The Simple Truth about Effects of Mutual Coupling in MIMO Arrays for Single and Multiple Bit Streams in Rich Isotropic Multipath

The Simple Truth about Effects of Mutual Coupling in MIMO Arrays for Single and Multiple Bit Streams in Rich Isotropic Multipath

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Abstract— **In this paper, we demonstrate the effects of mutual coupling on embedded radiation efficiency, correlation, diversity gain, and MIMO efficiency for single and multiple bit streams in rich isotropic multipath (RIMP) environment by using parallel dipoles and a compact wideband four-port antenna as examples.**

Index Terms—**antenna, propagation, measurement.**

I. INTRODUCTION

MIMO systems have drawn considerable interest for improving wireless communications via diversity and/or spatial multiplexing. For mobile communications compact multi-port antennas are usually desired, which makes mutual coupling between different antenna ports inevitable.

Mutual coupling tends to reduce the antenna correlations [1]. Hence, there has been a myth that mutual coupling improves MIMO capacity [2], where the adverse effect of mutual coupling on the embedded radiation efficiency has been overlooked. It is well known that the mutual coupling tends to degrade MIMO performance, provided that its effects on correlation and antenna efficiency are correctly taken care of [3].

In this paper, we first demonstrate the overall effect of the mutual coupling using two parallel dipoles as an example. Then, we use a compact wideband MIMO antenna as a more practical example to further illustrate the mutual coupling effects. Specifically, we will show the effect of the mutual coupling on MIMO systems with single bit stream (i.e., diversity) and multiple bit streams (i.e., spatial multiplexing). We will show that the mutual coupling effect is more severe on spatial multiplexing throughput than on the diversity gain. Besides numerical studies, we also provide measurement results in a reverberation chamber to further validate our analysis. The reverberation chamber emulates rich isotropic multipath (RIMP).

II. MUTUAL COUPLING

In this section, we demonstrate the effects of mutual coupling on MIMO performance using two example MIMO antennas: an array consisting two parallel dipoles and a compact wideband four-port antenna.

A. Parallel Dipoles

The equivalent circuit of the parallel dipoles is shown in Fig. 1. In this work, we assume three-dimensional (3-D) isotropic rich multipath (RIMP) environment [5]. We also assume that the half-wavelength dipoles are identical and located at y_1=-d/2 and y_2=d/2 along y-axis, respectively.

The isolated antenna patterns (without mutual coupling) of the half-wavelength dipoles are

$$\vec{G}_i(\theta,\phi) = -\hat{\theta}\frac{2C_k\eta\cos(\pi/2\cos\theta)}{k\sin\theta}\exp(jk\frac{d_i}{2}\sin\theta\sin\phi) \quad (1)$$

where i=1, 2, d_1=-d, d_2=d, $C_k = -jk/4\pi$, and η is free space wave impedance. The embedded radiation patterns (with mutual coupling) are

$$\begin{aligned}\vec{G}_{emb,1}(\theta,\phi) &= \vec{G}_1(\theta,\phi)I_1 + \vec{G}_2(\theta,\phi)I_2 \\ \vec{G}_{emb,2}(\theta,\phi) &= \vec{G}_1(\theta,\phi)I_2 + \vec{G}_2(\theta,\phi)I_1\end{aligned} \quad (2)$$

From the equivalent circuit, when the excitation current at port 1 is unity, I_1=1, I_2=-$Z_{12}/(Z_{11}+Z_s)$, where Z_{11}, Z_{12} and Z_S are input impedance, mutual impedance, and source impedance, respectively. We assume Z_S = 50 Ω (i.e., without matching network); the self-impedance of a half-wavelength dipole is 73 + 42.5j Ω whereas the expressions of Z_{11} and Z_{12} are given in [6].

The correlation including mutual coupling can be calculated as

$$\rho = \frac{\iint_{4\pi} G_{emb,1}(G_{emb,2})^* d\Omega}{\sqrt{\iint_{4\pi}|G_{emb,1}|^2 d\Omega \iint_{4\pi}|G_{emb,2}|^2 d\Omega}} \quad (3)$$

where $G_{emb,i}$ is the $\hat{\theta}$ component of $\vec{G}_{emb,i}(\theta,\phi)$. Since no ohmic loss is assumed in the dipoles, embedded element impedance can be readily calculated as

