Back Pressure Traffic Signal Control with Fixed and Adaptive Routing for Urban Vehicular Network

Decentralized Traffic Signal Control with Fixed and Adaptive Routing of Vehicles in Urban Road Networks

Ali A. Zaidi, Balázs Kulcsár, and Henk Wymeersch

Abstract—**City-wide control and coordination of traffic flow can improve efficiency, fuel consumption, and safety. We consider the problem of controlling traffic lights under fixed and adaptive routing of vehicles in urban road networks. Multi-commodity back-pressure algorithms, originally developed for routing and scheduling in communication networks, are applied to road networks to control traffic lights and adaptively reroute vehicles. The performance of the algorithms is analyzed using a microscopic traffic simulator. The results demonstrate that the proposed multi-commodity and adaptive routing algorithms provide significant improvement over a fixed schedule controller and a single-commodity back-pressure controller, in terms of various performance metrics, including queue-length, trips completed, travel times, and fair traffic distribution.**

I. INTRODUCTION

IN urban road networks, traffic congestion is a major problem leading to time loss, pollution, and accidents [1]. Vehicle flows in such networks are controlled by traffic lights and are affected by route choices that the drivers make. Traffic conditions can thus be improved by the development of efficient traffic signal control and route selection methods, see the papers e.g., [2], [3] for a survey to the field. Traditionally, traffic light controllers follow a pre-defined optimized schedule [4], which may result in a poor performance under time-varying traffic conditions and under very high traffic demands. This problem can be alleviated through adaptive traffic signal controllers, such as SCOOT, UTOPIA, SCATS, or RHODES [5]–[7]. In these adaptive traffic controllers, real-time measurements are collected using on-road detectors. Based on these measurements, either the parameters (splits, offsets, cycle-length) of the signal plans are adjusted on a cycle-to-cycle basis or a best signal plan is selected from a pre-defined set of signal plans. The implementation of these methods, however, requires centralized decision making for all intersections based on the traffic-related measurements. In addition to these traffic-adaptive signal control implementations, other centralized traffic signal control algorithms have recently been proposed [9]–[12] using different approaches from control theory, such as linear quadratic regulator, robust control, and model predictive control. The traffic signal control problem has also been studied under game theoretic formulations [13].

In contrast to the many centralized approaches for traffic signal control, the literature on decentralized solutions, which would be very useful especially for large urban areas, is scarce. Recently, researchers in the transportation and control communities have proposed different traffic-adaptive scalable and distributed methods [14]–[21], where the general idea is to solve a separate optimization problem for every intersection. These per-intersection optimization problems are loosely coupled via real-time traffic conditions. The implementation of these controllers requires either the knowledge of expected traffic load on the links associated with the intersection during the next cycle, or the difference between the traffic loads on the links associated with the network. Many of these schemes are inspired by scheduling and routing algorithms in wireless networks, in particular the well-known back-pressure scheme from [22]. Back-pressure is a decentralized scheme that can provide maximum network throughput under the assumption that all links in the network have infinite capacities (it is in fact optimal in the sense of supporting maximum traffic arrival rates that guarantee stability of queues in a stochastic sense). This idea was first adapted to urban road networks in [16], where it was shown that significant performance gains can be achieved in terms of network queue lengths by employing a back-pressure scheme for signal control. It was also shown to provide good performance compared to the fixed time schedule controllers, when the links have finite capacities. However, [16] does not dynamically re-route vehicles, leading to local bottlenecks in the road network.

In the literature, there exist different methods for route selection based on different performance metrics such as shortest path, shortest travel time, congestion minimization, etc. For vehicle routing, the fundamental challenge is that the traffic demand and vehicle departure times at different links in a road network are not known a priori. However, the real-time traffic information along with the historic traffic data can be used to anticipate traffic conditions and has been shown to be very useful in devising route selection methods [23]. Recent works on traffic-adaptive routing methods include [24], [25].

In this paper, we extend [16] by performing both traffic-adaptive signal control and routing, under back-pressure based control methods. In particular, rather than a single-commodity back-pressure scheme with fixed routes as in [16], we apply a multi-commodity (one commodity per destination) version of the back-pressure scheme [26], [27] under both fixed and

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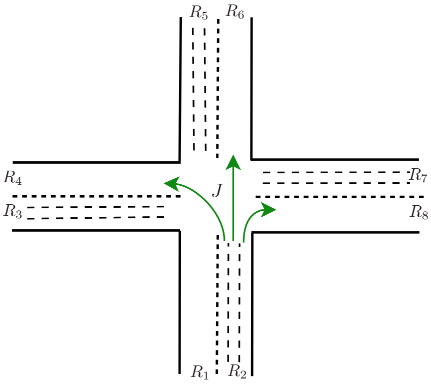

Fig. 1: A four-way junction with 8 roads and 12 possible traffic movements.

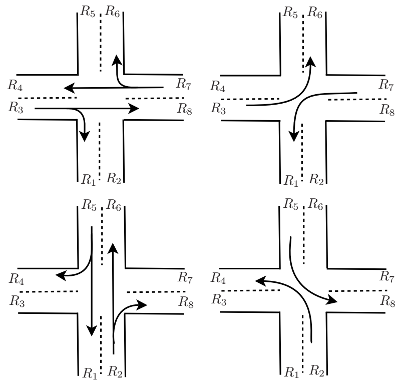

Fig. 2: Typical phases through a four-way junction.

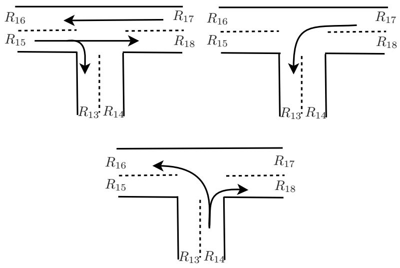

Fig. 3: Typical phases through a three-way junction.

adaptive route selection. Our results demonstrate that the proposed schemes can provide significant performance gains.

