THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Multidimensional Modulation Formats for Coherent Single- and Multi-Core Fiber-Optical Communication Systems

Tobias A. Eriksson



Photonics Laboratory Department of Microtechnology and Nanoscience (MC2) CHALMERS UNIVERSITY OF TECHNOLOGY Göteborg, Sweden, 2015

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Photonics Laboratory Department of Microtechnology and Nanoscience (MC2) Chalmers University of Technology, SE-412 96 Göteborg, Sweden Phone: +46 (0) 31 772 1000

Front cover illustration: Constellation in the x- and y-polarization for the four-dimensional modulation format $256-D_4$.

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Tobias A. Eriksson Photonics Laboratory Department of Microtechnology and Nanoscience (MC2) Chalmers University of Technology, SE-412 96 Göteborg, Sweden

Abstract

This thesis covers multidimensional modulation formats for coherent optical communication systems including spatial division multiplexed systems using multicore fibers. The single-mode optical signal has four dimensions which are spanned by the two orthogonal polarizations and the in-phase and quadrature components. By optimizing modulation formats in the four-dimensional (4D) signal space, formats with increased asymptotic power efficiency and/or spectral efficiency can be found. In this thesis, a range of 4D modulation formats are studied and several experimental realizations of different 4D formats are presented. This thesis also includes modulation formats with higher dimensionality. Two different experimental realizations of biorthogonal modulation in eight dimensions are presented where either two consecutive timeslots or two wavelength channels are used to span the eight dimensions. In the experiments, the multidimensional formats are compared to conventional twodimensional formats in terms of achievable transmission reach.

Multicore fibers systems are also investigated in this thesis and the impact of inter-core crosstalk on quadrature phase shift keying signals is studied. Further, multidimensional modulation formats over spatial superchannels consisting of signals in several cores are explored. Experimental demonstrations of low-complexity formats capable of increased transmission reach at a small reduction in spectral efficiency are presented.

This thesis also studies the impact on the achievable information rate using mutual information for different assumptions of the channel distribution in fiber-optical transmission experiments. It is shown that decoders operating in four-dimensions can achieve significant higher achievable information rates for highly nonlinear fiber channels.

Keywords: Fiber-optical communication, multidimensional modulation formats, spectral efficiency, power efficiency, 128-ary set-partitioning 16QAM (128-SP-16QAM), 16-ary quadrature amplitude modulation (16QAM), quadrature phase shift keying (QPSK), binary pulse position modulation QPSK (2PPM-QPSK), biorthogonal modulation in eight dimensions, polarization-switched QPSK (PS-QPSK), lattice based modulation, multidimensional position modulation, multicore fiber transmission, crosstalk, single parity check-coded modulation, four-dimensional (4D) estimates of mutual information (MI), achievable information rate.

List of Papers

This thesis is based on the following appended papers:

- [A] Tobias A. Eriksson, Martin Sjödin, Pontus Johannisson, Peter A. Andrekson, and Magnus Karlsson, "Comparison of 128-SP-QAM and PM-16QAM in Long-Haul WDM transmission," *Optics Express*, vol. 21, no. 16, pp. 19269–19279, Aug. 2013.
- [B] Martin Sjödin, Tobias A. Eriksson, Peter A. Andrekson, and Magnus Karlsson, "Long-Haul Transmission of PM-2PPM-QPSK at 42.8 Gbit/s," Proc. Optical Fiber Communication Conference (OFC), Anaheim, CA, USA, 2013, paper OTu2B.
- [C] Tobias A. Eriksson, Pontus Johannisson, Martin Sjödin, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson, "Frequency and Polarization Switched QPSK," Proc. European Conference on Optical Communication (ECOC), London, United Kingdom, 2013, paper Th.2.D.4.
- [D] Tobias A. Eriksson, Pontus Johannisson, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson, "Biorthogonal Modulation in 8 Dimensions Experimentally Implemented as 2PPM-PS-QPSK," Proc. Optical Fiber Communication Conference (OFC), San Francisco, CA, USA, 2014, paper W1A.5.
- [E] Tobias A. Eriksson, Saleem Alreesh, Carsten Schmidt-Langhorst, Felix Frey, Pablo Wilke Berenguer, Colja Schubert, Johannes K. Fischer, Peter A. Andrekson, Magnus Karlsson, and Erik Agrell, "Experimental Investigation of a Four-Dimensional 256-ary Lattice-Based Modulation Format," *Proc. Optical Fiber Communication Conference (OFC)*, Los Angeles, CA, USA, 2015, paper W4K.3.
- [F] Tobias A. Eriksson, Pontus Johannisson, Benjamin J. Puttnam, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson, "K-over-L Multidimensional Position Modulation," *Journal of Lightwave Technology*, vol. 32, no. 12, pp. 2254– 2262, Jun. 2014.
- [G] Tobias A. Eriksson, Benjamin J. Puttnam, Ruben S. Luís, Magnus Karlsson, Peter A. Andrekson, Yoshinari Awaji, and Naoya Wada, "Experimental Investigation of Crosstalk Penalties in Multicore Fiber Transmission Systems," *IEEE Photonics Journal*, vol. 7, no. 1, paper 7200507, Feb. 2015.

- [H] Benjamin J. Puttnam, Tobias A. Eriksson, José Manuel Delgado Mendinueta, Ruben S. Luís, Yoshinari Awaji, Naoya Wada, Magnus Karlsson, and Erik Agrell, "Modulation Formats for Multi-Core Fiber Transmission," *Optics Express*, vol. 22, no. 26, pp. 32457–32469, Dec. 2014.
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- [J] Tobias A. Eriksson, Tobias Fehenberger, Norbert Hanik, Peter A. Andrekson, Magnus Karlsson, and Erik Agrell, "Four-Dimensional Estimates of Mutual Information in Coherent Optical Communication Experiments," Proc. European Conference on Optical Communication (ECOC), Valencia, Spain, 2015, paper We.4.6.5.
- [K] Tobias A. Eriksson, Tobias Fehenberger, Peter A. Andrekson, Magnus Karlsson, Norbert Hanik, and Erik Agrell, "Impact of 4D Channel Distribution on the Achievable Rates in Coherent Optical Communication Experiments," Submitted to Journal of Lightwave Technology, 16 Nov. 2015.

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- [P1] Jianqiang Li, Ekawit Tipsuwannakul, Tobias A. Eriksson, Magnus Karlsson, and Peter A. Andrekson, "Approaching Nyquist Limit in WDM Systems by Low-Complexity Receiver-Side Duobinary Shaping," *Journal of Lightwave Technology*, vol. 30, no. 11, pp. 1664–1676, Jun. 2012.
- [P2] Ekawit Tipsuwannakul, Jianqiang Li, Tobias A. Eriksson, Magnus Karlsson, and Peter A. Andrekson, "Transmission of 3× 224 Gbit/s DP-16QAM Signals with (up to) 7.2 bit/s/Hz Spectral Efficiency in SMF-EDFA Links," Proc. Optical Fiber Communication Conference (OFC), Los Angeles, CA, USA, 2012, paper OW4C.6.
- [P3] Ekawit Tipsuwannakul, Jianqiang Li, Tobias A. Eriksson, Fredrik Sjöström, Johan Pejnefors, Peter A. Andrekson, and Magnus Karlsson, "Mitigation of Fiber Bragg Grating-Induced Group-Delay Ripple in 112 Gbit/s DP-QPSK Coherent Systems," Proc. Optical Fiber Communication Conference (OFC), Los Angeles, CA, USA, 2012, paper JW2A.69.
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- [P9] Benjamin J. Puttnam, José Manuel Delgado Mendinueta, Ruben S. Luís, Werner Klaus, Jun Sakaguchi, Yoshinari Awaji, Naoya Wada, **Tobias A. Eriksson**, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson, "Energy Efficient Modulation Formats for Multi-Core Fibers," Proc.m Optoelectronic and Communications Conference/Australian Conference on Opitcal Fibre Technology (OECC/ACOFT), Melbourne, Australia, 2014, pp. 694–696.

- [P10] Tobias A. Eriksson, Pontus Johannisson, Erik Agrell, Peter A. Andrekson, and Magnus Karlsson, "Experimental Comparison of PS-QPSK and LDPCcoded PM-QPSK with Equal Spectral Efficiency in WDM Transmission," Proc. European Conference on Optical Communication (ECOC), Cannes, France, 2012, paper Tu.3.3.3.
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Tobias A. Eriksson

Göteborg November 2015

 $^{^1\}mathrm{Please}$ note that both "fiber guys" and "opto boys" are terms often used in a gender neutral way.

List of Abbreviations

1D one-dimensional 2D two-dimensional 4D four-dimensional 4FPS-QPSK 4-ary frequency and polarization switched QPSK 8D eight-dimensional ADC analog-to-digital converter **AIR** achievable information rate APE asymptotic power efficiency ASE amplified spontaneous emission AWGN additive white Gaussian noise BCH Bose-Chaudhuri-Hocquenghem **BER** bit-error rate **BPSK** binary phase-shift keying **B2B** back-to-back **CFM** constellation figure of merit CMA constant modulus algorithm DAC digital-to-analog converter DCF dispersion-compensating fiber **DD-LMS** decision-directed least mean square **DSP** digital signal processing **EDC** electronic dispersion compensation EDFA erbium-doped fiber amplifier ENOB effective number of bits FEC forward error correction **FIR** finite impulse response GMI generalized mutual information HD hard decision I/Q-modulator in-phase and quadrature-modulator **IF** intermediate frequency

ISI intersymbol interference

K-SP-MQAM K-ary set-partitioning MQAM **KiMDPM** K-arv inverse-MDPM LDPC low-density parity check LLR log-likelihood ratio LO local oscillator MCF multicore fiber **MDPM** multidimensional position modulation MI mutual information MIMO multiple input multiple output **MMF** multimode fiber MPPM multi-pulse position modulation MZM Mach-Zehnder modulator NRZ non-return to zero **OFDM** orthogonal frequency division multiplexing **OOK** on-off keying **OSNR** optical signal-to-noise ratio **OTDM** optical time-division multiplexing PAM pulse amplitude modulation **PBC** polarization beam combiner **PBS** polarization beam splitter **PM** polarization multiplexed **PMD** polarization mode dispersion POL-QAM polarization-QAM **PPM** pulse position modulation **PS-QPSK** polarization-switched QPSK **QAM** quadrature amplitude modulation **QPSK** quadrature phase-shift keying **RF** radio frequency RS Reed-Solomon SD soft decision **SDM** space-division multiplexing SE spectral efficiency **SMF** single mode fiber SNR signal-to-noise ratio SO-PM-QPSK subset-optimized PM-QPSK SOP state-of-polarization **SPC** single parity-check **TPC** turbo product code WDM wavelength division multiplexing XOR. exclusive or

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Chapter 1

Introduction

We live in the *information age* where information, entertainment and communication is only a click away. Many people stay connected during all waking hours of the day and a large part of the world economy is governed by information technology. The communication network often referred to as *the Internet* has enabled services such as instantaneous information search, real-time communication using messaging softwares or video calls, online gaming, social networks and real-time streaming of high definition video.

