LDPC codes for optical channels: Is the "FEC limit" a good predictor of post-FEC BER?

LDPC Codes for Optical Channels: Is the "FEC Limit" a Good Predictor of Post-FEC BER?

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Abstract: We answer the question in the title negatively. More precisely, the FEC limit is invalid for soft decision decoding and low to medium code rates. A better predictor is the generalized mutual information.

OCIS codes: (060.4080) Modulation, (060.4510) Optical communications.

1. Introduction and Motivation

Soft-decision forward error correction (SD-FEC) and multilevel modulation formats are key technologies for realizing high spectral efficiencies in optical communications. The combination of FEC and multilevel modulation is known as coded modulation (CM), where FEC is used to recover the sensitivity loss caused by the nonbinary modulation.

Current digital coherent receivers are based on powerful digital signal processing (DSP) algorithms, which are used to detect the transmitted bits and to compensate for channel impairments and transceiver imperfections. The optimal DSP should find the most likely *coded sequence;* however, this is hard to realize in practice, and thus, most receivers are implemented suboptimally. In particular, detection and FEC decoding are typically decoupled at the receiver: soft information on the code bits is calculated first, and then, an SD-FEC decoder is used.

When optical communications research began to incorporate digital coherent receivers, it became impractical to implement realtime receivers. Instead, offline DSP was used, typically operating on a sample of data less than 1 μs long. In this scenario, the BER after FEC decoding, which is here called *post-FEC BER* or $\mathrm{BER}_{\mathrm{pos}}$, should be as low as 10^{-12} or 10^{-15} to meet higher-layer quality requirements. Since such low BER values cannot be reliably estimated by Monte Carlo simulations, the conventional design paradigm has been to simulate the system without FEC encoding and decoding, and optimize it for a much higher BER requirement, the so-called *FEC limit* or FEC threshold. The rationale is that a certain BER without coding, which is here called *pre-FEC BER* or $\mathrm{BER}_{\mathrm{pre}}$, supposedly can be brought down to the desired post-FEC BER by previously verified FEC implementations.

The use of FEC limits assumes that the decoder's performance is fully characterized by $\mathrm{BER}_{\mathrm{pre}}$, and that different channels with the same $\mathrm{BER}_{\mathrm{pre}}$ will result in the same $\mathrm{BER}_{\mathrm{pos}}$. Under some assumptions on interleaving the code bits, and that the FEC operates only on *hard decisions*, this assumption is justifiable. The use of FEC limits, however, did not change with the adoption of SD-FEC in optical communications.

For any given channel, $\mathrm{BER}_{\mathrm{pre}}$ *can* be used to predict $\mathrm{BER}_{\mathrm{pos}}$ of an SD-FEC decoder. This has been done for example for some of the SD-FEC decoders in the G.975.1 standard [1], where $\mathrm{BER}_{\mathrm{pos}}$ values are tabulated versus $\mathrm{BER}_{\mathrm{pre}}$. However, a problem arises when uncoded experiments or simulations rely on tabulated values and claim (without encoding and decoding information) the existence of an SD-FEC decoder that can deal with the measured $\mathrm{BER}_{\mathrm{pre}}$. The caveat with this approach is that it relies on the strong assumption that the same SD-FEC encoder and decoder pair will perform identically for two different channels with the same $\mathrm{BER}_{\mathrm{pre}}$. As we will see later, there is no information-theoretic basis for this assumption, and it is indeed often incorrect. This paper investigates the range of validity of this assumption, and discusses alternative metrics for estimating $\mathrm{BER}_{\mathrm{pos}}$ of SD-FEC decoders.

2. System Model and Achievable Rates

We consider 11 WDM channels of 32 GBaud in a 50 GHz grid over a single-span of standard single mode fiber (SSMF) shown in Fig. 1. Optical fiber transmission is simulated using the nonlinear Schrödinger equation (NLSE) via the split step Fourier method with a step size of 100 m, oversampling factor of 4 samples/symbol, and root-raised-cosine (RRC) filters with 1% rolloff. The detection process was modeled as an ideal phase- and polarization-diverse coherent receiver

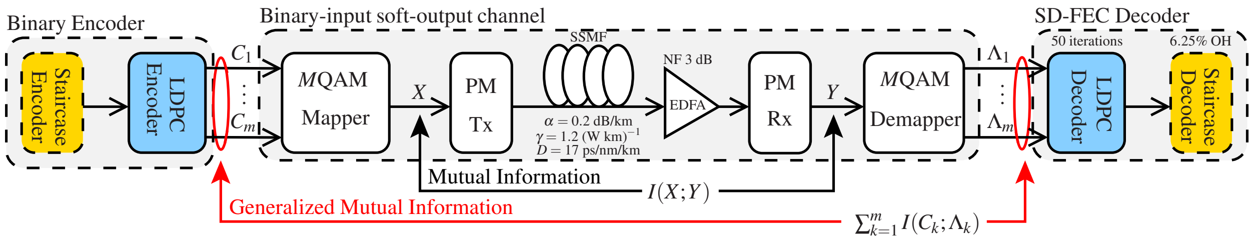

Fig. 1. Polarization-multiplexed single-span optical with 11 WDM channel under consideration.