The remainder of this paper is organized as follows. In Section II, we mathematically formulate the problem of traffic signal control and adaptive routing of vehicles. The algorithms based on back-pressure multi-commodity schemes are proposed in Section III. Performance of these algorithms is analyzed with a detailed discussion in Section IV. Finally, the key findings are summarized in Section V along with directions for future work on this topic.

II. Problem Formulation

A. Road Network

Consider an urban road network comprised of N links/roads and L junctions (signalized intersections). We model the network as a directed graph $\mathcal{G} = (\mathcal{R}, \mathcal{J})$, where $\mathcal{R} = \{R_1, R_2, \ldots, R_N\}$ is the set of links and $\mathcal{J} = \{J_1, J_2, \ldots, J_L\}$ is the set of junctions in the road network. A vehicle exogenously enters the network from a certain link (origin), travels along one or more links in the network and finally leaves the network at a certain link (destination). Thus, for each vehicle in the network, there is an associated origin and destination pair. All vehicles that have a common origin and destination pair constitute a flow f. Let $\mathcal{F}$ be the set of all flows in the network and let $(o(f), d(f))$ be the origin–destination pair for a flow $f \in \mathcal{F}$, where $o(f), d(f) \in \mathcal{R}$. Let $\lambda_f(t)$ be the rate at which vehicles associated with flow f exogenously enter $o(f)$ at discrete time slots $t \in \mathbb{N}$, with $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[\lambda_f(t)] = \lambda_f$. We assume that the flow arrival processes are independent of each other and also independent across time slots and have finite second moments. At any time t, let $Q_{ab}(t)$ be the number of vehicles queued in a link R_a to move to an adjacent link R_b and let $Q_a(t) = \sum_b Q_{ab}(t)$ be the the total queue length at link R_a.

B. Traffic Phase Switching

Each junction has certain traffic movements associated with it. A traffic movement through a junction corresponding to the vehicles exiting R_a and entering R_b is denoted by the pair (R_a, R_b). Let $\mathcal{M}_i$ be the set of all traffic movements through a junction J_i. Consider an example of a four-way junction in Fig. 1, where there are twelve possible traffic movements. The set of all possible traffic movements for this four-way junction is given by

$$\begin{aligned}\mathcal{M} = \{&(R_3, R_8), (R_3, R_1), (R_7, R_4), (R_7, R_6),\\ &(R_3, R_6), (R_7, R_1), (R_5, R_4), (R_5, R_1),\\ &(R_2, R_6), (R_2, R_8), (R_2, R_4), (R_5, R_8)\}. \end{aligned} \tag{1}$$

A subset of traffic movements that can occur simultaneously through a junction constitute a phase. Let $\mathcal{P}_i = \{p_1^i, p_2^i, \ldots, p_l^i\}$ be the set of all possible phases through a junction J_i. As examples, consider a four-way junction with four possible phases in Fig. 2 and a three-way junction with three possible phases in Fig. 3. Typical phases of the four-way junction are given by

$$\begin{aligned} p_1 &= \{(R_3, R_8), (R_3, R_1), (R_7, R_4), (R_7, R_6)\},\\ p_2 &= \{(R_3, R_6), (R_7, R_1)\},\\ p_3 &= \{(R_5, R_4), (R_5, R_1), (R_2, R_6), (R_2, R_8)\},\\ p_4 &= \{(R_2, R_4), (R_5, R_8)\}, \end{aligned} \tag{2}$$

and for the three-way junction are given by

$$\begin{aligned} p_1 &= \{(R_{15}, R_{13}), (R_{13}, R_{18}), (R_{17}, R_{16})\},\\ p_2 &= \{(R_{17}, R_{13})\},\\ p_3 &= \{(R_{14}, R_{16}), (R_{14}, R_{18})\}. \end{aligned} \tag{3}$$

Furthermore, we assume that with every possible movement (R_a, R_b) through a junction, there is a rate $s_{ab}(t)$ with which vehicles can flow through the junction. That is, $s_{ab}(t)$ is equal to the number of vehicles that can go from link R_a to R_b if a phase p is activated, where $(R_a, R_b) \in p$.

C. Routing of Vehicles

We consider that each vehicle that enters the network has a fixed destination but the route it takes towards the destination may be either fixed or variable. That is, we have two cases:

1) *Fixed Routing:* In fixed routing we assume that all vehicles that have a common origin and destination, follow the same route. That is, for all vehicles belonging to a certain flow f in the network, the route is fixed. Let $\mathcal{L}(f)$ be the set of links forming the route of flow f.
2) *Adaptive Routing:* In adaptive routing, the route of each vehicle is adapted with traffic conditions. Thus, the vehicles with a common origin and destination pair may not necessarily follow the same route. We consider that for every vehicle, the route is dynamically updated at every junction. Whenever a vehicle enters a link R_a, its next movement (R_a, R_b) through the upcoming junction is decided in real-time and the vehicle joins one of the possible queues (lanes) accordingly. For instance, in the example shown in Fig. 1, when a vehicle enters R_2, it can join one of the three possible vehicle queues $\{Q_{24}, Q_{26}, Q_{28}\}$.

D. Control Problem

At every junction J_i, there is a controller $\mathcal{C}_i$ that has to perform the following tasks at every time slot t.

1) Select a phase $p_k^i(t) \in \mathcal{P}_i$ (i.e., the traffic controller gives the right of way to certain traffic movements in every time slot).
2) Make a routing decision for the vehicles (i.e., assign queues to the vehicles) related to every flow f passing through the given junction.

The routing decisions are communicated to the corresponding vehicles and the vehicles in the network are assumed to follow the routing decision made by the traffic controller.

