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PAPER

Nonresonant high-frequency excitation of mechanical vibrations in a movable quantum dot

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Abstract

Nonresonant high-frequency electrostatic actuation of a movable quantum dot (QD) is investigated analytically. The electronic dot state is in tunneling contact with a continuum of electronic states in bulk electrodes. Gate electrodes induce an electric ac-field at the dot. Absorbtion of the field is accompanied by increment or decrement of one vibrational quantum in the QD. It is shown that the rate of increment overcomes the rate of decrement of vibrational quanta when the driving frequency exceeds the inverse tunneling time between the bulk electrodes and the QD. This results in a mechanical instability, leading to relatively large amplitude oscillations of the QD, which are saturated by nonlinear dissipation.

Introduction

Nanoelectromechanical resonators are attracting much attention because of their high quality factors [1], low mass and tunable dynamics [2]. Nanoelectromechanical oscillators are applicable for low mass sensing [3], electrical transducers [4] and charge sensing [5] among others. The general approach to control nanoelectromechanical systems is to couple external electric fields to the mechanical motion. The most straightforward method to excite mechanical vibrations is to apply an ac-field at a frequency comparable with the eigenfrequency of the mechanical vibration [4, 6–10]. Another way to actuate vibrations is to integrate the mechanical resonator in an electrical LC-circuit, and to drive the mechanical motion by using the side bands of the electrical resonance [11–13]. A similar approach is utilized in optomechanical cavities where the mechanical vibration is controlled by the detuning from the cavity resonance [14–17]. A new nonresonant actuation mechanism of mechanical vibrations in graphene based resonators was recently considered [18], where the capacitance model was used to describe the dynamics of the electrical subsystem. In this description, overdamped charge relaxation rather than resonant properties of the electronic subsystem was exploited to drive the mechanical vibrations. This finding gave rise to the question how the nonresonant excitation mechanism would manifest itself in a quantum mechanical system where the electronic subsystem is quantized. Further, if the mechanical dynamics is excited by this mechanism, what mechanisms then saturate the excitation? In the present article, these questions are addressed by investigation of a quantum mechanical model of the nonresonant excitation mechanism for a movable single-level QD. The QD is coupled to a continuum electronic reservoir by quantum tunneling. Systems of this kind have been extensively studied considering electromechanical backaction [19, 20] and nonlinear effects [5, 21, 22]. The electromechanical coupling has been investigated in both the strong [23] and weak [24] regime and the system can in many cases be described by a mechanical resonator coupled to an effective thermal bath [25–27]. Further, a critical parameter in these systems is the ratio between the vibrational frequency ω_m and the tunneling frequency Γ. The current noise, which is an important characteristic of the system, takes sub-Poisson values in the high-frequency [28] $\omega_m \gg \Gamma$ and coherent [29] $\omega_m \sim \Gamma$ regimes but exceeds the Poisson value in the low-frequency [24] regime. In the following article, strong feedback in the weak coupling and low-frequency regime due to a high-frequency electric field is studied. Special attention is paid to the nonlinear saturation mechanism in the pumping regime.

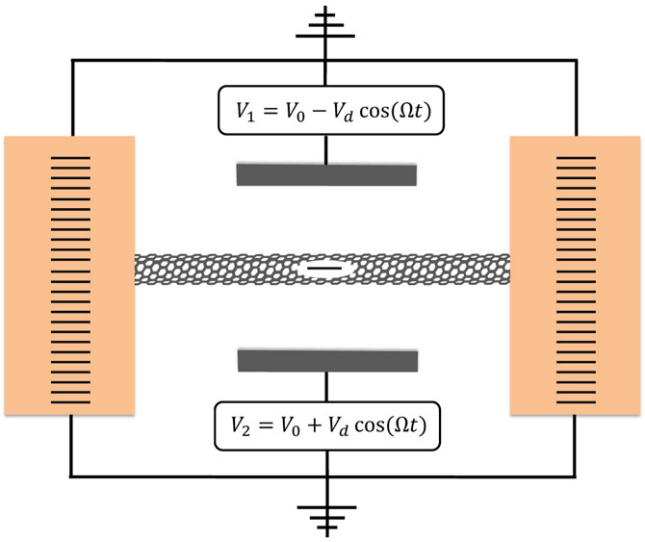

Figure 1. The QD on a suspended CNT is free to perform vertical oscillations. The driving ac-voltage V_d applied to the gate electrodes (dark gray) introduces a coupling between the mechanical vibration and electronic subsystem. The black bars indicate electronic states.

