THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

IN

SOLID AND STRUCTURAL MECHANICS

Structural Reliability and Identification with Stochastic Simulation

Application to Railway Mechanics

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Cover:
A sensing carpet consists of force cells to measure the sleeper-ballast interface load. The experiment was conducted outside Vislanda, a village in southern Sweden.

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ABSTRACT

System identification of structures based on measured response data can play a key role in improving reliability based structural designs. However, the experimental limitations of *in situ* tests and uncertainties of required model complexity together with the inverse nature of system identification give rise to a number of challenging issues. Common examples of these issues in spatially varying parameter identification problems are poor modelling or ill-conditioning problems that are created due to inappropriate discretization of the parameter field. Even with a proper discretization, the large number of uncertain parameters associated with these problems makes the standard optimization or sampling techniques computationally cumbersome and also more prone to the so called *curse of dimensionality* problem. An in-depth study of such modelling and computational issues is presented for finding appropriate methods to treat them in a railway ballast stiffness-field property identification, and for doing test planning of *in situ* experimental conditions. This is achieved by utilizing a recently proposed Bayesian approach, known as enhanced *Bayesian Updating with Structural reliability methods* through feasibility studies. By interpreting the Bayesian system identification problem as an equivalent reliability problem, this approach opens up the possibility to employ well-developed rare-event samplers, such as subset simulation, to efficiently draw samples from the posterior probability distribution in high-dimensional inference problems. Another topic of this thesis is to develop a time integration scheme for fast simulation of large finite element models with spatially localized nonlinear or stochastic properties. This is a prerequisite for the ballast-sleeper load characterization problem, in which the local nonlinear/uncertain properties of the spatially varying ballast bed (along the sleeper length) is of major concern. Briefly stated, the developed integration scheme computes the system response based on the solution of an underlying linear system augmented with a low-rank nonlinear pseudo-force vector that accommodates the local nonlinearity and uncertainty effects. This is achieved through an efficient correction-prediction method. The presented integration scheme is combined with a developed modal reduction method, which is enhanced to take into account the effect of pseudo-forces in its modal dominancy analysis. It has been successfully applied to the studied moving-load simulation problem where the sleeper response statistics is required for estimating the risk of failure. The problem of detecting non-minimality in modal reduction of systems with multiple or very close eigenvalues (as in the studied railway track structure with large clusters of neighboring eigenvalues) is described and two methods to circumvent this problem is proposed. The reduction method is enhanced to effectively treat systems under moving loads or distributed loading, by incorporating information from structural properties of the input force matrix into the modal dominancy analysis.

KEYWORDS: Bayesian system identification, stochastic simulation, time-integration, model reduction, finite element model, sleeper-ballast load characterization.

To my beloved family

PREFACE

The work presented in this doctoral thesis has been carried out during the years 2011–2016 at Department of Applied Mechanics at Chalmers University of Technology in Göteborg. It has been part of the project TS14 “Multicriterion optimization of track properties” within the Swedish National Competence Center CHARMEC (CHAlmers Railway MEChanics), with special support from Abetong AB and Trafikverket.

I would first like to gratefully acknowledge the efforts of my supervisor, Prof. Thomas Abrahamsson, whose guidance, knowledge, and unwavering support made this thesis possible. I am extremely appreciative of his valuable advices, but also for the amount of freedom he granted me for pursuing my own ideas. Thank you for being friendly, kind and patient.

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Finally, and most importantly, my heartfelt thanks goes to my dearest love, Fatemeh, for her patience, kindness and endless support during this period of our lives. My deepest gratitude goes to my dear parents whom I owe everything in my life, and to my dear sister and brother who have always been by my side. I love you all.

Gothenburg, March 2016

Sadegh Rahrovani

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A Sadegh Rahrovani, Thomas Abrahamsson. 'Ballast stiffness field parameter estimation using a Bayesian methodology,' submitted to *Engineering Structures*, 2016.

Paper B Sadegh Rahrovani, Majid Khorsand Vakilzadeh, Thomas Abrahamsson. 'Modal dominancy analysis based on modal contribution to frequency response function H_2-norm,' *Mechanical Systems and Signal Processing*, **48**:218-231, 2014.

Paper C Sadegh Rahrovani, Majid Khorsand Vakilzadeh, Thomas Abrahamsson. 'On Gramian-based techniques for minimal realization of large-scale mechanical systems,' *Conference proceedings of the Society for Experimental Mechanics Series, Springer*, **45**(7):797-805, 2014.

Paper D Sadegh Rahrovani, Thomas Abrahamsson, Klas Modin. 'Efficient dynamic analysis of structures with local nonlinear or stochastic properties,' submitted to *Journal of Sound and Vibration*, 2016.

Paper E Sadegh Rahrovani, Johanna Lilja, Thomas Abrahamsson, Jens Nielsen. 'On the accuracy and efficiency of reliability methods applied in railway sleeper design,' submitted to *Vehicle System Dynamics*, 2016.

Appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for most of the work in **Papers A-E**, i.e. was the prime responsible in planning the paper, took part in developing the theory, developed the numerical implementations, carried out the numerical simulations and wrote the paper. The following publication is a result from the work carried out in the PhD program and relates to **Paper A**.

COMPLEMENTARY PAPERS

Paper I Sadegh Rahrovani, Ivan Au, Thomas Abrahamsson. 'Bayesian treatment of spatially varying parameter estimation problems via Canonical BUS,' *Conference Proceeding of IMAC XXXIV*, Orlando (FL), USA, 2016.

CONTENTS

REVIEW AND SUMMARY

1 INTRODUCTION

A number of probabilistic analysis tools have been developed for uncertainty quantification and propagation during the last decades, but most complex structural and infrastructural systems are still designed with traditional simplified safety factor design. These traditional designs are usually based on deterministic analyses in combination with empirically based safety factors to account for uncertainties. Such safety factors are normally provided by design codes. The ever present uncertainties are only accounted for indirectly via safety factors that are specified more based on practical experience rather than on theory or experimental statistical evidence. This makes it difficult to determine the condition under which a given safety factor is valid. Therefore, the corresponding values given by the design code cannot be used for load situations that were not considered when a guideline was established. Another issue of concern is that a safety factor alone is not a consistent measure of safety and does not rigorously address the degree of reliability of the designed system. This often leads to an overly robust design, or in rare cases, a design solution that is too prone to failure. Moreover, safety factors do not necessarily constitute unique measures of safety. As an example, different mathematical expressions of a limit state, *i.e.* the system state at the boundary between the safe and failure domains, may lead to different estimates of safety levels for the structure under study [1–2].

In view of these issues, the use of structural reliability analyses for structural design is becoming more widespread as many industries are currently experiencing changes that push their products beyond the envelope of their safety regimes. In a reliability-based design approach, reliability is used as the measure of system safety and represents the probability that a system will perform its intended functionality under a specified service condition and over a specified period of time [3–5]. This approach enables explicit incorporation of given statistical data. Such data can for instance be collected during inspections or load monitoring and it can be used in reliability predictions in terms of updated failure probabilities conditioned on data. This is a desirable feature that can be used to avoid high degrees of conservatism in the design process while providing a good level of safety and serviceability of structures. In addition, this approach provides a formal link between a reliability based safety requirement and an ordinary design code whereby the factor values given by the design codes can be revisited and better understood before they are extended beyond their originally intended scope [6].