III. Proposed Methods

In this section, we will describe two novel methods for signal control: one with fixed routing and one with dynamic routing. These methods are based on the back-pressure algorithm [22], originally invented for scheduling and routing of packets in wireless networks. In [16], a back-pressure scheduling algorithm was used for traffic signal control, assuming all vehicles follow fixed routes but ignoring the fact that all vehicles in the network have different destinations (single-commodity back pressure scheme). We now propose to employ multi-commodity back-pressure schemes for traffic signal control with fixed as well as adaptive routing of vehicles. In contrast to the single-commodity scheme where the vehicle queue length information has to be known on a per-link basis, the operation of multi-commodity schemes under fixed and adaptive routing requires queue length information on a per-flow and per-destination basis. Since the numbers of origins and destinations in a road network are normally very big, it is not possible to maintain physically separate vehicle queues on a per-flow and per-destination basis. In order to tackle this issue, we utilize the concept of virtual queues, which is essential for the operation of the proposed multi-commodity back-pressure traffic control schemes in road networks.

A. Virtual Queues and Virtual Vehicles

Following the wireless networking approach in [26], [27], we introduce *virtual traffic* and *virtual queues* (referred to as shadow queues in [27]) in the road network. For each vehicle that exogenously enters a link in the network, we generate a virtual vehicle with probability one and another virtual vehicle with probability $\epsilon > 0$. Hence, for any flow f in the network, the arrival rate of virtual traffic is $(1+\epsilon)\lambda_f(t)$. The reason for introducing ϵ here is explained in Section III-D1. With the virtual traffic we can associate two virtual queues: we denote the number of virtual vehicles of flow f on link R_a by $\tilde{Q}_a^f$; similarly, we denote the number of virtual vehicles for destination d on link R_a by $\tilde{Q}_a^d$. We note that virtual traffic and queues are merely counters, which form a fictitious queuing system on which the signal control and route control algorithms are based. The real queues $Q_{ab}(t)$ containing real vehicles are maintained on a per movement basis, for every possible movement (R_a, R_b) through a given junction.

B. Signal Control Algorithm with Fixed Routing

The signal control algorithm for each junction is decentralized[1]. At each junction J_i, the algorithm works based on the *per flow virtual queue length* information $\tilde{Q}_a^f$ for all links R_a associated with the given junction. The algorithm works as follows for each junction J_i:

1) For all (a,b) such that $(R_a, R_b) \in \mathcal{M}_i$, determine the flow with maximum back-pressure and then assign a weight to that flow:

$$f_{ab}^\star(t) = \arg\max_{f\in\mathcal{F}_i}\{\tilde{Q}_a^f(t) - \tilde{Q}_b^f(t)\}, \tag{4}$$

$$W_{ab}(t) = \max\{\tilde{Q}_a^{f_{ab}^\star(t)}(t) - \tilde{Q}_b^{f_{ab}^\star(t)}(t), 0\}, \tag{5}$$

where $\mathcal{F}_i$ is the set of all flows passing through links R_a and R_b.

2) For each phase $p_k^i \in \mathcal{P}_i$, compute the pressure release as

$$S_{p_k^i}(t) = \sum_{(R_a,R_b)\in p_k^i} W_{ab}(t)s_{ab}(t). \tag{6}$$

3) The controller C_i at junction J_i activates the phase $p_{k^\star}^i$ with the highest pressure release, i.e., it selects

$$p_{k^\star}^i = \arg\max_{p_k^i\in\mathcal{P}_i} S_{p_k^i}(t). \tag{7}$$

When a certain phase is activated, the real vehicles in the network move according to the given rates and the queues

[1]The algorithm is decentralized in the following sense. A controller at a junction makes decisions based on *local* communication with vehicles associated with those links that are connected to the given junction.

of real vehicles evolve accordingly. The virtual queues evolve according to:

$$\tilde{Q}_a^f(t+1) = \tilde{Q}_a^f(t) - \sum_{c:(R_c,R_b)} I_{\{f=f_{ab}^\star\}} s_{ab}(t) + \sum_{c:(R_c,R_a)} I_{\{f=f_{ca}^\star\}} s_{ca}(t) + I_{\{a=o(f)\}} \tilde{\lambda}_f(t), \quad \text{for} \quad a \neq d(f), \tag{8}$$

where $I_{\{\cdot\}}$ is an indicator function (whose value is equal to 1 if the statement in its argument is true otherwise its value is equal to 0) and $\tilde{\lambda}_f(t)$ is the number of virtual vehicles associated with flow f that exogenously enter $o(f)$ at time t. We assume that $\tilde{Q}_a^f(t) = 0$ for all $a = d(f)$, i.e., a vehicle is not counted in any queue when it enters its destination link.

C. Signal Control Algorithm with Adaptive Routing

We follow [27], where an algorithm is proposed that decouples routing and scheduling in wireless networks. Adaptive routing operates by placing incoming vehicles in real queues according to a probabilistic routing, which signal control is based on back-pressure on virtual queues per destination.

1) Signal Control Algorithm: The signal control algorithm for each junction is again decentralized. At each junction J_i, the algorithm works based on the *per destination virtual queue length* information $\tilde{Q}_a^d$ for all links R_a associated with the given junction. The algorithm works as follows for each junction J_i:

1) For all (a,b) such that $(R_a, R_b) \in \mathcal{M}_i$, determine the destination with maximum back-pressure and then assign a weight to that destination:

$$d_{ab}^\star(t) = \arg\max_d \{\tilde{Q}_a^d(t) - \tilde{Q}_b^d(t)\}, \tag{9}$$

$$W_{ab}(t) = \max\{\tilde{Q}_a^{d_{ab}^\star(t)}(t) - \tilde{Q}_b^{d_{ab}^\star(t)}(t), 0\}. \tag{10}$$