What historical events triggered the dawn of the information age? The invention of the telegraph during the 1800's [1, Chapter 6] and the first transoceanic telegraph cables [1, Section 8.1.3] enabled communication between continents without the long latency related to mailing letters. The work on communication theory by Shannon in 1948 [2] and the invention of error control coding around 1950 [3, 4], lay the groundwork for digital communication systems as we know them today. Further the invention of the computer [5] and the start of what would later become the Internet in 1969 [6] are key elements in today's information-based society.

The demonstration of the first laser in 1960 by Maiman [7] followed by the invention of the semiconductor laser [8, 9], together with the invention of the optical fiber in 1966 [10, 11] established the most important building blocks of a fiber-optic link. Starting with the Atlanta Fiber System Experiment in 1976 [12] a plethora of fiber optical field experiments were conducted and in the middle of the 1980's the optical fibers became the dominating transmission medium in new telecom links [13]. The first transatlantic cable using optical fibers was installed in 1988 [14]. The invention of the erbium-doped fiber amplifier (EDFA) in the mid 1980's [15–17] has enabled amplification in the optical domain of hundreds of wavelength division multiplexing (WDM) channels. In recent times, the demonstration of a real-time coherent receivers using digital signal processing (DSP) to mitigate transmission impairments [18] made way for today's fiber optical systems applying spectrally efficient multilevel modulation formats.

Today, fiber optical links constitute the backbone of the Internet as well as the mobile communication network. In the next couple of years there is no foreseeable reduction in growth of the data traffic and the expected annual growth is 23 % [19]. The reasons for the continued growth are several. The number of users will increase, in the developed countries more than 80 % of the population are using the Internet while worldwide this number is 43 % which is expected to grow annually [20]. The bandwidth of the applications is expected to increase, especially real-time streaming videos, and so are the number of users for this type service.

1.1 The Outline of this Thesis

This thesis is devoted to modulation formats for coherent optical communication systems. Access to the full four-dimensional (4D) optical field, given by the in-phase and quadrature components of the two polarization sates, is typically detected in the coherent receiver to track polarization effects. This has opened up for the possibility of using modulation formats optimized in the four-dimensional space for fiber-optical communication [21, 22]. This thesis describes several two-dimensional (2D) and 4D modulation formats, as well as formats optimized in higher dimensions. Different methods of increasing the dimensionality of the signal spaces is discussed in the papers and in the thesis.

The outline of this thesis is as follows. Chapter 2 introduces the building blocks of long-haul coherent fiber optical communication systems. Chapter 3 introduces the basic notation and figures of merit needed for comparison of different modulation formats. Chapter 4 discusses conventional modulation formats and novel modulation formats optimized in a multidimensional space. The formats that are experimentally investigated in Papers A-I, as well as formats described in the literature, are introduced and explained. In Chapter 5 the different multidimensional modulation formats are compared for uncoded and coded systems. Finally, Chapter 6 provides a future outlook.

The included papers can be summarized as follows. In Paper A, an experimentally investigation of a 4D format based on set-partitioning of the polarization multiplexed (PM) 16-ary quadrature amplitude modulation (QAM) constellation is presented. This is an extension of [23] which presented the first experimental demonstration of this format. Paper B investigates an alternative experimental realization of the polarization-switched QPSK (PS-QPSK) symbol alphabet format using two consecutive timeslots to span four dimensions. Papers C and D investigate two different experimental realizations of biorthogonal modulation in eight dimensions using either two consecutive time-slots or two neighbouring WDM channels to realize the eight dimensions. Paper E presents an experimental study of a modulation format with the same spectral efficiency (SE) as PM-16QAM where the constellation points are selected from the most dense 4D lattice. Paper F is a theoretical paper, investigating pulse position modulation with multiple pulses per frame in combination with quadrature phase-shift keying (QPSK) and PS-QPSK. Papers G-I relate to multi-core fiber transmission where Paper G experimentally investigates the impact of different crosstalk levels on QPSK transmission. Papers H and I experimentally investigate different modulation schemes over spatial superchannels over several cores of a multicore fiber. Papers J and K investigate the impact of different channel assumptions on the achievable information rate for coded systems in the presence of fiber nonlinearities.

Chapter 2

Building Blocks of Long-Haul Coherent Fiber Optical Systems

In this chapter, the different building blocks of a coherent transmission system is introduced, starting with the channel itself followed by digital signal processing and forward error correction (FEC). The last part of this chapter introduces transmission schemes using spatial division multiplexing.

2.1 The Optical Fiber Channel

The basic principle of a fiber optical transmission system is depicted in Fig. 2.1. The aim is to transmit digital data from point A to point B without inflicting errors in the transmitted bit sequence. The channel consists of N amplified fiber spans constructing a link of desired length. The transmitter converts the binary data b to an analog optical signal which is transmitted over the channel. The receiver recovers the signal and makes decisions on the received signal to obtained the bit sequence \hat{b} . In the following sections, the different buildings blocks of the coherent transmission channel are introduced and discussed. This thesis focuses on point-to-point long-haul transmission systems, however it should be noted that optical networks with routing of different WDM channels is an important application for optical transmission systems [24–26].

2.1.1 The Optical Fiber

The optical fiber is one of the key components for long-haul optical transmission systems. The main feature is the low loss, typically around 0.2 dB/km at 1550 nm although recent records report 0.146 dB/km at 1560 nm [27] and 0.149 dB/km at 1550 nm [28]. The low loss enables transmission systems with span lengths typically



Figure 2.1: Basic schematic of a fiber-optical transmission channel.

between 50 km and 120 km, before the transmitted signal requires amplification. A signal propagating in the optical fiber can be modeled with the Manakov model [29] given by

$$\frac{\partial A_x}{\partial z} = -\frac{\alpha}{2}A_x - i\frac{\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A_x}{\partial t^3} + i\frac{8}{9}\gamma(|A_x|^2 + |A_y|^2)A_x, \qquad (2.1)$$

$$\frac{\partial A_y}{\partial z} = -\frac{\alpha}{2}A_y - i\frac{\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A_y}{\partial t^3} + i\frac{8}{9}\gamma(|A_x|^2 + |A_y|^2)A_y, \qquad (2.2)$$

where A_x and A_y are the complex amplitudes of the two orthogonal polarization components and α is the attenuation along the fiber. The terms containing β_2 and β_3 govern the effects of chromatic dispersion where β_2 is related to the dispersion coefficient as $D = -2\pi c \beta_2/\lambda^2$. For a single mode fiber (SMF), D is typically in the region between 16–17 ps/(nm km). The parameter β_3 governs the third-order dispersion and can often be neglected. The term containing the nonlinear coefficient γ gives rise to the nonlinear interactions such self-phase modulation, cross-phase modulation, and four-wave mixing [30]. The factor 8/9 is due to random birefringence and assumes long fiber lengths to achieve averaging. The nonlinear interactions are a limiting factor for long-haul transmission systems as the strength increases with optical power, preventing the system to operate at higher optical powers to achieve an increased link optical signal-to-noise ratio (OSNR) [31].

2.1.2 Optical Amplification Techniques

The most common type of optical amplifier in commercial systems as well as in research experiments today is the erbium-doped fiber amplifier (EDFA). The EDFA was invented in the mid 1980's and revolutionized fiber-optical transmission systems providing a solution capable of amplifying signals in the optical domain [15–17]. Before the EDFA, the optical signal had to be detected and retransmitted to combat the attenuation of long links. The EDFA is constructed from a fiber doped with the rare-earth element erbium which can be optically pumped to achieve population inversion and thus provide gain. The noise generated in the EDFA originates from spontaneously emitted photons due to the population inversion. These photons are emitted with random phase, polarization and wavelength (within the bandwidth of the EDFA), and are amplified as they propagate along the erbium-doped fiber. The generated noise is thus designated amplified spontaneous emission (ASE) noise. The most common gain bandwidth of the EDFA is the so-called "C-band" with the range



Figure 2.2: Illustration of the AWGN channel.

1530- -1565 nm. However, optical amplifiers for different wavelengths suitable for fiber-optical communication exist [32].

After the EDFA, the most common amplification technology for transmission links is the Raman amplifier [33, 34]. The gain arises from stimulated Raman scattering which transfer energy from a pump wave that can be co- or counter-propagating with the signal. One key difference between Raman amplification and the EDFA is that the amplification in the Raman case is distributed along the transmission fiber [35]. Higher order Raman amplifiers [36] use additional pumps that amplify the first-order pump which pushes the distributed gain region further into the fiber [35]. Raman amplifiers are widely available today and deployed in commercial systems. Other less common optical amplifiers includes semiconductor optical amplifiers (SOAs) [37, 38] and phase-sensitive amplifiers [39].

2.1.3 Modeling of Fiber Optical Transmission Links

The coupled differential equations in Eqs. (2.1)-(2.2) require a solution using numerics, which is typically done using the split step Fourier method [40]. This is computationally complex, i.e. time consuming, and since the solution depends on many input parameters such as pulse shape, input power, channel separation, transmission distance, etc., it is hard to draw general conclusions from numerical simulations [30]. Hence, simpler models which resemble the solution to Eqs. (2.1)-(2.2) under certain simplifications or conditions are of interest.

The AWGN Channel

The most simple model for a fiber optical communication system is shown in Fig. 2.2 where additive white Gaussian noise (AWGN) is considered as the only impairment. The transmitted bits b are mapped to discrete symbols x_i drawn from the constellation \mathscr{C} and converted to a continuous signal $x(t) = \sum x_i p(t - iT)$, where p(t) is the pulse shape. The signal is transmitted over the continuous channel which adds uncorrelated white Gaussian noise n(t) with zero mean. The receiver is modeled as a perfect matched filter and ideal sampling with one sample per symbol. The received symbols are then given as $y_i = x_i + n_i$ where n_i is zero-mean Gaussian variable with variance σ^2 . Neglecting the nonlinear effects of the transmission system, the variance can be estimated from knowledge of the pulse shape, the loss in each fiber span, and the noise figure of the EDFAs. Decisions are made on y_i to find the most likely transmitted bits \hat{b} .

