

# Visualizing Single-Camera Reprojection Errors Using Diffusion

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## Abstract

*This paper reports on early-stage research into using dynamic scale-space representation of image point reprojection error data obtained during calibration of a single camera. In particular, we employ time-dependent simulation of the heat equation, diffusing the point reprojection errors over the entire image plane. Initial experiments show the expected effect of an originally large number of point reprojection error measurements being coalesced into a smaller number of relatively larger regions. We round off the paper by presenting ongoing work aiming to exploit the time-progression of simulations to convey further information and thereby assisting manual visual analysis.*

Categories and Subject Descriptors (according to ACM CCS): I.4.1 [Image Processing and Computer Vision]: Digitization and Image Capture—Camera calibration

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## 1. Introduction

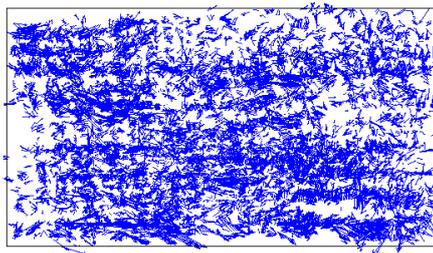
Non-photogrammetric and semi-photogrammetric camera calibration procedures in computer vision often aim to minimize a reprojection error [Zha00]. This type of error describes how well the camera projection model captures the mapping from object points in space to points on the image captured by the camera. Visualizing the reprojection error measure in an intuitive way is often not easy, especially if many object points have been acquired by the camera. Using only the spatial domain for visualization, that is, basing the visualization on the two-dimensional image plane, and possibly extending the representation into the third dimension, can lead to confusing graphical representations of the error.

In this early-stage work we aim to investigate the addition of a temporal dimension to two dimensional spatial visualization. Although not physically meaningful for a cumulative view of the point data set, we can still formulate a time-dependent transformation that leads to a scale-space interpretation [Lin93] of the data. Similar approaches have been examined, for example, in medical image processing to highlight hotspots in point-like images at progressive zoom levels [MDWL14]. However, here we do not consider zooming, but rather view the series of transformed images as snapshots of a time series. Such time series data may be viewed

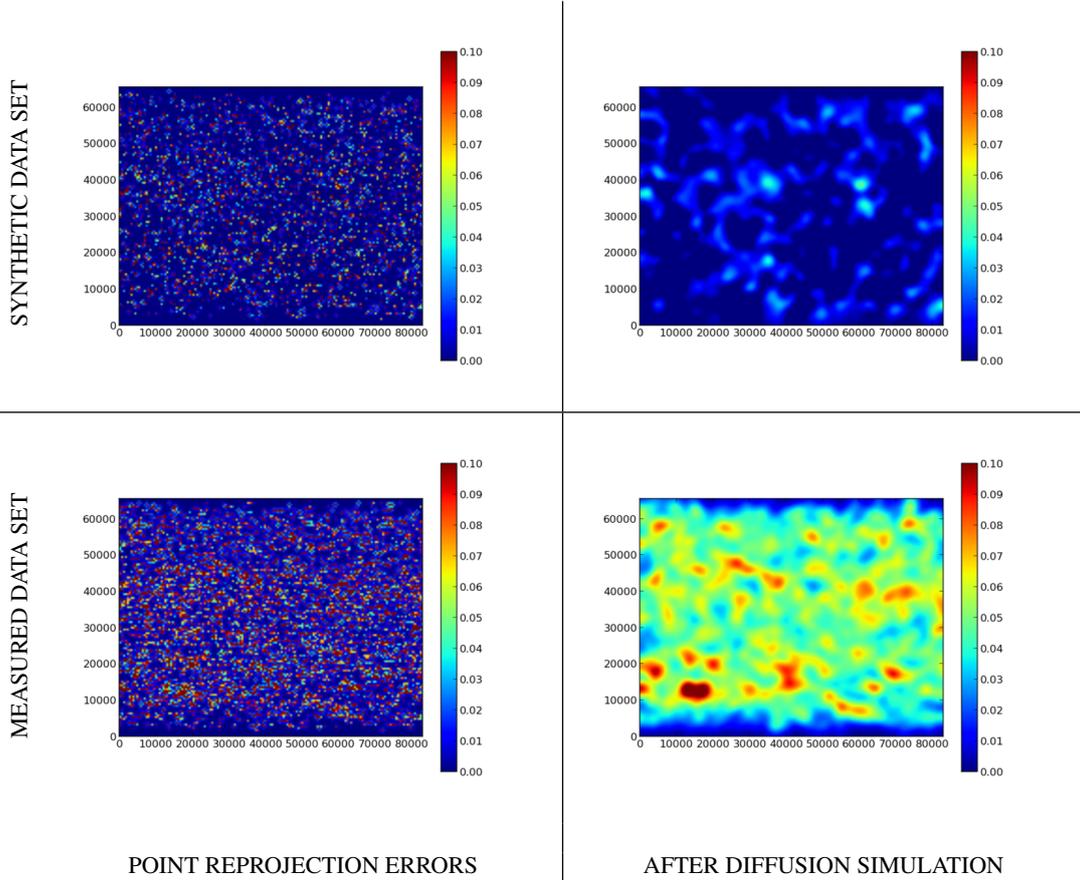
as three-dimensional data sets with two spatial and one temporal dimension.