A critical preparatory step needed before commencing a structural reliability analysis for the modifications of guideline safety factors is to establish an accurate model to obtain predictions of the system state (or system performance). This requires answering the following two questions: First, which deterministic model structure should be chosen for the system analyses? Second, which probability distribution function does best represent the model's parameter uncertainty? Addressing these model selection issues is not always a trivial task when treating complex structures and loading conditions [7]. As such, this study examines a complex railway ballast-sleeper load characterization problem in which the spatially varying ballast property is of major concern. It necessitates the use and adoption of appropriate methods that are suitable for establishing a valid model of the railway track structure. Such a validated model is a requirement for the accurate probabilistic prediction of railway sleeper failure.

2 RESEARCH MOTIVATION

The ballasted track plays a significant role in railway transportation systems and it consists of two major parts: the track superstructure and its substructure [8–10]. The track superstructure contains rails, rail pads and sleepers, as shown in Fig. 1. The sleepers are considered fundamental components of the track superstructure that are used to maintain the gauge between the rails and transmit the wheel/rail contact loads from a passing train down to the uppermost part of the substructure, the ballast bed media. The focus of this work is the most common Swedish railway sleeper, a pre-stressed concrete monobloc.

Different criteria and strategies are used for the design of sleepers and depend on the train traffic (train speed, axle load, etc.) and the maintenance status of the track. The design is mostly made on response prediction of the bending moment at critical cross-sections along the length of the sleeper. The current practice for sleeper design is based on guidelines. Two examples of include the UIC leaflet [11] and the European standard [12]. They both refer to the minimum allowable capacity of the bending moment at cross-sections in the sleeper midpoint and under the rail seats. The characteristic bending moment represents the required capacity at the end of sleeper's service life (*i.e.*, 50 years) [10]. This traditional sleeper design is based on simplified deterministic calculation models used in combination with safety factors that account for uncertainties, in order to determine the characteristic bending moments.

One simplification that has been made in traditional safety factor based sleeper design concerns how the dynamic interaction of the vehicle and the components in the track superstructure are accounted for. The dynamic loading is mostly caused by irregularities of the rail head and wheel tread (including wheel flats, corrugated rails, non-smooth rail joints, etc.) and is accounted for via a magnification multiplier on the nominal static wheel load. This factor is either given as one fixed value or as a linear or non-linear function of train speed. In the European standard, the magnification factor depends on whether the train speed is below or above 200 km/h. However, the motivation for this specific splitting speed is not given by the standard or found in other known references.

Another simplification is the way in which the sleeper support distribution has been developed. The assumed ballast distribution along the sleeper used for determining the characteristic bending moment is one of the most influential parameters in the design code calculation models. The real-world embedding ballast conditions are neither uniformly distributed nor fixed in time but vary randomly from site to site and from sleeper to neighbouring sleepers. However, most guidelines use one or two representative ballast distributions; *e.g.*, a uniform distribution of ballast stiffness along the complete length of the sleeper or a piece-wise constant stiffness limited to the neighbourhood of the rail seats and no ballast support elsewhere. The differences of sleeper responses that are either calculated based on guidelines or determined from data given by *in situ* studies are unknown [13].

The above-mentioned safety factors given by the design codes should ultimately be revisited, better understood and possibly updated before they are used in simulations with higher speeds and axle loads than those present at the time of establishing the current codes. Accurate reliability prediction for the sleepers based on sleeper bending moment response and statistical track test data could help in this understanding. Thus, developing valid models of the track structure is highly relevant for the prediction of sleeper reliability. This is, however, challenged by the lack of knowledge about the ballast bed variation along one sleeper and the corresponding load transferring mechanism through which the train's axle-load is transferred into the ballast media, as shown in Fig. 2–3. The scientific challenges in mathematical modelling of complex structural and infrastructural systems, *e.g.*, railway track structures, is the main focus of this research.

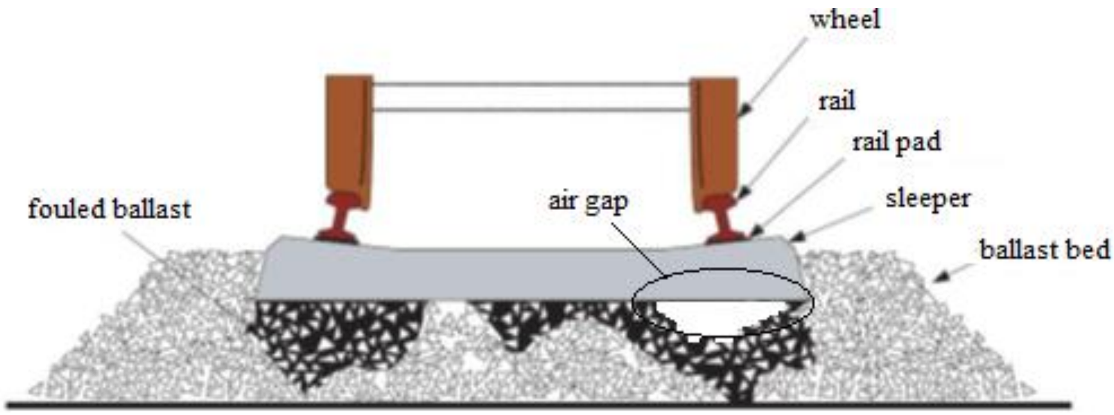

Fig. 1. An illustration of ballast configuration variation along a sleeper (from [14]). A significant variation of ballast density along sleeper-ballast interface may occur due to different loading conditions, poor maintenance or quality of ballast.

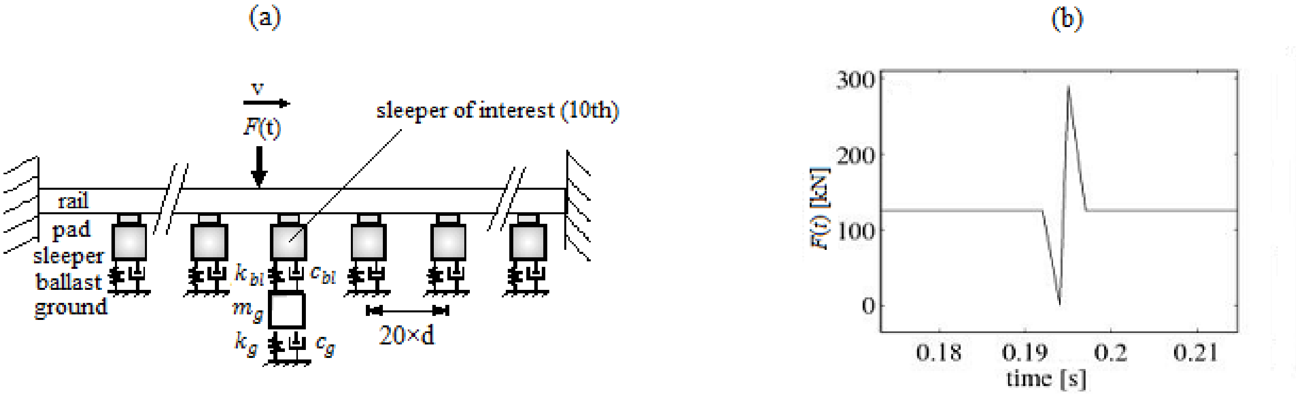

Fig. 2. A moving load with velocity *v* on a railway track with local nonlinearity or uncertainty due to the fact that sleepers put on a non-uniform ballast bed create state/parameter dependent interaction forces (a) Studied track model consisting of 20 bays of equal length. The ballast variation along the centre sleeper in bay no. 10 is of particular interest. (b) Track is subjected to a time-varying moving load $F(t)$. Prescribed load peaks at the passage of the center sleeper.