The Gaussian Noise Model

For transmission systems without inline dispersion compensation, i.e., where the accumulated dispersion is large, the total distortion including the ASE noise and the nonlinear interaction has been shown to resemble AWGN. The received signal for these systems can be modeled with the so-called *Gaussian noise model* [41–44] where the nonlinear interactions are approximated as an extra degradation in the received OSNR as

$$OSNR_{tot} = \frac{P_{ch}}{P_{ASE} + \alpha_{NLI} P_{sig}^3},$$
(2.3)

where $P_{\rm ch}$ is the power of the channel, $P_{\rm sig}$ is the total average signal power of the full WDM spectra, $P_{\rm ASE}$ is the power of the ASE noise and $\alpha_{\rm NLI}$ depends on the transmission fiber and the transmitted signal [45]. As mentioned, this model is a good approximation for systems with large amounts of accumulated dispersion, which depends on the symbol rate that has to be sufficiently high to ensure that the pulses are broadened quickly. It is also only valid for signal powers that are not unreasonably higher than the optimal power. This model has shown to predict the performance of various 2D and 4D modulation formats in links without inline dispersion compensation, with good precision when compared to experimental results at a symbol rate of 28 Gbaud [46].

It is important to note that this model does not take memory effects into account as it treats the nonlinear effects as AWGN. However, the interplay between signal nonlinear interactions and dispersion are in fact deterministic and if compensated for can increase the throughput of the system [47–49].

2.2 The Transmitter

In this section the important components of the transmitter for a coherent optical communication system are discussed.

2.2.1 The Laser

For long-haul fiber optical communication, a coherent light source is needed, i.e., a laser, to enable communication using the phase of the carrier. Lasers can be constructed in many different ways and the type of laser that is used is strongly dependent on requirements such as price, output power, wavelength range and tunability, linewidth, compatibility with integration, etc. For coherent optical communication systems, the most important property of the laser is the amount of phase noise, since information is modulated on both the amplitude and the phase of the carrier. Ideally, the spectral shape of the laser would be a delta function at the carrier frequency. However, the presence of phase noise means that the spectral shape is broadened, which is often modeled with a Lorentzian shape [50, 51]. The amount of phase noise



Figure 2.3: Schematics for an optical I/Q-modulator.

is typically described by the linewidth which is a measure of how stable the phase is over time. The phase noise is typically modeled as a Wiener process

$$\Phi_i = \Phi_{i-1} + \Delta n_i, \tag{2.4}$$

where Φ_i is the phase of the *i*th sample and Δn_i is an independent and identically distributed Gaussian random variable with zero mean and variance given as $\sigma_{\Phi}^2 = 2\pi\Delta\nu T_s$, where $\Delta\nu$ is the linewidth of the laser and T_s is the sampling interval [52]. For a coherent receiver the phase noise is given by the mixing of the free running signal and local oscillator (LO) lasers and $\Delta\nu = \Delta\nu_{\rm sig} + \Delta\nu_{\rm LO}$. The mitigation of phase noise using DSP is discussed in section 2.4.5.

2.2.2 The I/Q-Modulator

In order to use multilevel modulation formats such as 16-ary quadrature amplitude modulation (16QAM), access to modulate both the amplitude and the phase of the signal is needed. In legacy direct-detection systems, Mach-Zehnder modulators (MZMs) were used to modulate on-off keying (OOK) and binary phase-shift keying (BPSK) signals or phase-modulators were used for phase shift keying signals. For coherent systems, in-phase and quadrature-modulators (I/Q-modulators), for which a schematic overview is shown in Fig. 2.3, are typically used. The optical carrier is split into two arms, each containing an MZM. The relative phase shift between the arms is $\pi/2$ which results in the two signals being added orthogonally and thus arbitrary modulation over both phase and amplitude is possible. To enable the use of the polarization for modulation, typically a second I/Q-modulator is utilized where the two modulated signals are combined with orthogonal polarizations using a polarization beam combiner (PBC). This can be done with discrete components, as has been done in the papers included in this thesis, or more conveniently using an integrated dual-polarization I/Q-modulator.

I/Q-modulators are almost exclusively used for coherent fiber optical communication systems. However, alternative techniques exist such as using directly-modulated injection-locked lasers for MQAM generation [53]. Before high-speed digital-toanalog converters (DACs) were widely available, cascading of I/Q-modulators to enable MQAM generation with binary driving signals was investigated [54].

2.2.3 Electrical Signal Generation

In direct-detection systems, the modulators are typically driven by binary signals, or multilevel signals generated by combining binary signals in the electrical domain. Non-return to zero (NRZ) pulse shapes are obtained by simply using the binary input sequence for modulation. Pulse shaping is performed in the optical domain where return-to-zero pulse shapes are obtained using a pulse carver [55] or by using mode-locked lasers [56]. Early coherent experiments used NRZ driving signals as binary pattern generators at high bit-rates were available with this pulse shape. To generate 4-ary pulse amplitude modulation (PAM) driving signals to modulate 16QAM, binary patterns were combined with different amplitudes levels [57]. This is also the case for most papers included in this thesis, except the most recent papers which use a high-speed arbitrary waveform generator.

The development in high-speed electronics has made DACs available with sufficient bandwidth and effective number of bits (ENOB) for the symbol rates required in optical communication systems. This has opened up the possibility of tailored driving signals that can for instance apply pulse shaping in the electrical domain using raised-cosine or root-raised cosine pulses [58, 59]. Further, pre-equalization to compensate for the transfer characteristics of the transmitter components can be applied [60]. Using, DACs in the transmitter, it is also possible pre-compensate for dispersion in the electrical domain [58]. Another possibility is nonlinear precompensation [61, 62] of the system. The high-speed DAC has enabled transmission systems with high SE since the pulse shaping allows channels with narrow spectral widths, thus permitted channel separation close to Nyquist spacing (channel spacing close to the symbol-rate) [63, 64].

2.3 The Coherent Receiver

Legacy systems use direct-detection techniques, where a photo-detector is used to convert the optical signal to the electrical domain. The photo-detector is a squarelaw detector, i.e. the output current is proportional to the optical power while the phase information of the signal is lost in the detection. This method is used to detect OOK signals. Another common format for direct detection system is BPSK, which can be detected using a delay interferometer to find the relative phase between received symbols [65]. For more advanced modulation formats, the receiver structure becomes more complicated when using direct detection. However, different formats with 3 bits [66] and 4 bits [67] per symbol have been demonstrated.

During the 1980's, a lot of research was focused on coherent detection systems due to the increased sensitivity over direct detection [68–70]. However, these coherent



Figure 2.4: Schematics for a polarization diverse coherent receiver.

systems were hard to implement since they required an optical phase-locked loop to synchronize the LO phase to the signal phase. As the main goal of these systems were to increase the sensitivity over direct detection techniques, the interest in coherent detection was lost with the invention of the EDFA [15–17], which enabled the receiver sensitivity to be significantly increased by optical pre-amplification.

In more recent times, the interest in coherent detection was renewed when gamechanging real-time measurement using DSP-aided coherent receivers with free-running LOs were demonstrated [18, 71]. At this point in time, the electronics had reached speeds where DSP could be used to track the phase difference between the signal and LO lasers, thus rendering the complicated hardware used for phase tracking unnecessary. Slowly varying polarization-rotations could also be tracked in the DSP, enabling much simpler realizations of polarization-multiplexed formats without the use of optical hardware-based polarization tracking. The recent interest in the coherent receiver was as a method to enable detection of spectrally efficient modulation formats. The introduction of DSP also opened up a new research field where for instance dispersion compensation, nonlinear mitigation, equalization and polarization demultiplexing and tracking could be performed in the digital domain.

2.3.1 Polarization Diverse Coherent Optical Front-End

Today, coherent detection is almost exclusively used in experiments and new commercial long-haul systems. A polarization-diverse coherent receiver is illustrated in Fig. 2.4. An optical signal E_{sig} is split into two orthogonal components using a polarization beam splitter (PBS) and the two components are sent to two separate 90° optical hybrids. The LO signal is also split into two orthogonal polarization states using a second PBS and the two projections are mixed with the signal in the hybrids. Balanced photo-detectors are typically used after the hybrids to obtain four photocurrents that are proportional to the in-phase and quadrature components of the two polarizations [72]. It is important to note that the receiver structure in Fig. 2.4 maps the full 4D optical signal to the electrical domain, regardless of the polarization state of E_{sig} . All the appended papers in this thesis are based on polarization-diverse coherent detection schemes.



Figure 2.5: Illustration of a typical DSP flow showing the center sample in the xand y-pol. at different positions for a PM-QPSK signal.

Analog-to-Digital Converters

The four analog photocurrents in Fig. 2.4 are digitized using analog-to-digital converters (ADCs) that sample the signal in time using a fixed time-base which converts the analog signal into a discrete-time signal. The ADC quantizes the signal into a finite set of values which is determined by the resolution [58]. The ADC is characterized by the bandwidth, which determines the maximum symbol rate that can be used, and the ENOB, which gives the effective amplitude resolution that will determine how many signalling levels that can be used [73]. It should be noted that the ENOB is also dependent on the ADC clock timing jitter [74]. The required ENOB per increased bit for MQAM constellations [74]. For instance, to receive 64QAM, an ENOB of approximately 6 bits is required [74]. To realize high-speed ADCs, time-interleaving of lower speed ADCs is often used and digital circuits perform the interleaving of the sampled signals and compensates for any mismatch between the different ADCs [75].

2.4 Digital Signal Processing

Much of the progress for coherent optical communication systems can be attributed to the use of DSP which allows the use of free-running LOs where the phase-tracking is done in the digital domain rather than with complex hardware implementations based on phase-locked loops. Furthermore, the access to both the amplitude and the phase of the optical signal allows the use of spectrally efficient modulation formats as well as the possibility to mitigate different transmission impairments in the DSP. This section introduces the different building blocks and algorithms of a typical DSP structure, for which an overview is shown in Fig. 2.5 illustrated for a PM-QPSK signal.

2.4.1 Optical Front-End Correction

The optical signal can be distorted by imperfections in the transmitter and the receiver. In the transmitter, impairments such as non-ideal bias for the I and Q signals as well as the 90° phase shift in the I/Q-modulator, imperfect splitting ratio of the optical signal, different amplitudes of the radio frequency (RF) driving signals or different gain of the drive-amplifiers as well as non-ideal polarization splitting can distort the signal. The impairments in the receiver are mainly 90° hybrid imperfections and unequal responsivities of the photo-detectors [76].

The in-phase and quadrature parts of the detected optical signal are ideally orthogonal. However, imperfections in the 90° hybrid can cause the received signal to lose orthogonality. To compensate for this an orthogonalization algorithm is used, typically Gram-Schmidt [76, 77]. Another solution is to use the Löwdin algorithm which rotates both signal components (compared to Gram-Schmidt which leaves one vector as it is) and creates an orthogonal set of vectors which are closest in a least square distance sense to the original set of vectors [78]. This algorithm, or other symmetric methods, are suited for high-order modulation, as the quantization noise is more equally distributed over both signal components [77].