The resulting physics resembles of a mechanical resonator driven at the nonresolved side bands of an optomechanical cavity [30].

Hamiltonian model

The model system under consideration is a movable QD suspended between two grounded electrodes and symmetrically placed between two voltage biased capacitor plates, figure 1. The QD is free to perform vertical oscillations and is in tunnel contact with the continuous density of states of the electrodes. If the QD is deflected, its energy level is shifted due to the electric field $E(t) = E_0 \cos(\Omega t)$, with driving frequency Ω, induced by the gates. This affects the charge occupation on the QD. Simultaneusly, the charge in the electric field generates a backaction force on the mechanical subsystem. The aim is to investigate this feedback when the mechanical frequency $\omega_{\rm m}$ is much slower than is the tunneling frequency Γ.

The electronic spectrum of the electrodes is described as a continuous spectrum of states with chemical potential μ. The electronic states on the QD are reduced to a single level with energy ϵ_d. The energy ϵ_d is assumed to be close to the chemical potential μ of the electrodes. A physical realization of such a system can be achieved by suspending a semiconducting carbon nanotube and to tune its electronic bands into a single level by a static electric field between the gate and lead electrodes. The QD is symmetrically tuned by two gate electrodes at the same static potential V_0 relative to the bulk electrodes, to reduce the Joule heating in the system. Further effects due to asymmetry has been studied in similar systems [31] but will be disregarded here. The energy level on the QD couples to all the electronic states in the electrodes with tunneling strength $\mathcal{T}$. Both $\mathcal{T}$ and the density of states in the electrodes ν are assumed to be constant.

The mechanical motion of the QD is described as a harmonic oscillator with effective mass m, amplitude of zero-point fluctuation $a_0 = \sqrt{\hbar/(2\, m\omega_{\rm m})}$ and raising (lowering) operator $c^\dagger$ ($\hat{c}$).

The Hamiltonian of the coupled electronic and mechanical subsystems takes the form

$$\hat{H} = \int {\rm d}\tilde{\epsilon}\nu\tilde{\epsilon}\hat{l}_{\tilde{\epsilon}}^\dagger\hat{l}_{\tilde{\epsilon}} + \epsilon_d({\rm t})\hat{d}^\dagger\hat{d} + \int {\rm d}\tilde{\epsilon}\nu\mathcal{T}\left(\hat{l}_{\tilde{\epsilon}}^\dagger\hat{d} + \hat{l}_{\tilde{\epsilon}}\hat{d}^\dagger\right) + \hbar\omega_{\rm m}\hat{c}^\dagger\hat{c},$$
$$\epsilon_d(t) = \epsilon_0 + eEa_0\cos(\Omega t)\left(\hat{c}^\dagger + \hat{c}\right), \qquad (1)$$

where $\tilde{\epsilon}$ are the energies of the electrode states with corresponding creation (annihilation) operators $\hat{l}_{\tilde{\epsilon}}^\dagger$ ($\hat{l}_{\tilde{\epsilon}}$) and ν is the density of states at μ in the electrodes. The creation (annihilation) operator $\hat{d}^\dagger$ ($\hat{d}$) corresponds to the single state on the dot. The electric field with field strength E shifts the energy of the dot level. The shift is proportional to the mechanical displacement which gives an electromechanical coupling. The energy of the level at zero ac-electric field is given by ϵ_0 and e is the elementary charge. Diagonalizing the time-independent part of the electronic subsystem into a hybridized Fermi sea transforms the Hamiltonian (1) into the form

$$\hat{H}(t) = \hat{H}_0 + \left(e^{i\Omega t} + e^{-i\Omega t}\right)\hat{H}_{\rm int},$$
$$\hat{H}_0 = \int d\epsilon\nu\epsilon\hat{\psi}_\epsilon^\dagger\hat{\psi}_\epsilon + \hbar\omega_{\rm m}\hat{c}^\dagger\hat{c},$$
$$\hat{H}_{\rm int} = \frac{eEa_0}{2}\left(\hat{c}^\dagger + \hat{c}\right)\hat{d}^\dagger\hat{d}, \tag{2}$$

with creation (annihilation) operators of the hybridized Fermi sea $\hat{\psi}_\epsilon^\dagger(\hat{\psi}_\epsilon)$ corresponding to the level with energy ϵ. These operators relate to the dot state operators according to

$$\hat{d}^\dagger = \int d\epsilon\nu\frac{\mathcal{T}}{\epsilon - \epsilon_0 + i\hbar\Gamma}\hat{\psi}_\epsilon^\dagger, \tag{3}$$

where $\Gamma = \pi\mathcal{T}^2\nu/\hbar$ is the tunneling frequency between the dot and electrodes.