2) For each phase $p_k^i \in \mathcal{P}_i$, compute the pressure release as

$$S_{p_k^i}(t) = \sum_{(R_a,R_b)\in p_k^i} W_{ab}(t) s_{ab}(t). \tag{11}$$

3) The controller C_i at junction J_i activates the phase with the highest pressure release, i.e., it selects

$$p_{k^\star}^i = \arg\max_{p_k^i \in \mathcal{P}_i} S_{p_k^i}(t). \tag{12}$$

When a certain phase is activated, the real vehicles in the network move according to the given rates and the queues of real vehicles evolve accordingly. The virtual queues evolve according to:

$$\tilde{Q}_a^d(t+1) = \tilde{Q}_a^d(t) - \sum_{b:(R_a,R_b)\in\mathcal{M}_i} I_{\{d_{ab}^\star(t)=d\}} s_{ab}(t) + \sum_{c:(R_c,R_a)\in\mathcal{M}_i} I_{\{d_{ca}^\star(t)=d\}} s_{ca}(t) + \sum_{f\in\mathcal{F}} I_{\{o(f)=a,d(f)=d\}} \tilde{\lambda}_f(t), \quad \text{for} \quad a \neq d, \tag{13}$$

where $I_{\{\cdot\}}$ denotes the indicator function and $\tilde{\lambda}_f(t)$ is the number of virtual vehicles associated with flow f that exogenously enter $o(f)$ at time t. We assume that $\tilde{Q}_d^d = 0$, i.e., a vehicle is not counted in any queue when it enters its destination link.

2) Adaptive Route Control Algorithm: Let $\sigma_{ab}^d(t)$ be the number of virtual vehicles transferred from link R_a to link R_b for destination d under the above signal control algorithm during the time slot t, $\bar{\sigma}_{ab}^d$ its expected value in stationary regime, and $\hat{\sigma}_{ab}^d(t)$ the estimate at time t of this expected value.

1) At every junction, compute $\hat{\sigma}_{ab}^d(t)$ for every feasible movement $(R_a, R_b) \in \mathcal{M}_i$ associated with that junction using an exponential averaging method:

$$\hat{\sigma}_{ab}^d(t) = (1-\beta)\hat{\sigma}_{ab}^d(t-1) + \beta\sigma_{ab}^d(t), \tag{14}$$

where $0 < \beta < 1$ is a smoothing factor.

2) Compute the routing probabilities:

$$P_{ab}^d(t) = \frac{\hat{\sigma}_{ab}^d(t)}{\sum_{c:(R_a,R_c)\in\mathcal{M}_i} \hat{\sigma}_{ac}^d(t)}. \tag{15}$$

3) A vehicle entering link R_a joins real queue Q_{ab} with probability $P_{ab}^d(t)$. That is, the vehicle entering R_a destined for R_d will be routed to R_b with probability $P_{ab}^d(t)$ at time t through the junction J_i.

The routing information is communicated to vehicles in terms of probabilities or percentages. For example, consider that for the four-way junction illustrated in Fig. 1 if $[P_{24}^d(t) = 0.1, P_{26}^d(t) = 0.2, P_{28}^d(t) = 0.7]$, then among all those vehicles that enter link 2 having destination d, approximately 10 percent should join queue Q_{24}, 20 percent should join queue Q_{26}, and 70 percent should join queue Q_{28}. In this setup, the routing probability is governed by the BP scheme, implicitly. In fact, we first estimate the mean of the virtual vehicles transferred from link a to b heading destination d, i.e. the local flows to destination d. The estimation of the mean uses a recursive method, taking into account the latest known vehicle number at every instant t and the estimated mean from the previous sample time. By means of the the average valued local flows (towards a destination d) in stationary regions, we split vehicles according to the probability calculated. Moreover, we route vehicles in a local, decentralized context (V2I communication is required though). Finally, BP only enables traffic phase activation, while the proposed routing solution distributes the virtual flows.

3) Enhancing the Performance of Adaptive Routing Algorithm: It will be shown in Section IV that the proposed back-pressure routing algorithm is suitable for heavily loaded networks but can lead to unnecessarily long routes in a low load situation. This is also the case in wireless networks, where several methods have been proposed to improve the delay performance of back-pressure routing [27], [28]. These methods are usually based on including bias terms in the calculation of queue backlogs. That is, if one wants to encourage (discourage) traffic flow to a certain link, then one can add bias terms in the the calculation of queue backlog differences. For instance, one can modify (9) and (10) as follows:

$$d^{\star}_{ab}(t) = \arg\max_{d}\{\tilde{Q}^{d}_{a}(t) - \tilde{Q}^{d}_{b}(t) + \alpha(V^{d}_{a} - V^{d}_{b})\}, \quad (16)$$

$$W_{ab}(t) = \max\{\tilde{Q}^{d^{\star}_{ab}(t)}_{a}(t) - \tilde{Q}^{d^{\star}_{ab}(t)}_{b}(t) + \alpha(V^{d^{\star}_{ab}(t)}_{a} - V^{d^{\star}_{ab}(t)}_{b}), 0\}, \quad (17)$$

where V^{d}_{a} is equal to the minimum number of links that exist between link a and link d (destination d) and α is a non-negative real number that can be optimized. A higher value of α forces the vehicles to follow shorter paths, which is good for low-load situations but may not be good in a high load situation, as shown in Section IV.

D. Stability and Optimality

1) Infinite Length Links: In this section we discuss optimality of the proposed methods in the sense of supporting maximum traffic arrivals in a road network under the assumption that all links are infinitely long. Although in practice all links in a network have finite lengths, the BP scheme is originally inspired by its proven throughput optimality under the assumption of links with infinite lengths. The earlier papers [16], [19] that study single-commodity traffic signal control under real queues guarantee optimality under the assumption of infinite length links. In order to complement the existing literature, we also provide this discussion. To be precise, we first define stochastic stability and capacity region of a network and then discuss the optimality.

***Definition* 1.** *A queue $Q(t)$ with stochastic arrival and departure processes is said to be strongly stable if*

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}[Q(\tau)] < \infty. \quad (18)$$

Furthermore, if all queues in a network are stable, then the network is said to be stable.