It should be noted that impairments in the transmitter that cause the signal to lose orthogonality are not possible to mitigate in this stage of the DSP using these orthogonalization methods since the I- and Q-components are constantly rotating due to the remaining intermediate frequency (IF) from LO, and also the polarization states are not yet demultiplexed.

2.4.2 Electronic Dispersion Compensation

With the coherent receiver, the dispersion compensation can be moved from the optical domain to the DSP [79–81]. One of the main benefits of not using inline dispersion compensation is that the accumulation of dispersion changes the nonlinear interaction of the system as the transmitted pulses are broadened by the dispersion. It has been shown that this can lead to increased optimal launch powers and increased achievable transmission distances as the nonlinear distortion manifests in a way that has less impact on the bit-error rate (BER) [41, 82, 83]. Increased nonlinear tolerance can also be achieved by optically compensating for all the accumulated dispersion at the receiver, or pre-compensating in the transmitter. However, as it is harder to vary the amount of dispersion using hardware, this method is less flexible. Another, advantage of using electronic dispersion compensation (EDC) is that the attenuation of the dispersion-compensating fibers (DCFs) is removed from the system. This also means that the extra EDFA per span that is typically used for compensation of the loss of the DCF can be avoided, possibly also reducing the cost of the system. However, it should be noted that chirped fiber Bragg gratings for

dispersion compensation, which themselves, in contrast to DCFs, do not induce any nonlinear distortion, is a possible alternative to the DCFs [84].

The electronic dispersion compensation is performed by applying the inverse transfer function of the dispersion. This is ideally done by applying an all-pass filter with quadratic phase which is typically implemented in the frequency domain [81]. However, implementation can also be done as a finite impulse response (FIR) filter in the time domain [85]. The biggest difference between the two implementations is the computational complexity. For small amounts of accumulated dispersion the time domain method is faster and vice versa [86]. With modern transmitters, which rely on DACs, signal processing is typically also present in the transmitter. This means that the electronic dispersion compensation can, fully or partly, be moved to the transmitter side.

For flexible systems, the dispersion coefficient D and the transmission length might not be known. To find the amount of dispersion that needs to be compensated, several blind estimation techniques exist that finds the amount of accumulated dispersion of the received signal [87–89].

2.4.3 Digital Back Propagation

Instead of only compensating the linear effect of dispersion, linear and nonlinear effects can be compensated jointly by solving the nonlinear Schrödinger equation or the Manakov model (Eqs. (2.1)-(2.2)) with inverse signs of the dispersion and the nonlinear coefficient [90–92]. In other words, the received signal is calculated as propagating backwards through the transmission link to undo effects of dispersion and nonlinear interactions. This is typically done with the split-step Fourier method [92] or by perturbation analysis [93, 94]. The nonlinear effects can also be precompensated for in the transmitter [61, 93].

Digital back propagation allows increased optical launch powers which translates into a higher OSNR and thus increased transmission distances. However, the complexity is extremely high compared to other parts of the DSP architecture. Further, in WDM transmission it has been shown that for significant gain from the back propagation, the full set of WDM channels has to be considered since a major part of the nonlinear interference arises from the WDM channels. Although complex, multichannel nonlinear compensation has been demonstrated in experiments using back propagation of superchannels detected by a single coherent receiver [95] or detected with a spectrally sliced receiver [96]. Further, nonlinear pre-compensation of 3 WDM channels with 16 Gbaud PM-16QAM with frequency-locked carriers has been demonstrated [62].

2.4.4 Adaptive Equalization and Polarization Demultiplexing

One of many advantages of using coherent detection and DSP is that the timevarying polarization rotation and polarization mode dispersion (PMD) can be tracked and compensated for in the digital domain, avoiding complicated hardware. The compensation is typically performed using four adaptive FIR-filters in a butterfly configuration as illustrated in Fig. 2.6, with complex valued filter taps [77, 85, 97].



Figure 2.6: Butterfly FIR-filter configuration for adaptive equalization and polarization demultiplexing.

It should be noted that when this filter is adapted it will typically also approximate a matched filter which can mitigate time-invariant impairments such as intersymbol interference (ISI) and, to some extent, residual chromatic dispersion.

The method for adaptation of the filter taps depends on the system and especially on the modulation format that is used. The most common blind adaption algorithm used for PM-QPSK is the constant modulus algorithm (CMA), originally introduced by Godard in 1980 [98]. This algorithm has been modified for the four-dimensional signal space spanned by PM-QPSK [77, 99]. The error function for the CMA is constructed based on the fact that the QPSK symbols have a constant power. This also makes it possible to apply the adaptive equalization before the phase noise and frequency offset is tracked. For PS-QPSK, the CMA designed for PM-QPSK does not work for polarization de-multiplexing which might seem surprising since PS-QPSK can be expressed as a subset of PM-QPSK (described in section 4.2.2). However, for PS-QPSK the constant modulus criterion is ambiguous, thus rendering polarization demultiplexing impossible [101]. Instead it has been shown that a modified cost function can be applied for proper demultiplexing of the polarization states [101].

For higher order QAM constellations, other methods of updating the filter taps are employed as these systems do not have a constant modulus. Interestingly though, the CMA does in fact work for 16QAM signals, although with suboptimal convergence and steady-state performance [102, 103]. The most common blind method for adaptive equalization for higher order QAM formats is the decision-directed least mean square (DD-LMS). The filter coefficients are updated by minimizing the distance to the closest constellation point for the received symbols. This means that polarization demultiplexing as well as frequency and phase estimation has to be performed before or within the DD-LMS loop. Typically, the CMA is used for preconvergence [102], i.e. to perform a rough polarization demultiplexing, before the equalizer is switched to DD-LMS operation. Other methods for updating the FIR filters include the radius-directed equalizer [57, 77, 104] and independent component analysis [105].

2.4.5 Carrier-Frequency and Phase Estimation

For a coherent receiver based on baseband down-conversion with a LO that is freerunning in relation to the transmitter laser, the intermediate frequency between these



Figure 2.7: Illustration of the frame structure for data-aided transmission schemes.

two lasers has to be estimated and compensated for. Further, the transmitter and LO both exhibit independent phase noise. In principle the frequency offset and the phase noise could be compensated jointly, although for practical reasons these two are often separated [77]. The frequency offset is typically found with schemes based on detecting a peak in the spectrum of the received signal [77].

For phase-shift keying modulation such as BPSK and QPSK, as well as for PS-QPSK, the phase tracking is typically performed using the Viterbi-Viterbi algorithm [106] which works as follows. For an MPSK signal, the modulation can be removed by raising the signal to the Mth power [106], e.g. for QPSK the signal is raised to the 4th power [77]. The phase is found after the modulation has been removed, typically taking the argument averaged over a block of symbols to reduce the impact of AWGN noise. This block can be implemented as a moving window and the length as well as the weighting function can be optimized depending on the signal-to-noise ratio (SNR) and the laser linewidth [107, 108]. If the same laser is used for both polarizations, the Viteri-Viterbi estimate can be extended to jointly estimate the phase using the signal in both polarizations [109] which has been shown to relax the linewidth requirement for PS-QPSK systems significantly [110].

For QAM constellations larger than QPSK, the Viterbi-Viterbi algorithm is suboptimal since the modulation cannot be removed by raising the constellation to the *M*th power and other methods are typically preferred. One possible method is partitioning of the QAM constellation into rings with constant amplitude on which the Viterbi-Viterbi is applied [103, 111, 112]. Another method is the blind phase search based on test angles [113]. This method tries a fixed set of angles and selects the angle which minimizes the distances to the closest constellation points over a block of symbols.

2.4.6 Data-Aided Digital Signal Processing

For modern systems using DACs in the transmitter, data-aided signal processing is an interesting alternative to blind DSP, specially in the context of systems which aim for a flexible choice of modulation format [46]. The data-aided DSP relies on pilot symbols inserted in-between the payload symbols carrying data, as shown in Fig. 2.7. The pilot symbols are exploited to aid the signal processing algorithms [107, 114, 115]. The data-aided scheme in [115], which was also used in Paper E, relies on BPSK symbols for frame synchronization [116]. Further, carrier offset estimation is performed with the aid of QPSK training symbols. A second set of QPSK training symbols are used for equalization and channel estimation [115]. The phase tracking typically has to be performed blindly since the phase evolution is rapid which would require a too frequent insertion of pilot symbols for a reasonable pilot overhead. The blind phase search based on test-angles can be used for format-flexible systems as it is based on decision on the transmitted constellation [113]. Pilot symbols can however be used for cycle-slip mitigation [117, 118].

2.5 Forward Error Correction

The field of error control coding was introduced around 1950 with the first publication on coding theory by Golay [3], Shannon's seminal work [2], and the introduction of the Hamming codes [4]. The use of FEC made its way to fiber optical communication systems in the early 1990's using mainly Bose-Chaudhuri-Hocquenghem (BCH) codes [119–121] and later Reed-Solomon (RS) codes after ITU-T recommendations [122, 123].

The FEC coding is an essential part of modern optical communication systems and trades a small part of the spectral efficiency, due to the overhead from the code, into a large gain in sensitivity. Without the use of FEC, higher order modulation formats such as PM-16QAM cannot be transmitted more than a few kilometers before the received signal is no longer "error free" [124], [Paper A, I], which typically is defined as BER $< 10^{-15}$. However, using advanced FEC schemes, transmission over transoceanic distance has been demonstrated with PM-16QAM [124–128]. In fiber-optical systems, it is also important that the FEC has resilience towards error bursts [129].

The traditional coding schemes are based on hard decision (HD) decoding which means that the received samples are demodulated and detected into a finite alphabet of bits or symbols before information is passed to the decoder. Modern coding schemes relies on soft decision (SD) decoding, which passes soft information of the received samples, typically in the form of log-likelihood ratios (LLRs), to the decoder. Examples codes typically used with SD decoding are low-density parity check (LDPC) codes [130, 131], polar codes [132, 133] and turbo product codes (TPCs) [134, 135]. Note that many recent coding schemes rely on a concatenation of an inner SD iterative coding scheme that might suffer from an error floor [136] and a HD outer code that cleans up the output from the inner code such that the BER levels approaches error free (BER < 10^{-15}). Turbo product codes have been considered for optical communication in [137–139] [Paper E]. The most common SD coding schemes considered in recent fiber optical communication experiments are based on LDPC codes [140–142], which has also been used in real-time field experiments [143].

The choice of coding scheme depends on many parameters such as the desired coding gain, tolerance against burst errors, choice of modulation format, complexity of real-time implementation, tolerated overhead, flexibility of error correcting capability, etc. The comparison of different coding solutions for optical communication systems has been studied in plenty of publications [133, 139, 144–149]. Many of the recent publications suggests LDPC codes [145, 148], often from the family of spatially-coupled LDPC codes [133, 146, 149, 150], for future fiber optical systems.