Tunneling rates and excitation of mechanical quanta

To investigate the possibility of an excitation mechanism caused by the electromechanical interaction, the aim of this section is to derive and analyse the rate equation for mechanical quanta. The interaction term induces Stokes- and anti-Stokes-like processes in which one electron and one quantum from the external and mechanical fields, respectively, are involved. At zero temperature, only stimulated absorption of the field is induced. During the absorption an electron is lifted from the hybridized Fermi sea above its surface and simultaneously one vibrational quantum is either absorbed or emitted. The excited state of the electronic subsystem is assumed to relax to its ground state at a time scale much faster than the time between two stimulated absorption processes. In the considered limit $\Omega \gg \omega_{\rm m}$, no substantial stimulated emission to the electromagnetic field takes place as long as $k_{\rm B}T \ll \hbar\Omega$.

The interaction changes the total number of vibrational quanta in the QD. If the rate of increment of vibrational quanta overcomes the rate of decrement, the mechanical motion becomes unstable. To analyse this possibility let us investigate the average stationary occupation of mechanical quanta $\mathcal{N}$.

We calculate $\mathcal{N}$ by solving the quantum Liouville equation

$$i\hbar\frac{\partial}{\partial t}\hat{\rho} = \left[\hat{H}_0 + \left(e^{i\Omega t} + e^{-i\Omega t}\right)\hat{H}_{\rm int},\ \hat{\rho}\right] + i\hbar\hat{\mathcal{L}}\left(\hat{\rho}\right), \tag{4}$$

with density operator $\hat{\rho}$ and Lindblad superoperator $\hat{\mathcal{L}}$ describing the mechanical dissipative coupling to an environmental bath which will be discussed in detail later. Let us assume the electronic subsystem to always be in its equilibrium distribution due to fast internal relaxation. The density operator then takes the form $\hat{\rho}(t) = \hat{\rho}_m(t)\otimes\hat{\rho}_{\rm eq}$, where $\hat{\rho}_{\rm m}(t)$ is the density operator of the mechanical subsystem and the electronic subsystem fulfills $\left[\hat{H}_0,\ \hat{\rho}_{\rm eq}\right] = 0$. We expand the mechanical denisty operator $\hat{\rho}_m(t) = \sum_{n=-\infty}^{\infty}\hat{\rho}_n\exp(in\Omega t)$, where $\hat{\rho}_n = \sum_{k=0}^{\infty}\hat{\rho}_n^{(k)}$ and $\hat{\rho}_n^{(k)} \propto \varepsilon^{|n|+2k}$ with $\varepsilon = eEa_0/(2\hbar\Omega) \ll 1$. Further, let us assume the stationary term $\hat{\rho}_0^{(0)}$ to be diagonal in the $|n\rangle$ basis due to dephasing processes not governed by equation (4). We include terms to second order in ε and trace over the electronic subsystem. The stationary mechanical density matrix $\hat{\rho}_{\rm st} = {\rm Tr}_{\rm el}\left(\hat{\rho}_0^{(0)}\otimes\hat{\rho}_{\rm eq}\right)$ fulfills the stationary equation

$$\Gamma^-\left(\hat{c}\hat{\rho}_{\rm st}\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\hat{\rho}_{\rm st}\right) + \Gamma^+\left(\hat{c}^\dagger\hat{\rho}_{\rm st}\hat{c} - \hat{c}\hat{c}^\dagger\hat{\rho}_{\rm st}\right) = -\hat{\mathcal{L}}\left(\hat{\rho}_{\rm st}\right), \tag{5}$$

with $\Gamma^\pm = \Gamma_{+\Omega}^{\pm\omega_{\rm m}} + \Gamma_{-\Omega}^{\pm\omega_{\rm m}}$ and

$$\Gamma_{\pm\Omega}^{\pm\omega_{\rm m}} = \frac{\left(eEa_0\right)^2}{2\pi\hbar^2\Gamma}\int d\epsilon d\epsilon'\nu^2 n_{\rm f}(\epsilon)\left(1 - n_{\rm f}\left(\epsilon'\right)\right) \times \frac{(\hbar\Gamma)^3\delta\left(\epsilon' - \epsilon \pm \hbar\Omega \pm \hbar\omega_{\rm m}\right)}{\left|\epsilon - \epsilon_0 + i\hbar\Gamma\right|^2\left|\epsilon' - \epsilon_0 + i\hbar\Gamma\right|^2}, \tag{6}$$