***Definition* 2.** *The capacity region Λ associated with the network is the closure of the set of all flow arrival rates that can be stably supported by the network. That is, for a network to be stable, we must have $\{\lambda_f\}_{f\in\mathcal{F}} \in \Lambda$. Furthermore, a scheme that can stabilize all flows that have arrival rates in the capacity region is known as a throughput-optimal scheme.*

If the route of every flow f is fixed, then the capacity region is the set of all flow arrivals that are supportable given the set of flows and their corresponding routes. We note that the capacity region of a network under fixed flow routes cannot be larger than the capacity of the same network where routes of the flows are not fixed, since capacity region cannot decrease by removing path constraints on flows.

The traffic control and routing algorithms proposed above are based on the algorithms for scheduling (rate allocation) and routing of packets in a communication network. These algorithms are throughput-optimal according to [26, Theorem 4] and [27, Theorem 1], where the authors show that if a network is stable under the back-pressure scheme based on real queues (which is already known to be optimal) with flow arrival rate λ_f, then it is also stable (i.e., real queues are stable) under the back-pressure scheme based on virtual queues with flow arrival rate $\lambda_f(1+\epsilon)$ for any $\epsilon > 0$. In the proposed traffic control algorithm, the phase activation (activation of a set of movements) procedure is equivalent to scheduling over a communication network. In contrast to a wireless communication network, this scheduling procedure is decentralized at every junction because the activation of links associated with one junction does not affect the activation of links at any other junction if one assumes links to be infinitely long. The possible traffic movements associated with a given junction can be interpreted as connections in a wireless network. Similar to a wireless network where packets cannot be simultaneously transmitted with high rates over neighboring links due to cross-link interference, here in the road network, some movements at a junction cannot be activated simultaneously. With this interpretation and equivalence in mind, we establish optimality of the proposed traffic control and routing algorithms. The algorithms are optimal in the sense that they can stably support any flow arrival rate which is in the interior of the capacity region.

2) Finite Length Links: When a network has links with finite lengths, the issue of stability (according to Def. 1) does not arise because the queues can never be unstable due to finite length links. In this situation, stability corresponds to maintaining bounded queue backlogs in the links where traffic is being input to the network (ingress buffers in the context of communication networks [29]), assuming that origin links can be infinitely long. It is not known if the back-pressure based schemes are throughput-optimal in this context. In the following section, we analyze performance of the proposed back-pressure based algorithms over a network having links of finite lengths.

IV. Performance Analysis

We analyze performance of the proposed algorithms in terms of queue lengths, trips completed, and travel time using PTV VISSIM [30], which is a microscopic traffic simulator. Within VISSIM, every vehicle is simulated individually and several useful properties related to every vehicle can be accessed dynamically. We will consider and compare four distinct methods:

- *Fixed time (FT) schedule signal controller:* The possible phases at each intersection are activated in a predetermined periodic fashion. All vehicles are assumed to follow shortest routes to their respective destinations.
- *Single-commodity back-pressure (SC-BP) controller:* As proposed in [16], each junction i maintain queues $Q_a(t)$ for all connected links. For each pair $(R_a, R_b) \in \mathcal{M}_i$, the back pressure $W_{ab}(t) = Q_a(t) - Q_b(t)$ is computed. For each phase $p^i_k \in \mathcal{P}_i$, the pressure release is computed as $S_{p^i_k}(t) = \sum_{(R_a,R_b)\in p^i_k} W_{ab}(t)s_{ab}(t)$. Finally, the phase giving rise to the maximum pressure release is selected. This approach is similar to Section III-B, but does not distinguish between different flows. All vehicles are assumed to follow shortest routes to their respective destinations.

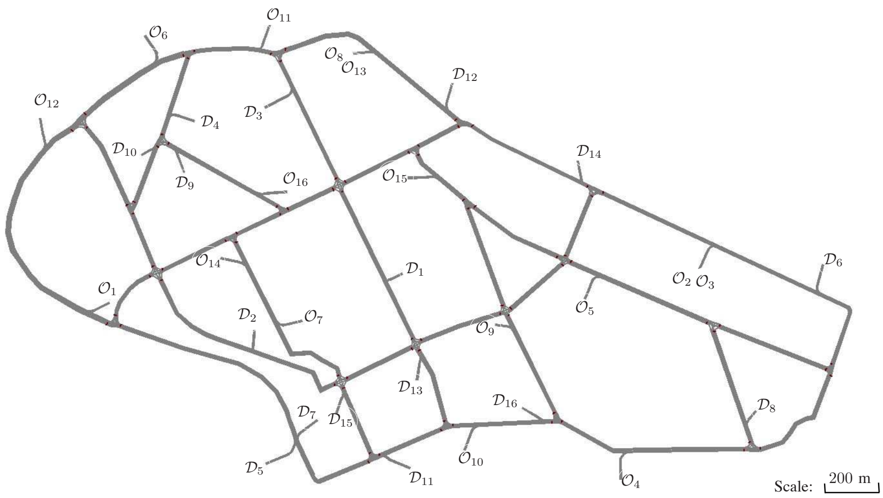

Fig. 4: Road network with 24 intersections, 84 links, 16 origins and 16 destinations.

- *Multi-commodity back-pressure control (MC-BP) with fixed routing:* The method described in Section III-B. Moreover, all vehicles are assumed to follow shortest routes to their respective destinations.
- *Adaptive routing back-pressure control (AR-BP):* The method described in Section III-C.

For the sake of simplicity, we assume that $s_{ab}(t) = s_{ab}$ for all t, i.e., flow rate through a junction does not depend on time or any other state[2] in the road network. However, the traffic signal control schemes presented above also applicable to the situations where traffic movement rates are time varying.