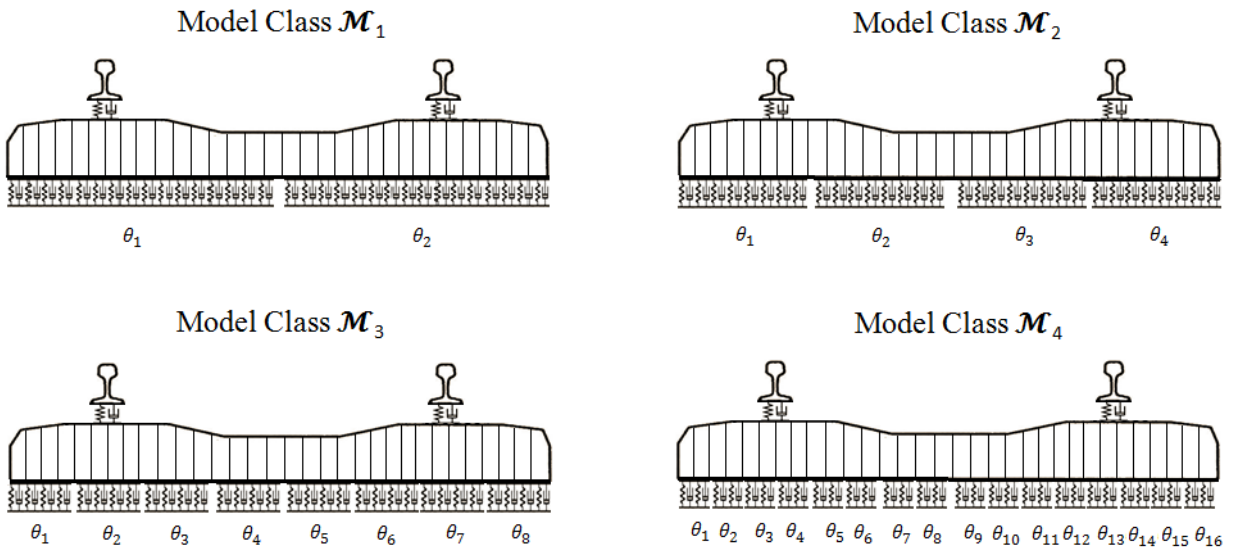

Fig. 3. Four candidate model classes with 2, 4, 8 and 16 free independent parameters for representing stochastic ballast bed conditions under the sleeper of interest in bay number 10. Subdivision of the sleeper indicates that the sleeper of interest is divided into 32 Euler-Bernoulli elements. Sleepers in other bays are represented by 10 Euler-Bernoulli beam elements.

3 CHALLENGES IN MATHEMATICAL MODELLING OF COMPLEX STRUCTURES

Constructing mathematical models that produce accurate predictions of system response is a fundamental element in science and has great potential in applications for improving reliability-based structural designs. However, the problem of characterizing and predicting the response of real-world structural and infrastructural systems that are subjected to different loading and environmental conditions is a formidable challenge. Such challenges exist in many railway and seismic engineering applications.

First of all, the variability in material properties and environmental conditions together with the complex constitutive behaviour of structural materials make it practically infeasible to construct reliable deterministic models based solely on first principles. Thus, knowledge on statistical distributions and dependencies regarding various uncertain features must be obtained by testing and be incorporated into reliability predictions. Obtaining this knowledge requires conducting effective *in situ* experiments and a patient collection of data [15–17]. Such experimental activities do more often than not lead to system identification activities to provide mathematical black-box input/output models that can be used for predictions.

Secondly, the experimental limitations of *in situ* tests and uncertainties of required model complexity together with the inverse nature of the system identification give rise to a number of challenging issues. Examples of such issues include the issue of model selection (how to address uncertainties in a model structure), the parameter identifiability issue (whether the test data is rich in information with respect to the chosen parameters) and the parameter uncertainty issue (how precisely the uncertain values of the parameters can be pinned down by the measurement data). These challenges can be alleviated to a certain extent by performing an experimental design study prior to conducting the *in situ* experiments. This contains an optimal selection of experimental conditions in order to increase the information value of data in regards to the uncertain parameter set. This, however, may not completely resolve the modelling issues listed above, particularly in non-trivial identification problems such as in *nonlinear system identification* or *spatially varying parameter estimation* [18]. Therefore, it is important to evaluate the performance of available methods through feasibility studies using simulated data and to find the appropriate ones that are suitable for a problem of interest.

Lastly, the computational costs associated with probabilistic system identification and reliability prediction of complex dynamic systems are of major concern [19]. The coupling between system modelling, probabilistic analysis and optimization makes the problem computationally intense since it often involves the evaluation of a large number of simulations as shown in Fig. 4. This motivates the development and use of efficient algorithms for a deterministic and stochastic analysis.

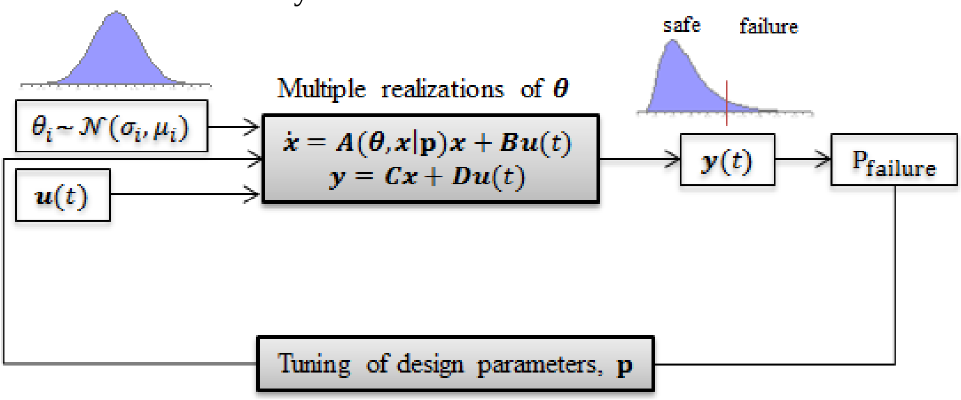

Fig. 4: A schematic view of the process of reliability-based design optimization of design parameter vector $\boldsymbol{p}$. Optimization with multiple iterations to converge demands a large number of realizations of random parameter vector $\boldsymbol{\theta}$ and associated simulations to compute the failure probability P_{failure} at every major step of optimization iteration.

The above mentioned conceptual and computational issues are amplified in spatially varying parameter estimation problems. Such is the case in the ballast stiffness identification problem, since the variation properties of the parameter field are usually unknown. Therefore, inappropriate discretization of the random field can lead to either poor modelling (due to using inaccurate models) or an ill-conditioned problem (due to the use of over-parameterized models). Even with a proper discretization of the ballast stochastic field, the large number of uncertain parameters associated with these problems make the standard optimization or sampling schemes computationally cumbersome [20–21]. This problem becomes more challenging when nonlinear effects are accounted for in the identification models. This thesis explores the treatment of such challenges.