Figure 2.8: Basic layout of a transmission system with FEC encoding and decoding.

However, as discussed in several of these papers the extra gain from SD codes comes at the cost of significantly increased encoder and decoder complexity [139, 146, 147].

A very basic outline of a transmission system using FEC is shown in Fig. 2.8. Note that for optimal performance, the modulation format and the FEC coding scheme should be jointly optimized. For instance, assuming an AWGN channel, a capacity-achieving system should apply a "Gaussian-like" modulation scheme [151, 152]. However, for a practical case, although assuming capacity-achieving coding schemes, the resolution of the DACs, discussed in section 2.2.3, will limit the type of constellation that can be used. Further, the linewidth of the lasers used as well as the capability of the DSP to recover the constellation at extremely low SNR will further limit what constellation type that can be used in practice. However, in general the larger the constellation the better is the sensitivity with optimal coding, compared at the same achievable information rate, i.e. the rate at which error free transmission is possible assuming capacity achieving codes. This can be seen in Fig. 2.9 where the achievable information rate for the AWGN channel (calculated using mutual information (MI) which is explained in section 3.4) for conventional MQAM constellations are plotted. As seen, compared at the same SNR, a larger QAM constellation can operate at a higher information rate. In theory this means that, neglecting complexity, the optimal system should operate with the maximum number of signaling levels per dimension (i.e. per input to the I/Q-modulator) that the DACs and DSP can handle.

For practical systems however, due to the complexity of the circuit design, it may be desirable to keep the FEC coding scheme fixed. This means that for systems such as the one illustrated in Fig. 2.8, optimization is instead moved to the choice of modulation format. For practical reasons, fiber-optical communication systems are often assumed to use a fixed FEC scheme, as for instance in the modulation format-flexible systems studied in [46]. A fixed FEC scheme is also often assumed when different modulation formats for optical communication systems are compared, as is the case for the papers appended in this thesis.



Figure 2.9: Achievable information rates for MQAM-constellations.

2.6 Space-Division Multiplexing

With modern fiber optical communication systems applying technologies as for example Nyquist spaced WDM, nonlinear compensation techniques, Raman amplification, and advanced optimized FEC schemes, the maximum throughput that a single fiber system can sustain is expected to be reached in the near future. The gain seen for new technologies in recent times is small compared to the era when WDM was introduced with the invention of the EDFA where more and more channels could be used to increase the throughput. Today, a situation is approached where the bandwidth supported by a system using a sole SMF is fully occupied. This is sometimes referred to as the "capacity crunch" [31, 153–155].

One possibility of overcoming this imminent limitation is by introducing a new physical dimension to the systems, i.e. space. Systems using space-division multiplexing (SDM) basically comes in two variants; transmitting signals over several fiber modes in a multimode fiber or over several cores of a multicore fiber (MCF). However, note that SDM can also realized using parallel SMFs, which has been discussed for upgrades scenarios in combination with SDM-compatible amplifiers [156]. The concept of mode division multiplexing dates back to the early 1980's with the first experimental realization [157], demonstrating transmission over two separate modes. Fibers supporting more than one mode are typically divided into few-mode fibers [158–161], which are designed to support only the lowest order modes and multimode fibers [162, 163] which guides many modes (72 modes at 1550 nm is stated in [163]). Although the support of a large number of spatial channels is attractive from a throughput point of view, it comes at the cost of complexity. As the modes propagate along the fiber, they couple and multiple input multiple output (MIMO) equalization is required to untangle the modes at the receiver [160, 162, 164, 165].



Figure 2.10: Illustration of different layouts of the cores of a (a) 7-core fiber [166], (b) 12-core fiber [167], (c) 19-core fiber [168], and (d) 22-core fiber [169].

This is further complicated by the different propagation speeds of different modes.

MCFs have several cores within the same cladding, where the size of the fiber itself is similar to that of a single-core fiber. MCFs for communication purposes date back to 1979 [170], when a seven core fiber was experimentally investigated. The MCFs are fabricated with different number of cores where the most common layouts are the following: Seven cores with one core in the middle and six on an outer ring [166, 170, 171], [Papers G, H, I], 12-cores in a ring-like structure [167, 172] and 19 cores on a hexagonal lattice [168, 173]. Illustrations of the cross-sections of these fibers are shown in Fig. 2.10a-c. Note that other core layouts exist such as 6 core fibers [174] and 10 core fibers [175]. Recently, a 22-core fiber was used in an experiment reporting a record throughput per fiber of 2.15 Pb/s [169]. The cross-section of this fiber is illustrated in Fig. 2.10d.

The biggest challenge in MCF systems is the crosstalk between the cores. The most straightforward method of reducing the crosstalk is to place the cores with large separation. However, the cross-section area of the fiber is limited due mechanical properties, which means that other methods of reducing the crosstalk have to be employed, where two major techniques exists today. The first technique reduces the coupling by either using trench-assisted structures where a low refractive index is introduced around each core to suppress the field overlap [167, 176] or by introducing air holes around the cores in so-called hole-assisted MCFs [174]. The second technique designs the cores with slightly different refractive indices in adjacent cores as the coupling strength is dependent on the propagation constant [176, 177]. These types of fibers are usually referred to as heterogeneous MCFs. Another option is to allow strong coupling between the cores and, similar to multimode transmission, apply MIMO equalization in the receiver to compensate for this coupling [155, 164].

To select between MCF or multimode fibers for future systems is not trivial and both technologies have drawbacks and advantages. MCFs can be constructed with low crosstalk but with increasing core numbers the crosstalk will increase and MIMO processing might be needed [155, 164]. Multimode fibers scale easily to a large number of spatial channels but requires MIMO processing and possibly large amounts of buffering in the receiver as the modes have different propagation speeds. The large effective area of the multimode fiber could possibly reduce the impact of fiber nonlinearities [158] while for MCF the effective area is similar to that of an SMF. Splicing of MCF is more difficult compared to multimode fibers, as the crosstalk depends on the rotation of the fiber [178]. Further practical issues, such as techniques for coupling of light into the spatial channels and amplification technologies, will determine which SDM technology that will emerge as the most practical solution [165]. In fact, maybe the most practical solution will be MCF where each core is multimode [179] as the complexity of the MIMO equalizers might be kept reasonable. These types of fibers have been used to demonstrate record number of spatial channels [180, 181].

It should be noted that other spatial division multiplexing techniques exist, such as simply using a bundle of SMFs [156, 165] or multi-element fibers which are individual fibers bundled together in the same coating [182]. Further, photonic bandgap fibers are a promising technology which guides the light in air, thus having the potential to achieve lower loss, latency, and nonlinear interference as well as a larger low-loss bandwidth [183–185].
Chapter 3

Modulation Formats and Performance Metrics

In this chapter, different metrics used for comparing modulation formats are discussed.

3.1 Basic Concepts and Notation

In this section modulation formats are studied assuming a discrete-time memoryless channel with AWGN as the only impairment (see section 2.1.3). The kth symbol of a symbol alphabet is denoted as the vector

$$\mathbf{c}_{k} = (c_{k,1}, c_{k,2}, \dots, c_{k,N}), \tag{3.1}$$

where N is the number of dimensions. Traditionally, modulation formats are considered in the two-dimensional space spanned by the in-phase and quadrature parts of the signal. In that case, the kth symbol can written as $\mathbf{c}_k = (\operatorname{Re}(E_{x,k}), \operatorname{Im}(E_{x,k}))$, where $E_{x,k}$ is the discrete sampled optical field in the x-polarization. As the full 4D field is required in the receiver for polarization-demultiplexing, modulation over four dimensions has attracted significant research attention. A 4D symbol, where the four dimensions are given by in-phase and quadrature components of the two polarization states, can be denoted as $\mathbf{c}_k = (\operatorname{Re}(E_x), \operatorname{Im}(E_x), \operatorname{Re}(E_y), \operatorname{Im}(E_y))$ where E_x and E_y denote the sampled optical field in the x- and y-polarization state, respectively.

The symbol alphabet, or constellation, of a modulation format with M symbols is given by the set of vectors

$$\mathscr{C} = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M\}. \tag{3.2}$$

With this notation, the constellation of QPSK is given as $\mathscr{C}_{QPSK} = \{(\pm 1, \pm 1)\}$ and PM-QPSK as $\mathscr{C}_{PM-QPSK} = \{(\pm 1, \pm 1, \pm 1, \pm 1)\}$. The notation " \pm " indicates all possible sign selections, i.e. the constellation for \mathscr{C}_{QPSK} is given as $\{(\pm 1, \pm 1)\}$ $\{(1,1),(1,-1),(-1,1),(-1,-1)\}$. Note that " \pm " will also be used in a different way as $\{\pm(1,1)\}$ to indicate $\{(1,1),(-1,-1)\}$. Further, the cartesian product, "×", denotes the set of all ordered pairs. In particular, this is used to generate a constellation with increased dimensionality. For instance, using $\mathscr{C}_{\text{BPSK}} = \{(\pm 1)\}$, QPSK can be described as $\mathscr{C}_{\text{QPSK}} = \mathscr{C}_{\text{BPSK}} \times \mathscr{C}_{\text{BPSK}}$. Further, PM-QPSK can be written as $\mathscr{C}_{\text{PM-QPSK}} = \mathscr{C}_{\text{QPSK}} \times \mathscr{C}_{\text{QPSK}}$. Note that $\mathscr{C} \times \mathscr{C}' \neq \mathscr{C}' \times \mathscr{C}$, where \mathscr{C} and \mathscr{C}' are two different sets.

For convenience the operator $\mathscr{P}(\cdot)$ is introduced to denote all unique permutations. As an example

$$\{\mathscr{P}(\pm 1,0)\} = \{(1,0), (-1,0), (0,1), (0,-1)\}.$$
(3.3)

The average symbol energy, E_s , of a modulation format with M symbols, assuming uniform probability of the symbols, is given by

$$E_s = \frac{1}{M} \sum_{k=1}^{M} ||\mathbf{c}_k||^2, \qquad (3.4)$$

where $||\mathbf{c}_k||^2$ is the energy of the *k*th symbol. The average energy per bit is $E_b = E_s / \log_2(M)$.

The Euclidean distance between two symbols of a constellation is given by $d_{k,l} = ||\mathbf{c}_k - \mathbf{c}_l||$. The minimum Euclidean distance [22] of a constellation is given by

$$d_{\min} = \min_{l \neq k} d_{k,l}.$$
(3.5)

This minimum Euclidean distance is used in the following sections in the calculation of different metrics for comparing modulation formats. The robustness to additive white Gaussian noise (AWGN) of a constellation is dependent on d_{\min} , as the majority of the symbol errors will occur between symbols that are closest to each other in the space where decisions are taken.

