^{\text{cons}}|\eta = y\}$, i.e., the minimum per-BS consumed power which guarantees a given throughput $\eta = y$. Here, considering different codeword lengths, the results are obtained for $\lambda = 0.1, \alpha = 4$, $K = 500$ nats and $\eta = 0.2$ npcu. Finally, Fig. 5 evaluates the effect of the path loss exponent α on the system performance. According to the figures, the following conclusions can be drawn:

1) The approximation approach of (15) and the analytical derivations of (16)-(17) are very tight for a broad range of parameter settings, and the results can be used for analytical investigation of PPP-based networks (Figs. 1-3, 5).

2) With the considered parameter settings, the network outage probability (resp. ASE) decreases (resp. increases) with the per-BS consumed power. However, the outage probability and the ASE converge to constant values at high powers, because the network becomes interference-limited (Figs. 1, 3).

3) For a given number of nats per codeword, there is an optimal codeword length maximizing the throughput (Fig. 2). Intuitively, this is because the code rate (resp. decoding probability) decreases (resp. increases) with the codeword length. Thus, there is a tradeoff and the maximum throughput is achieved by a finite value of the codeword length[3].

4) The data transmission efficiency of dense networks is considerably affected by the inefficiency of the PAs, and the sensitivity of the throughput to the PAs inefficiency increases with the codeword length (Fig. 4). For instance, with codewords of length $L = 1000$ channel uses, throughput requirement $\eta = 0.2$ npcu and the parameter setting of Fig. 4, increasing the PAs efficiency from 0.25 to 0.6, decreases the required per-BS consumed power by (almost) 15 dB (Fig. 4).

5) In harmony with [3] which studies PPP-based networks with asymptotically long codewords, the per-user throughput increases with the path loss exponent (Fig. 5). This is intuitively because, compared to the useful signal, the interference signal is more affected as the path loss exponent increases.

6) Finally, with realistic assumptions on the codewords length/PA properties and the practical range of SNRs, the outage probability requirements of the vehicle-to-vehicle communication are not satisfied in dense conditions (Fig. 1 and [14]). Thus, the results emphasise the importance of using performance-enhancing techniques for dense networks.

[3]The results of Fig. 2 are given for Gaussian codes which support any transmission rates [24]. In practical codes, however, the coding constraints should be considered in the codeword length optimization such that the data rate does not exceed 1.

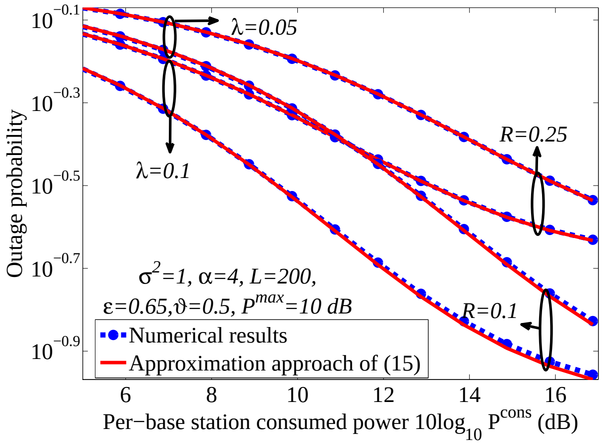

Figure 1. The outage probability versus the per-BS consumed power $10\log_{10} P^{\text{cons}}$ dB for different densities λ and rates R. The parameters are set to $\alpha = 4$, $L = 200$, $\vartheta = 0.5$, $\epsilon = 0.65$, $P^{\max} = 10$ dB, $\sigma^2 = 1$.

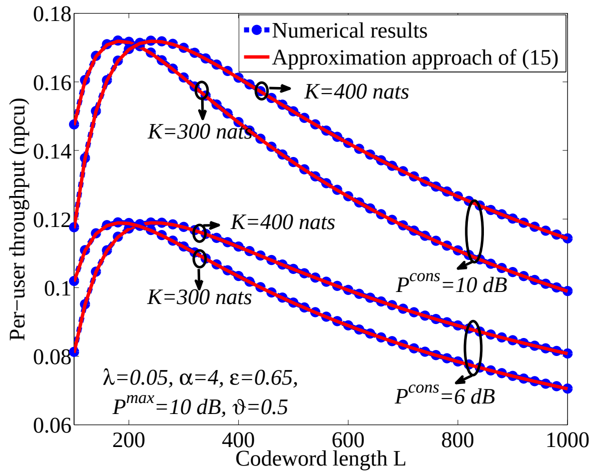

Figure 2. The per-user throughput versus the codeword length L. The parameters are set to $K = 300, 400$ nats, $P^{\text{cons}} = 6, 10$ dB, $\lambda = 0.05, \alpha = 4, \vartheta = 0.5, \epsilon = 0.65$, $P^{\max} = 10$ dB, $\sigma^2 = 1$.