4 RELATED AND PREVIOUS WORK

Previous work by the research group was conducted on finding a minimum pressure sensor density by studying the effect of embedding ballast stiffness statistical properties on stress prediction in the sleeper [22]. The hypothesis was that the minimum pressure sensor density corresponds to the spatial discretization needed to resolve the smallest correlation length of the stochastic ballast stiffness field that affects the sleeper bending stress prediction accuracy. This critical correlation length was considered to be the shortest correlation length that gives a clearly different sleeper bending stress distribution than a fully uncorrelated stiffness field would give. Based on that study, the authors determined that the number of pressure sensors required to capture the embedding pressure field with acceptable accuracy need not be more than 32 for a 2.5 m long sleeper under the conditions given. That determination led to the conclusion that the distance between pressure sensors should not be longer than about 8 cm. Based on this conclusion, an *in situ* test was performed in 2008. The measurement technique and results are described in [23]. Basically it consisted of 96 strain gauges distributed over 32 triangular load cells (*i.e.*, three strain gauges in each load cell) along the sleeper. Based on the results from several train passages at several sites, a significant scatter in the ballast pressure distribution was observed. The clear tendency was that the ballast pressure at the centre of the sleeper was lower compared to the ballast pressure under the rail seats. This observation was confirmed by a visual inspection of the sleepers placed in track for up to 38 years after sleeper removal and bottom surface examination [10,24]. However, it was concluded that the measurement system needed to be of better quality for giving high quality results and an improved understanding of the sleeper-ballast interface loads. In comparison, measuring sleeper acceleration is more practically feasible and economically less expensive but the informativeness of these two data sets, with respect to ballast stiffness parameters of different resolutions needs to be investigated. Based on these previous numerical and experimental studies this study focuses on performing feasibility studies on identifying ballast models down to spatial resolutions of 16 cm. The feasibility to base a test on acceleration measurements and/or sleeper-ballast pressure data in a test planning setting is studied. The simulated data is generated according to the likely ballast scenarios that were observed in [23].

This research work is part of the on-going research activities in the national railway center of excellence CHARMEC. The project is called TS14 and it is in the category of interaction of train and track research program. The project is connected to the finalized CHARMEC projects MU5 (concerning the mechanical properties of concrete sleepers) [26], TS1 (concerning the calculation models of track structure [25]) and TS9 (concerning the track dynamics and sleepers [22]). Other experimental and theoretical related studies on modelling of the ballast/sleeper interaction and the effect on train/track dynamics can be found in [27–31] and associated articles are referred to in **paper A**.

5 RESEARCH AIM AND SCOPE

The present work has been carried out with a focus on conceptual and computational issues that typically arise in the spatially varying parameter estimation problems. These are also known as field parameter identification problems. The specific application of focus originates from a moving load problem in railway mechanics in which the ballast distribution along a sleeper is of particular interest because its variation causes variation in the sleeper stress levels. The perspective has been from a structural dynamics point of view instead of a mathematical statistics point of view and the aims of this research project have been on the following.

I. **Alleviating modelling issues in ballast stiffness field parameter identification and planning effective in situ experiments:**
A significant effort must be put on the test-driven spatially varying ballast stiffness identification problem for the purpose of gaining more insight into the load transfer mechanism in the sleeper-ballast interface. In addition, more information about the stochastic ballast variation along the sleeper length is needed. This requires an in-depth study of the modelling issues for finding appropriate methods to tackle these problems and for doing test planning of *in situ* experimental conditions. Through feasibility studies, the performance of candidate identification methods for robust and efficient treatment of conceptual as well as computational challenges that typically arise in high-dimensional inference problems are investigated. The obtained results of this preliminary study can be used as the basis for design of effective test experiments aimed at measuring informative data for the purpose of model calibration.

II. **Treating computational issues in ballast stiffness field parameter identification:**
A main focus of this research is on computational efficiency because future work is planned for a structural reliability analysis that will either be combined with optimisation in reliability based design optimization or be used for motivating updates of safety factors in design guidelines. Thus, a significant effort must be made for efficient assessment of risk of failure in moderate-to-large size finite element (FE) dynamic models, such as for the studied railway track. This can be done by algorithmic development of reliability methods and/or by adopting approximate solutions for evaluating the time response of the system. Focusing on this problem is strongly needed in the reliability assessment (forward) problem, the system identification (inverse) problem and in system design optimization as these fields are known to be notoriously computationally demanding and their efficiency relies on fast simulations.

6 RESEARCH METHODOLOGY

Broadly speaking, establishing models for accurate prediction of system response is achieved through model driven approaches or data driven (statistical) approaches. So-called white-box models are fully derived by extensive physical modelling from first principles with parameters that have a clear physical meaning. The finite element (FE) modelling approach falls into this category. In the data-driven approaches that result in black-box models one instead relies solely on recorded measurement data to identify both the model structure and the corresponding uncertain parameter values [32–34].

Combining prior physical knowledge with information content of test data (known as the grey-box approach) is sometimes an advantageous strategy in terms of better statistical accuracy because prior physical knowledge reduces the model space that must be searched and the number of parameters that need to be estimated [35]. A grey-box approach is considered a suitable modelling strategy for this study, since it can combine our physical knowledge about the track superstructure with the information given by test data, as shown in Fig. 5. The test data is then used to infer a model structure that particularly regards the load transfer mechanism through the sleeper-ballast interface. Thus, the interest is not only in identifying the ballast stiffness parameters but also in the variation of the ballast stiffness along the length of one sleeper of interest. In this regard, an *in situ* test is planned for quantifying the uncertainties about statistical distributions and dependencies regarding the ballast stiffness parameters. See Fig. 6 for a schematic view of the system identification anatomy.

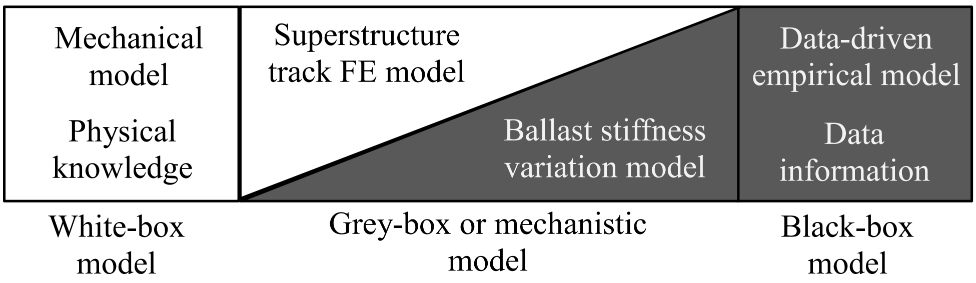

Fig. 5: Statistical modelling of physical systems known as grey-box modelling. This strategy has been utilized for establishing predictive models of railway track structure.