where $n_{\rm f}(\epsilon)$ is the Fermi distribution. The rates $\Gamma_{\pm\Omega}^{\pm\omega_{\rm m}}$ are equivalent to the rates obtained by applying Fermi's golden rule to the electronic transitions. The plus signs in $\Gamma_{\pm\Omega}^{\pm\omega_{\rm m}}$ correspond to one quantum being added to the mechanical or external oscillation, respectively. Analogously a minus sign corresponds to the removal of one quantum from the relevant oscillation. The rate $\Gamma^+ = \Gamma_{+\Omega}^{+\omega_{\rm m}} + \Gamma_{-\Omega}^{+\omega_{\rm m}}$ describes the combined processes of stimulated absorption and emission which increase the number of vibrational quanta. Similarly, we have the rate of decrease of vibrational quanta $\Gamma^- = \Gamma_{+\Omega}^{-\omega_{\rm m}} + \Gamma_{-\Omega}^{-\omega_{\rm m}}$. In the low temperature limit of the Fermi sea $k_{\rm B}T \ll \hbar\Omega$ and in the case $\mu = \epsilon_0$ the rates take the form

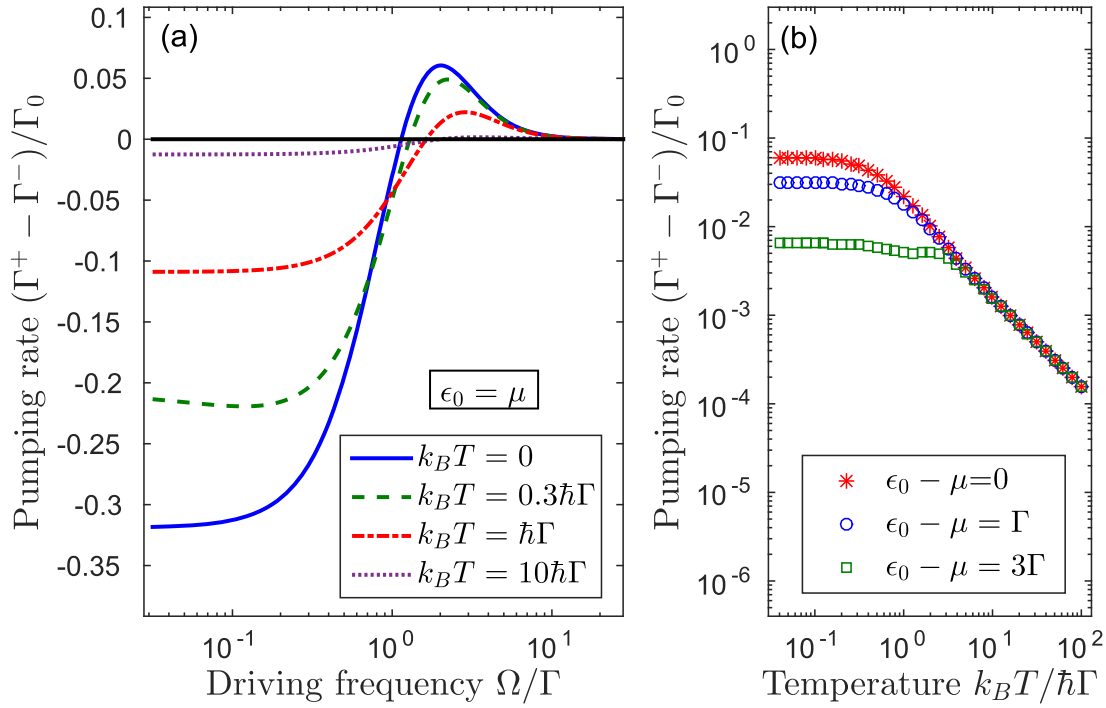

Figure 2. Pumping rate of the mechanical motion introduced by the electric field with $\Gamma_0 = \omega_m (eEa_0)^2/(\hbar^2\Gamma)$. Above $\Omega > \Gamma$ in (a), the induced pumping region is reached where the net effect is that vibrational quanta are pumped into the vibration. For temperatures $k_B T > \hbar\Gamma$ in (b), the largest achievable pumping diminishes with increasing temperature. The optimal driving frequency to achieve this pumping depends slightly on temperature but is of the order of Γ. The plots were created with the ratio $\omega_m/\Gamma = 10^{-2}$ but the plots can be used whenever $\omega_m \ll \Omega \sim \Gamma$.