$$e_{emb} = 1 - |S_{11}|^2 - |S_{21}|^2 \quad (4)$$

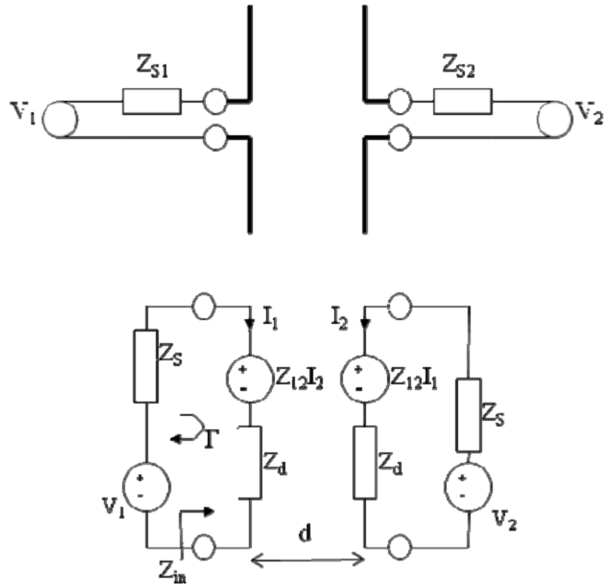
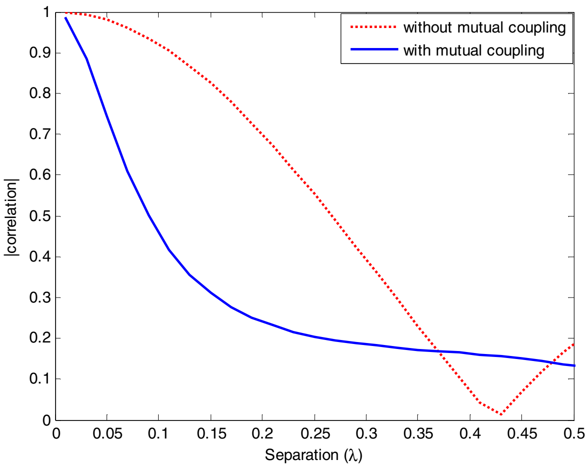
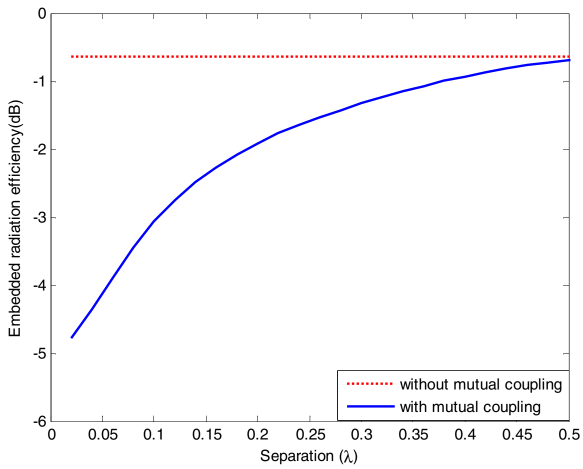


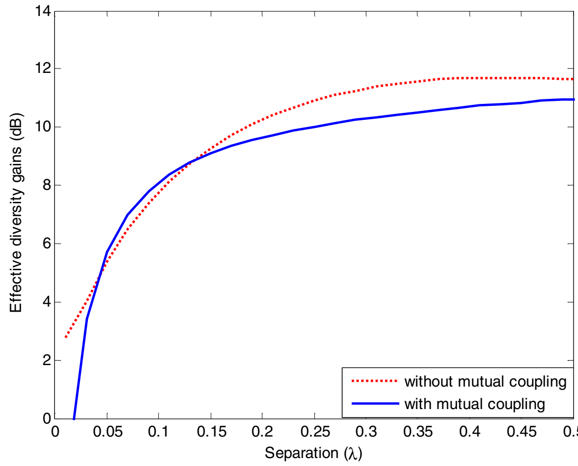
Fig. 1 Equivalent circuit for parallel dipoles [4].



(a)



(b)



(c)

Fig. 2. Correlation (a), embedded radiation efficiency (b), and MRC diversity gain (c) of two parallel dipoles in RIMP with/without mutual coupling.

where the S-parameters can be easily converted from the impedance parameters. Actually, for lossless antennas the correlation in RIMP can also be expressed in terms of S-parameters, i.e., [7], [8]

$$\rho = \frac{-(S_{11}^{*}S_{12} + S_{21}^{*}S_{22})}{\sqrt{(1-|S_{11}|^2-|S_{21}|^2)(1-|S_{22}|^2-|S_{12}|^2)}} \tag{5}$$