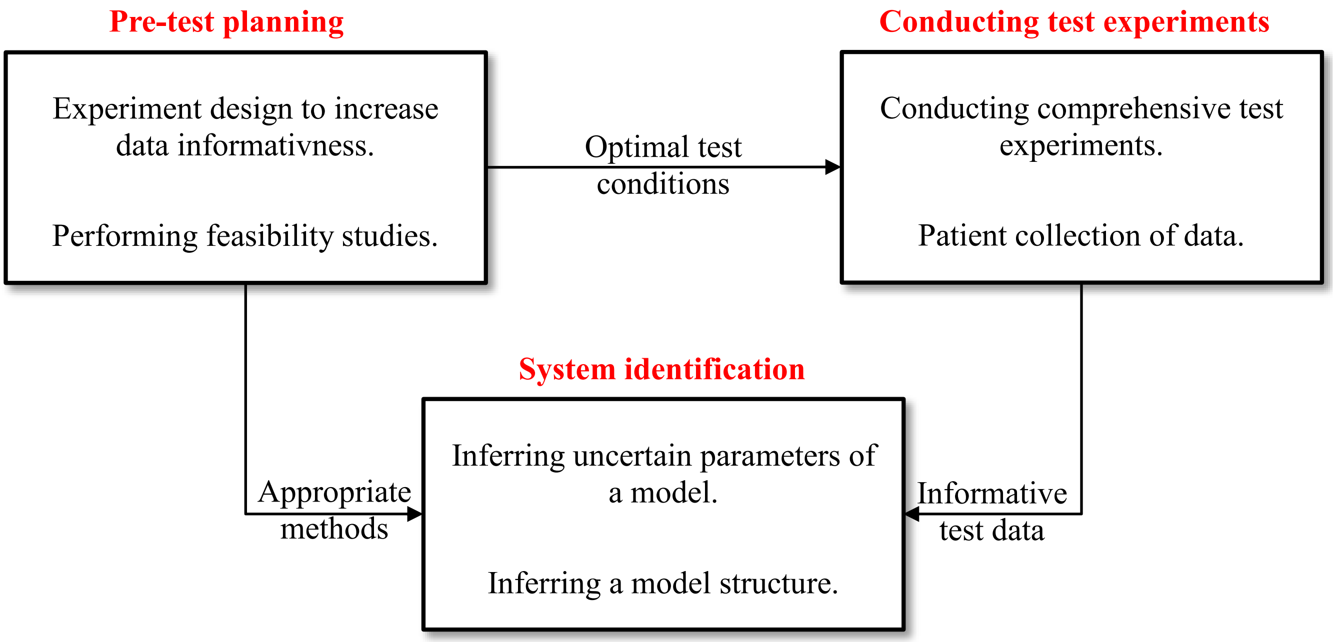

Fig. 6: A schematic view of the system identification process

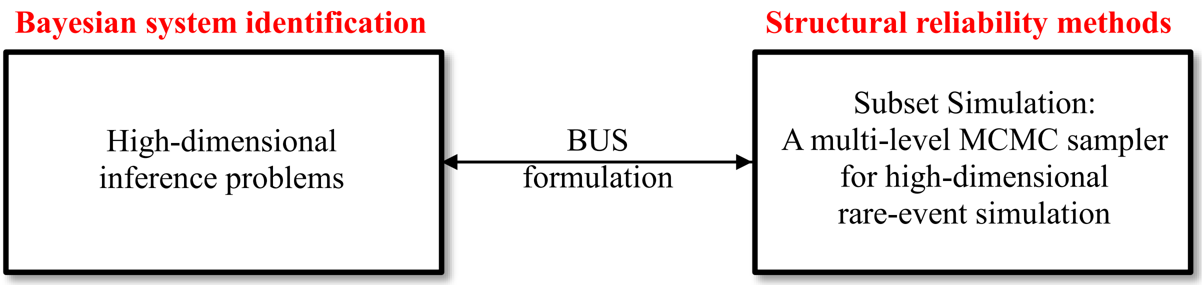

Fig. 7: BUS solves the Bayesian identification problem by interpreting it as an equivalent reliability problem. This solution enables us to use rare-event samplers, such as subset simulation, to alleviate the problematic issues in high-dimensional inference problems.

In a field property identification problem treated by numerical procedures, there is a need to discretise the target field and represent the parameter field in a finite-dimensional parameter subspace [20]. Since the variation of the ballast stiffness along the length of a sleeper is not known *a priori*, a suitable strategy is to use a set of probable candidate ballast model classes of different structural complexity. At the model class selection level it is considered best practice to penalize more complicated model classes such that the simpler models (with a reasonable consistency to data) are preferred over too heavily parametrized models that are more prone to noise and may lead to over-fitting to data [18]. Otherwise, if a model class assessment is based purely on some metric of model fit with respect to the data, the best model class will always be the most complicated one. This might result in unreliable over-parameterized predictive models.

To alleviate the modelling challenges in a field property identification problem, a Bayesian probabilistic framework is utilized for both the model calibration and the model selection levels of the system identification problem. Among the different advantages offered by the Bayesian approach is that it does not require the uncertain parameters to be identifiable and it has a built-in penalty term against over-parameterized model classes at the model-selection level. This leads to a principled means of model selection [36]. These advantages make the probabilistic Bayesian framework a suitable candidate for the grey-box system identification of the railway ballasted track in which one aim is to gain physical insight into the load transferring mechanism. However, many sampling techniques typically suffer from algorithmic issues known as the *curse of dimension* and *application robustness* when they are applied to real-world inference problems. To alleviate these issues that typically arise in a field parameter identification problem, a recently proposed framework called the enhanced Bayesian Updating using Structural reliability (BUS) is employed [37–39]. This approach opens up the possibility to directly apply well-developed available reliability methods, such as the subset simulation (SS) algorithm, to the Bayesian updating problems for the purpose of obtaining the posterior statistics, as shown in Fig 7. Briefly stated, subset simulation is a robust and efficient multi-level Markov Chain Monte Carlo (MCMC) based stochastic simulation technique that has been developed in reliability context for rare event simulation [40–41]. The performance of enhanced BUS as a candidate methodology for the spatially varying Bayesian inference problem has been investigated through feasibility studies. The result has been used for pre-test planning of effective *in situ* experiments to reduce time and cost. To achieve higher levels of efficiency, an efficient time-integration and reduction scheme has been developed for fast deterministic simulation of the studied railway track system with local nonlinearity, property uncertainty and moving load. Employed approaches, methodologies and algorithms that are considered appropriate candidates for efficient and robust tackling of the ballast stiffness field parameter identification, are discussed in the following subsections.

6.1 On Bayesian system identification using reliability methods

Structural system identification is concerned with extracting from measurement data the information about the uncertain parameters of a representative model of that structure. This study adopts a Bayesian probabilistic framework to tackle the spatially varying random field parameter inference problem. A main challenge in a Bayesian updating problem, is to robustly and efficiently handle a problem with a high-dimensional parameter space and large measurement data sets. This is still under intense research and it is discussed here.

The primary target in a Bayesian model updating problem is to get the posterior probability distribution of the unknown parameter set $\boldsymbol{\theta}$ in the context of a model class $\mathcal{M}_i$ [18]. This is commonly referred to as the first level of Bayesian system identification. Given measurement data $\mathcal{D}$, *a priori* probability distribution function over the parameters $p(\boldsymbol{\theta}|\mathcal{M}_i)$ is updated to obtain an *a posteriori* distribution $p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M}_i)$ using Bayes´ rule

$$p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M}_i) = P(\mathcal{D}|\boldsymbol{\theta},\mathcal{M}_i)p(\boldsymbol{\theta}|\mathcal{M}_i)/P(\mathcal{D}|\mathcal{M}_i) \tag{1}$$

in which $P(\mathcal{D}|\boldsymbol{\theta},\mathcal{M}_i)$ denotes the likelihood function. The posterior distribution is given by the use of the denominator normalizing constant

$$P(\mathcal{D}|\mathcal{M}_i) = \int P(\mathcal{D}|\boldsymbol{\theta},\mathcal{M}_i)p(\boldsymbol{\theta}|\mathcal{M}_i)\mathrm{d}\boldsymbol{\theta} \tag{2}$$