3.2 Lattices and Sphere Packing

Finding modulation formats with high uncoded sensitivity can be formulated as *sphere packing* in N dimensions [186]. This reasoning assumes an AWGN channel with the same noise variance in every dimension. For this type of channel the noise is circular (in two dimensions), spherical (in three dimensions) or hyper-spherical in higher dimensions. As an example, this problem can be formulated in two dimensions as how to optimally pack M coins on a table. Using computer-aided searches with different algorithms, optimized sphere packings have been found in two and three-dimensions [187] as well as in four dimensions [188]. It should be noted that the problem of finding the optimal packing for a large number of spheres [186] is quite different from finding the optimal packing of only a few spheres [22, 188–190]. In coherent optical communication, two, four and eight dimensions are of specific interest due to the fact that the phase and amplitude of one polarization constitutes two dimensions. However, for other types of systems other dimensionalities might be



Figure 3.1: $2 \times 2D$ -projections of two different representations of the optimal packing of 8 points in 4 dimensions showing (a) sphere-packing results from [188] and (b) the same 8 points rotated into a subset of the D₄ lattice.

of interest such as for intensity modulated systems where modulation formats have been optimized with sphere packing in three dimensions [191].

The constellations obtained by sphere packing are often irregular, which makes implementation and detection of modulation formats based on such structures complicated. In low number of dimensions, the most densely sphere-packed constellations can also often be expressed as constellation points on regular periodic structures, often referred to as *lattices*. Constellations based on selection of M points from a lattice are more regular which possibly simplifies implementation. This is illustrated in Fig. 3.1, showing projections in two 2D planes of two realizations of the same constellation [192] where Fig. 3.1a shows a polarization rotated version of the constellation from [188] and Fig. 3.1b the optimal way to choose eight points from the D₄ lattice (explained later in this section). As seen, due to the symmetry of Fig. 3.1b, that form of the constellation would be easier to realize in an experiment which is a reason for investigating constellations with points on lattices.

Most conventional modulation formats consist of points from the 2D rectangular lattice, denoted as Z_2 and shown in Fig. 3.2a. If four points are cut out in a rectangular fashion QPSK is obtained and if 16 points are cut out in the same way, 16QAM is obtained. However, this lattice is not the most dense in two dimensions. By using the hexagonal lattice A_2 , shown in Fig. 3.2c, a denser packing can be achieved and this has been studied as a base for coded modulation in wireless systems [193].

For coherent fiber optical systems, modulation formats are often optimized in the 4D space. One commonly used lattice is denoted D_4 . To illustrate this lattice, a 2D example with the D_2 lattice is plotted in Fig. 3.2b. This lattice can be derived from the Z_2 lattice by removing half the points such that the minimum distance between any pair of points is increased. This can be done in two ways, corresponding to the two subsets of the Z_2 lattice and as seen the obtained pattern is in analogy with a "checkerboard pattern". However, in two dimensions Z_2 and D_2 have the same properties, which can be understood from the fact that Z_2 can be obtained from D_2 by a simple 45° rotation and re-scaling. In four dimensions however, the Z_4 and D_4 latices no longer have the same properties and D_4 is more dense than Z_4 . Further,



Figure 3.2: Illustration of the (a) Z_2 and the (b) D_2 lattices. Note that these are the same lattice just rotated and scaled. This figure serves as an illustration of the difference between the Z- and D-lattices which is used in for instance four dimensions to derive different modulation formats. (c) Shows the A_2 lattice.

the D_4 is denser than the A_4 lattice, and is in fact the most dense lattice in four dimensions. Many of the well-known 4D formats, such as PS-QPSK [22] and the setpartitioning QAM formats [194], have their constellation points on the D_4 lattice. Different lattices for optical communication has been studied in [195].

3.3 Metrics for Comparing Modulation Formats

To compare the performance of different modulation formats, the appropriate figure of merit depends on the intended scenario. For legacy systems where the WDM spacing is much larger than the symbol rate, as illustrated in Fig. 3.3a, the bandwidth of the channels can be increased when using formats with lower SE to achieve the same overall bit rate as when a format with higher SE is used. For this type of system, modulation formats can be compared at the same bit rate. In modern systems, pulse shaping is applied to achieve compact spectra, thus allowing channel spacing close to or at the symbol rate [196] as illustrated in Fig. 3.3b. For this type of system, the bandwidth is fixed and the formats must be compared at the same symbol rate. Note that even if the channel spacing is changed to allow for wider channels, it is not possible to increase the overall bit rate of the full WDM system in this way since the



Figure 3.3: (a) legacy WDM spacing (b) Nyquist-spaced WDM.

ultimate limit is the bandwidth provided by the EDFAs. Depending on which type of FEC the targeted system applies, different figure of merits should be use. In the following section, different metrics suitable for scenarios without FEC or with hard decision (HD) FEC are introduced. Metrics suitable for systems with soft decision (SD) FEC will be introduced in section 3.4. These different metrics will be used later for comparing different formats.

3.3.1 Uncoded Spectral Efficiency

Comparing modulation formats without FEC, the SE is typically defined as the number of transmitted bits per polarization, i.e. per pair of dimensions, as

$$SE = \frac{\log_2(M)}{N/2},\tag{3.6}$$

where M is the number of symbols in the constellation and N the dimensionality [22]. Further, $\log_2(M)$ is the number of bits per symbol and the unit of SE is "bit/symbol/dimension-pair" (bit/2D). When comparing 4D formats, the unit "bit/symbol/four-dimensions" (bit/4D) will sometimes also be used. Note that in many experimental investigations, another measure of SE is often given, namely "bit/second per bandwidth use", bit/s/Hz. However, this measure requires information about channel spacing and bandwidth.

3.3.2 Asymptotic Power Efficiency

The asymptotic power efficiency (APE) is a relevant measure when comparing modulation formats at the same bit rate, i.e. targeting legacy WDM with large channel separation where bandwidth expansion is possible. Another relevant scenario is elastic optical networks where the channel bandwidth can be varied [197]. The APE gives the sensitivity gain over QPSK at asymptotically high SNR. The APE is given as [198, Section 5.1.2]

$$APE = \frac{d_{\min}^2}{4E_b} = \frac{d_{\min}^2 \log_2(M)}{4E_s}.$$
 (3.7)

The factor 1/4 normalizes the APE to 0 dB for BPSK and QPSK [22]. The APE is often given in dB and it should be noted that it is also common to use the *asymptotic power penalty* which is defined as 1/APE.

3.3.3 Constellation Figure of Merit

The constellation figure of merit (CFM) is a relevant measure when comparing modulation formats at the same symbol rate, i.e. with the same bandwidth. In the same way as for the APE, the CFM gives the sensitivity difference between two formats at asymptotically high SNR. The CFM is given as [199, 200]

$$CFM = \frac{d_{\min}^2 N}{2E_s}.$$
(3.8)



Figure 3.4: Examples of two different bit-to-symbol mappings for QPSK showing (a) natural mapping and (b) a Gray-coded constellation. (c) Shows an example of a 2-bit constellation which is not possible to Gray-code.

This is a more applicable measure for modern systems operating with a channel spacing close to Nyquist spacing, as illustrated in Fig. 3.3b. In the same way as for the APE, CFM is typically given in dB. As an example, the CFM for QPSK is 3.0103 dB.

3.3.4 Monte Carlo Simulations

The APE and CFM both give a sensitivity comparison between modulation formats at asymptotically high SNR. However, today's coherent communication systems rely on FEC and if hard decision (HD) coding schemes are used, typically the pre-FEC BER target is in the region around 10^{-3} [122, 201]. In this region, APE and CFM are no longer valid and to find the sensitivity, simulations have to be used. To find the theoretical predicted BER at a certain SNR, simulation with AWGN as the only impairment and minimum Euclidean distance decoding is performed. The results will be dependent on the bit-to-symbol mapping. An example of natural and Gray mapping of a QPSK constellation is shown in Fig. 3.4. Note that for the Gray-coded constellation, making an error to a symbol at distance d_{\min} results in exactly one bit error. For the natural mapping, a transition between some nearest neighbours results in two bit errors. For many of the higher-dimensional modulation formats, it is impossible to Gray-code the constellation as the close packing results in more nearest neighbours than bits. An example of such a constellation is shown in Fig. 3.4, where four constellation points have been placed around a point at the origin. As seen, the center point has three symbols at distance d_{\min} and carries two bits and can thus not be Gray coded.

3.4 Mutual Information

For modern coherent optical communication systems, FEC (see section 2.5) is an essential component which increases the sensitivity of the system significantly. A simple schematic of a coherent transmission system applying FEC is depicted in



Figure 3.5: (a) simple schematic of a coherent transmission system. For hard-decision codes, decision on the received symbols are made as shown in (b). For soft-decision decoding, soft information, such as the log-likelihood ratios (LLRs), is passed to the decoder as indicated in (c).

Fig. 3.5a. The information bits are encoded, transmitted over the channel and detected. After the DSP, the received symbols are sent to the FEC decoder. Codes such as RS [122] and BCH [201] with HD decoding are widely used for long-haul fiber optical transmission systems. As shown in Fig. 3.5b, for HD decoding, decisions to bits or symbols are made on the received samples before information is passed to the decoder. Recently, FEC schemes based on SD have gained significant interest and a major part of recent system experiments assume SD FEC. For SD decoding, the symbols are not detected into a finite set of bits or symbols before decoding, but rather the soft information describing the reliability of each symbol or bit, often in the form of LLRs, is passed to the decoder.

For the HD codes, there exists a relation between the input BER and the BER after decoding [139]. Hence, the term "FEC limit" is typically used and this limit often lies somewhere around BER $\approx 10^{-3}$. Thus, for this type of systems, estimating the BER from the received constellation gives a good estimate of the post-FEC BER and this is typically what is done in experiments today. For SD decoders, there exists no direct relation between the pre-FEC BER and the post-FEC BER. This can intuitively be understood from Fig. 3.5c as the decoder works on soft input and not bits, and information is lost if decisions are made on the received symbols. In other words, the decoding performance can depend on the modulation scheme used. However, the tradition of using the "FEC limit" has transferred to systems where SD coding schemes are assumed. The best solution to find the post-FEC BER would be to implement the full system, including encoder and decoder. However, as most experiments uses off-line processing it is not feasible to process enough samples for good statistics at a post-FEC BER of 10^{-15} .

For systems assuming SD FEC, it is clear that a better figure of merit than the pre-FEC BER is needed. It has been shown for optical communication systems



Figure 3.6: Illustration of the input and output variables as well as the distributions used with mutual information.

that the MI is a more reliable figure of merit [202]. MI has been studied in various scenarios for the fiber optical channel. Assuming that the nonlinear effects are small and/or only considering specific propagation effects, lower bounds on the MI have been found [203–206]. A lower bound on the MI has been estimated in simulations for different fiber-optical links [207], and for ring-like constellations [31]. Further, MI has also been studied for constellation shaping for the nonlinear fiber optics channel [208, 209]. In [210], a lower bound on the MI using mismatched decoding is found when the nonlinear interference of a WDM system is modeled as slowly varying ISI. In fiber-optical experiments, MI has been used as a figure of merit in transmission experiments of PM-QPSK [202], when investigating transceiver induced limits on the constellation order [211, 212], and to investigate the reach increase with digital back propagation [213].