Once the correlation and the embedded radiation efficiency are known, the effective diversity gain with maximum ratio combining (MRC) can be readily calculated according to

$$DG = F^{-1}(\gamma) / F_{ideal}^{-1}(\gamma)\Big|_{1\%} \tag{6}$$

where $(\cdot)^{-1}$ denotes functional inversion, F is the cumulative distribution function of the MRC output signal-to-noise ratio (SNR), and $F_{ideal}(\gamma) = 1-\exp(-\gamma)$ is the CDF of the output SNR of the ideal reference antenna in the Rayleigh fading environment. Since two identical dipole antennas are assumed, the CDF F in (6) is [9]

$$F(\gamma) = 1 - \frac{\lambda_1 \exp(-\gamma/\lambda_1) - \lambda_2 \exp(-\gamma/\lambda_2)}{\lambda_1 - \lambda_2}. \tag{7}$$

where $\lambda_1 = e_{emb}(1+|\rho|)$ and $\lambda_2 = e_{emb}(1-|\rho|)$.

Fig. 2 shows the correlation, embedded radiation efficiency, and MRC diversity gain of the two parallel dipoles in RIMP with/without mutual coupling as a function of dipole separation d. As can be seen, mutual coupling reduces the correlation as well as embedded radiation efficiency. If its adverse effect on antenna efficiency is overlooked, one could be misled to conclude that mutual coupling improves diversity gain. However, the diversity gain in RIMP will always degrade if the antennas have mutual coupling. Thus, reduction of mutual coupling will always help to improve diversity gain. Fig. 2 shows some elevation spacing for which this apparently is not true. However, here the curves marked “without mutual coupling” are unphysical. The coupling is there, but it is not included in the simulations, so only the curve marked “with mutual coupling" is correct. This curve can of course be improved by redesigning the antennas (without introducing lossy materials) so that the mutual coupling vanishes.

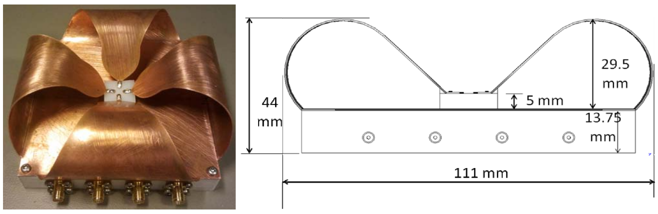
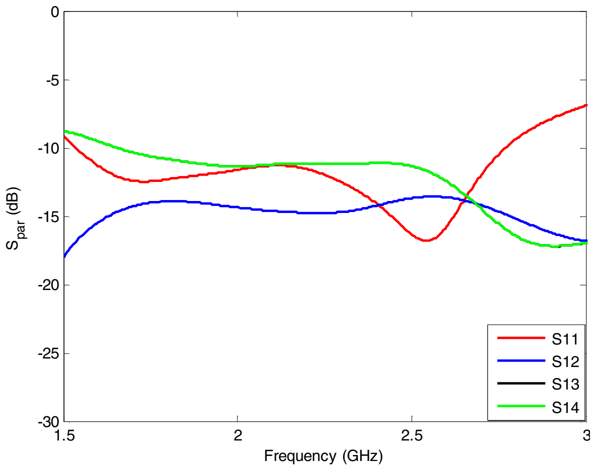
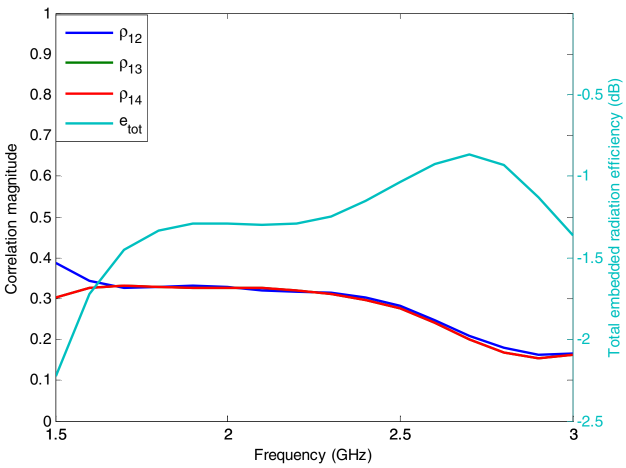


Fig. 3. Photo (left) and profile drawing (right) of the bowtie antenna.



(a)



(b)

Fig. 4. S-parameters (a), and correlation and embedded radiation efficiency (b) of bowtie antenna.