with electronic chromatic dispersion compensation. Data for the central channel was recorded. Gray-mapped MQAM constellations with $M = 2^m = 4, 16, 64, 256$ equally likely symbols are considered.

Irregular repeat-accumulate low-density parity-check (LDPC) codes with rates $R_c \in \{1/3, 1/2, 3/4, 9/10\}$ were used. This leads to FEC overheads (OHs) of $\{200, 100, 33.3, 11.1\}$%. An outer staircase code with 6.25% OH that gives a final BER_{pos} of 10^{-15} for a post-LDPC BER of $4.7 \cdot 10^{-3}$ [2, Table I] is assumed (see Fig. 1). Each transmitted frame consists of 64,800 code bits $C_1, \ldots, C_m$, which are assigned cyclically to the modulating bits, with no interleaver. At the receiver, *soft* information on the code bits is calculated in the form of logarithmic likelihood ratios (LLRs), which we denote by Λ_k with $k = 1, \ldots, m$. These LLRs Λ_k are then passed to the LDPC decoder.

An achievable rate for the receiver in Fig. 1 is the generalized mutual information (GMI) [3–6]

$$\text{GMI} \triangleq \sum_{k=1}^{m} I(C_k; Y) = \sum_{k=1}^{m} I(C_k; \Lambda_k), \tag{1}$$

where $I(\cdot;\cdot)$ is the mutual information (MI) between two arbitrary random variables. The GMI is then the sum of *bit-wise* MIs between code bits and LLRs. Fig. 2 shows the GMI as a function of the span length, for MQAM constellations. For each distance and M, we used the launch power that gave the highest GMI. In this figure, we also show the distance required for the SD-FEC decoder in Fig. 1 to give $\text{BER}_{\text{pos}} = 10^{-15}$ for each combination of 4 constellations and 4 LDPC codes. The vertical position of these 16 markers represent the resulting achievable rates and clearly show that the results follow the GMI curves. This agrees perfectly with the results in [6]. The penalties due to the suboptimality of the LDPC and staircase code are between 5 and 15 km and are highest for high code rates.

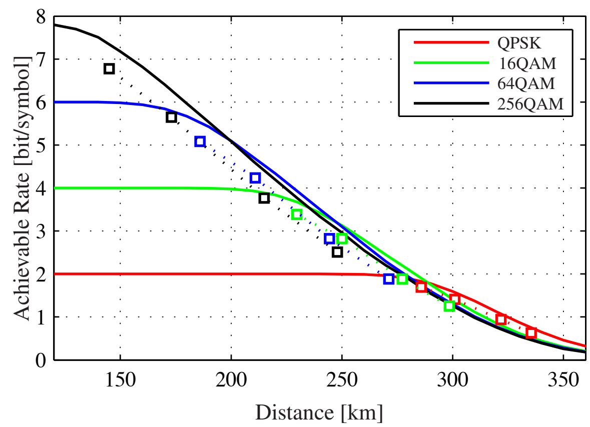

Fig. 2. Achievable rates versus span length.

3. Post-FEC BER Prediction

To study the robustness of the pre-FEC BER as a metric to predict post-FEC BER, we show in Fig. 3 the post-LDPC BER as a function of BER_{pre} for the same 16 cases (markers) as in Fig. 2. Ideally, all the lines for the same rate should fall on top of each other, indicating that measuring BER_{pre} is enough to predict BER_{pos} when the channel (in this case, the modulation format) changes. The results in this figure show that this is indeed the case for high code rates. For low and moderate code rates, however, BER_{pre} fails to predict the performance of the SD-FEC decoder. An intuitive explanation for this is that the SD-FEC in Fig. 1 does not operate on bits, and thus, it is not surprising that a metric that is based on bits (i.e., the pre-FEC BER) cannot be used to predict the performance of the decoder.

Using BER_{pre} to predict the performance of SD-FEC decoders has no information-theoretic justification. To remedy this, it was proposed in [7] to use the symbol-wise MI $I(X;Y)$ (see Fig. 1) as a metric to better predict BER_{pos}. The values of BER_{pos} as a function of the normalized MI $I(X;Y)/m$ are shown in Fig. 4. Again, the prediction works well only for high code rates. An explanation for this is that the MI is an achievable rate for the *optimum* receiver, but not for the (suboptimal) receiver in Fig. 1.

