Thesis for the Degree of Licentiate of Engineering

Mitigation of nonlinear fiber distortion using phase conjugation

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Abstract

This thesis is devoted to the study of fiber-optic communication systems that mitigate nonlinear distortion due to the Kerr effect by transmitting a phase-conjugated copy alongside the signal followed by coherent superposition. This kind of communication systems are referred to as phase-conjugated twin waves systems when the phase-conjugated copy is transmitted on the orthogonal polarization. The nonlinear distortion due to the Kerr effect is one of the main limiting factors in modern coherent fiber-optic communication systems and it is of high interest to design communication systems that have a higher tolerance for nonlinear distortion at high optical launch powers.

With a perturbation analysis and numerical modelling, we investigate a time domain implementation, transmitting the signal and the phase-conjugated copy in different time slots. We show that even though this scheme is predicted by the perturbation analysis to perform worse compared to regular phase-conjugated twin waves, the numerical results show that it performs comparably. Also it is argued that the time domain scheme could be easier to implement in practice. We then make an experimental comparison between a link performing coherent superposition in digital signal processing and a link performing coherent superposition all-optically in a phase-sensitive amplifier. The results from that comparison show that the performance in the nonlinear regime is comparable with both approaches. Last, we investigate numerically the impact of the span power map in single- and multi-span links that perform the coherent superposition inline with the use of phase-sensitive amplifiers. It is shown that by employing distributed Raman amplification in a multi-span phase-sensitive amplifier link, it is possible to achieve significant increases in tolerance against nonlinear distortion leading to increased transmission reach, in some cases by as much as a factor of 8.

Keywords: fiber-optic communication, nonlinear optics, phase-sensitive amplifiers, phase conjugation

List of papers

This thesis is based on the following appended papers:

[A] **H. Eliasson**, P. Johannisson, M. Karlsson and P. A. Andrekson, “Mitigation of nonlinearities using conjugate data repetition,” *Optics Express*, vol. 23, no. 3, pp. 2392-2402, 2015.

[B] **H. Eliasson**, S. L. I. Olsson, M. Karlsson and P. A. Andrekson, “Comparison between coherent superposition in DSP and PSA for mitigation of nonlinearities in a single-span link,” Proceedings European Conference on Optical Communication (ECOC), Cannes, France, 2015, paper Mo.2.5.2.

[C] **H. Eliasson**, S. L. I. Olsson, M. Karlsson and P. A. Andrekson, “Mitigation of nonlinear distortion in hybrid Raman/phase-sensitive amplifier links,” *Optics Express*, vol. 24, no. 2, pp. 888-900, 2016.

Related publications and conference contributions by the author not included in the thesis:

[D] **H. Eliasson**, P. Johannsson, H. Sunnerud, M. Westlund, M. Karlsson and P. Andrekson, "Transmitter mask testing for 28 GBaud PM-QPSK," Proceedings European Conference on Optical Communication (ECOC), London, UK, 2013, paper Tu.3.C.2.

[E] C. Lundström, **H. Eliasson**, I. Fatadin, P. Johannisson, P. Andrekson and M. Karlsson, "Mask testing of 28 GBaud 16-QAM transmitters using time-resolved error vector magnitude," Proceedings Signal Processing in Photonic Communications (SPPCom), Boston, USA, 2015, paper SpS4D.3.

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Henrik Eliasson

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Abbreviations

ASIC application-specific integrated circuit

AWG arbitrary waveform generator

CD chromatic dispersion

CDR conjugate data repetition

CMA constant modulus algorithm

CS coherent superposition

CW continuous wave

DAC digital-to-analog converter

DBP digital back-propagation

DCF dispersion compensating fiber

DCM dispersion compensating module

DSP digital signal processing

EDC electronic dispersion compensation

EDFA Erbium-doped fiber amplifier

ENOB effective number of bits

EVM error vector magnitude

FBG fiber Bragg grating

FEC forward error correction

FWM four-wave mixing

GN Gaussian noise

GVD group velocity dispersion

HNLF highly nonlinear fiber

IQM IQ modulator

LO local oscillator

MI mutual information

MSSI mid-span spectral inversion

NF noise figure

NLPS nonlinear phase shift

OFDM orthogonal frequency division multiplexing

OOK on-off keying

OP optical processor

OPC optical phase conjugation

OSNR optical signal-to-noise ratio

PAM pulse-amplitude modulation

PAPR peak-to-average power ratio

PBC polarization beam combiner/splitter

PC phase conjugation

PCTW phase-conjugated twin waves

PDF probability density function

PM polarization multiplexed

PMD polarization mode dispersion

PPLN periodically poled lithium niobate

PSA phase-sensitive amplifier

QAM quadrature amplitude modulation

QPSK quadrature phase-shift keying

RFL Raman fiber laser

RIN relative intensity noise

RPU Raman pump unit

RZ return-to-zero

SBS stimulated Brillouin scattering

SE spectral efficiency

SMF single-mode optical fiber

SNR signal-to-noise ratio

SPM self-phase modulation

SRS stimulated Raman scattering

SSFM split-step Fourier method

WDM wavelength-division multiplexing

Contents

Chapter 1

Introduction

The idea to use glass fibers with a core and cladding of slightly different refractive indices as the transmission medium for optical signals originates from an article by Kao and Hockham published in 1966 [1]. The invention of the optical fiber together with the first demonstration of a functioning laser by Maiman in 1960 [2] and later the semiconductor laser [3, 4] are maybe the most important inventions that still form the technological basis for any fiber-optic communication system. A later invention which also proved important is the Erbium-doped fiber amplifier (EDFA) by Mears et al. in 1987 [5] which allowed for broadband amplification of light removing the need for electrical repeaters in long communication links.

The first fiber-optic communication systems used simple modulation formats like return-to-zero (RZ)-on-off keying (OOK) [6] and inline dispersion compensation with dispersion compensating fibers (DCFs) [7]. Later, systems also made use of wavelength-division multiplexing (WDM) to transmit several channels in one fiber at different wavelengths, something which was made possible by the invention of the EDFA. Modern coherent long-haul links can transmit more than 60 Tb/s over transoceanic distances in a single fiber [8]. These systems use more advanced modulation formats [9, 10] such as polarization multiplexed (PM)-quadrature phase-shift keying (QPSK), PM-16-ary quadrature amplitude modulation (QAM) or even PM-64QAM together with soft-decision forward error correction (FEC) [11] in order to achieve higher spectral efficiency (SE). The link itself contains fiber spans of 50-120 km length with EDFAs between fiber spans in order to recover the fiber loss. In the receiver, the signal is mixed with a local oscillator (LO) laser in a 90 degree hybrid so that both the amplitude and phase of the optical signal can be detected. After detection the signal goes through digital signal processing

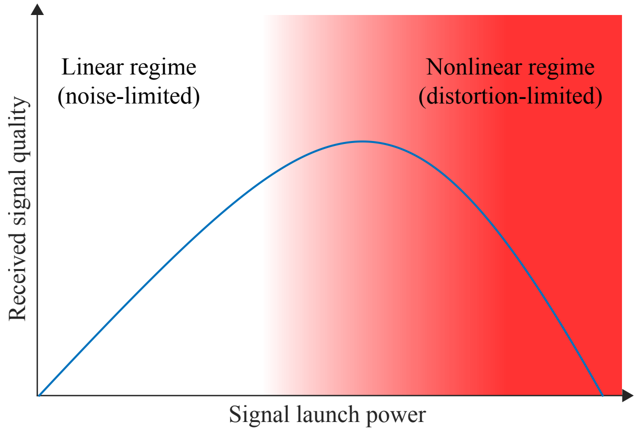

Figure 1.1: Received signal quality after propagation in an optical fiber as a function of signal power launched into the fiber.

(DSP) [12] that can e.g. remove the effect of chromatic dispersion (CD) [13] and track the relative phase drift between the transmitted signal and the LO laser.

On a fundamental level, the factors that limit the amount of information that can be reliably communicated over a fiber-optic communication link are the noise [14] added by the inline optical amplifiers [15] and the nonlinear distortion due to the Kerr effect [16] in the optical fiber itself. In order to reduce the impact of noise, one can increase the power of the light launched into the fiber. As the optical power is increased, the impact of the nonlinear distortion increases and eventually dominates over the noise meaning that the signal quality is made worse by further increasing the signal launch power. This is sometimes referred to as the nonlinear Shannon limit [17, 18]. This behaviour is also illustrated in Fig. 1.1. The two launch power regimes are often referred to as the linear and nonlinear regime. This behaviour at different launch powers leads to there being an optimum launch power that gives the best received signal quality. This optimum launch power lies in an intermediate regime where neither noise nor nonlinear distortion can be neglected, this intermediate regime is sometimes called the pseudo-linear regime [17]. This is in contrast to e.g. wireless communication systems where the transmission medium is air, which has negligible nonlinear properties.

Over the years, many methods have been suggested in order to mitigate the nonlinear distortion due to the Kerr effect at high launch powers. The first method suggested was to use optical solitons in fiber-optic communication systems [19]. A soliton is an optical pulse that forms when the self-phase modulation (SPM) phase shift exactly counterbalances the group velocity dispersion (GVD). This leads to the pulse propagating unaltered in the fiber since the effects of dispersion and the Kerr effect cancel each other. This was a hot research topic in the 1990s but it was eventually found that such systems were not competitive because of e.g. soliton interaction [20] and the effect of Gordon-Haus jitter [21].

The second method suggested for this purpose was optical phase conjugation (OPC) which was suggested by Fisher et al. in 1983 [22]. In that paper, it was shown analytically, that by performing an OPC operation on the signal at the center point of a fiber-optic link, it is possible to undo the nonlinear distortion generated in the first half of the link in the second half of the link. This method was later experimentally demonstrated in [23]. In recent years there has been a renewed interest in the topic with the investigation of OPC links employing distributed Raman amplification and/or several inline OPC devices [24–26].