$$\Gamma_{-\Omega}^{\pm\omega_m} = \frac{\left(eEa_0\right)^2}{\pi\hbar^2\Gamma}\chi\left(\frac{\Omega \mp \omega_m}{\Gamma}\right), \tag{7}$$

$$\chi(x) = \frac{x\,\arctan(x) + \ln\left(1 + x^2\right)}{x\left(4 + x^2\right)}. \tag{8}$$

We start by analysing the influence of these rates in the case of no intrinsic mechanical dissipation. If the rate of decrease of vibrational quanta Γ^- exceeds the rate Γ^+, the steady state probability to find the system with n vibrational excitations is given by the Boltzmann distribution with an effective temperature $T_{\rm eff}$. However, if $\Gamma^+ - \Gamma^- > 0$, the vibrational motion becomes unstable and grows exponentially with the rate $\Gamma^+ - \Gamma^-$. Figure 2(a) displays $\Gamma^+ - \Gamma^-$ and shows that the vibrational motion becomes unstable when $\hbar\Omega/\Gamma$ is slightly larger than 1. The pumping mechanism is a nonresonant phenomenon since the condition on Ω to give effective pumping is independent on ω_m in the considered limit $\Gamma \gg \omega_m$, figure 2(a). Hence, a movable quantum dot coupled to a continuum via tunneling elements can be actuated mechanically by nonresonant high-frequency electric fields if the induced pumping overcomes the intrinsic mechanical damping.

The influence of a finite temperature is understood by noticing that the matrix elements in equation (6) exhibit detailed balance for the processes $-\Omega - \omega_m \leftrightarrow +\Omega + \omega_m$ and $-\Omega + \omega_m \leftrightarrow +\Omega - \omega_m$, respectively. If the temperature of the environment is increased to $k_B T \sim \hbar\Omega$, the occupation of the electronic states are smoothed, which opens the transition channels with emission of $+\hbar\Omega$ back to the electric field. For temperatures $k_B T \gg \hbar\Omega$, the probabilities to absorb and emit vibrational quanta equalizes and the efficiency of the pumping phenomenon is supressed as $1/T$, figure 2(b).

Saturation mechanism

When the mechanical vibration becomes unstable, the amplitude of oscillations will be saturated by some mechanism which prevents $\mathcal{N}$ from diverging. Many resonant oscillator ecitations are saturated by a nonlinear potential proportional to e.g. the Duffing nonlinearity $(\hat{c}^\dagger + \hat{c})^4$ since it decreases the efficiency of the pumping mechanism at larger amplitudes of oscillation. However, the mechanism discussed here is a nonresonant phenomenon and an anharmonic mechanical potential will not saturate the vibration. One possible saturation mechanism can occur if the electronic subsystem is considerably heated by the driving field, which would

decrease the pumping efficiency. However, we will assume the bulk electrodes to efficiently keep the system at low temperature. Further, a linear Lindblad dissipative coupling to a thermal bath at zero temperature

$$\mathcal{L}_{\rm L}\left(\hat{\rho}_{\rm st}\right) = \gamma_{\rm L}\left(\hat{c}\hat{\rho}_{\rm st}\hat{c}^{\dagger} - \frac{1}{2}\left\{\hat{c}^{\dagger}\hat{c},\, \hat{\rho}_{\rm st}\right\}\right) \tag{9}$$

with strength $\gamma_{\rm L}$ will only shift the rate in figure 2(a) downwards, $\Gamma^{-} \to \Gamma^{-} + \gamma_{\rm L}$ for the diagonal $\hat{\rho}_{\rm st}$, reducing the unstable region to a frequency window but not saturate the vibrations within the window. However, a nonlinear Lindblad super operator with strength $\gamma_{\rm NL}$

$$\mathcal{L}_{\rm NL}\left(\hat{\rho}_{\rm st}\right) = \gamma_{\rm NL}\left(\hat{c}\hat{c}\hat{\rho}\hat{c}^{\dagger}\hat{c}^{\dagger} - \frac{1}{2}\left\{\hat{c}^{\dagger}\hat{c}^{\dagger}\hat{c}\hat{c},\, \hat{\rho}\right\}\right) \tag{10}$$

will saturate the vibration. For simplicity, let us limit the analysis to zero temperature of the environmental bath.