The denominator constant is known as the model evidence in the model selection context. In a Bayesian model selection problem it is difficult to avoid the explicit evaluation of the model evidence $P(\mathcal{D}|\mathcal{M}_i)$ since it is the primary quantity of interest based on which the competing models are compared [18]. The relative model evidence is then used to compare the posterior probability of two competing models by the quotient

$$P(\mathcal{M}_i|\mathcal{D})/P\left(\mathcal{M}_j\middle|\mathcal{D}\right) = (P(\mathcal{D}|\mathcal{M}_i)P(\mathcal{M}_i|\mathbf{M}))/(P\left(\mathcal{D}\middle|\mathcal{M}_j\right)P\left(\mathcal{M}_j\middle|\mathbf{M}\right)) \tag{3}$$

where $P(\mathcal{M}_i|\mathbf{M})$ expresses the user judgement on the initial probability of model $\mathcal{M}_i$ of a candidate set $\mathbf{M}$ that includes N_M models. $P(\mathcal{M}_i|\mathcal{D})$ represents the posterior probability of model $\mathcal{M}_i$ relative to other candidates, given data. Assuming the same initial probability for models, the model with (relatively) higher model evidence is the more probable.

Unfortunately, computing the posterior statistics and model evidence for complex models is not always trivial. The difficulty primarily results from the complexity of the likelihood function $P(\mathcal{D}|\boldsymbol{\theta},\mathcal{M}_i)$ which makes the integrals given by Eq. 2 either too complex to be analytically integrated or of too high dimension to be efficiently treated with conventional numerical integration methods [42]. The most common approach is to use MCMC based stochastic simulation techniques to solve for it [42]. While MCMC stochastic simulation techniques provide powerful tools for Bayesian computations, difficulties are often encountered in applications. This happens particularly often when treating problems with high dimensional parameter space or large sets of data. One common example of these difficulties is the so called *curse of dimensionality*, a situation in which the algorithm only works effectively in low dimension but systematically breaks down in problems with high dimensional parameter spaces [38]. Another issue of concern that typically arises when dealing with large amounts of informative data is that the *a posteriori* distribution takes on significant values only in a small region of the parameter space whose size generally shrinks in an inverse square root law with the data size. This can cause efficiency problems due to slow convergence of the Markov chain and give estimation bias [38]. Recent developments have been focused on adopting rare event stochastic simulation algorithms (*e.g.* subset simulation) to treat Bayesian updating problems [38,43,44], see **Paper A** for a deeper discussion.

6.2 On reliability prediction using stochastic simulation

It is often important to predict the system reliability or performance considering modelling uncertainties. Such predictions can be made based on *a priori* knowledge of modelling uncertainties (as is the case in reliability-based design or life-cycle cost optimization) or, better yet, on *a posteriori* knowledge of uncertainties conditioned on test data (as is the case in system identification, control and health monitoring). In either case, the interest in system reliability is directed towards simulation of failure events that rarely occur in reality. They therefore correspond to the tail distribution of a response quantity of interest, denoted by Y. The motivation to apply stochastic simulation to efficiently and robustly capture these rare failure events is discussed in brief below.

Basically, the failure probability P_F is defined as the probability that the response Y exceeds a pre-specified threshold value b. Assuming that prior knowledge about the uncertain parameters $\boldsymbol{\theta} \in \Re^n$ (assuming a pre-specified model) is given by the probability distribution function $p(\boldsymbol{\theta})$, the probability P_F can be determined as

$$P_F = \int_F p(\boldsymbol{\theta}) = \int I_F(\boldsymbol{\theta}) p(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} \tag{4}$$

where the indicator function $I_F(\boldsymbol{\theta})$ is unity when its argument is true, that is for parameter realizations in the failure region $\boldsymbol{\theta} \in F$, and zero otherwise. The region of the parameter space that corresponds to unsatisfactory system performance (or failure) is denoted by $F = \{Y > b\}$. Note that for simplicity the conditioning on the pre-specified model is omitted. Based on a given *a posteriori* statistics $p(\boldsymbol{\theta}|\mathcal{D})$ for a pre-specified model, the reliability predictions can be updated through updated failure probabilities conditioned on that data is

$$P_F^{\text{updated}} = \int I_F(\boldsymbol{\theta}) p(\boldsymbol{\theta}|\mathcal{D})\, \mathrm{d}\boldsymbol{\theta} \tag{5}$$

Evaluating multi-dimensional integral in Eq. (4) is the main concern in a reliability problems. Analytical methods or direct numerical integration schemes can be utilized only for treatment of simple reliability problems. However, this is not the case when the failure region has a complex geometry in the parameter space or when the parameter space is of high dimension. To cope with these problems, one can use standard Monte Carlo simulation (MC), which is a numerical integration scheme in a statistical setting. Using this approach, the integral is viewed as an expectation, leading logically to estimation by means of "statistical averaging" based on independent identically distributed (i.i.d.) samples [41]. According to Eqs. (4–5) the failure risk is then computed through the following expectations.

$$P_F = \mathrm{E}(I_F(\boldsymbol{\theta})) \tag{6a,b}$$
$$P_F^{\text{updated}} = \mathrm{E}(I_F(\boldsymbol{\theta})|\mathcal{D})$$

Standard MC simulation is known to be robust to the application (the type and complexity of the failure limit state is problem/application dependent) and dimension of the problem. These are well recognized issues that are known as *application robustness* and *dimension sustainability* within the reliability community [38]. However, the MC approach becomes extremely time consuming when one intend to find small probabilities, *i.e.* failure probabilities such that P_F <0.001. The reason resides in the fact that the coefficient of variance (*i.e.* c.o.v. = estimation standard deviation /mean) for the MC estimator approaches $1/(NP_F)^{1/2}$, where N is the MC simulation sample size. Consequently, the required number of system analyses in MC, for a given estimation variance, is proportional to $1/P_F$ [40]. This leads to extensively high computational costs in engineering applications, in which the targeted failure reliability is often in the range of 10^{-3}~10^{-6}.

Recognizing the efficiency problem of the standard MC when treating rare event simulation problems, advanced MC methods aim at reducing the estimation variance for a given computational costs. Examples of these so-called variance reduction techniques are importance sampling and stratified sampling (on example is the Latin Hypercube sampling). The challenge is, however, to beat MC on efficiency (smaller c.o.v.) without losing out on 'application robustness' [40]. The Subset Simulation (SS) is a technique for rare event sampling that has been developed for this purpose in the reliability literature. The SS strategy is to break down the rare (failure) event problem into a sequence of more frequent nested events and determine the failure probability as a product of conditional probabilities, each of them being estimated by MCMC simulation [45]. The variance for the SS estimator is proportional to $\ln(1/P_F)^{r/2}$, ($2 \leq r \leq 3$), which increases at a much slower rate compared to P_F^{-1} that is the convergence rate of the variance of the MC estimator. Moreover, the conditional samples produced by subset simulation algorithm can also be used for estimating the conditional expectation in probabilistic failure analysis, a feature not shared by conventional variance reduction techniques.