Another, more recently suggested method for mitigation of nonlinear distortion is digital back-propagation (DBP). This method was first suggested by Essiambre et al. in 2005 [27]. The idea behind DBP is to propagate the received optical field backward through the fiber by a brute force numerical solution of the nonlinear partial differential equation describing light propagation in an optical fiber. DBP has been a "hot" research topic ever since its introduction due to the potential for significant performance gains [28]. There are two big problems associated with the use of DBP, first, the complexity of the numerical calculations required is immense [29], second, in order to back-propagate a whole WDM spectrum we need to detect all channels simultaneously. In recent years there has been progress on the topic in several directions, e.g. with pre-distortion of several WDM channels simultaneously at the transmitter side [30] or with DBP of several WDM channels with the use of a spectrally sliced coherent receiver [31].

A fourth method that was proposed recently by Liu et al. [32] is phase-conjugated twin waves (PCTW). The idea behind the concept is to transmit a phase-conjugated copy of the signal alongside the signal on the orthogonal polarization. In the receiver DSP, the two fields are coherently superposed leading to mitigation of nonlinear distortion if certain conditions are fulfilled, more on this in section 4.2. This concept was also generalized to utilize other signalling dimensions than polarization for the phase-conjugated copy in [33]. In [34], a modification of PCTW in order to improve SE was investigated and in [35], a PCTW subcarrier coding scheme was investigated in an orthogonal frequency division multiplexing (OFDM) system. A scheme with similarities to PCTW that transmitted the phase-conjugated copy on a separate wavelength followed by coherent superposition (CS) in DSP was investigated in [36].

The fifth method which will be discussed here is the use of two-mode phase-sensitive amplifiers (PSAs) that has been shown to offer improved performance in a nonlinear transmission regime [37] as well as in a linear regime because of their 0 dB quantum limited noise figure (NF) [38]. There are similarities between the use of two-mode PSAs and the concept of PCTW since in both cases, a phase-conjugated copy is transmitted alongside the signal. An important difference between PCTW and a PSA link is that the CS operation is performed all-optically in a PSA link, opening up the possibility to perform the CS inline at each amplifier site.

All of these methods have been experimentally demonstrated and shown to improve performance in the presence of nonlinear distortion. Still to the best of my knowledge, there are no commercial fiber-optic communication systems in place which employ any of these technologies. The reason for these methods not being used in commercial communication systems are different for each method but in general one can say that the performance gain does not yet outweigh the increased system complexity. In the case of OPC, the optical complexity of the link itself is significantly increased because of the inline OPC devices. If using DBP, the required computation power at the receiver side is too high [28] to be feasible with todays semiconductor technology even though this could change as application-specific integrated circuit (ASIC) technology improves. In the case of PCTW it is different since it is not the question of complexity which is stopping the method from being used. Instead, it is the fact that the SE of the system is reduced by 50 % due to the transmission of the phase-conjugated copy. In most cases it is not acceptable to reduce SE by 50 % compared to e.g. PM-QPSK. The research on long-haul links using PSAs is still in it's early stages but it is clear that system complexity is an issue in such systems as well. In order for all-optical methods for mitigation of nonlinear distortion like OPC or the use of PSAs to become attractive in commercial systems it is important to reduce cost/complexity and optimize the mitigation of nonlinear distortion.

1.1 This Thesis

This thesis focusses on understanding and optimizing the methods that mitigate nonlinear distortion by transmitting a conjugated copy of the signal alongside the signal followed by CS in receiver DSP or all-optically in a PSA. In Paper A, we investigate a certain time-domain implementation of generalized PCTW which we call conjugate data repetition (CDR). We provide a novel time-domain perturbation analysis showing why it is possible to mitigate nonlinear distortion with CDR and perform numerical simulations comparing systems using CDR to systems using conventional PCTW. The two following papers investigate different aspects of the mitigation of nonlinear distortion in PSA links. In Paper B, we perform an experimental comparison between the performance of a single-span system that performs the CS operation in receiver DSP and a system that performs the CS all-optically using a PSA. It is found that the two ways of coherently superposing the optical fields are essentially equivalent in terms of performance in a nonlinear transmission regime. In Paper C, we investigate numerically the impact of the span power map in single-span and multi-span PSA links. The span power map is modified using distributed Raman amplification and it is shown that by optimizing both the span dispersion map and power map, significant increases in transmission distance of a multi-span PSA system can be achieved.

Thesis Outline

In Chapter 2, a summary of the theoretical and numerical models of light propagation in an optical fiber is presented. Chapter 3 begins with a historic overview of the research topic of phase conjugation in fiber optics followed by a mathematical description and a summary of the different methods that can be used to phase conjugate an optical signal. In Chapter 4, some of the different ways of using phase conjugation in order to mitigate nonlinear distortion are explained in more detail and they are then compared to each other to highlight the weaknesses and strengths of the different methods.

Chapter 2

Light Propagation in Fibers

The physical effects that alter the optical field during propagation in an optical fiber are commonly divided into linear effects such as attenuation and dispersion, and nonlinear effects due to the Kerr effect as well as Raman and Brillouin Scattering. At low optical power, the nonlinear effects can be neglected and the evolution of the field is well described by linear effects. As the optical power of the field is increased, the nonlinear effects become stronger and will at high enough powers distort the signal beyond recognition. Linear and nonlinear effects will be discussed separately below.

2.1 Fiber Attenuation and Dispersion

As a lightwave propagates in an optical fiber, power is lost, mainly due to material absorption and Rayleigh scattering. The evolution of the optical power $P(z)$ in the presence of linear attenuation is governed by Beer's law and can be written as

$$P(z) = P(0)\exp(-\alpha z), \tag{2.1}$$

where α is the attenuation coefficient. For modern optical fibers designed to achieve low attenuation, α is typically 0.17 dB/km at 1550 nm [39], meaning that less than 4 % of the light is lost per km. Associated with the attenuation, the effective length is defined as

$$L_{\text{eff}} = \frac{1}{\alpha}(1 - \exp(-\alpha L)), \tag{2.2}$$

where L is the length of the fiber. For a long standard single-mode optical fiber (SMF), the effective length is approximately 21 km. The physical meaning of the effective length is the length over which the accumulated nonlinear phase shift

$$\phi_{\mathrm{NL}}(z) = \gamma \int_0^z P(z')dz', \tag{2.3}$$

where γ is the nonlinear coefficient [1/(W km)], equals $\phi_{\mathrm{NL}}(L) = \gamma L_{\mathrm{eff}} P(0)$, i.e. the length over which a constant input optical power gives the same ϕ_{NL}.

Light of different wavelength propagate with different group velocity in an optical fiber. This is referred to as group velocity dispersion (GVD) or chromatic dispersion (CD). Linear propagation of an optical field is described in frequency domain by

$$\tilde{E}(z,\omega) = \tilde{E}(0,\omega)\exp(i\beta(\omega)z), \tag{2.4}$$

where $\beta(\omega)$ is the propagation constant. The real part of β describes dispersive properties while the imaginary part describes attenuation. In order to analyze the frequency dependence of the propagation constant, $\beta(\omega)$ is Taylor expanded around the carrier frequency ω_0 according to

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \frac{\beta_3}{6}(\omega - \omega_0)^3 + ..., \tag{2.5}$$

where β_1 is the group velocity at the carrier frequency, β_2 the GVD coefficient and β_3 describes the frequency dependence of the GVD. The GVD coefficient is given in units of [ps^2/km] and is related to the dispersion parameter $\mathrm{D} = -2\pi c\beta_2/\lambda^2$ which is given is units of [ps/(nm km)].

Yet another linear propagation effect is polarization mode dispersion (PMD) which is due to the randomly varying birefringence of the optical fiber. The random birefringence of subsequent segments of an optical fiber is due to non-uniformities in fabrication, e.g. deformed core geometry, fiber stress or bending. This leads to pulse spreading through frequency dependent polarization rotations during propagation. This effect is often neglected in computer models of nonlinear propagation of light in a fiber, not because it is not important but because of the increased complexity due to having to deal with statistical analysis on a large collection of random fibers.

2.2 Nonlinear Effects

The Kerr effect, discovered by scottish physicist John Kerr in 1875 [40, 41], is a change in the refractive index of any material when an electromagnetic field is applied. At a more fundamental level this is due to the nonlinear polarization response of the material. This effect is commonly described by introducing a power-dependent part n_2 of the refractive index

$$n(\omega, P) = n_0(\omega) + n_2(P/A_{\mathrm{eff}}) \tag{2.6}$$

where n_2 is the nonlinear-index coefficient and A_{eff} is the effective area of the fiber mode. The strength of the nonlinearity in a fiber is specified by the nonlinearity coefficient

$$\gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}}, \tag{2.7}$$

where λ is the wavelength of the light. Nonlinear propagation of light in an optical fiber is commonly modeled using the two coupled equations of the Manakov model [42]

$$\frac{\partial E_{\text{X}}}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 E_{\text{X}}}{\partial t^2} + \frac{g(z)-\alpha(z)}{2}E_{\text{X}} + i\gamma(|E_{\text{X}}|^2 + |E_{\text{Y}}|^2)E_{\text{X}}, \tag{2.8}$$

$$\frac{\partial E_{\text{Y}}}{\partial z} = \underbrace{-\frac{i\beta_2}{2}\frac{\partial^2 E_{\text{Y}}}{\partial t^2}}_{\text{dispersion}} \underbrace{+\frac{g(z)-\alpha(z)}{2}E_{\text{Y}}}_{\text{local loss/gain}} + \underbrace{i\gamma(|E_{\text{X}}|^2 + |E_{\text{Y}}|^2)E_{\text{Y}}}_{\text{nonlinearities}}, \tag{2.9}$$

where E_{X} and E_{Y} are the electrical fields of two orthogonal polarization states and $g(z)$ and $\alpha(z)$ are the local gain and attenuation. In these equations higher-order dispersive effects due to β_3 as well as the effect of Raman and Brillouin scattering have been neglected. One can however point out that e.g. the gain from distributed Raman amplification still can be modeled by Eqs. (2.8) and (2.9) through the parameter $g(z)$. The last term in the equations is what gives rise to nonlinear effects.