With this nonlinear dissipation mechanism the stationary mechanical occupation is given by the stationary occupation probabilies $P_n = \langle n\,|\hat{\rho}_{\rm st}|\,n\rangle$ following

$$\begin{aligned}(n+1)(n+2)P_{n+2} + \frac{\Gamma^{-}+\gamma_{\rm L}}{\gamma_{\rm NL}}(n+1)P_{n+1} + \frac{\Gamma^{+}}{\gamma_{\rm NL}}nP_{n-1} \\ - \left(n(n-1) + \frac{\Gamma^{-}+\gamma_{\rm L}}{\gamma_{\rm NL}}n + \frac{\Gamma^{+}}{\gamma_{\rm NL}}(n+1)\right)P_n = 0.\end{aligned} \tag{11}$$

To solve this difference equation we will follow the method used by Nord and Gorelik [32]. We introduce $\mathcal{P}(z) = \sum_{n=0}^{\infty} z^n P_n$, where z is a complex number inside the unit circle. This turns equation (11) into the second order differential equation in

$$(1+z)\frac{\partial^2}{\partial z^2}\mathcal{P}(z) + \frac{\Gamma^{-}+\gamma_{\rm L}}{\gamma_{\rm NL}}\frac{\partial}{\partial z}\mathcal{P}(z) - \frac{\Gamma^{+}}{\gamma_{\rm NL}}\left(1 + z\frac{\partial}{\partial z}\right)\mathcal{P}(z) = 0, \tag{12}$$

where the constants of integration are given by the absolute convergence criterion at $\mathcal{P}(-1)$ and normalization condition $\mathcal{P}(1) = 1$. The solution takes the form

$$\begin{aligned}\mathcal{P}(z) &= c\left(1 + \frac{\Gamma^{+}}{\gamma_{\rm NL}}\int_{-1}^{z} {\rm d}z'\, g\left(z, z'\right)\right), \\ c &= \left(1 + \frac{\Gamma^{+}}{\gamma_{\rm NL}}\int_{-1}^{1} {\rm d}z'\, g\left(1, z'\right)\right)^{-1},\end{aligned} \tag{13}$$

$$g\left(z, z'\right) = {\rm e}^{\frac{\Gamma^{+}}{\gamma_{\rm NL}}\left(z-z'\right)}\left(\frac{1+z'}{1+z}\right)^{\left(\frac{\Gamma^{+}+\Gamma^{-}+\gamma_{\rm L}}{\gamma_{\rm NL}}-1\right)}. \tag{14}$$

The average number of mechanical quanta in the oscillator is then given by $\mathcal{N} = \partial_z \mathcal{P}(z)$ evaluated at $z = 1$,

$$\mathcal{N} = \frac{1}{2}\left(1 + c\left(\frac{\Gamma^{+}+\Gamma^{-}+\gamma_{\rm L}}{\gamma_{\rm NL}} - 1\right) + \frac{\Gamma^{+}-\Gamma^{-}-\gamma_{\rm L}}{\gamma_{\rm NL}}\right). \tag{15}$$

In the induced damping region $\Gamma^{-} > \Gamma^{+}$, the electromechanical coupling introduces an effective temperature of the mechanical oscillator. In the absence of an external field $E = 0$, the oscillator is damped to its ground state $\mathcal{N} = 0$. With increasing external field, both $\gamma_{\rm L}$ and $\gamma_{\rm NL}$ in equation (12) can be neglected and the average number of mechanical quanta approaches

$$\mathcal{N} \approx \frac{\Gamma}{2\omega_{\rm m}}\frac{\chi\left(\frac{\Omega}{\Gamma}\right)}{\chi'\left(\frac{\Omega}{\Gamma}\right)}, \tag{16}$$

which is independent of the external field strength and intrinsic mechanical damping processes. In the induced pumping region $\Gamma^{+} > \Gamma^{-}$ there is no saturation of $\mathcal{N}$ with increasing field strength.

Cooling of the mechanical oscillator is possible in the induced damping region if the bath temperature T is higher than the effective temperature characterizing the Boltzmann distribution

$$T_{\rm eff} = \frac{\hbar\omega_{\rm m}}{k_{\rm B}\ln(1 + 1/\mathcal{N})}. \tag{17}$$