Evaluation of the multi-dimensional integral given by Eq. (5) cannot usually be evaluated analytically nor evaluated numerically if the number of parameters is not high is not very small [36]. This can be tackled by using the BUS formulation [37–39]. It opens up the possibility to transform the Bayesian problem into an equivalent reliability problem. Then robust rare event samplers, such as subset simulation, can be employed to obtain the posterior statistics and to estimate the conditional probabilities, given by Eq. (5). This is done by combining the rejection sampling with the subset simulation algorithm. An enhancement of the BUS approach has been used for ballast stiffness field parameter identification. This is discussed in **Paper A** and [39].

6.3 On fast dynamic analysis of complex dynamic systems

Efficient stochastic simulation is an essential ingredient in structural reliability and identification problems. For these, often a large number of computed perturbed solutions are needed in order to study the effect of parameter variation on system performance. The above discussed algorithmic development in rare event stochastic simulation aims at increasing the statistical efficiency by reducing the number of forward simulation runs while treating the input-output descriptor of the system as black-box. The efficiency therefore relies on fast deterministic simulations. This is of particular value when treating large nonlinear dynamic systems. A part of this study focuses on achieving high efficiency in simulation of a moving load on a railway track for problems that include local nonlinearity or uncertainty. These nonlinearities and uncertainties are due to that sleepers are put on a stochastic non-uniform ballast bed which creates state and parameter dependent interaction forces.

In simulation of linear time invariant systems the problem complexity manifests itself both as model size (number of states) as well as model dynamic range (ratio of the largest eigenvalue to the lowest one). The problem complexity is often high in many high fidelity finite element analyses, see [46–47] for a literature survey. The reason is that the simulation duration is usually determined by the slow dynamics, to which we often focus our interest, while the integration step size used is dictated by the fastest dynamics of the system due to stability and accuracy restrictions. This leads to that a large number of time steps are required for the observation of the slow dynamics and thus the time discretization of forces needs to be made to a very fine resolution. In structural dynamics problems these high fidelity models often arise when finite element analysis is used to obtain an accurate discrete model of the governing equation and often results in models with over a million degrees of freedom [46–47]. The corresponding equation is given by the *state equation*

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f}(t) \tag{7}$$

which may be integrated for the solution of the system states $\boldsymbol{x}(t)$. From these the system outputs $\boldsymbol{y}$ can be computed through an algebraic relation called the *observation equation* or *prediction equation* based on the problem of interest as

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{8}$$

The state, input force and the output vectors are $\boldsymbol{x} \in \Re^{N\times 1}$, $\boldsymbol{f} \in \Re^{N\times 1}$, $\boldsymbol{y} \in \Re^{n_y\times 1}$ respectively. The dynamic matrix $\boldsymbol{A}$, the input matrix $\boldsymbol{B}$ and the output matrix $\boldsymbol{C}$ are given by $\boldsymbol{A} \in \Re^{N\times N}$, $\boldsymbol{B} \in \Re^{N\times n_u}$ and $\boldsymbol{C} \in \Re^{n_y\times N}$, respectively. In this study, the experimental observables $\boldsymbol{y}$ are the ballast-sleeper interface force and the sleeper acceleration. The predictions $\boldsymbol{y}$ are for the bending moments at two critical cross-sections along the sleeper that are used for reliability prediction.

The central idea in model reduction techniques is to systematically capture the main input–output properties by a much simpler model than what is necessary for a very precise description of the entire system state [48–49]. Many of these techniques are based on domain decomposition computation. Common such spatial domain decomposition methods are the Proper Orthogonal Decomposition and the Principle Component Analysis that aim at identifying a subspace of high energy modes onto which the dynamics is projected (through Galerkin Projection) [50–51]. These methods, which have been extensively used in computational dynamics, implicitly assume that all degrees of freedom can actually be observed or measured. The balanced truncation, which has been derived in a control context, accounts for incomplete observability of system states. By exploiting the input-output relation, it aims at reducing the system dynamics into the subspace of states which make significant output contribution [52–55]. However, these states (modes) may not carry the clear physical interpretation as the system eigenmodes do. This is one major motivation for the structural dynamics community to instead employ modal reduction approach. A main restriction in the general applicability of modal reduction techniques has been the lack of a proper dominancy analysis as well as the lack of a guaranteed bound for the approximation error [56].

A part of this study has been mainly on developing a modal technique for model reduction of linear time-invariant dynamic systems. Particular interest has been put on structures that are subjected to moving and distributed input loadings. A reduction has been developed based on modal contribution to the system input-output relation, see **Paper B**. Briefly stated, a quadratic dominancy metric given in a closed form formulation is presented based on the modal contributions to the $\mathcal{H}_2$ norm of the observed frequency response function (FRF) matrix. It is thus related to the root-mean-square of the prediction. However, one issue for many of the modal dominancy metrics is to detect the non-minimality and to effectively handle systems with multiple or close eigenvalues. Such is the case in reduction of track FE model with dense clusters of neighbouring eigenvalues. A QR-decomposition based technique is presented in **Paper B** to circumvent this issue. It is done by detecting the uncontrollable and unobservable modal coordinates. A modally balanced solution for this problem is presented in **Paper C**. Another issue of concern is that many of the input-output based reduction techniques become ineffective when they are applied to systems subjected to a moving or distributed loading input. In this regard, the presented method is an improvement to these since it incorporates information extracted from the structural and spectral properties of the input force in the modal dominancy analysis. The performance of the proposed method is validated in the moving load problem of interest. For more details see **Paper B** and **Paper C**.

One major challenge for the developed modal reduction method in inverse inference problems or in a probabilistic problem setting is to make the reduced model capable of efficiently and accurately taking the effect of parameter variation into account [57–61]. In view of this, a part of this study is focused on developing an integration scheme for fast simulation of structural systems with local nonlinearity and uncertainty to which the modal reduction method can be directly applied. The system response is computed based on the solution of an underlying linear system augmented with a low rank nonlinear pseudo-force vector $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{\theta})$ that accommodates the local nonlinearity or parameter variation effects, given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}^{\mathrm{lin}}\boldsymbol{f}(t) + \boldsymbol{B}^{\mathrm{res}}\boldsymbol{g}(\boldsymbol{x},\boldsymbol{\theta}) \tag{9}$$

A prediction-correction time integration schemes has been considered as a suitable approach. By this one can take the advantages of both explicit and implicit schemes to achieve a high level of efficiency. Using an exponential integrator exploits the extremely fast computation of a proper underlying linear model´s response that boost the overall efficiency. This feature is not provided by general-purpose solvers, such as Runge-Kutta (see **Paper D**).

7 THESIS CONTRIBUTIONS

To the author's knowledge, the most important part of the work that is novel can be summarized as:

I. The development of a method for modal reduction of large scale Linear Time Invariant (LTI) systems, based on accurate preservation of the input/output relationship. In treating problems with a high dimensional input space, such as in moving load or distributed loading problems, the presented method is an improvement to existing methods as it incorporates information about the structural properties of the input force in the modal dominancy analysis. Moreover, the presented method is able to treat the metric non-uniqueness that is a problematic issue in reduction of systems with multiple eigenvalues such as those present in the case of railway track structures with clusters of nearly coalescing eigenvalues (**Paper B**, **Paper C**).

II. The development of an efficient time integration scheme for fast simulation of structural systems with local nonlinearity or uncertainty to which the developed modal reduction techniques can be directly applied. The presented scheme overcomes an obstacle towards for the use of the developed modal reduction technique in inverse inference problems or in a probabilistic problem setting. This is done by an efficient manner of taking the local nonlinearity and local uncertainty effects into account. Such is the case in simulation of a moving load on a railway track for the purpose of characterizing the unknown sleeper-ballast load along the length of the sleeper of interest. (**Paper D**).