2.3 Distributed Raman Amplification

Raman scattering, named after physicist Chandrasekhara Raman [43] is an inelastic scattering process meaning that the scattered light has a different frequency than the incoming light. An important application of Raman scattering in fiber-optic communication systems is distributed Raman amplification [44]. In a fiber span with distributed Raman amplification, the signal propagates in the transmission fiber together with one or more pump waves and power is transferred from the pump waves to the signal through stimulated Raman scattering (SRS). In this manner it is possible to improve noise properties with the use of distributed Raman amplification. For a 100 km span, the span NF can be improved by 6 dB compared to a EDFA-amplified span through the use of backward pumped Raman amplification [45, Table 3.1]. The pump waves can be copropagating, counterpropagating or both with two or more pump waves. An illustration of a bidirectionally pumped Raman-amplified span is shown in Fig. 2.1. The Raman gain that the signal experiences depends on e.g. the wavelength separation between the signal and the pump and the pump power. In regular SMF, the peak in the Raman gain spectrum is located at

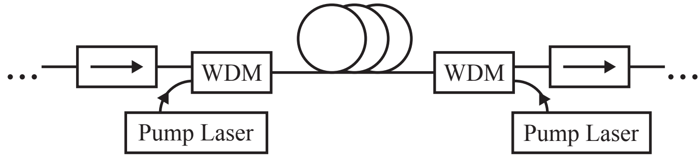

Figure 2.1: Illustration of a bi-directionally pumped Raman-amplified span. The signal is coupled together with the pump into the fiber span with a WDM-coupler. At the end of the span, a backward travelling pump is coupled into the fiber span with a second WDM-coupler. Optical isolators are placed before and after each span to stop Rayleigh backscattered signal light from accumulating.

frequency shifts around 13.2 THz, meaning that in order to amplify a signal at 1550 nm, the pump should be located around 1450 nm [45, Chapter 1.2].

The reason why distributed Raman amplification is interesting in the context of mitigation of nonlinear fiber distortion with all optical methods, e.g. using OPC or PSAs, is because it is the most straightforward way of manipulating the span power map. In the case of OPC, it has long been known that in order to optimize the mitigation of nonlinear distortion, a symmetric power map around the OPC point is needed [22]. In the case of PSAs, it was shown in Paper C that the inline mitigation of nonlinear distortion through CS in PSAs becomes more efficient by also using distributed Raman amplification.

There are several ways of using distributed Raman amplification, the least complex is to have a single backward pump in each span. In this way it is possible to have local gain in the latter parts of each span. As a consequence, the noise generated in each span is reduced. If the purpose is to achieve a highly symmetric or flat power map, higher-order pumps propagating in both directions are needed [46]. Higher-order pumping refers to the use of pumps that amplify the first-order pump, leading to the first-order Raman pump experiencing distributed gain and providing a more evenly distributed gain to the signal. In [47], second-order bidirectional pumping was used to realize a 80 km Raman amplified span with power excursions of only ± 0.4 dB. Second-order pumps can also be used in a Raman fiber laser (RFL) based scheme where fiber Bragg gratings (FBGs) with high reflectivity at the first-order pump wavelength are placed at both ends of each span leading to a first-order pump being generated in a manner similar to lasing [48]. Such RFL amplification was demonstrated in [49] where it was found that the system was limited by relative intensity noise (RIN) transferred from the copropagating pumps.

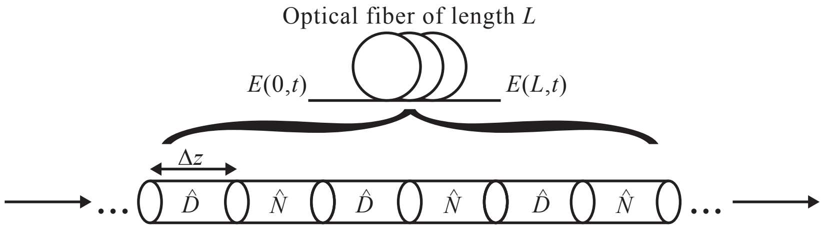

Figure 2.2: Illustration of the split-step Fourier method (SSFM), the fiber length L is divided into several segments of length Δz and the field $E(0,t)$ is propagated alternating between linear steps according to Eq. (2.10) and nonlinear steps according to Eq. (2.11) in order to find the solution $E(L,t)$.

2.4 Split-step Fourier Method

The most commonly used method for computer modelling of light propagation in a fiber is the SSFM [50]. The basic idea behind the method is to divide each step ∂z of Eqs. (2.8) and (2.9) into two parts, one step propagating with linear effects, dispersion and attenuation and one nonlinear step applying an SPM rotation. The linear step is described by the operator [45, Chapter 2.4.1]

$$\hat{D} = -\frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2} + \frac{g(z) - \alpha(z)}{2} \tag{2.10}$$

while the nonlinear step is described by the operator

$$\hat{N} = i\gamma(|E_{\mathrm{X}}|^2 + |E_{\mathrm{Y}}|^2). \tag{2.11}$$

The propagation is then performed in steps of length Δz according to

$$E_{\mathrm{X,Y}}(z + \Delta z, t) \approx \exp(\Delta z \hat{D}) \exp(\Delta z \hat{N}) E_{\mathrm{X,Y}}(z, t), \tag{2.12}$$

where the two operators are evaluated consecutively, the linear operator in frequency domain and the nonlinear operator in time-domain. Of course, in reality the linear and nonlinear effects occur simultaneously and not in a consecutive manner. Because of this it is important to choose the step-size Δz carefully in order to obtain an accurate approximation of Eqs. (2.8) and (2.9) using the SSFM [51]. The idea behind the SSFM is illustrated in Fig. 2.2.

In this thesis, the main application of the SSFM is to evaluate the optical field after propagation in an optical fiber with the purpose of estimating the performance of different schemes for nonlinear distortion mitigation. Another important application of the SSFM that should be mentioned is DBP [28].

Although there are other ways of propagating the received optical field backward through the fiber, DBP based on SSFM is often considered as a reference when benchmarking alternative methods that can be based on e.g. a perturbation approach [52], Volterra series [53, 54] or stochastic DBP [55]. As was mentioned in the introduction, the main problem with using the SSFM for the purpose of DBP is the computational complexity and both of these alternative approaches are used in an attempt to reduce complexity.

2.5 Perturbation Theory

Perturbation theory is a useful tool for theoretical analysis of nonlinear distortion in the weakly nonlinear regime. Perturbation theory is based on the assumption that

$$|\delta_{NL}| \ll |E_S|, \tag{2.13}$$

where $|\delta_{NL}|$ is the amplitude of the nonlinear distortion and $|E_S|$ is the amplitude of the signal. Under this assumption, the nonlinear distortion δ_{NL} is generated by a source field E_S propagating only taking β_2 and α into account. These assumptions also make it possible to do analytical calculations of the nonlinear distortion generated by e.g. a Gaussian pulse train [56]. Perturbation theory can be used e.g. to make a first-order theory for the mitigation of nonlinear distortion for some of the methods discussed in this thesis. This has been done for the case of PCTW in [32] with a frequency domain perturbation analysis and for the case of CDR in Paper A with a time domain perturbation analysis. In the frequency domain analysis, the spectrum of the nonlinear distortion $\tilde{\delta}_{NL}(\omega)$ is calculated, while in the time domain analysis, the time domain representation $\delta_{NL}(t)$ is calculated.

In section 4.4, the frequency domain perturbation analysis will be used to derive the nonlinear distortion mitigation properties of a single-span PSA link. An important aspect to point out about these derivations is that since they are based on a perturbation theory, they are formally not valid in a highly nonlinear regime, i.e. when $|\delta_{NL}|$ cannot be neglected compared to $|E_S|$. The derivations can not safely predict what will happen and how well nonlinear distortion will be mitigated in a strongly nonlinear regime. The best approach for evaluating performance in a strongly nonlinear regime is to perform numerical simulations using the SSFM or by performing experiments.

Another important application of perturbation theory in recent years is the development of the Gaussian noise (GN) model [57]. The basic idea behind the GN model is to apply perturbation theory in order to find the nonlinear distortion generated in a highly dispersive regime. The conclusion of that work is that under certain assumptions, reasonably valid for modern coherent fiber-optic communication systems, the nonlinear distortion will appear as random additive white Gaussian noise, even though the nonlinear distortion is deterministic.

Now, a solution of Eq. (2.8) based on a single-polarization frequency domain perturbation analysis will be presented. This solution will be used in section 4.4 in the derivation of the mitigation of nonlinear distortion in a single-span PSA link. We start by expanding Eq. (2.4) with a first-order nonlinear perturbation term $\tilde{u}^{(1)}(z,\omega)$ according to [58]

$$\tilde{E}(z,\omega) = \sqrt{P_0}\exp\left(\frac{G(z)+iC(z)\omega^2}{2}\right)\left[\tilde{u}^{(0)}(\omega)+\tilde{u}^{(1)}(z,\omega)\right], \tag{2.14}$$

where P_0 is the average launch power and $\tilde{u}^{(0)}(\omega)$ is the transmitted signal spectrum normalized such that $< |u^{(0)}(t)|^2 >= 1$. The logarithmic power evolution $G(z)$ is given by

$$G(z) = \int_0^z [g(z') - \alpha(z')]dz', \tag{2.15}$$

where $\alpha(z)$ is the local attenuation and $g(z)$ is the local gain which can be achieved by e.g. Raman amplification. The expression for the accumulated dispersion is

$$C(z) = \int_0^z \beta_2(z')dz'. \tag{2.16}$$

The expression for the first-order perturbation term is then [58]

$$\begin{aligned}\tilde{u}^{(1)}(L,\omega) = i\gamma P_0 L_{\mathrm{eff}} &\int_{-\infty}^{\infty} \mathrm{d}\omega_1 \int_{-\infty}^{\infty} \mathrm{d}\omega_2 \eta(\omega_1\omega_2) \\ &\times \tilde{u}^{(0)}(\omega+\omega_1)\tilde{u}^{(0)}(\omega+\omega_2)\tilde{u}^{(0)*}(\omega+\omega_1+\omega_2),\end{aligned} \tag{2.17}$$

where $\eta(\omega_1\omega_2)$ is the nonlinear transfer function [58]

$$\eta(\omega_1\omega_2) = \frac{1}{L_{\mathrm{eff}}}\int_0^L \exp[G(z) - i\omega_1\omega_2 C(z)]dz, \tag{2.18}$$

which fully defines the nonlinear properties of the link in the first-order approximation. The expression for the nonlinear perturbation, Eq. (2.17), is very general and can be used for many purposes other than deriving the mitigation of nonlinear distortion in PSA links which will be done in section 4.4. Eq. (2.17) is e.g. also the starting point in the derivation of the GN model [57]. In order to have a more intuitive understanding of Eq. (2.17), one can interpret the double integral over ω_1 and ω_2 as the calculation of all four-wave mixing (FWM) terms for which the FWM product end up at ω.