B. 4-Port Bowtie Antenna

The diversity gain study in the prior section corresponds to single bit stream transmission. It is of more interest to study the MIMO performance for the transmission of multiple bit streams (i.e., spatial multiplexing). Thus in this section we examine the mutual coupling effects on MIMO performances for both diversity and spatial multiplexing. And we use a so-called bowtie antenna [10] as an example. The bowtie antenna is designed to cover a wide frequency range from 1.7 to 2.7 GHz. Fig. 3 shows its photo and drawing. The bowtie antenna was designed to serve as a multi-port micro base station antenna. From its dimensions shown in Fig. 3, it can be seen that it is very compact. With relaxed size requirement, its performance can be further improved.

Fig. 4 shows its S-parameters, correlation, and embedded radiation efficiency obtained based on CST simulations. As can be seen, the bowtie antenna is compact and wideband. Its correlations are reasonably low, yet its mutual coupling fairly high (around -10 dB for most of the frequency band) due to the compact design. And due to the mutual coupling, the embedded radiation efficiency of the bowtie antenna is below -1 dB for most of the frequency.

Next, we use the spatial multiplexing throughput to study the case of transmission of multiple bit streams. A simple throughput model has been presented in [11], which was extended for the spatial multiplexing case for open-loop MIMO systems [12] and closed-loop MIMO systems [13]. Based on the model, the average throughput of a system with fixed modulation and coding scheme (MCS) in a fading channel can be approximated by

$$T_{\text{put}}\left(\bar{\gamma}\right) = T_{\text{put,max}}\left(1 - F\left(\gamma_{th}/\bar{\gamma}\right)\right) \qquad (8)$$

where $\bar{\gamma}$ represents the average γ, γ_{th} is the threshold value, F denotes the CDF of γ, and $T_{\text{put,max}}$ denotes the maximum data rate. We define the relative throughput as $T_{\text{put}}/T_{\text{put,max}}$ and use the relative throughput hereafter. Note that the relative throughput can be regarded as the probability of detecting (PoD) multiple streams.

For simplicity, we assume open-loop MIMO systems with ZF receivers and that transmit antennas are uncorrelated. The MIMO channel is given by

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{H}_w \qquad (8)$$

where $\mathbf{H}_w$ denotes the spatially white MIMO channel with i.i.d. and unit variance complex Gaussian variables, $\mathbf{R}$ is the correlation matrix at the receive side, and $\mathbf{R}^{1/2}$ is the Hermitian square root of $\mathbf{R}$. For simplicity and without loss of generality, we assume unity input signal power and noise variance, the SNR of the ith stream is then [14]

$$\gamma_i = 1\Big/\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{i,i} \qquad (9)$$

where $[\mathbf{X}]_{i,i}$ denotes the ith diagonal element of the matrix $\mathbf{X}$.

Fig. 5 shows the probability of detecting two and four streams of using the bowtie antenna at 2.2 GHz. For reference purpose, the corresponding i.i.d. (independent and identically distributed) ideal cases (without any correlation or mutual coupling) are also presented. As will be shown in Fig. 6, the degradation from the i.i.d. case is rather severe at low frequencies; as the correlation reduces at higher frequencies, the degradation diminishes at high frequencies.

Defining the degradation at 95% PoD level due to the bowtie antennas with respect to the i.i.d. case as MIMO efficiency, the MIMO efficiency is calculated and shown in Fig. 6, where M×N implies M ports of bowtie antennas and N bit streams. As can be seen, a single bowtie antenna is good in detecting 2 bit streams, yet very bad for detecting 4 bit streams. This is because a single bowtie antenna receives rank-3 MIMO channel most of the time, especially at low frequencies.

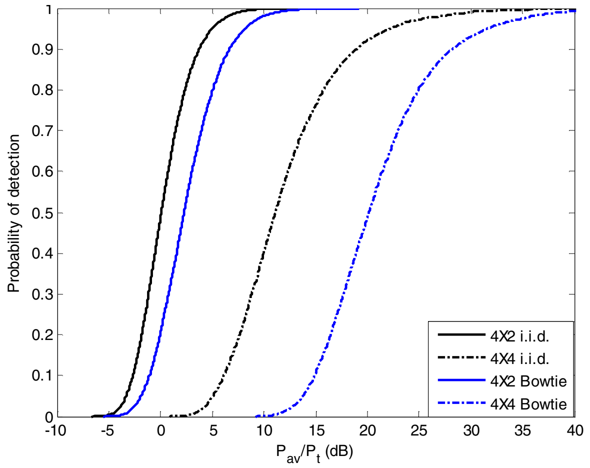


Fig. 5. Probability of detection by ZF algorithm of 2 and 4 streams in RIMP of the 4-port bowtie antenna at 2.2 GHz.