III. An in-depth study of these issues is performed for finding appropriate methods to treat them in the ballast stiffness field property identification and for doing test planning of *in situ* experimental conditions. The performance of the candidate identification method for alleviating the problematic issues in (ballast) field property identification problems are investigated through feasibility studies. The obtained results of this preliminary study are used as the basis for design of effective test experiments, aiming at measuring informative data for the purpose of model calibration **(Paper A)**. Validating the performance of the *enhanced BUS* Bayesian framework to alleviate the mentioned problematic issues in a real-world high-dimensional inference problem is novel.

8 SUMMARY OF APPENDED PAPERS

Paper A:

Safety factors, used for design of railway sleepers and given by the railway sleeper design codes, need to be set and revisited as both higher train speeds and heavier axle loads is the trend. To achieve this, establishing valid predictive models of railway track structure, for reliable prediction of the bending moment at specific locations along the sleeper, is a necessity. This is, however, challenged by the lack of knowledge about the ballast bed variation along each sleeper and the load transferring mechanism, through which the train axle-load is transferred into the ballast media. In this regard, an *in situ* experiment needs to be conducted for quantifying the uncertainties about statistical distributions and dependencies, regarding the ballast stiffness parameters. However, the experimental limitations of *in situ* tests and uncertainties of required model complexity together with the inverse nature of the system identification give rise to a number of challenging issues. First, the variation of the ballast stiffness random field is not known a priori and thus inappropriate discretization of the parameter field typically leads to either inaccurate modelling or an ill-conditioned problem. Secondly, even with a proper discretization, the large number of uncertain parameters associated with these problems make the standard optimization or sampling schemes computationally cumbersome and also more prone to the issue of *curse of dimensionality*. An in-depth study of such issues is presented for finding appropriate methods to treat them in a railway ballast stiffness field property identification and for doing test planning of *in situ* experimental conditions. A recently proposed Bayesian approach, known as enhanced BUS, is utilized to handle such high-dimensional inference problems. This approach employs rare-event samplers, such as subset simulation, to efficiently draw samples from the posterior probability distribution in high-dimensional problems. Performance of the method is investigated and the feasibility to base a test on acceleration measurements or sleeper-ballast pressure data in a test planning setting is studied.

Paper B:

A main restriction in the general applicability of modal reduction techniques has been the lack of a proper dominancy analysis as well as the lack of a guaranteed bound for the approximation error. In this study, a modal dominancy approach for reduction of dynamic systems is presented. A quadratic metric introduced based on modal contribution to the $\mathcal{H}_2$-norm of the frequency response function matrix is given in closed-form. A performance and error analysis of the proposed modal dominancy procedure is carried out and the problems of metric non-uniqueness and structural non-minimality for a class of systems with multiple eigenvalues are described. A method to circumvent these problems is proposed. In treating problems with high dimensional input space, such as in moving and/or distributed loading problems, the presented method is an improvement as it incorporates knowledge about the structural and spectral properties of the input force in the modal dominancy analysis. In addition, the method´s performance is validated for the reduction of a large-scale finite element model that originates from a moving load problem in railway mechanics. The results are compared with results of the reduction-after-balancing approach.

Paper C:

In this paper, a review of Gramian based minimal realization algorithms is presented and several comments regarding their properties are given. The ill-conditioning and efficiency problems that typically arise in balancing of large scale realizations are addressed. A new algorithm to treat non-minimal realization of very large second order systems with dense clusters of close eigenvalues is proposed. The method benefits from the effectiveness of balancing techniques in treatment of non-minimal realizations in combination with the computational efficiency of modal techniques to treat large-scale problems.

Paper D:

Treating the entire structure as being nonlinear in simulation of structural systems with local nonlinearity or uncertainty can increase the computational efforts drastically. This could be a chief obstacle particularly in an uncertainty quantification problem or in a probabilistic problem setting since both require a large number of forward simulations. In response, this paper presents an integration scheme that combine with reduction for fast computation of the dynamic response of structures with local nonlinearity or stochasticity. Briefly stated, the system response is computed based on the solution of an underlying nominal linear system. This is augmented with a low-rank nonlinear pseudo-force vector that accommodate the local nonlinearity and uncertainty effects. The procedure uses a two-stage prediction-correction time integration scheme in which the response of the underlying linear system is computed efficiently using an exponential integrator and the corrected solution that correspond to the contribution of the pseudo-forces is computed using a fixed-point iteration technique. This is made without need to solve a nonlinear algebraic equation set. In deriving the discrete time system matrices, the triangular-hold filter is used for approximate representation of the external force and the nonlinear pseudo-force kernels. Approximations of the convolution integrals involved are given by analytical expressions. In order to achieve higher levels of computational efficiency for large-scale problems, the time integration scheme is combined with the developed modal reduction technique as presented in **Paper B** and **Paper C**. It is enhanced to take into account the effect of the corresponding local nonlinearity or uncertainty in its modal dominancy analysis. Distinct features of the method are discussed in detail through illustrative numerical examples and its computational performance is investigated through a moderate size real-world problem that originates from a moving load problem in railway mechanics.

Paper E:

Due to the highly stochastic environment of railway tracks, a probabilistic approach is used to estimate the probability of track component failure for design purpose. The accuracy and required computational effort of both analytical and stochastic simulation reliability methods with application to sleeper design are investigated. Two hierarchical models, based on a detailed finite element analysis to represent the track dynamics, are used for estimation of the failure surface. The accuracy of the different methods is evaluated by comparison to the result of a standard Monte-Carlo simulation. Subset simulation algorithm is used for probabilistic failure analysis to investigate the ballast distribution that can cause the sleeper failure.

9 CONCLUDING REMARKS AND FUTURE WORK

The overall aim of the work presented in this thesis is to provide an in-depth study of the modelling and computational problematic issues in ballast field property identification problems. These problems are investigated through feasibility studies for finding appropriate methods to alleviate these issues and for doing test planning of *in situ* experimental conditions. One possible sensor configuration is to use accelerometers for measuring the dynamic response to train traffic. Although using only acceleration data is economically efficient, the collection of sleeper-ballast interface force data in the planned in-field experiment is shown to be more informative. The obtained results of the feasibility study confirm the informativeness of force cell data with respect to the local ballast stiffness parameters, even for high resolution ballast models with 16 discrete stiffness parameters for each sleeper. The work indicates that a successful stiffness calibration outcome can be expected when sleeper-ballast interface forces are measured by force cells and no other sensors need to be engaged. It is evidenced by the Bayesian approach that using only measured acceleration data lacks the sufficient information required for identification of the ballast stiffness distribution with a spatial resolution that is finer than 325 mm for the investigated sleeper configuration. However, it is argued that a number of accelerometers can be added to the instrumented sleeper for the purpose of model validation. A test set-up including a thin-film sensor system that can sense the pressure distribution between the sleeper and its ballast bed has been exposed to some preliminary *in situ* testing, and field testing to provide data on ballast/sleeper interface properties is planned for the near future. It was found that the implemented Bayesian framework gives reasonable results and a successful calibration outcome can be expected in a future calibration based on real test data. In this regard, the Enhanced BUS formulation is considered as a suitable candidate for the ballast stiffness field parameter identification and its use can gain physical insight into how the train axle-load is transferred into the ballast media.

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