Chapter 3

Phase Conjugation of Lightwaves

The first suggested application of optical phase conjugation in fiber-optic communication systems was dispersion compensation. In 1979, Yariv et al. showed analytically that by performing OPC using FWM on a pulse in the center point of a fiber-optic link, the effects of dispersion due to β_2 in the first half of the link can be reversed and the pulse is returned to its original shape after propagation over the second half of the link [59]. The process of performing OPC once at the centerpoint of a link is sometimes also referred to as mid-span spectral inversion (MSSI). This was long before the age of coherent detection with CD compensation in DSP and the issue of how to perform dispersion compensation all-optically was an open question and OPC was considered as a possible option. The next big step in the study of OPC in fiber optics came in 1983 when Fisher et al. showed that when performing MSSI, not only are the effects due to CD cancelled, but it is also possible to compensate for the combined effect of CD and SPM due to the Kerr effect [22]. A brief theoretical explanation of the reversal of GVD and SPM will be provided in section 4.1. Since then, the study of OPC in fiber optics has been an ongoing topic, being popular in the 1990s, with experimental studies of the polarization dependence of OPC devices [60, 61], CD compensation [62] and numerical studies of nonlinearity mitigation [63, 64]. In the 2000s, OPC in WDM systems was studied numerically [65] and in experiments [66]. In recent years, fiber-optic communication systems using both distributed Raman amplification and OPC [25, 26, 67] as well as systems employing several inline OPC devices [24, 26, 68] have received attention, these topics will be discussed more in section 4.1.

Another way that phase conjugation can be used in order to mitigate nonlinear distortion is to generate a phase-conjugated copy of the signal wave at the transmitter and transmit both waves alongside each other. By performing

CS of the waves it is then possible to mitigate nonlinear distortion due to the Kerr effect [32]. This method has received attention in recent years with demonstrations of mitigation of nonlinear distortion in PSA links [37] in 2012 and in PCTW systems [32] in 2013. These topics will be discussed in sections 4.2, 4.3 and 4.4.

One can also mention that the process of phase conjugation (PC) is not unique for lightwaves. It is also possible to perform PC on e.g. acoustic waves [69] or with electromagnetic waves in the GHz range [70]. In applications of PC outside of fiber optics, the purpose is often to achieve spatial self-focussing [71], where e.g. an electromagnetic or acoustic wave is focussed back on its source after reflection in a phase conjugating mirror. In the context of lightwaves, the process of PC is referred to as OPC.

3.1 Mathematical Description

The process of PC can be described mathematically in either the time domain or in the frequency domain. In the time domain, phase conjugation is a complex conjugation of the electrical field in phasor notation which will be notated as

$$E(t) \xrightarrow{\hat{\mathrm{PC}}} E^*(t). \tag{3.1}$$

The PC process in frequency domain will then be the Fourier transform of both sides which is

$$\tilde{E}(\omega) \xrightarrow{\hat{\mathrm{PC}}} \tilde{E}^*(-\omega). \tag{3.2}$$

In Eq. (3.2) we see the reason why OPC sometimes is referred to as MSSI, since ω changes sign. The PC process in time and frequency domain is illustrated in Fig. 3.1. In time domain the signal is mirrored with respect to the imaginary axis, flipping the sign of the imaginary part. In frequency domain, the Fourier transform of the signal is inverted with respect to ω and complex conjugated. The inversion of the signal in frequency domain provides an intuitive understanding of why it is possible to reverse the effects of β_2 by performing MSSI since the spectral components of the transmitted signal with higher group velocity will have a lower group velocity after OPC. With the same reasoning we also understand why the effects of odd orders of dispersion, e.g. β_3, cannot be reversed.

3.2 Methods for Phase Conjugation of Light

There are several methods that can be used to generate a phase-conjugated lightwave. The methods range from the use of nonlinear effects such as FWM

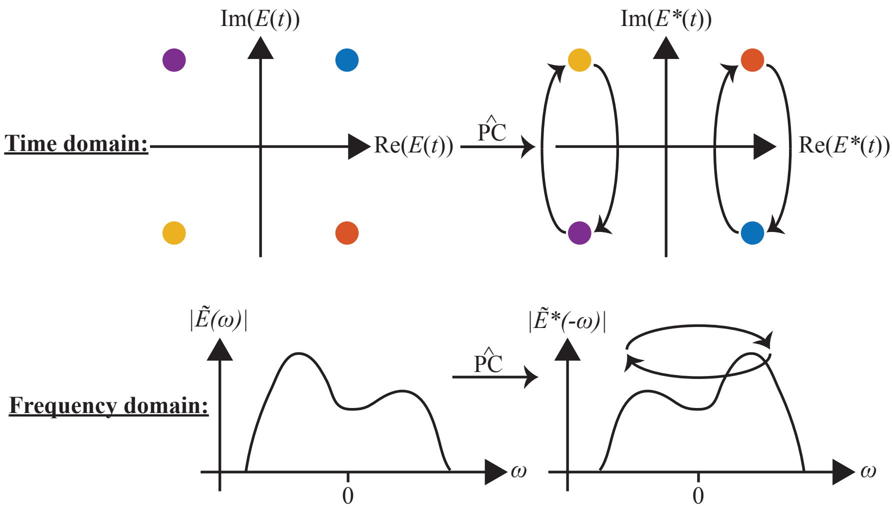

Figure 3.1: An illustration of PC of a QPSK constellation diagram in the time domain (top row) and PC of an arbitrarily shaped asymmetric spectrum in the frequency domain (bottom row).

and stimulated Brillouin scattering (SBS) to electrical generation of a conjugated waveform in transmitter DSP which is then modulated on a continuous wave (CW) using a digital-to-analog converter (DAC)-driven IQ-modulator. These different methods and their strengths and weaknesses for different applications will now be discussed.

3.2.1 Methods Based on Fiber Nonlinearities

One common method to generate the phase-conjugated copy in fiber optics is through FWM between a signal wave and a high-power pump wave in a highly nonlinear fiber (HNLF) [72]. When using this method the phase-conjugated wave propagates in the same direction as the original wave and the carrier frequencies of the three waves fulfill $\omega_S + \omega_C = 2\omega_P$ where ω_S, ω_C and ω_P are the frequencies of the signal wave, phase-conjugated wave and pump wave, respectively. An important property of the signal and phase-conjugated waves generated through FWM is that they are phase-locked. Phase-locked waves are a requirement if the waves are to be amplified by a PSA with stable gain. A drawback of this method is that the only way of applying dispersion pre-compensation on both the signal and phase-conjugated wave is all-optically using e.g. DCFs or FBGs, limiting the amount of dispersion pre-compensation

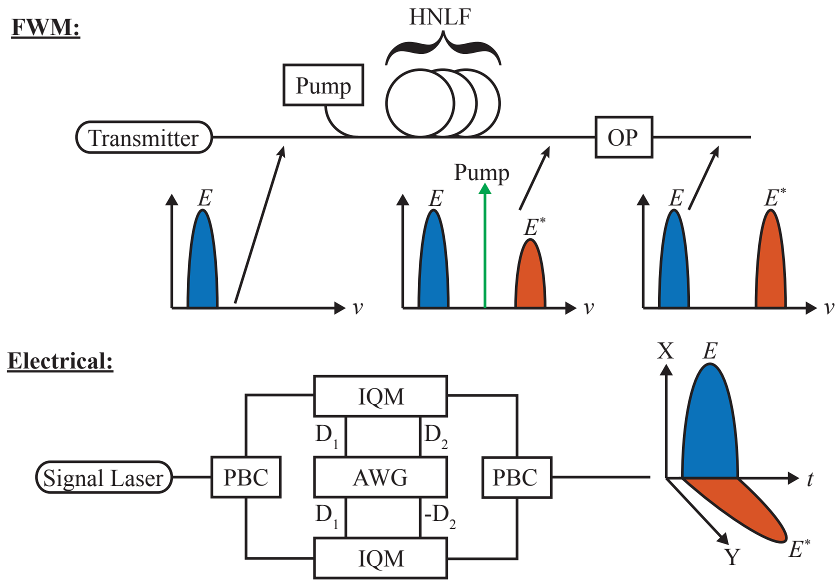

Figure 3.2: An illustration of generation of a phase-conjugated copy using FWM (top row) and electrical generation of a phase-conjugated copy (bottom row). For the electrical case it is assumed that no dispersion pre-compensation is applied. D_1 and D_2 are the electrical driving signals to the IQ modulators. Abbreviations denote optical processor (OP), polarization beam combiner/splitter (PBC), arbitrary waveform generator (AWG), IQ modulator (IQM)

that can realistically be applied. Another way of saying this is that the dispersion operator $\hat{D}$ and phase conjugation operator $\hat{PC}$ do not commute, i.e. changing the order that they are applied also changes the resulting output. This makes FWM-based methods incompatible with an antisymmetric dispersion map in a long-haul link, which is desired e.g. in a PCTW system, see section 4.2. An important advantage of FWM-based phase conjugation is that it makes it possible to generate phase-conjugated copies of e.g. WDM signals with large total bandwidth. This was done using a polarization-insensitive fiber-optic parametric amplifier in [73]. An illustration of the generation of a phase-conjugated copy or idler using FWM is illustrated on the top row in Fig. 3.2.