V. Conclusion

This paper studied the data transmission efficiency of dense networks in the presence of finite-length codes and non-ideal PAs. We derived closed-form expressions for the outage probability, per-user throughput and ASE of the network. Our results indicate that the inefficiency of the PAs affects the performance of dense networks remarkably. Moreover, the system performance is substantially affected by the finite length of the codewords. Finally, for a given number of information nats per codeword, there is an optimal codeword length maximizing the throughput/ASE of the network.

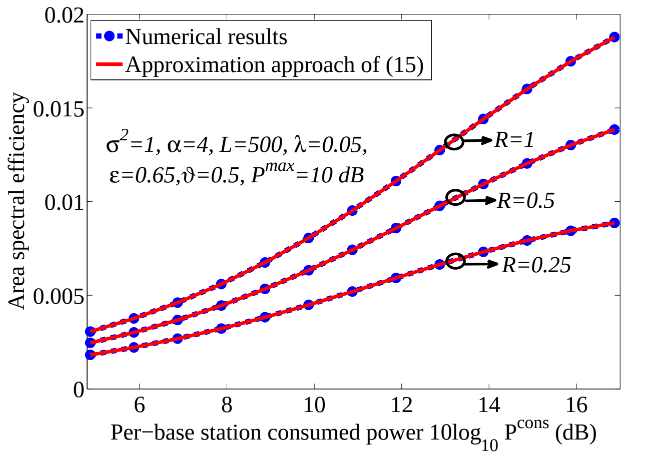

Figure 3. ASE versus the per-BS consumed power $10\log_{10} P^{\text{cons}}$ dB. The parameters are set to $\alpha = 4, L = 500, \vartheta = 0.5, \epsilon = 0.65, P^{\max} = 10$ dB, $\lambda = 0.05$, $\sigma^2 = 1$.

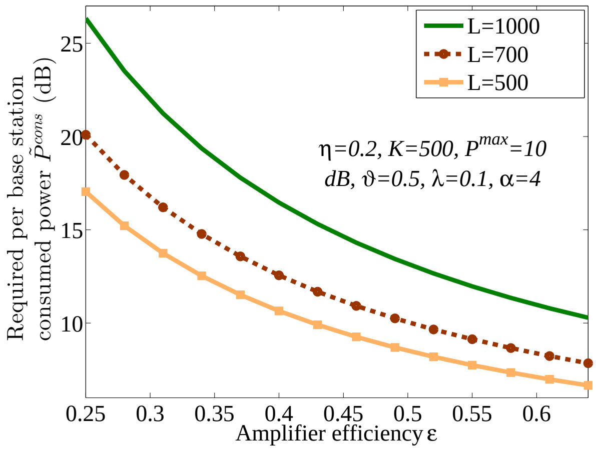

Figure 4. The required per-BS consumed power for a given throughput $\eta = y$. The parameters are set to $\alpha = 4, K = 500$ nats, $\vartheta = 0.5, \lambda = 0.1, y = 0.2$, $P^{\max} = 10$ dB, $\sigma^2 = 1$.

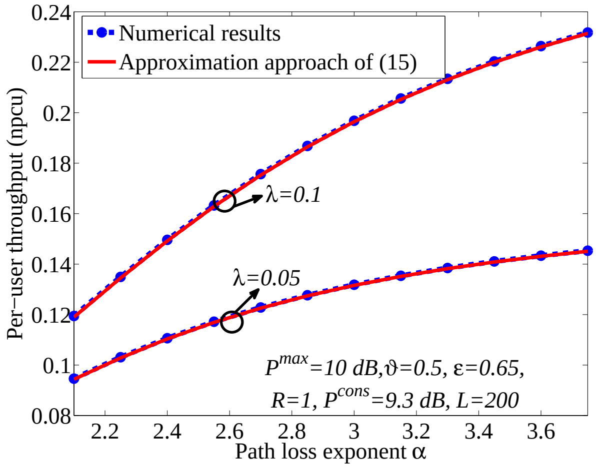

Figure 5. Per-user throughput for different path loss exponents, $R = 1$ npcu, $\vartheta = 0.5, \epsilon = 0.65, L = 200, P^{\text{cons}} = 9.3$ dB, $P^{\max} = 10$ dB, $\sigma^2 = 1$.

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