A. Network and Simulation Parameters

The simulations are performed using a road network from a central region in the Stockholm area, comprising 24 signalized intersections (16 three-way intersections and 8 four-way intersections) and 84 links. The network is depicted in Fig. 4. The lengths of the longest and the shortest links are approximately 1980 meters and 333 meters. All links are assumed to have three lanes, where each lane is 3.5 meters wide. There are 16 traffic origins $\{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_{16}\}$ and 16 destinations $\{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_{16}\}$ in the network. The traffic associated with an origin-destination pair $\mathcal{O}_i - \mathcal{D}_i$ forms a flow f_i. Hence, there are 16 traffic flows in total, $\{f_1, f_2, \ldots, f_{16}\}$. We perform simulations with cars of dimensions 4.11 meters $\times$ 1.5 meters and 4.76 meters $\times$ 1.5 meters. The maximum speed of all vehicles is set to 70 km/h, as some of the links on the boundary of the network in Fig. 4 are highways. We assume that a car is in a queue if its speed is below a certain threshold (here set to 5 km/h). For the fixed time schedule control (FT), we assume the time period of each cycle equal to 60 seconds at all intersections (both three-way and four-way) according to the signal plan (phase distribution) given in Table I[3]. For the back-pressure methods, we consider that

[2]For example, the state of the network may change in case of an accident.

[3]The time-loss due to amber or yellow signals is not considered in the simulations, however, it can be incorporated easily in VISSIM.

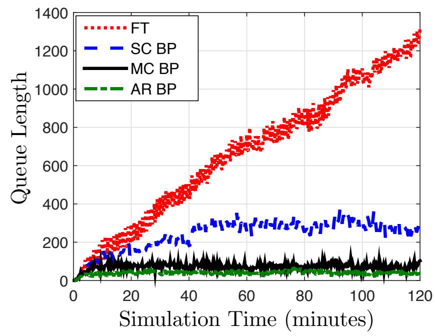

Fig. 5: Evolution of total queue length in the network over time under different control methods with vehicle input rate equal to 350 vehicles/hour.

a phase is activated after every 15 seconds. Moreover, all the traffic related measurements are also taken after every 15 simulation seconds in order to update the signal phases and the routing decisions under the back-pressure methods. All simulations are performed for 7200 simulation seconds (i.e., 2 hours). Within VISSIM, we have set the simulation speed to 10 simulation seconds per second and the simulation resolution is set equal to 1 in order to generate fastest simulation results. Further details on how the simulations were performed using VISSIM are given in Appendix A.

TABLE I: Phase distributions

Intersection Type	p_1	p_2	p_3	p_4
Four-way	18 sec.	12 sec.	18 sec.	12 sec.
Three-way	24 sec.	12 sec.	24 sec.	–

B. Simulation Results and Discussions

In Fig. 5, we fix vehicle arrival rate to 350 vehicles/hour at all traffic origins and plot the evolution of queue length over time under different signal control methods. Here, queue length refers to the total number of vehicles that are queued in the network. An arrival rate of 350 vehicles/hour per origin

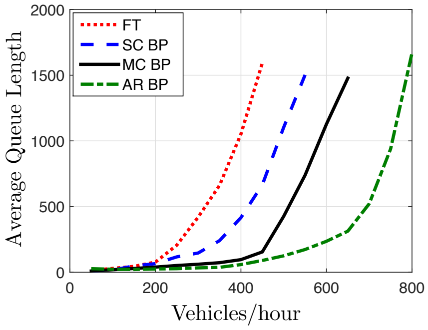

Fig. 6: Average queue lengths over a 2 hours long simulation time period as a function of the vehicle input arrival rate under different control methods.

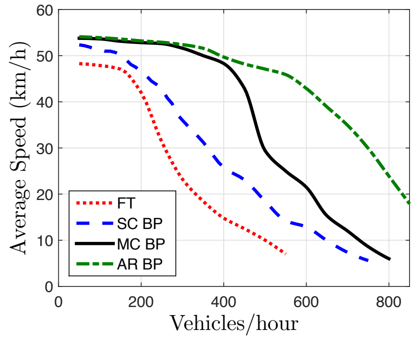

Fig. 8: Average vehicle speeds computed over a 2 hours simulation time period as a function of the vehicle input arrival rate under different control methods.

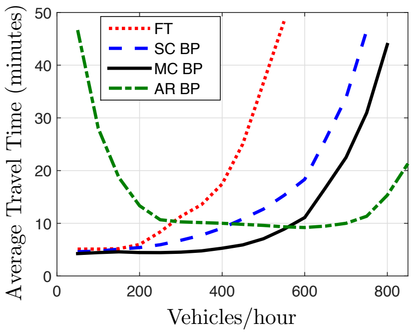

Fig. 7: Average travel times computed over a 2 hours simulation period as a function of the vehicle input arrival rate under different control methods.

means that approximately 5600 vehicles enter the network per hour since there are 16 origins. As we observe in Fig. 5, AR-BP yields smallest queue length followed by MC-BP, SC-BP, and FT respectively. Queue length under FT is increasing approximately linearly over time, whereas for the BP based methods the queue lengths remain bounded. It was already shown in [16] that SC-BP outperforms SCATS (where a signal plan is optimized over a set of fixed signal schedules) in terms of queue lengths. An additional observation is that the multi-commodity back-pressure schemes (MC-BP, AR-BP) have the potential to provide considerable improvement over single-commodity scheme.