When discussing the generation of a phase-conjugated copy using FWM in a HNLF in this thesis, it will be assumed that there is one pump and one co-polarized signal present at the input to the HNLF. It is also possible to use dual-pump configurations where the two pumps are separated in frequency and on orthogonal polarization states as in [74, 75]. An in-depth discussion of such configurations is outside the scope of this thesis.

Other nonlinear effects such as e.g. SBS and SRS can also be used to perform phase conjugation [76, Chapter 7]. Such methods are often used in other fields than fiber optics but can not be considered as practical in the context of fiber optical communication with the purpose of phase conjugating a signal at high symbol rate. Such methods will not be discussed at depth in this thesis.

3.2.2 Electrical Phase Conjugation

A third method of generating a phase-conjugated wave is the one recently demonstrated in [32] where the signal and phase-conjugated waves were generated by modulating CW carriers with IQ-modulators driven by software-controlled DACs or an AWG. This method is illustrated on the bottom row in Fig. 3.2. The benefit of this method is that it opens up the possibility to simultaneously apply dispersion pre-compensation for thousands of kilometers through transmitter side electronic dispersion compensation (EDC). The large amount of dispersion pre-compensation is needed in order to achieve an anti-symmetric dispersion map over a long link which is required to optimize the mitigation of nonlinear distortion in PCTW systems.

This approach has its drawbacks as well, one of them being that the peak-to-average power ratio (PAPR) of a signal increases with the amount of dispersion pre-compensation applied in the transmitter DSP. Generating electrical drive signals with a high PAPR requires a large effective number of bits (ENOB) in the DAC in order to have good signal fidelity. Another potential drawback of generation of a phase-conjugated wave in a software-defined transmitter is that the waves are not necessarily phase-locked. If e.g. two waves at different wavelengths are required to be phase-locked this can be solved by modulating different lines of e.g. an electro-optic comb.

OPC can also be performed inline by performing coherent detection of the signal followed by inversion of the sign of the complex part of the signal and then re-modulating the phase-conjugated signal on a new carrier using an IQ-modulator. This was experimentally demonstrated in [77] and also later investigated in [78]. An advantage of this method is that it allows for free control of the signal wavelength after OPC by tuning the CW laser used in the OPC device. A downside of electro-optical OPC is that the bandwidth of the phase conjugation is limited by the bandwidth of the electrical components, i.e. photodetectors, electrical amplifiers and IQ-modulators making it challenging to perform OPC on several WDM channels with a single device.

Chapter 4

Mitigating Nonlinear Distortion using Phase Conjugation

There are several different ways that phase conjugation can be used to mitigate nonlinear distortion due to the Kerr effect [79]. The method which was proposed first was OPC or MSSI in 1983 [22]. More recently, mitigation of nonlinear distortion using PCTW [32], CDR [Paper A] and PSAs [[37], Paper B, Paper C] has been proposed. These three methods transmit a conjugated copy alongside the signal in different ways, mitigating nonlinear distortion at the expense of reduced SE. In this chapter we will first give a broad overview of the different methods followed by a more in-depth analysis of each separately. All of the methods which will be covered are illustrated in a simplified manner in Fig. 4.1. The first method is OPC or MSSI where the signal is first propagated over N/2 fiber spans, where N is the total number of spans. At the center point of the link, the signal is phase conjugated leading to reversal of nonlinear distortion in the second half of the link. The second method is PCTW where a phase-conjugated copy of the signal is transmitted on the orthogonal polarization. CS is then performed in receiver DSP leading to mitigation of nonlinear distortion due to anticorrelated nonlinear distortion on the signal and conjugate wave. The third method is CDR which means transmitting the conjugated copy in different time slots than the signal followed by CS of the the two pulse trains in receiver DSP. The last method is mitigation of nonlinear distortion using non-degenerate two-mode PSAs. In this case, a phase-conjugated copy of the signal wave is transmitted on a separate wavelength and the CS operation is performed all-optically in each inline amplifier.

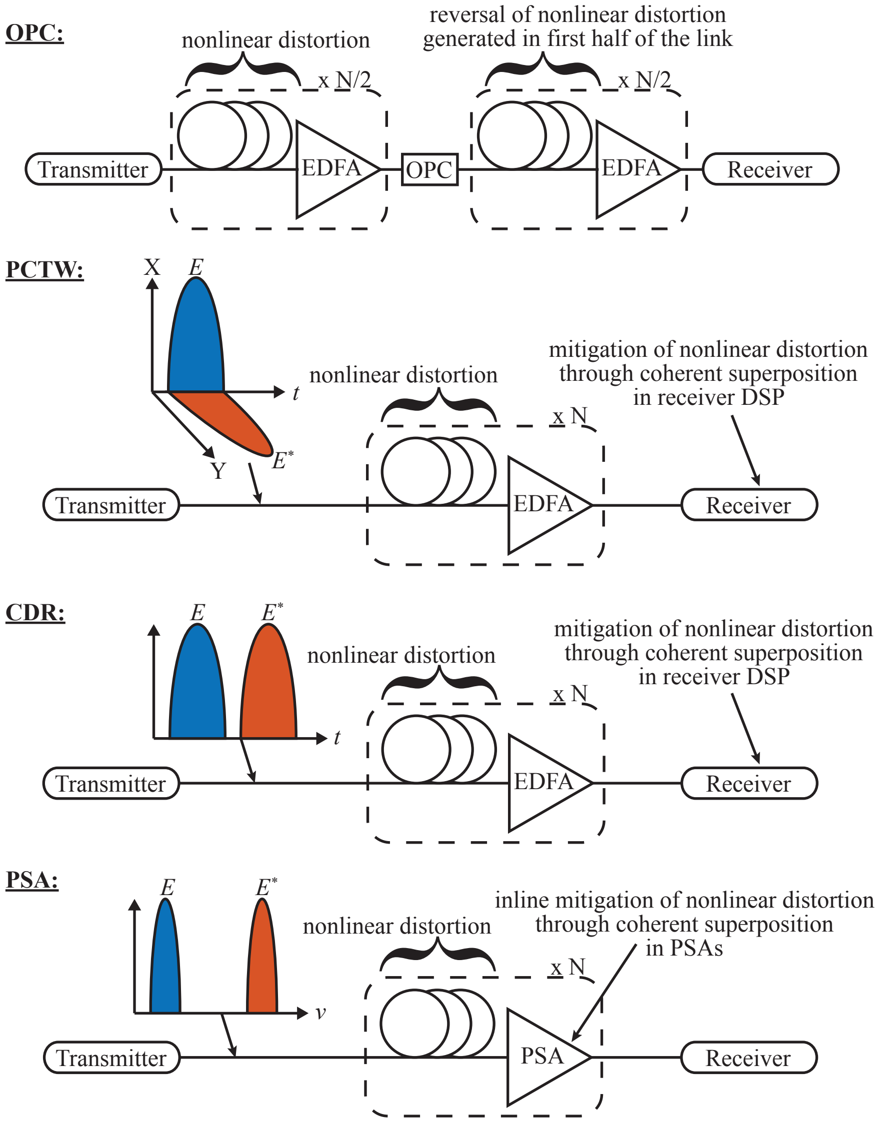

Figure 4.1: A schematic illustration of the different methods for mitigation of nonlinear distortion that are covered in the thesis. The techniques are optical phase conjugation (OPC) (first row), phase-conjugated twin waves (PCTW) (second row), conjugate data repetition (CDR) (third row, investigated in paper A), phase-sensitive amplifiers (PSAs) (fourth row, investigated in papers B and C).

4.1 Optical Phase Conjugation

Optical phase conjugation for the purpose of mitigating nonlinear distortion due to the Kerr effect was first suggested by Fisher et al. in [22]. In that paper it was shown with a theoretical and numerical analysis that the combined effect of GVD and SPM on a pulse propagating in a dispersive nonlinear medium, e.g. an optical fiber, could be reversed by performing optical phase conjugation on the pulse followed by retraversal of the medium. In order to understand this phenomenon we look at the propagation equation for the waveform before and after OPC. The propagation equation for a single polarization signal, i.e. setting $E_{\mathrm{Y}} = 0$ in Eq. (2.8) is

$$\frac{\partial E_{\mathrm{X}}}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 E_{\mathrm{X}}}{\partial t^2} + \frac{g(z)-\alpha(z)}{2}E_{\mathrm{X}} + i\gamma|E_{\mathrm{X}}|^2 E_{\mathrm{X}}. \tag{4.1}$$

By complex conjugating the propagation equation we get the propagation equation for the conjugated waveform

$$\frac{\partial E_{\mathrm{X}}^*}{\partial z} = \frac{i\beta_2}{2}\frac{\partial^2 E_{\mathrm{X}}^*}{\partial t^2} + \frac{g(z)-\alpha(z)}{2}E_{\mathrm{X}}^* - i\gamma|E_{\mathrm{X}}^*|^2 E_{\mathrm{X}}^*. \tag{4.2}$$

We see that the signs of the GVD and SPM terms are changed, meaning that the effects of both GVD and SPM can be reversed.

The term containing α is real-valued and thus does not change sign. In order for GVD and SPM to be reversed perfectly, we need a transparent fiber with $g(z)-\alpha(z) = 0$ or alternatively a power map which is symmetrical around the point where the OPC is performed. In practice we cannot have lossless fibers, however it is possible to have a power map with a higher degree of symmetry around the OPC point by means of e.g. distributed Raman amplification. In recent years there has been a renewed interest in OPC, to a large extent sparked by investigations of Raman-amplified OPC systems. In [67], the performance of long-haul systems employing different Raman amplification schemes in conjunction with mid-link OPC was studied numerically. In recent years, multi-span links employing both distributed Raman amplification and OPC have been demonstrated experimentally [25, 26]. A detailed numerical study of different Raman amplifier configurations in OPC systems was performed in [80]. A RFL-based approach with FBG reflectors at both ends of each span together with second-order Raman pumps was demonstrated in an OPC system in [81].