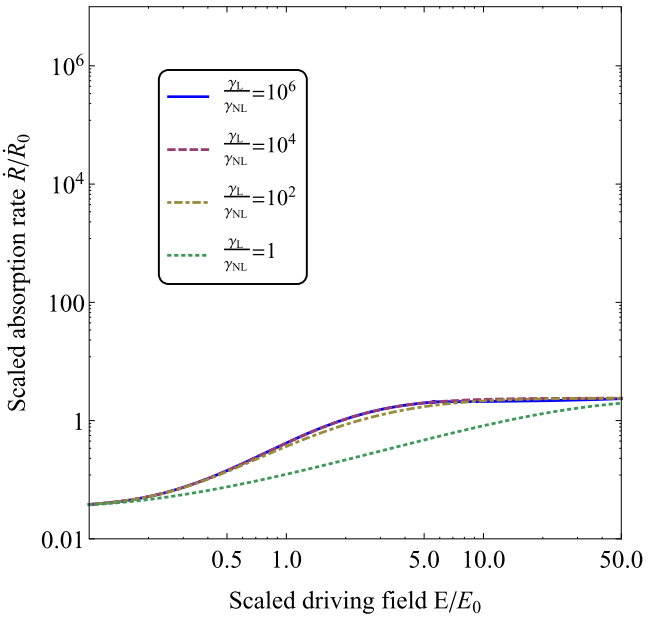

Figure 3. The scaled absorption rate of the driving field in the induced damping region saturates at high-amplitude driving fields, here plotted for $\Omega = \Gamma/2$. The scaled absorption rate increases with increasing field strength. This is due to the increased effective temperature of the mechanical oscillator which increases the average number of mechanical quanta $\mathcal{N}$. The number $\mathcal{N}$ is saturated at high electric fields $E \gg E_0$ where the effective temperature and the fluctuations are determined by the induced processes and the intrinsic processes can be neglected. Hence, at the saturation plateau $\mathcal{N}$ as well as $\dot{R}/\dot{R}_0$ are independend of both the linear and nonlinear intrinsic damping. Here $\omega_m/\Gamma = 10^{-2}$ and $T = 0$ was used.

The effective temperature at the high field plateau when $\Gamma^- - \Gamma^+ \gg \gamma_L$, see figure 3, can be approximated by

$$T_{\rm eff} \approx \frac{\chi(\Omega/\Gamma)}{2\chi'(\Omega/\Gamma)}\frac{\hbar\Gamma}{k_B}, \tag{18}$$

under the assumptions that $\Omega \gg \omega_m$, $\mathcal{N} \gg 1$ and $\hbar\Gamma \gg k_B T$ which gives an effective temperature independent of the intrinsic mechanical damping. For the case $\Omega = \Gamma/10$ and $k_B T = \hbar\Gamma/10$ and $\chi(x)$ is given in equation (8), the temperature of the mechanical oscillator can be lowered by a factor of $T/T_{\rm eff} \approx 2$. Cooling of nanomechanical resonators, by different mechanisms which result in an effective thermal bath, have also been observed in similar systems [25–27], as mentioned in the introduction.

Detection of the instability

One way to experimentally observe one manifestation of the nonresonant excitation phenomenon is to measure the average energy absorption rate $\dot{R}$ from the external electric field into the system. The energy absorption rate can be expressed as

$$\begin{aligned}\dot{R} &\approx \frac{\hbar\Omega}{2}\left(\Gamma_{-\Omega}^{+\omega_m} - \Gamma_{+\Omega}^{+\omega_m} + \mathcal{J}\mathcal{N}\right),\\ \mathcal{J} &= \left(\Gamma_{-\Omega}^{+\omega_m} + \Gamma_{-\Omega}^{-\omega_m} - \Gamma_{+\Omega}^{+\omega_m} - \Gamma_{+\Omega}^{-\omega_m}\right),\end{aligned} \tag{19}$$

in the non-equilibrium stationary state. For simplicity let us again consider the case of zero temperature for both the electrodes and the mechanical damping reservoir. At low external fields $E/E_0 \ll 1$, where $E_0 = \hbar\Gamma\sqrt{\gamma_L/\omega_m}/(ea_0)$, the mechanical subsystem is close to its ground state and $\dot{R}$ is of the order of $\Gamma_{-\Omega}^{+\omega_m}$. An increase of E increases the number of vibrational quanta, which enhances the absorption rate.

The qualitative behavior of the absorption rate in the induced damping and induced pumping region is conveniently analysed by introducing the scaling $\dot{R}_0 = \Omega(eEa_0)^2/(\hbar\Gamma)$. The scaled absorption rate $\dot{R}/\dot{R}_0$ then only depends on the external field strength via the average mechanical quanta $\mathcal{N}$.

In both regions, the effective temperature of the mechanical oscillator is increased at $E \sim E_0$, figures 3 and 4. If E is enhanced further in the induced damping region, the scaled absorption rate saturates due to the saturation of $\mathcal{N}$, figure 3. The saturated value of $\dot{R}/\dot{R}_0 \propto \Gamma/\omega_m$ is independent of both the linear and nonlinear intrinsic mechanical damping. However, in the induced pumping region no saturation of the scaled absorption rate takes place.