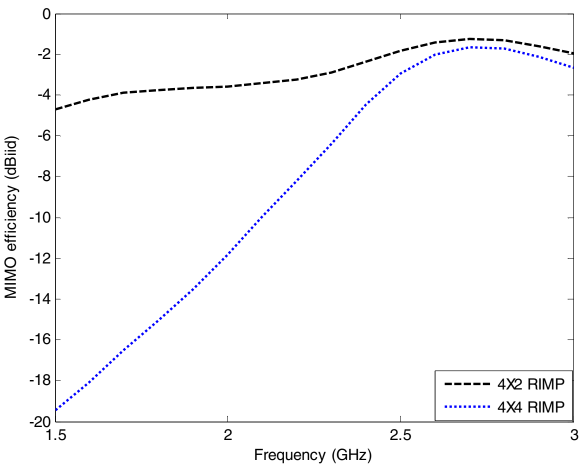


Fig. 6. MIMO efficiencies of 2 and 4 streams in dBiid for the bowtie antenna in RIMP as a function of frequency at 95% PoD level. The i.i.d. reference for the M×N bowtie configuration is the M×N MIMO system for the i.i.d. case, where M denotes the number of ports of the bowtie antenna and N denotes the number of bit streams.

III. REVERBERATION CHAMBER MEASUREMENT

To further examine the bowtie antenna, we measured it in the Bluest reverberation chamber (RC) [15]. Fig. 7 shows the drawing of the Bluetest RC and a photo of the bowtie antenna in the RC. Fig. 8 shows its measured and simulated MRC diversity gains against the 3-port and 4-port i.i.d. cases. As can be seen, there are in general good agreements between the RC measurements and simulations. The diversity gain is close to the 3-port i.i.d. case at low frequency. We believe that this is due to the fundamental limitation discussed in [16] of a maximum of 6 independent ports of a small antenna radiating into full space. The bowtie radiates on one side of a ground plane, so therefore the maximum number of independent ports should be 6/2 = 3.

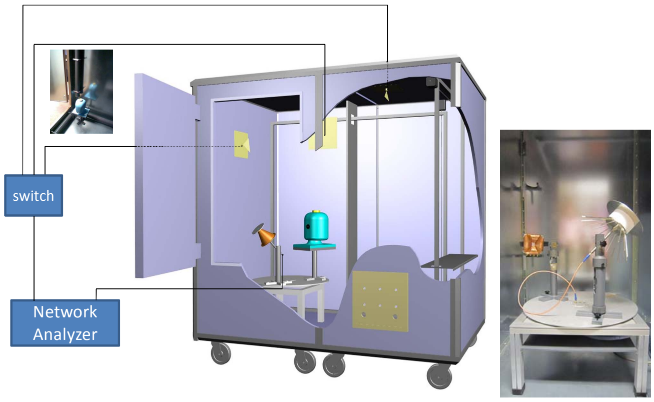


Fig. 7. Drawing of the Bluetest RC and a photo of the bowtie antenna in the RC.

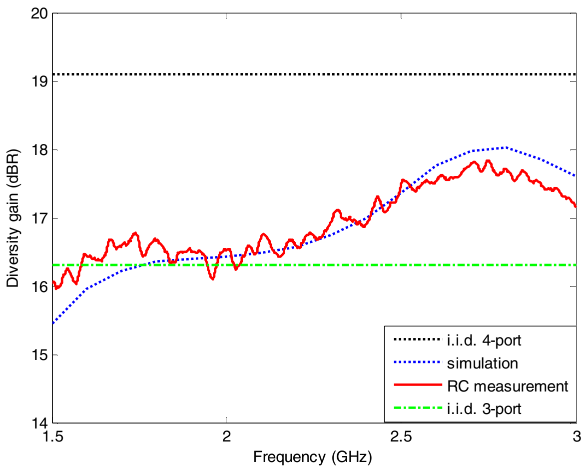


Fig. 8. Measured and simulated MRC diversity gains of bowtie antenna.

IV. CONCLUSIONS

In this work, we examine the antenna mutual coupling effects on correlation, embedded radiation efficiency, diversity gain, and MIMO throughput. It is shown that mutual coupling reduces both correlation and embedded radiation efficiency; and mutual coupling tends to degrade MIMO performance in general, especially for PoD of multiple streams (i.e., MIMO efficiency).

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