Another important limiting factor for the efficiency of the mitigation of nonlinear distortion in OPC systems is PMD. To the best of my knowledge, no comprehensive study has been published on this topic but in [82] some preliminary results were presented. It was found that PMD is an important limiting factor and that the impact on the efficiency of the mitigation of nonlinear distortion can be modelled as a bandwidth limitation of the nonlinear compensation.

Another recent area of research is systems that perform the OPC operation several times along the transmission path [68]. In [24] it was shown with a semi-analytic approach that by using $\mathrm{N_{OPC}}$ phase conjugators in the transmission path, the signal-to-noise ratio (SNR) can be improved compared to a system performing ideal DBP by

$$\Delta\mathrm{SNR} = \sqrt{\mathrm{N_{OPC}} + 1}, \tag{4.3}$$

which becomes a significant increase with many inline OPC devices. In a numerically modelled 9,600 km link with 119 inline OPC devices, this was shown to improve the SNR compared to a system employing ideal DBP by approximately 10 dB [24] due to the suppression of parametricall amplified noise. The approach of using several inline OPC devices in many ways resemble the approach of using inline PSAs which will be covered in section 4.4. One important difference compared to a system employing inline PSAs is that a system employing multiple inline OPC devices does not per definition require that we sacrifice 50 % SE, however this is often still the case since in many experimental implementations, the phase conjugation also shifts the frequency of the signal so that the signal occupies different wavelengths before and after OPC.

4.2 Phase-Conjugated Twin Waves

The concept of PCTW was introduced by Liu et al. in 2013 [32] and is illustrated in a simplified manner on row two in Fig. 4.1. In that paper they used a frequency domain perturbation analysis to show that the nonlinear distortion on a signal and its conjugate copy transmitted on the orthogonal polarization are anticorrelated assuming a symmetric power map and an antisymmetric dispersion map. The conclusion from the analysis was also confirmed experimentally in a recirculating loop experiment. In that experiment, the antisymmetric dispersion map was performed by applying EDC in transmitter and receiver DSP. CS was then performed in receiver DSP, leading to the cancellation of nonlinear distortion to first order. An antisymmetric dispersion map has also been shown to reduce the complexity of perturbation-based nonlinearity pre-compensation [52]. The reason for this is that an antisymmetric dispersion map effectively reduces the memory length of the fiber-optic channel meaning that fewer symbols have to be taken into account when calculating the generated nonlinear perturbation. In the context of PCTW, CS is the addition of the two fields E_{X} and E_{Y} according to

$$E_{\mathrm{CS}} = E_{\mathrm{X}} + E_{\mathrm{Y}}^{*}, \tag{4.4}$$

where E_X is the signal on the X-polarization and E_Y is the conjugated copy of the signal transmitted on the Y-polarization. The theory of PCTW was then generalized in [33] to include other signalling dimensions for the conjugated copy such as time [Paper A], SDM modes [83] or wavelength [36] for the conjugate copy. In [33], the concept was also extended to include phase conjugation of an entire WDM spectrum leading to mitigation of inter-channel nonlinear effects as well, this was demonstrated in [73] in a system with inline dispersion compensation. It is challenging to simultaneously phase conjugate and dispersion pre-compensate an entire WDM spectrum. Using the approach of the experimental demonstration in [32] with dispersion pre-compensation applied in the electrical domain in transmitter DSP requires that all channels are modulated simultaneously. This requires a very high transmitter bandwidth, effectively covering the whole WDM spectrum bandwidth. Such a transmitter can e.g. be built by using several synchronized software-defined transmitters modulating different lines of an optical comb. This was demonstrated with the purpose of doing transmitter-side pre-compensation for both intra-channel and inter-channel nonlinear effects in [30].

When considering the mitigation of nonlinear distortion in PCTW systems it is important to note that the theory in [32] is only valid to first order. The derivation does not take signal-noise interaction or higher-order signal-signal interaction into account. This is an important difference compared to OPC since the theoretical explanation for the mitigation of nonlinear distortion in OPC systems show that also higher-order nonlinear distortion can be reversed. However, this does not mean that PCTW does not mitigate higher-order nonlinear effects but that it is unknown what happens with higher-order nonlinear effects.

The two remaining methods for mitigation of nonlinear effects, CDR and PSAs can be formulated in the generalized sense as PCTW systems [33]. CDR can be formulated as a certain implementation of generalized PCTW in time domain while two-mode PSA links can be formulated as generalized PCTW in frequency domain. For a multi-span PSA system it is important to point out that the CS operation is performed not once in receiver DSP like in a PCTW system but several times all-optically in each of the inline PSAs.

An interesting property of a PCTW signal with the phase-conjugated wave transmitted on the orthogonal polarization is that it under certain polarization rotations is a PM-pulse-amplitude modulation (PAM) signal. This is the same thing as saying that the signal is real-valued under certain polarization rotations. This can be seen by considering the polarization rotation by a matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} E_\mathrm{X} \\ E_\mathrm{Y} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} E_\mathrm{X} \\ E_\mathrm{X}^* \end{pmatrix} = \sqrt{2}\begin{pmatrix} \mathrm{real}(E_\mathrm{X}) \\ \mathrm{imag}(E_\mathrm{X}) \end{pmatrix}, \tag{4.5}$$

where the field after the rotation is real valued in both polarizations. Assum-

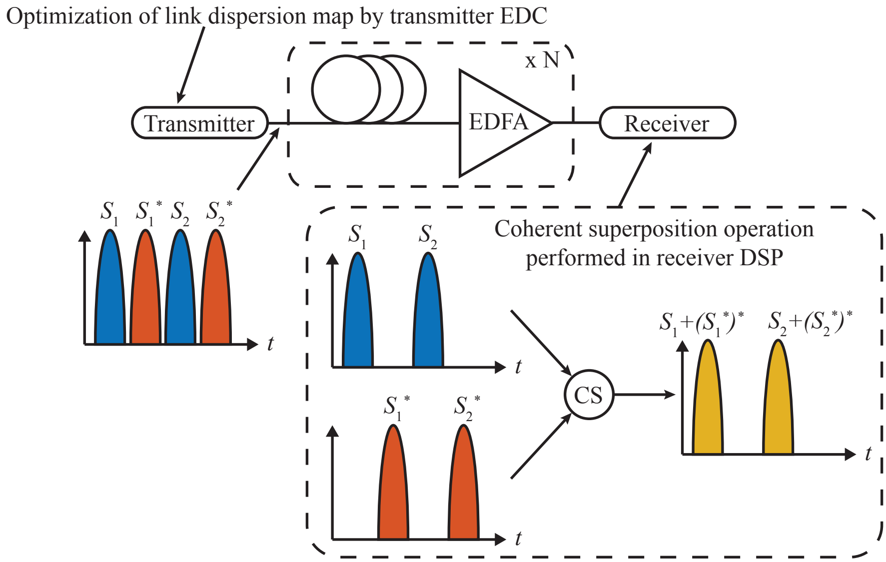

Figure 4.2: Schematic of the CDR link that was investigated in Paper A. A conjugated copy of each symbol is transmitted in the consecutive symbol slot. In receiver DSP, the signal pulse train is coherently superposed with the conjugate copy pulse train and part of the nonlinear distortion generated during propagation is mitigated.

ing that the signal in E_{X} is 4^M-ary QAM, the real valued signals after the polarization rotation will be 2^M-ary PAM. Under this polarization rotation, the nonlinear distortion, which is seen as anti-correlated on the two polarizations in the PCTW picture, becomes purely imaginary and thus orthogonal to the infomation-carrying quadratures [33].

4.3 Conjugate Data Repetition

The concept of conjugate data repetition (CDR) can be formulated as PCTW in a generalized form [33] where the conjugated copy is transmitted in different time slots instead of on the orthogonal polarization. The concept is illustrated in Fig. 4.2. In transmitter DSP, one signal pulse train modulated with e.g. QPSK and one phase-conjugated copy of the signal pulse train is generated. The two pulse trains are then interleaved in time and pre-EDC giving an antisymmetric link dispersion map is applied. In receiver DSP, one

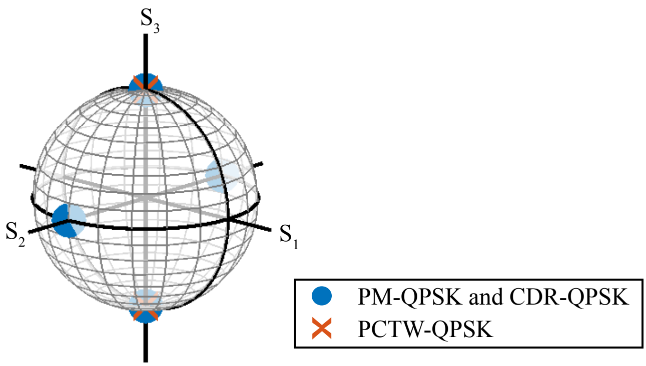

Figure 4.3: The points occupied on the Poincare sphere by PM-QPSK, CDR-QPSK and PCTW-QPSK.

of the pulse trains is delayed one symbol, phase-conjugated and then the two pulse trains are added. For a PM CDR signal, independent CDR pulse trains are transmitted on orthogonal polarization states. This concept was investigated numerically and with a time domain perturbation analysis in Paper A. With the time domain perturbation analysis it was shown that the nonlinear distortion on a train of CDR Gaussian pulses can be mitigated to a large extent through CS of the signal and conjugate pulse train in receiver DSP. This is in contrast to the case of PCTW, where the mitigation of nonlinear distortion is complete in a first-order perturbation analysis [32]. Despite this, it was found in numerical simulations that CDR has almost exactly the same performance as PCTW. Still, it is not possible to make an argument for CDR based on performance.