3.4.1 Definition of Mutual Information

Consider a memoryless discrete input, discrete output channel as shown in Fig 3.6 for N=2. The channel input X is an N-dimensional real input random variable that is drawn from the constellation \mathscr{C} with probability $P_X(x)$. The N-dimensional real output of the channel is denoted Y and it is dependent on X and the channel. The MI is given by

$$I(\boldsymbol{X};\boldsymbol{Y}) = \sum_{\boldsymbol{x}\in\mathscr{C}} P_{\boldsymbol{X}}(\boldsymbol{x}) \int_{\mathbb{R}^N} p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}|\boldsymbol{x}) \log_2 \frac{p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}|\boldsymbol{x})}{p_{\boldsymbol{Y}}(\boldsymbol{y})} d\boldsymbol{y},$$
(3.9)

where \mathbb{R}^N denotes the *N*-dimensional space, $p_{Y|X}(y|x)$ is the channel transition distribution and $p_Y(y)$ is the channel output distribution [2, 31, 214]. The MI gives the highest information rate at which reliable communication is possible, i.e. where the post-FEC BER approaches zero. In other words, at this rate there exists a code where the BER after decoding approaches 0. If X is drawn from \mathscr{C} with uniform probability, the maximum MI over all possible channels is $\log_2 M$, where M is the

cardinality of the constellation. In information theory, the term *capacity* is defined as the maximum MI for the specific channel over any possible input distribution. It is important to note that Eq. (3.9) gives the MI for a memoryless channel and that the fiber-optical channel does exhibit memory from the interplay between dispersion and nonlinear effects. This means that Eq. (3.9) gives a lower bound on the MI calculated over sequences of symbols [31, 215]. It has also been shown that considering a finite memory from nonlinear effects in the fiber-optical channel can have a large effect on the achievable information rate [216].

In many cases, the channel is assumed to be memoryless AWGN such that $\mathbf{Y} = \mathbf{X} + \mathbf{Z}$, where \mathbf{Z} is AWGN with zero mean and variance $\sigma_N^2 = N_0/2$ in each dimension [31]. It can be shown that for the AWGN channel, capacity is reached when \mathbf{X} has a Gaussian distribution [217, Ch. 10]. This gives the famous AWGN capacity [2] which is given as

$$C = \frac{N}{2}\log_2(1 + \text{SNR}), \qquad (3.10)$$

where C is given in bits per N-dimensional symbol and SNR = $E_S/(N \cdot N_0/2)$ [218].

3.4.2 Estimating Mutual Information for a Fiber Optical Channel

Assuming a memoryless AWGN channel, the MI given by Eq. (3.9) can be estimated using a Monte Carlo approach of K received symbols as

$$\frac{1}{K} \sum_{i=1}^{K} \log_2 \frac{p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}_i|\boldsymbol{x}_i)}{p_{\boldsymbol{Y}}(\boldsymbol{y}_i)} \xrightarrow{p} I(\boldsymbol{X}; \boldsymbol{Y}), \qquad (3.11)$$

where \xrightarrow{p} denotes the convergence in probability. Note that as K increases, the accuracy of the MI estimate also increases. For some channels this is a straightforward approach. Assuming a multivariate Gaussian distribution, $p_{Y|X}$ is given as

$$p_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^N |\boldsymbol{\Sigma}|}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right), \quad (3.12)$$

where Σ denotes the covariance matrix and $|\Sigma|$ the determinant. Assuming an AWGN channel, the noise in each dimension is independent and identically distributed, e.g., $\Sigma = \sigma_N^2 \mathbb{I}$, where \mathbb{I} denotes the $N \times N$ identity matrix.

However, for the fiber optical channel, $p_{Y|X}$ is not known, as the interplay between nonlinear effects and dispersion, especially in WDM transmission, makes it hard to analytically express $p_{Y|X}$. Instead, the concept of mismatched decoding can be applied. Following [219], it can be shown that evaluating the output symbols from the channel as if they were transmitted over an auxiliary channel with transition probability, $q_{Y|X}$, yields a lower bound on the MI. Note that since MI is an achievable rate, a lower bound will also be an achievable rate. The better $q_{Y|X}$ can be described to resemble the true channel $p_{Y|X}$, the tighter the bound is and the higher the achievable rate becomes.

In order to estimate the achievable information rate (AIR) of a fiber optical channel using MI, several assumptions will be made. It is assumed that all symbols of \mathscr{C} are independent and equiprobable. Note that if probabilistic shaping [220] is used this is no longer true. The channel is assumed to be memoryless and the main reason for this is that the decoders that are used today assumes this. Any linear memory of the channel such as residual dispersion or ISI is typically compensated for in the DSP. However, note that memory arising from the combined effects of large amounts of dispersion and the fiber nonlinearities do add memory and if such effects are compensated for, the AIR can be increased [47–49]. With these assumptions and using the mismatched decoding approach a lower bound on the MI can be estimated as

$$\frac{1}{K} \sum_{i=1}^{K} \log_2 \frac{q_{\boldsymbol{Y}|\boldsymbol{X}}(y_i|x_i)}{q_{\boldsymbol{Y}}(y_i)} \xrightarrow{p} \text{AIR}, \qquad (3.13)$$

where the term AIR is introduced to distinguish this lower bound from the true MI. Note that this gives an achievable rate for a decoder that uses the same assumptions as stated here. This approach has been used in Papers J and K.

3.4.3 Generalized Mutual Information

In many systems, in particular coherent fiber optical systems, the receiver structure that is used separates the decoder from the DSP where the input to the decoders is the soft information describing the reliability of the coded bits or symbols. This type of receiver structure keeps the complexity of the receiver reasonable and it is the most commonly used type of SD receiver [221]. This type of receiver is typically referred to as *bit-wise decoder* or *bit-interleaved coded modulation decoder* [222]. For this type of decoder, the generalized mutual information (GMI) has been shown to give a good estimate of the post-FEC BER [218, 222]. Assuming a uniform distribution of the transmitted symbols, the GMI can be estimated as [222]

GMI
$$\approx m - \frac{1}{K} \sum_{k=1}^{m} \sum_{i=1}^{K} \log_2 \left(1 + e^{(-1)^{b_{k,i}} \operatorname{LLR}_{k,i}} \right),$$
 (3.14)

where $\text{LLR}_{k,i}$ the log-likelihood ratio for the k^{th} bit position of the i^{th} received symbol and $b_{k,i}$ denotes the transmitted bit sequence. Note that the GMI is dependent on the bit-to-symbol mapping. For more details see [218, 222] and Paper K.

Chapter 4

Multidimensional Modulation Formats

In the era of direct detection systems, simple modulation formats based on binary signals such as OOK or differential BPSK were dominating. The reason was that such formats could be generated and detected with low complexity. To keep up with the demand for bandwidth, more WDM channels were simply added [223, 224] or electrical time-division multiplexing was applied to generate signals with higher bandwidths [225, 226]. Another technique that attracted significant research attention was optical time-division multiplexing (OTDM), where the multiplexing was performed by interleaving short pulses in the optical domain [227, 228]. However, since multiplexing and demultiplexing is significantly more complex for OTDM systems, WDM became the dominating multiplexing technology.

Eventually the bandwidth supported by the EDFA (C-band $\approx 1530-1565$ nm [32]) would be fully used and research on how to more efficiently use the available spectrum was initiated. More spectrally efficient modulation formats can be found by modulating both the amplitude and the phase of the signal. Further, the two polarizations can be used for multiplexing. Multilevel modulation formats can be implemented using differential detection to resolve the phase [66, 67, 229] and polarization tracking can be implemented using an all-optical approach [230]. However, it was first when the interest in the coherent receiver was resumed that the research on multilevel modulation formats really gained momentum. The main reason for this is that the phase tracking can be performed in the digital domain as explained in section 2.4, avoiding the use of complicated phase-locked loops or the use of differential detection. Further, polarization tracking can also conveniently be performed in the DSP (see section 2.4.4). With easy access to both the amplitude and phase information of the optical field in the receiver, transmission systems that use multilevel modulation formats can more easily be constructed.

The most commonly used modulation format for coherent transmission systems is PM-QPSK, for several reasons: The transmitter complexity is low and it can be implemented with binary driving signals which relaxes the complexity, especially before the use of high-resolution DACs became conventional. With PM-QPSK, the nonlinear transfer characteristics of the I/Q-modulator can also be exploited for better noise performance. Further, the algorithms in the DSP, namely Viterbi-Viterbi phase tracking and the CMA, are of reasonable complexity. Lastly, but perhaps the most important reason is that the required OSNR for PM-QPSK is suitable for transoceanic distances. PM-QPSK is conventional in modern commercially deployed systems [231, 232].

For transmission systems targeting a higher SE, PM-16QAM is often considered. It can be generated with 4-ary PAM signals and the required DSP architecture is well developed [113]. Furthermore, PM-16QAM is well suited for high overhead SD FEC, enabling high SE transmission over more than 3000 km [63, 233] as well as over transoceanic distances [234]. Today, PM-16QAM is also available for commercially deployed systems [235].

4.1 One- and Two-Dimensional Modulation Formats

Many of the conventionally used modulation formats can be expressed as one-dimensional (1D) formats, such as BPSK which is given as

$$\mathscr{C}_{\text{BPSK}} = \{(\pm 1)\}.$$
 (4.1)

However, as the coherent receiver is typically constructed as $2 \times 2D$ where the two dimensions in one polarization are given by the in-phase and quadrature components of the optical signal, these modulation formats are often given in two dimensions. With such 2D notation, BPSK is given as $\mathscr{C}'_{BPSK} = \{(\pm 1, 0)\}$ or $\mathscr{C}''_{BPSK} = \{(0, \pm 1)\}$. BPSK in one dimension gives SE = 2 bit/2D, APE = 0 dB and CFM = 3.0103 dB. However, if considered in two dimensions, these numbers change to SE = 1 bit/2D, APE = 0 dB and CFM = 6.0206 dB.

QPSK is the most commonly used modulation format in coherent optical communication systems. It can be described as multiplexing BPSK on the in-phase and quadrature part of the signal. It is given as

$$\mathscr{C}_{\text{QPSK}} = \mathscr{C}_{\text{BPSK}} \times \mathscr{C}_{\text{BPSK}} = \{(\pm 1, \pm 1)\}.$$
(4.2)

The QPSK constellation has SE = 2 bit/2D, APE = 0 dB and CFM = 3.0103 dB. (Note that it is the same values as for 1D BPSK).