In order to investigate further, we plot the average queue lengths and average travel times of vehicles from their origins to their respective destinations as functions of vehicle arrival rates in Fig. 6 and Fig. 7 respectively. Here the averages are taken over simulation time in the case of queue length and over both simulation time and number of vehicles in the case of travel time. According to Figs. 6–7, MC-BP is significantly superior to SC-BP in terms of both average queue length and average travel time at all vehicle arrival rates. The behavior of AR-BP is not straightforward – it provides relatively much smaller queue lengths but the travel times are very high at low traffic volumes. Normally, one expects that a larger queue length should lead to a higher travel time. In order to study what makes the average travel times so high under AR-BP, we must consider the average speed of vehicles under all schemes. Fig. 8 shows that average vehicle speeds are always highest under AR-BP. This implies that the vehicles travel longer distances on average to reach their destinations under AR-BP, especially when the vehicle arrival rates are low. Under MC-BP, a path for every vehicle from its origin to its destination is pre-defined, whereas in AR-BP a next hop route is chosen at every intersection. When the network is under-saturated, the pressure terms (queue backlog differences) are very low and a vehicle may traverse several links before arriving its destination, thus taking a route that is unnecessarily long. However, it is this adaptive routing that forces the vehicles to distribute in the network more uniformly and thus reduces congestion queue lengths when the network is heavily loaded. In a saturated network, although vehicles may follow a longer route on average under AR-BP, the travel time is significantly lower on average compared to the fixed routing methods as shown in Fig. 7. This reduction in average travel time happens due to a smaller queue lengths in the network as observed in Fig. 6.

In Section III-C3 we presented a modified version of AR-BP scheme that can force vehicles to avoid unnecessarily long routes in a load situation. This modified AR-BP method can be optimized for a given network. In Fig. 9 and Fig. 10, we plot average travel time and average queue lengths under the modified AR-BP scheme with different values of α. (Note that $\alpha = 0$ gives the original AR-BP scheme.) A higher value of α forces the vehicles to follow shorter path, which is good for low-load situations but may not be good in a high load situation. According to Fig. 9 and Fig. 10, there exits a value of α (equal to 1.5) for the given network that provides good performance in both low load and high load scenarios in terms of travel time as well as congestion.

Next we investigate the network throughput in terms of the number of vehicles exiting the network (number of completed trips) under different signal control methods. In Fig. 11, we plot the total number vehicles that exit the given network in two hours when the traffic is continuously arriving at a fixed rate. Interestingly, FT provides higher throughput than SC-BP at very high input traffic load, despite the fact that FT always gives rise to a higher time averaged queue length than

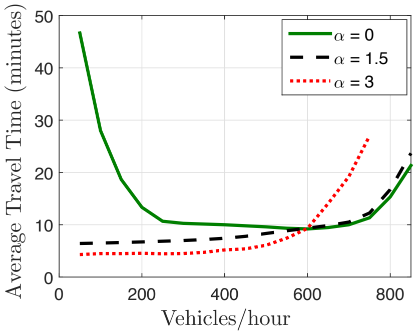

Fig. 9: Average travel time computed over a 2 hours simulation time period as a function of the vehicle input arrival rate under the modified AR-BP method.

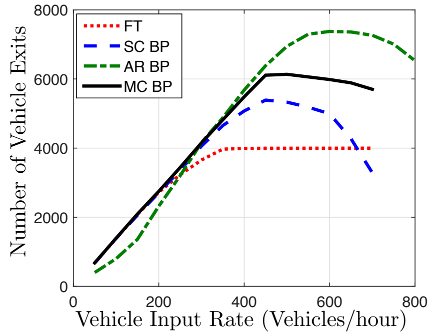

Fig. 11: Total number of vehicles that exit the network, i.e., reach their destinations during a 2 hour simulation period under different control methods.

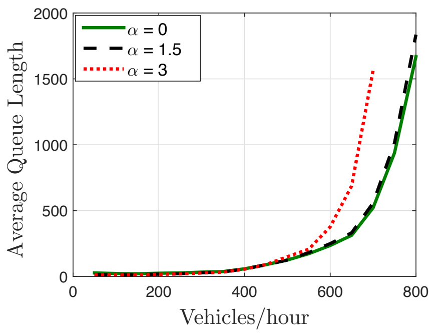

Fig. 10: Average queue length computed over a 2 hours simulation time period as a function of the vehicle input arrival rate under the modified AR-BP method.

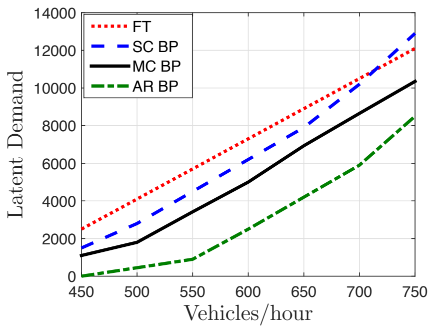

Fig. 12: Total number of vehicles that could not enter the network until the end of 2 hour simulation period under different control methods.

SC-BP according to Fig. 6. This happens due to the fact that when back-pressure schemes are employed over a network with finite length links, some links can experience deadlock situation, as observed in [19]. Deadlocks make the controllers non-work conserving[4] and may cause congestion propagation to other links in a network. The deadlocks occur at very high traffic loads depending on the network topology and especially when there is a significant mismatch between lengths (or capacities) of adjacent links. Note that our simulated network is quite asymmetric in terms of lengths of different links and therefore it is also more susceptible to deadlocks. One way of resolving deadlocks under SC-BP is to use normalized pressure functions [19]. Interestingly, the proposed back-pressure schemes MC-BP and AR-BP are robust against deadlocks because their control decisions are based on virtual queues that keep growing irrespective of the lengths (capacities) of their corresponding links.

Finally, we analyze performance of the proposed methods under the following two measures that are relevant in high load situations: i) latent demand and ii) latent delay. Latent demand refers to the total number of vehicles that are waiting till the end of simulation to enter the network. Latent delay refers to the total waiting time of all vehicles that are not able to immediately enter the network. This also includes waiting time (outside the network) of the vehicles which were later able to enter the network before the end of the simulation time. In Fig. 12 and Fig. 13, we have plotted latent demand and latent delay as functions of vehicle arrival rates, respectively. These simulation results also indicate the benefits of using virtual queues in back-pressure methods and adaptive routing in general for saturated networks.