The strongest argument for the use of CDR is that the conventional constant modulus algorithm (CMA) often used in coherent PM-QPSK systems will work for CDR-QPSK but not for PCTW-QPSK. This could lead to easier implementation in existing PM-QPSK systems since the same equalizer can be used for both PM-QPSK and CDR-QPSK. The reason behind this is the number of polarization states that the signal occupies, PM-QPSK and CDR-QPSK occupy four points on the Poincare sphere while PCTW-QPSK occupies only two. In order to show this we calculate the Stokes parameters [84]

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} |E_\mathrm{X}|^2 + |E_\mathrm{Y}|^2 \\ |E_\mathrm{X}|^2 - |E_\mathrm{Y}|^2 \\ 2\,\mathrm{Re}(E_\mathrm{X} E_\mathrm{Y}^*) \\ 2\,\mathrm{Im}(E_\mathrm{X} E_\mathrm{Y}^*) \end{pmatrix}, \tag{4.6}$$

for CDR-QPSK and PCTW-QPSK signals. For CDR-QPSK we find all symbols by considering all permutations of $E_\mathrm{X} = \frac{1}{2}(\pm 1 \pm i)$ and $E_\mathrm{Y} = \frac{1}{2}(\pm 1 \pm i)$, this is the same symbol set as for PM-QPSK. For a CDR-QPSK signal, this gives us the Stokes parameters

$$\begin{pmatrix} S_{0,\mathrm{CDR}} \\ S_{1,\mathrm{CDR}} \\ S_{2,\mathrm{CDR}} \\ S_{3,\mathrm{CDR}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix}. \tag{4.7}$$

For PCTW-QPSK we have, $E_\mathrm{X} = \frac{1}{2}(\pm 1 \pm i)$ and $E_\mathrm{Y} = E_\mathrm{X}^*$ which gives us the Stokes parameters

$$\begin{pmatrix} S_{0,\mathrm{PCTW}} \\ S_{1,\mathrm{PCTW}} \\ S_{2,\mathrm{PCTW}} \\ S_{3,\mathrm{PCTW}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{pmatrix}. \tag{4.8}$$

The points occupied on the Poincare sphere by the signal using the different schemes is illustrated in Fig. 4.3. The conventional CMA commonly used in coherent PM-QPSK systems has a cost function which is designed for a signal occupying four polarization states and will not function properly if there are only two. It is however possible to alter the cost function of the CMA so that it converges with two polarization states instead of four [85].

4.4 Phase-Sensitive Amplifiers

Phase-sensitive amplifiers are amplifiers that transfer energy from a pump wave to signal and idler waves through FWM. This is done by inserting the signal, idler and a high-power pump into a nonlinear medium, e.g. a HNLF. PSAs have also been demonstrated in other nonlinear media, e.g. periodically poled lithium niobate (PPLN) [86], but in this thesis it will be assumed that the nonlinear medium is a HNLF.

In this thesis, only two-mode PSAs will be discussed, two-mode meaning that the signal and idler are located at two different wavelengths with the pump centered between them. There are other variants of PSAs [87], e.g. one-mode with signal and idler at the same frequency and four-mode with two pump waves and two signal-idler pairs. One-mode PSAs can be used to perform phase- and amplitude regeneration. By regenerating the phase of a signal it is also possible e.g. to mitigate nonlinear phase noise due to the Kerr effect. Studies of such phase and amplitude regeneration can be found in e.g. [88, 89]. Not much work has been done on the performance in a nonlinear transmission regime of systems utilizing four-mode PSAs. The study of one-mode and four-mode PSAs is outside the scope of this thesis.

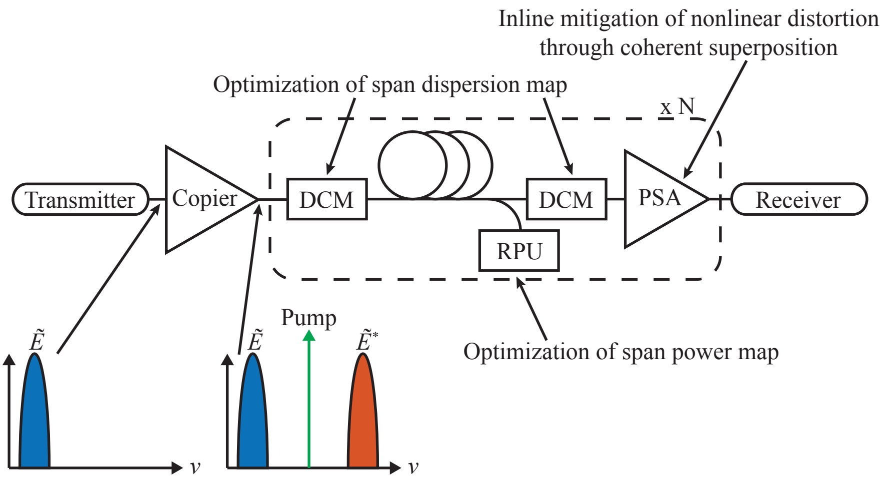

Figure 4.4: Schematic of the PSA link that was investigated in Paper C. Dispersion compensating module (DCM), Raman pump unit (RPU).

In two-mode PSA links, a phase-locked idler wave at a different wavelength than the signal is generated through FWM at the transmitter side by inserting the signal together with a high power pump into a HNLF, this device is often called a copier. Both the signal, idler and pump are then propagated in the link. The PSA itself is very similar to the copier, also consisting of a HNLF, the difference being that both the signal and the phase-locked idler are inserted into the HNLF together with the pump which is what gives the PSA its unique properties.

In the context of fiber-optic communication systems, an interesting property of PSAs is that they they perform CS of the signal and idler waves, as a consequence of this, PSAs have a 0 dB quantum-limited NF [38]. That PSAs can mitigate nonlinear distortion due to the Kerr effect also comes from the fact that they perform CS. CS is the simultaneous phase conjugation and addition of the signal field E_s and the idler field E_i according to

$$\begin{pmatrix} E_{\mathrm{s,out}}(t) \\ E^*_{\mathrm{i,out}}(t) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} E_{\mathrm{s,in}}(t) \\ E^*_{\mathrm{i,in}}(t) \end{pmatrix}. \tag{4.9}$$

where the transmitted signal and idler fields fulfill $E_\mathrm{i} = E^*_\mathrm{s}$. In the following, the focus will be on the CS property of PSAs since that is the property which has a big impact on the performance in a nonlinear transmission regime. Since the focus in this thesis is the properties of nonlinear distortion mitigation in PSAs, not much attention will be focussed on the advantageous NF.

As was mentioned, the property that improves the performance of a PSA link in a nonlinear transmission regime is the fact that PSAs perform CS. In the following, an explanation of why nonlinear distortion can be mitigated in a single-span PSA link, as depicted in Fig. 4.4 (with N = 1), will be provided. A frequency domain perturbation approach will be used, similar to the analysis performed in [32] but instead of having the phase-conjugated copy propagating on the orthogonal polarization, the two waves will be considered to be propagating independently of each other since they are typically separated by approximately 10 nm.

It is important to choose the span dispersion map carefully in order to mitigate nonlinear distortion efficiently in a PSA link. This was investigated in [90], where it was shown that optimizing the dispersion map in a single-span PSA link improved the tolerance against nonlinear distortion leading to higher launch powers for a 1 dB Q-factor penalty, and in [91], where the optimum dispersion map was found for different span lengths as well as different fiber loss coefficients. When discussing dispersion maps one can also point out that the accumulated dispersion on the signal and idler has to be compensated close to zero before each PSA in order to achieve phase-sensitive operation [92].

Now follows a derivation of the mitigation of nonlinear distortion in a single-span PSA link based on the perturbation analysis presented in section 2.5. The first step of the derivation is to calculate the nonlinear perturbation generated on the signal and idler waves. The transmitted signal and idler waves $u_{\mathrm{s}}^{(0)}$ and $u_{\mathrm{i}}^{(0)}$ fulfill the following relations

$$u_{\mathrm{i}}^{(0)}(t) = u_{\mathrm{s}}^{(0)*}(t) \text{ and } \tilde{u}_{\mathrm{i}}^{(0)}(\omega) = \tilde{u}_{\mathrm{s}}^{(0)*}(-\omega), \tag{4.10}$$

by inserting these expressions into Eq. (2.17) we find the perturbation generated on the signal and idler waves after propagation over a fiber of length L to be

$$\begin{aligned}\tilde{u}_{\mathrm{s}}^{(1)}(L,\omega) = i\gamma P_0 L_{\mathrm{eff}} \int_{-\infty}^{\infty} \mathrm{d}\omega_1 \int_{-\infty}^{\infty} \mathrm{d}\omega_2 \eta(\omega_1\omega_2) \\ \times \tilde{u}_{\mathrm{s}}^{(0)}(\omega+\omega_1)\tilde{u}_{\mathrm{s}}^{(0)}(\omega+\omega_2)\tilde{u}_{\mathrm{s}}^{(0)*}(\omega+\omega_1+\omega_2),\end{aligned} \tag{4.11}$$

and

$$\begin{aligned}\tilde{u}_{\mathrm{i}}^{(1)}(L,\omega) = i\gamma P_0 L_{\mathrm{eff}} \int_{-\infty}^{\infty} \mathrm{d}\omega_1 \int_{-\infty}^{\infty} \mathrm{d}\omega_2 \eta(\omega_1\omega_2) \\ \times \tilde{u}_{\mathrm{i}}^{(0)}(\omega+\omega_1)\tilde{u}_{\mathrm{i}}^{(0)}(\omega+\omega_2)\tilde{u}_{\mathrm{i}}^{(0)*}(\omega+\omega_1+\omega_2).\end{aligned} \tag{4.12}$$

Next we assume an antisymmetric dispersion map such that $C(z) = -C(L-z)$ and a symmetric power map such that $G(z) = G(L-z)$. Under these assumptions, we see upon close inspection of Eq. (2.18) that $\eta(\omega_1\omega_2)$ becomes real-valued. If $\eta(\omega_1\omega_2)$ is real-valued we have $\eta(\omega_1\omega_2) = \eta^*(\omega_1\omega_2)$. Using this together with the relationships in Eq. (4.10) we find that

$$\tilde{u}_{\mathrm{i}}^{(1)}(L,\omega) = -\tilde{u}_{\mathrm{s}}^{(1)*}(L,-\omega), \tag{4.13}$$

which is equivalent to

$$u_{\mathrm{i}}^{(1)}(L,t) = -u_{\mathrm{s}}^{(1)*}(L,t). \tag{4.14}$$

If we then perform the CS of the nonlinearly perturbed signal and idler fields according to Eq. (4.9) and use Eq. (4.14) we get

$$\begin{aligned} E_{\mathrm{s,out}} &= E_{\mathrm{s,in}}(L,t) + E_{\mathrm{i,in}}^*(L,t) \\ &= \sqrt{P_0}\exp\left(\frac{G(z)}{2}\right)\left[\tilde{u}_{\mathrm{s}}^{(0)}(t) + \tilde{u}_{\mathrm{s}}^{(1)}(L,t) + \tilde{u}_{\mathrm{i}}^{(0)*}(t) + \tilde{u}_{\mathrm{i}}^{(1)*}(L,t)\right] \\ &= \sqrt{P_0}\exp\left(\frac{G(z)}{2}\right)\left[2\tilde{u}_{\mathrm{s}}^{(0)}(t)\right], \end{aligned} \tag{4.15}$$

and we see that the nonlinear perturbation terms disappear in the CS operation.