## 2. Reprojection error visualization

Given reprojection errors for a set of image points captured by a single camera, the task of manually finding patterns in such a collection can be very challenging in terms of human effort required, especially if the set contains thousands of image points (Figure 1). Here, we investigate alternative



**Figure 1:** Deviations of measured image points from the image points predicted by a projection model for several thousand points accumulated from a few hundred images (frames).



**Figure 2:** Heat equation simulation in the image plane for synthetic (top row) and measured data (bottom row). We apply diffusion simulation transforming the source images (left column) into filtered images (right column) to assist manual pattern recognition. The  $x$  and  $y$  axes represent the coordinates in the image plane in number of sub-pixels relative to the bottom left corner of the plane.

techniques that can help reducing the level of complexity introduced by larger data sets, such as clustering [JMF99] or principal component analysis [Jol05]. Once the large data set has been filtered or otherwise compressed, visualization of the processed data can render manual recognition of patterns more feasible. Heatmaps are a widespread visualization method [WF09]; we choose to focus on this technique for our early-stage research. We opt for ignoring directionality of reprojection error measurements and solely consider the magnitude of the errors.

### 3. Diffusion simulation

Motivated by results from scale-space theory [Lin93] we decided to explore diffusion-based simulation and visualization of the reprojection error magnitudes. Specifically, we chose a simple form of diffusion simulation based on the heat equation. The two-dimensional heat equation is given

by

$$\frac{\partial e(x,y,t)}{\partial t} = D\nabla^2 e(x,y,t) \quad (1)$$

where  $e$  is the temperature at location  $(x,y)$  at time  $t$ ,  $D$  is the thermal diffusivity, and  $\nabla^2$  is the Laplace operator. In our case,  $e$  represents the reprojection error magnitude, and the location  $(x,y)$  corresponds to the pixel coordinates in the image plane. By simulating the heat equation, we obtain a multi-scale representation, with finer details damped with time [Lin93]. Hence, features that span larger areas of the image plane can potentially be revealed more easily than in the more detailed original data.

### 4. Preliminary results

We captured several thousand image points of a planar pattern and their reprojection error by performing calibration with a single camera [Zha00]. Figure 1 depicts the calculated

point reprojection errors as deviations of the measured image points from the points predicted by the calibrated projection model. We then simulated the heat equation according to Equation (1) setting the magnitude of the calculated point reprojection errors as initial point temperature values. In order to compare the simulation results against a baseline, we ran a simulation on artificial data. For this purpose, we generated synthetic reprojection errors around each image point by sampling random values from a standard normal distribution with adjusted standard deviation, where standard deviation was set to the maximum reprojection error magnitude obtained from the calibration. We then ran a simulation on the resulting artificial reprojection errors.

Figure 2 shows the start (left column) and end states (right column) of diffusion simulation performed for both the synthetic and the calibrated reprojection error data<sup>†</sup>. For the synthetic data the visualization of the simulation indicates a rather fast convergence to an equilibrium (top row). For the data from the calibration measurements the convergence is less pronounced after the same simulated elapsed time (bottom row), with larger and less evenly distributed coalesced errors. Based on this phenomenon of fusion of point errors into smoother, more visible error spots, we conjecture that diffusion-based simulation could possibly facilitate efficient visual identification of systematic reprojection errors.

## 5. Ongoing work

As a next step of this work, we plan to examine the effects temporal simulation may have on visualization. A possible effect could be that localizing the exact points contributing to an “error hotspot” formation as the simulation progresses in time is facilitated. Another effect could be that simulation allows for identification of similar patterns within and across reprojection error data sets based, for instance, on concepts related to heat kernel signatures [BK10]. Furthermore, we intend to conduct a user study to assess the usefulness of visualizing reprojection errors through diffusion simulations.

Specifically, we aim to investigate whether such visualizations actually could help a user in detecting patterns in the point reprojection error data set not easily recognized using traditional visualizations.

## 6. Acknowledgments

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<sup>†</sup> A video of the simulation together with a link to this paper can be found at <http://t2i.se/publications/>.

## References

- [BK10] BRONSTEIN M. M., KOKKINOS I.: Scale-invariant heat kernel signatures for non-rigid shape recognition. In *Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on* (2010), IEEE, pp. 1704–1711. 3
- [JMF99] JAIN A. K., MURTY M. N., FLYNN P. J.: Data clustering: a review. *ACM computing surveys (CSUR)* 31, 3 (1999), 264–323. 2
- [Jol05] JOLLIFFE I.: *Principal component analysis*. Wiley Online Library, 2005. 2
- [Lin93] LINDBERG T.: *Scale-space theory in computer vision*. Springer, 1993. 1, 2
- [MDWL14] MOLIN J., DEVAN K. S., WÅRDELL K., LUNDSTRÖM C.: Feature-enhancing zoom to facilitate ki-67 hot spot detection. In *SPIE Medical Imaging* (2014), International Society for Optics and Photonics, pp. 90410W–90410W. 1
- [WF09] WILKINSON L., FRIENDLY M.: The history of the cluster heat map. *The American Statistician* 63, 2 (2009). 2
- [Zha00] ZHANG Z.: A flexible new technique for camera calibration. *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 22, 11 (2000), 1330–1334. 1, 2