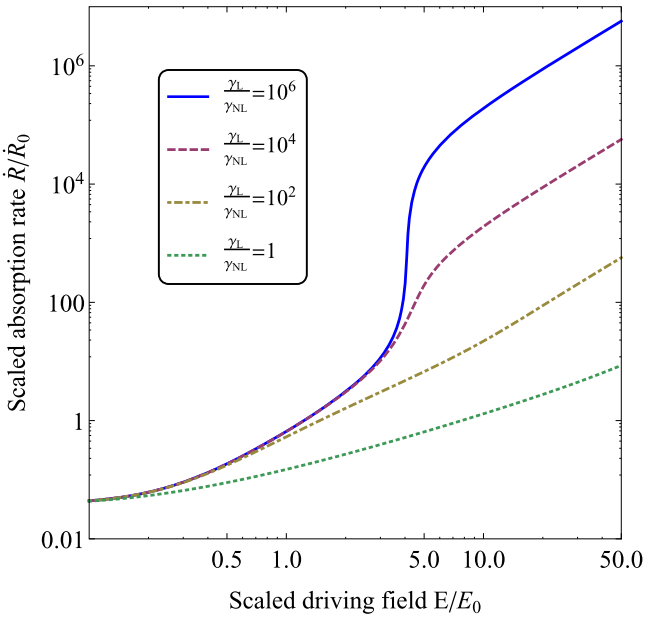

Figure 4. A mechanical instability may be achieved in the induced pumping region, here $\Omega = 2\Gamma$. The instability manifests itself by a huge increase in the absorption rate of the driving field. The scaled absorption rate increases with increasing number of vibrational quanta $\mathcal{N}$. For large nonlinear damping (dotted), $\mathcal{N}$ is limited by the nonlinear dissipation. Small nonlinear damping (solid) allows the effective temperature of the mechanical oscillator to increase at $E/E_0 \sim 1$. At $E/E_0 \sim 4$, the mechanics becomes unstable leading to a huge increase of the number of mechanical quanta $\mathcal{N}$. Below the instability point $\mathcal{N}$ is of the order Γ/ω_m and slightly above the instability $\mathcal{N} \approx \gamma_L/\gamma_{NL}$. Above the instability the vibration is saturated due to the nonlinear intrinsic damping. The width of the transition decreases with decreasing ω_m/Γ, here plotted for $\omega_m/\Gamma = 10^{-2}$ and $T = 0$.

Instead, when the electric field is enhanced, an instability is reached at $\Gamma^+ = \Gamma^- + \gamma_L$ in the induced pumping region. Above this point, a large number of vibrational quanta will be actuated and their distribution will no longer be close to the Boltzmann distribution. In the case of relatively small nonlinear damping $\gamma_{NL}/\gamma_L \ll \omega_m/\Gamma$, the mechanical instability is reached at $E \approx 4E_0$. The nonlinear damping only plays a minor role at the corresponding value of $\mathcal{N}$. Hence, for field strengths above this point, the number of vibrational quanta, and therefore the absorption $\dot{R}$, is drastically increased, figure 4 (dashed and solid line). However, at large nonlinear damping $\gamma_{NL}/\gamma_L > \omega_m/\Gamma$, it is not reasonable to talk about a mechanical instability. This is because the dynamics is saturated by the nonlinear damping already before the instability point is reached.

The electric field needed to reach the instability point is $\sim$0.1 mV μm^{-1} for an oscillator with the parameters $\Gamma = 1$ GHz, $\omega_m = 10$ MHz, $m = 10^{-16}$ g and quality factor $Q = \omega_m/\gamma_L = 10^5$ at optimal driving frequency $\Omega = 2\Gamma$ and $k_B T \ll \hbar\Gamma$. The moderate value of the estimated field strength suggests that the considered phenomenon can be detected with the available experimental techniques.

Conclusions

To conclude, it has been shown that a single-level movable QD in tunneling contact with an electronic continuum can be actuated mechanically by a nonresonant high-frequency electric field. This is because the electromechanical coupling induces transitions which increase and decrease the number of mechanical quanta in the QD. When the driving field frequency exceeds the tunneling rate, the net effect of the interaction favours increment of mechanical quanta in the QD, i.e. the mechanical motion is effectively pumped. For a strong enough external field, this pumping overcomes the intrinsic mechanical damping and the mechanical motion becomes excited. The dynamics is then saturated by nonlinear mechanical dissipation. The instability manifests itself as a large increase in the absorption rate of the external field for relatively small nonlinear dissipation. This is because the absorption rate is proportional to the average occupation of mechanical quanta.

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