Several approximations and assumptions have been made to reach this conclusion. First, it is assumed that the signal and idler waves do not interact nonlinearly with each other during propagation, this is reasonable since they are assumed to be separated by approximately 10 nm. Second, it is assumed that β_2 is the same at the signal and idler wavelength, in reality, the value of β_2 has a wavelength dependence. Third, it is assumed that the power evolution during propagation $G(z)$ is identical for the signal and idler waves as well as symmetric. In reality, the attenuation is slightly wavelength dependent. Also, a perfectly symmetric power map can not be achieved in practice even with bi-directional higher-order Raman pumps, it is however possible to come close, in [47] a 80 km Raman amplified span was demonstrated with optical power excursions of only ± 0.4 dB. Another important thing to point out is that this derivation is valid for a single-span link that uses one PSA before the receiver, performing CS only once. In a strict sense, the derivation is not valid for a multi-span PSA link with a PSA after each span. However, it is not unreasonable to approximate a multi-span PSA link as the concatenation of many single-span links, still this derivation will not tell us, e.g., what the optimal dispersion map in a multi-span PSA link is.

4.5 Comparison Between the Different Methods

Now a comparison between the different methods that were discussed in sections 4.1 to 4.4 will be provided. The strengths and weaknesses of the different

Table 4.1: Comparison of important properties for the different methods for mitigation of nonlinear distortion that have been covered in this thesis.

	OPC	PCTW	CDR	PSA
Quantum-limited amplifier NF [dB]	3	3	3	0
Number of hybrid receivers per channel	1	2	1	1
Spectral efficiency reduction [%]	0	50	50	50
Requires phase-locked waves	No	No	No	Yes
Requires inline dispersion compensation	No	No	No	Yes
Requires transmitter/receiver EDC	No	Yes	Yes	No

methods will be highlighted and discussed. This comparison is also summarized in Table 4.1.

The first important aspect to consider is the limitations on the SE with the different approaches. In this regard, it is most beneficial to use OPC since that method does not fundamentally require any reduction of SE. It is, however, important to point out that in many specific implementations of OPC, e.g. [23], SE is still reduced because of wavelength conversion. For all the other methods, PCTW, CDR and PSA, SE is reduced by 50 % since a phase-conjugated copy is transmitted alongside the signal.

When comparing the performance in the linear transmission regime, a system employing PSAs has an advantage because of the lower quantum limited NF. With all the other methods, there is a NF of 3 dB associated with the inline amplification. Also one can point out that the process of OPC itself has an inherent quantum-limited NF of 3 dB [93]. When discussing linear noise properties, it is important to note that for the scenario studied in Paper C, i.e. hybrid links using both distributed Raman amplification and PSAs, the 0 dB NF of the PSAs is not a significant advantage since the noise properties of the link will be mostly dictated by the noise from the distributed Raman amplification. The relationship between the noise added by the distributed Raman amplification and the inline PSAs will depend on e.g. the span length and the attenuation at Raman pump and signal wavelengths.

Another important issue is the required complexity of transmitters and receivers for the different schemes. In this regard, OPC- and PSA-systems are advantageous since they only require one hybrid receiver per channel as well as not requiring large amounts of transmitter and receiver EDC. In the case of OPC, EDC is not required since the inline OPC operations can also compensate for CD while for the case of PSAs it is required to use inline optical dispersion compensation using DCFs or FBGs.

An experimental comparison between the nonlinear distortion mitigation properties of mid-link OPC, PCTW, PSA and DBP systems was done recently in [94]. In that experiment it was found that the system performing CS in the digital domain performed best out of the four schemes. However, one can point out that e.g. the dispersion maps were not optimized in that experiment, leaving room for further optimization. The maybe most important question is which of these methods that has the potential to offer best performance in the nonlinear transmission regime, this is however so far an unanswered question and is left as one of the questions which will be discussed in the future outlook.

Chapter 5

Future Outlook

Experimental demonstration of hybrid Raman/PSA link
In Paper C it was concluded that the span power map is an important parameter in multi-span PSA links. The results in the paper are based on numerical simulations and it would be of great interest to demonstrate a hybrid Raman/PSA link experimentally. This is interesting for many reasons, first of all in order to see if the scheme is feasible to implement in practice. Second, some approximations and simplifications were made in the computer models, e.g. the effects of PMD and parametric noise amplification were not properly taken into account. Thus it is interesting to see how well the scheme can perform in practice.

Comparison between OPC and PSA systems
There are similarities between systems that perform OPC several times along the link and systems that perform CS of a signal and its phase-conjugated copy several times using PSAs. Still no comparative studies have been made between such multi-span OPC- and PSA systems regarding their ability to mitigate nonlinear distortion. Such a study would be of great interest and would improve the understanding of both concepts. In [94], an experimental comparison between 2-span systems using one OPC device or one PSA before the receiver was made. It is however difficult to draw conclusions from the results in that paper regarding systems with several inline OPC devices or PSAs. Questions of importance here could be e.g. the impact of PMD and of different realistic span power maps in relation to signal bandwidth as well as the fundamental limits of all-optical nonlinear distortion mitigation.

What is the optimum dispersion map in a long-haul PSA system?
In paper C, the optimum dispersion maps for a single-span PSA system, both with and without distributed Raman amplification were found. It was then assumed that by viewing a multi-span system as the concatenation of several single-span systems, the optimum span dispersion map for the single-span case would perform well in the multi-span case as well. This was an assumption and it was not shown in any way that this actually gives optimum performance in a multi-span PSA link. To do an exhaustive search for the optimum dispersion map in a multi-span PSA system is only feasible with a couple of spans because of the computation complexity involved with a multi-dimensional search with SSFM simulations. Thus it would be of interest to develop a theory, similar to the one presented in section 4.4, that takes multiple PSA-amplified spans into account. Such a theory could probably predict what the optimum dispersion map is for a multi-span PSA link.

Combining PSAs with transmitter pre-distortion
Another possible research topic is transmitter pre-distortion in PSA links. In Paper C it was shown that the nonlinear phase shift (NLPS) at optimal launch power in a long-haul PSA link can be significantly increased by employing ideal distributed Raman amplification. An interesting question is whether it is possible to achieve similar increases in tolerable NLPS by applying some form of pre-distortion on the transmitted waveform. In order for such a scheme to be attractive, the calculation of the pre-distorted waveform should be low-complexity, possibly based on a perturbation approach.

Using concepts from information theory
It is possible to study e.g. the mutual information (MI) [95] of a channel that uses inline PSAs or OPC devices to mitigate nonlinear distortion. To the best of my knowledge, such a study has not been done before. Interesting research questions could be e.g. if and how the mutual information of a channel changes by including phase conjugating elements or what the probability density function (PDF) of the output signal is. A study of the fundamental limits of OPC systems was done in [24], however in that study other metrics than MI were used.

Chapter 6

Summary of Papers

Paper A

"Mitigation of nonlinearities using conjugate data repetition," *Optics Express*, vol. 23, no. 3, pp. 2392-2402, 2015.

In this paper, we investigate a scheme for mitigation of nonlinear fiber distortion. The scheme is based on the idea to send a phase-conjugated copy of each symbol in the consecutive symbol slot followed by CS of the two pulse trains in receiver DSP. We call the scheme CDR. The mitigation of nonlinear distortion is derived using a time domain perturbation analysis and compare the efficiency of the mitigation in comparison to regular phase-conjugated twin waves in numerics. The results show that the performance in terms of transmission reach is comparable even though the perturbation analysis predict worse performance for CDR. It is then argued that since CDR is compatible with the common CMA algorithm, which PCTW is not, it could be easier to implement CDR in existing systems.

Paper B

"Comparison between CS in DSP and PSA for mitigation of nonlinearities in a single-span link," *Proceedings European Conference on Optical Communication (ECOC)*, Cannes, France, 2015, paper Mo.3.5.2.

An experimental comparison between the mitigation of nonlinear distortion in a single-span system performing CS all-optically in a PSA or in receiver DSP is presented in this paper. The performance in the nonlinear transmission regime is quantified with measurements of error vector magnitude (EVM) at different launch powers and optical signal-to-noise ratio (OSNR) penalty with noise loading. It is shown that the performance in the nonlinear regime is comparable with both approaches. Last, the fundamental differences between the two approaches in terms of required system hardware are discussed.

Paper C

"Mitigation of nonlinear distortion in hybrid Raman/phase-sensitive amplifier links," *Optics Express*, vol. 24, no. 2, pp. 888-900, 2016.

A numerical investigation of the impact of the span power map in single-span and multi-span PSA links is presented in this paper. Three different link amplifier configurations are investigated, PSA only, PSA with first-order backward-pumped Raman amplification and PSA with ideal distributed Raman amplification. First, the optimum dispersion maps are found in the single-span scenario and the OSNR penalty as a function of launch power and NLPS is evaluated. Then, multi-span PSA links with the three different amplifier configurations are modelled using the optimum single-span dispersion maps. It is found that the transmission reach of a multi-span PSA link can be increased by as much as a factor of 8.1 by including ideal distributed Raman amplification and using the span dispersion map found to be optimal in the single-span scenario.

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