The main contribution of this paper is to put forward the idea of using the GMI to predict the post-FEC BER. The rationale behind this is that an SD-FEC decoder is fed with LLRs, and thus, the GMI (see (1)) is a better metric than BER_{pre} and MI. The values of BER_{pos} as a function of the normalized GMI are shown in Fig. 5.

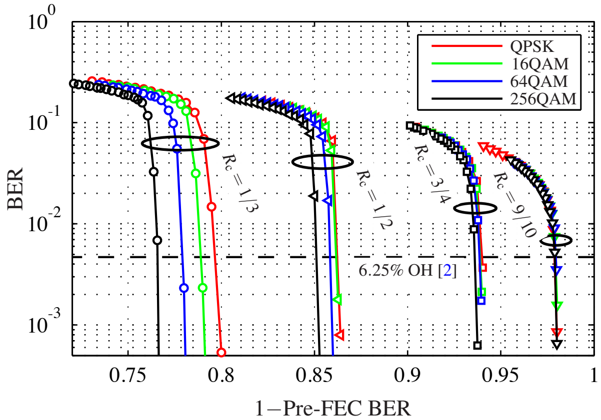

Fig. 3. Post-FEC BER vs. pre-FEC BER.

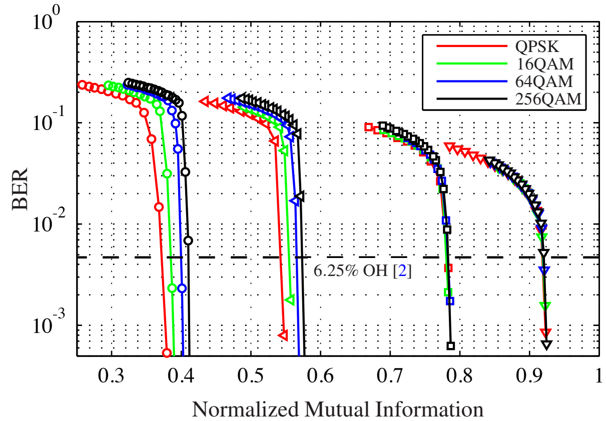

Fig. 4. Post-FEC BER vs. (normalized) MI

These results show that for a given code rate, changing the constellation does not greatly affect the post-FEC BER prediction based on the GMI. More importantly, and unlike for $\mathrm{BER}_{\mathrm{pos}}$ and MI, the prediction based on the GMI appears to work across *all* code rates. For high rates, however, the obtained results based on pre-FEC BER, MI, and GMI are very similar. We believe this could be explained using to the recently discovered asymptotics relationships between these quantities [8].

In Fig. 5, we also show the GMI values needed by an "ideal" SD-FEC code (vertical lines). The horizontal differences between these lines and the results obtained by the LDPC codes represent the rate penalty caused by the use of a suboptimal code. For this particular family of LDPC codes, this penalty is around 4% (0.04m bit/symbol) at the staircase code threshold. Combined with the 6.25% OH of the staircase code, an overall penalty of about 10% with respect of the GMI prediction was observed.

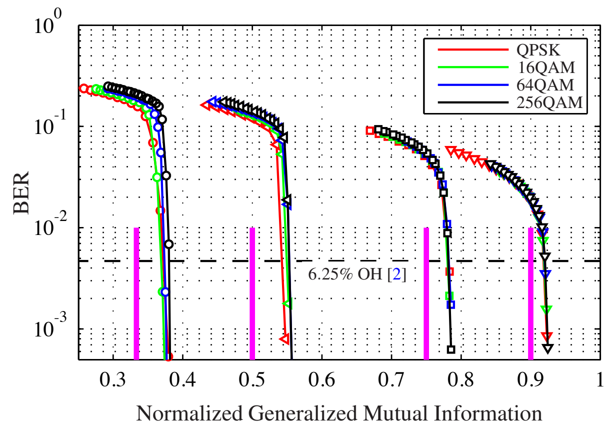

Fig. 5. Post-FEC BER vs. (normalized) GMI.

4. Conclusions

The pre-FEC BER and mutual information were shown to be poor predictors of the performance of soft-decision FEC decoders, except in the special case of very high code rates and square QAM. The so-called FEC limit is hence an unreliable design criterion for optical communication systems with soft-decision FEC. On the other hand, the generalized mutual information was found to give very good results for *all* code rates. We believe that these results are valid for other capacity-approaching soft-decision FEC codes, not only the LDPC codes studied in this paper.

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