From the above discussion, we conclude that the multi-

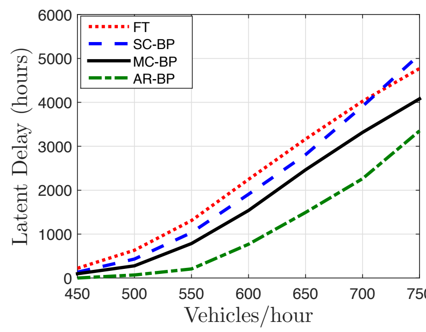

Fig. 13: Total waiting time of all vehicles that could not enter the network immediately during 2 hour simulation period under different control methods.

[4] A control is work-conserving if the two conditions, i) $Q_{ab}(t) > 0$ and ii) $Q_b(t) < C_b$, are sufficient to ensure that the server of the junction works during slot t, where C_b is capacity of link b. Non-work conservation is a sign of inefficiency [19].

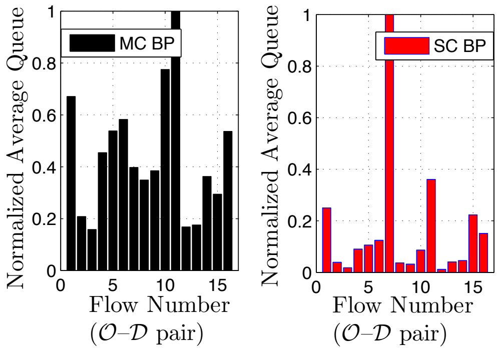

Fig. 14: Normalized queue lengths at vehicle arrival rate equal to 350 vehicles/hour.

commodity back-pressure control methods are significantly superior to the single-commodity back-pressure traffic method in terms of travel time, queue length, and trips completed. Another advantage of employing multi-commodity schemes is that it allows for relatively fair distribution of vehicle queues associated with different origin-destination pairs (flows) within the network. As an example, in Fig. 14 we have shown the normalized average queue lengths of all flows (origin-destination pairs) in the network under SC-BP and MC-BP, with a traffic arrival rate equal to 350 vehicles/hour. We can see that the queue length distribution among different traffic flows is more fair when the multi-commodity scheme is employed.

Remark on Communication Requirements: The performance gains discussed under the proposed back-pressure based methods (fixed and adaptive routing) are achieved assuming perfect communication between vehicles and controllers. In MC-BP and AR-BP, every vehicle has to communicate with upcoming controller and/or the adjacent controllers, depending on how these schemes are realized in practice. Moreover, under AR-BP the controllers have to communicate the routing information to the vehicles. The routing probabilities calculated according to (15) can be broadcast to all vehicles in the form of a look-up table and the vehicles would then adapt their routes depending on the received routing probabilities. We believe that the results presented in this paper provide motivation for analyzing back-pressure schemes under imperfect vehicle-to-infrastructure communication and exploring relevant communication protocols.

V. Conclusions and Future Directions

We studied the problem of decentralized traffic signal control and adaptive routing of vehicles under different back-pressure control schemes, namely, single-commodity back-pressure (SC-BP), multi-commodity back-pressure (MC-BP), and adaptive routing back-pressure (AR-BP). The proposed back-pressure methods address network level traffic control by means of the interacting queue dynamics on adjacent roads. Note, however, that the algorithms are applied locally, following decentralized control policies, relying on knowledge of adjacent queues. We observed that MC-BP always outperforms SC-BP in terms of average queue lengths, average vehicle travel time, and the number of trips completed. In addition, multi-commodity methods allow for relatively fair distribution of vehicle queues associated with different origin-destination pairs (flows) within the network. Due to the use of virtual queues, MC-BP and AR-BP are more robust to deadlock situations than SC-BP.

In SC-BP and MC-BP, all vehicles follow fixed routes. Fixed routing is not appropriate when a road network is heavily loaded with vehicles, since links will get more congested. AR-BP is able to to distribute vehicles more uniformly across the network, thereby significantly improving congestion, throughput, and travel times (on average). For low load situations, AR-BP may lead to unnecessarily long route selections for some vehicles, giving very high travel times on average. In such situations, a simple fixed-time control can have better performance. Alternatively, a modified version of AR-BP can be used to reduce travel times by restricting route selection from a set of fewer paths. In practice, the proposed routing method can be complemented with additional intelligence to avoid unnecessarily long routes in the case of very low traffic demand.

The implementations of MC-BP and AR-BP require communication from every vehicle to the traffic controller located at the upcoming intersection and/or between adjacent controllers, depending on how these schemes are realized in practice. For AR-BP, the controller also needs to broadcast routing information comprised of routing probabilities (or turning percentages) to the vehicles. An interesting direction in future would be to investigate MC-BP and AR-BP schemes subject to uncertain and delayed queue information.It will be useful to devise suitable protocols for vehicle-to-infrastructure communication. The proposed methods have been shown to be optimal in the sense of supporting maximum traffic arrival rate while maintaining stable queues under the assumption of infinite length links. The stability regions of a general network under the BP schemes are only known under the assumption of infinite length links. Thus, another useful direction would to characterize the stability regions of a road network with finite-length links under different BP schemes. Here, stability of a network would mean that all queues at the traffic origins are stable. All back-pressure based signal control algorithms proposed so far assume a fixed (pre-defined) signal phase duration. It will be interesting to study how much we can gain by keeping both phase duration and phase activation as functions of queue length information.

Appendix A
Simulation Details

The simulations are performed by allowing data exchange between MATLAB and VISSIM using the following procedure: Create the network file (.inp file) using VISSIM GUI, based on a network image taken from Google Earth. Drop all relevant objects (vehicle inputs, controllers, routing decisions, data collection points) in the network with desired initial parameters/settings. Save these settings in a file (.ini file). Create a MATLAB script, where first activate the VISSSIM COM server, and then load the network file and the settings file. Within the MATLAB script, one can access all relevant objects and change the control signals on state of the traffic.

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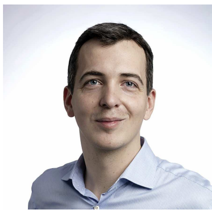

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