Both BPSK and QPSK belong to the family of M-ary QAM. Higher order QAM formats are commonly used for systems aiming at higher SE. After QPSK, 16QAM is the most common QAM format in coherent optical systems and it is constructed by 4PAM signals in the in-phase and quadrature components of the signal. The 1D constellation for 4PAM is given as

$$\mathscr{C}_{4\text{PAM}} = \{(\pm 1), (\pm 3)\},\tag{4.3}$$

which can be used to generate the constellation for 16QAM as

$$\mathscr{C}_{16QAM} = \mathscr{C}_{4PAM} \times \mathscr{C}_{4PAM}. \tag{4.4}$$



Figure 4.1: Constellation diagrams for different conventional modulation formats.

Note that even though 16QAM is usually treated in two dimensions, it can in fact be seen as a 1D format as it is constructed by 4PAM signals in the two quadratures. 16QAM has SE = 4 bit/2D, APE = -3.9794 dB and CFM = -3.9794 dB.

Higher order MQAM formats are also constructed from PAM-signals, however only the formats with even k, given $M = 2^k$, use all combinations of the PAM levels. In other words, 4QAM, 16QAM, 64QAM, ..., can be decomposed into 1D formats constructed by kPAM signals in the quadratures. However, for odd k, such as 8QAM, 32QAM, 128QAM, ..., only a subset of all possible combinations are used to achieve an integer number of bits per symbol. Thus these formats are purely two-dimensional and cannot be described using 1D vectors. The constellation diagrams for a selection of MQAM formats are plotted in Fig. 4.1. Note that 2QAM, i.e. BPSK, is a special case as it uses 2PAM in one dimension and 0 in the other. Further, 8QAM is also a special case since the implementation using a subset of 3-PAM signals, as shown in Fig. 4.1c and from here on denoted rectangular 8QAM, is not the most common.



Figure 4.2: Constellation for (a) 3-PSK and (b) 4D-3-PSK with 3 bits mapped to 8 four-dimensional symbols. Note that one of the constellation points has a lower probability.

The reason is that the sensitivity can be significantly increased by optimizing the constellation without much extra complexity [236]. Instead, circular 8QAM is often implemented as shown in Fig. 4.1d, here denoted as circular 8QAM. However, as shown in [236], further adaptation can yield even better sensitivity at BER = 10^{-3} .

Circular 8QAM has SE = 3 bit/2D, APE = -1.9793 dB, and CFM = -0.7299 dB which can be compared to the rectangular 8QAM which has APE = -3.0103 dB and CFM = -1.7609 dB. When increasing the modulation order the sensitivity is decreased which can easily be understood from the fact that for the same mean energy of the constellation, the symbols are packed more densely and thus are more affected by noise. For 32QAM which has SE = 5 bit/2D, the corresponding APE is -6.0206 dB and the CFM is -6.9897 dB. Going to 64QAM the SE is increased to 6 bit/2D. However, the sensitivity is further reduced to APE = -8.4510 dB and CFM = -10.2119 dB.

One of the reasons for the popularity of the MQAM formats is the fact that the required resolution of the DACs is kept reasonable. This can be seen comparing rectangular 8QAM (Fig. 4.1c) and circular 8QAM (Fig. 4.1d). For rectangular 8QAM, the required number of levels in one quadrature is 3. For circular 8QAM this number is increased to 5 that are not equally spaced, which puts further demands on the required resolution.

4.1.1 Optimized 2D Modulation Formats

For coherent optical communication, optimization of modulation formats is typically performed in four dimensions or higher as described in the next sections of this chapter. However, for comparison it is interesting to evaluate modulation formats optimized in two dimensions. The most dense lattice in two dimensions is the hexagonal lattice, shown in Fig. 3.2c [237]. This lattice has been studied for communication [238, 239] including wireless applications [193].



Figure 4.3: Constellation for (a)7-A₂ (b) 16-A₂ with a constellation point at the origin and recentered for zero mean. (c) 16-A₂ with three constellation points centered around the origin and recentered for zero mean. (d) The most optimally packed 2D 16-points constellation known.

The most power efficient 2D modulation format is phase-shift keying with 3 states, i.e. 3-PSK [240]. The constellation for this format is shown in Fig. 4.2a. The 3-PSK format has SE = 1.5850 bit/2D, APE = 0.7508 dB and CFM = 4.7712 dB. However, since the constellation has 3 points, it is not possible to map an integer number of bits to this format. To circumvent this problem, the same method as described for polarization-QAM (POL-QAM) in [22] can be used, where bits are mapped to concatenated symbols (POL-QAM is described in section 4.2.1). Two consecutive symbols of 3-PSK have $3^2 = 9$ possible symbols of which eight can be used to achieve an integer number of bits per 4D symbol. This format, designated 3-PSK-4D, has SE = 1.5 bit/2D, APE = 0.5115 dB and CFM = 4.7712 dB. As seen, at the same SE as PS-QPSK, this and the conventional 3-PSK format have lower APE and CFM, showing that optimization of modulation formats in four and higher dimensions is more powerful. The constellation for the 3-PSK-4D format is shown in Fig. 4.2b where it can be seen that one symbol is less probable than the other two.

For comparison purposes, a format using 7 points from the A₂ lattice is included [239]. This format uses the origin and then the 6 surrounding lattice points as shown in the constellation diagram in Fig. 4.3a. This format has SE = 2.8074 bit/2D, APE = -0.8682 dB and CFM = 0.6695 dB. Since this format has seven constellation points, it is not possible to map an integer number of bits.

An interesting problem is how to optimally choose 16 points from the A₂ lattice and the corresponding gain APE or CFM over 16QAM. In Figs. 4.3(b)-(c), two different placements of 16 constellation points are shown. In Fig. 4.3b, the constellation points are chosen around a point placed at the origin and this format is denoted 16- $A_{2,0}$. In Fig. 4.3c the points are instead chosen such that 3 points are on equal distance from the origin, which is denoted 16- $A_{2,3}$. Note that after the 16 points have been chosen from the A₂ lattice, the constellations are recentered to have zero mean. Due to this, the constellations in Fig. 4.3b-c are slightly asymmetric. The constellation 16- $A_{2,0}$ has SE = 4, APE = -3.4916 dB and CFM = -3.4916 dB. The 16- $A_{2,3}$ is a slightly better constellation with SE = 4 bit/2D, APE = -3.4381 dB and CFM = -3.4381 dB. Note that there exists other possible methods of choosing 16 points from the A₂ lattice with similar performance [241]. The best known 16 points constellation in two dimensions [192, 237, 242] is shown in Fig. 4.3d has APE = -3.3995 dB and CFM = -3.3995 dB.

4.2 Four-Dimensional Modulation Formats

To track the evolving polarization rotations that occurs during transmission, the DSP in the coherent receiver consists of an adaptive equalization stage which typically consists of four FIR filters, as explained in section 2.4.4. This operation is inherently 4D, which means that the samples originating from the full 4D optical field is accessible in the receiver. The four dimensions are spanned by quadratures of the optical signal and the two orthogonal polarization states. Conventionally, the two polarizations are seen as independent channels on which data is *multiplexed*. Thus, in the same fashion as 2D formats are optimized in the complex plane, it is natural to extend this optimization to four dimensions of the transmitted optical signal. The idea of 4D modulation in optical communication was first introduced during the 1990's [243–246], when the coherent receiver was introduced. However, without the use of DSP, which was not available at that time, these formats were hard to realize experimentally. It should also be noted that in information theory, higher dimensional modulation formats have been extensively studied. For instance, already in the 1970's 4D modulation formats were studied theoretically [247, 248].

In 2009 the research on modulation formats optimized in the 4D space for coherent fiber optical communication systems was initialized by Bülow [21] followed by Agrell and Karlsson [22, 249]. In the following, several 4D modulation formats for coherent optical communication systems will be introduced.

4.2.1 POL-QAM

The modulation format that Bülow introduced to the optical communication community was POL-QAM which extends the number of state-of-polarizations (SOPs) of QPSK from four to six [21]. Interesting to note is that this format, also called the 24-cell in geometry [250], was studied for communication in 1977 [248]. Each SOP contains the four phase states of a QPSK signal. This format can be expressed as

$$\mathscr{C}_{\text{POL-QAM}} = \{ (\pm 1, \pm 1, \pm 1, \pm 1), \mathscr{P}(\pm 2, 0, 0, 0) \}.$$
(4.5)

This can be seen as adding the extra symbols given by $\mathscr{P}(\pm 2, 0, 0, 0)$ to a conventional PM-QPSK symbol alphabet. Another representation of this format is $\mathscr{C}'_{\text{POL-QAM}} = \{\mathscr{P}(\pm 1, \pm 1, 0, 0)\}$. The constellation for the latter representation is plotted in Fig. 4.4. POL-QAM has SE = $\log_2(24)/2 = 2.2925$ bit/2D, APE = 0.5928 dB and CFM = 3.0103 dB. This format was also studied theoretically in [247, 248]. Note that since POL-QAM has 24 symbols in four dimensions, it does not carry an integer number of bits per symbol. Obviously this is a problem for uncoded transmission, as well as for conventional systems with FEC which typically assumes integer number of bits per symbol. A solution to avoid this problem was proposed in [22], and experimentally implemented in [251], which maps 9 bits on



Figure 4.4: Constellation for POL-QAM.

two consecutive POL-QAM symbols. The reasoning for this is that two consecutive POL-QAM symbols gives $24^2 = 576$ symbols which is close to $2^9 = 512$ symbols. However, a more accurate description of this method is to consider the two consecutive POL-QAM symbols as an eight-dimensional (8D) format from which 64 symbols have been removed. By doing so, this new format has SE = 2.25 bit/2D, APE = 0.5115 dB and CFM = 3.0103 dB. As seen the SE is still higher than that of QPSK, with a simultaneously increased APE by roughly 0.5 dB. Comparing the two formats with CFM, the SE can be increased without decreasing the CFM. The POL-QAM format has also been compared in experiments to PS-QPSK [252] and to $\mathcal{C}_{\text{Opt},16}$ (this format will be introduced in section) [253].

4.2.2 Polarization-Switched QPSK

PS-QPSK was found to be the most power efficient modulation format in four dimensions [22]. It is the 4D modulation format that has attracted the most research attention in fiber optical communication. Although this format was new to the fiber optical community, it was studied already in 1977 by Zetterberg and Brändström where it was called the 16-cell [248]. It was also studied in terms of APE for satellite communications [254]. The symbol alphabet of PS-QPSK is given by

$$\mathscr{C}_{\text{PS-QPSK}} = \{ (\pm 1, \pm 1, 0, 0), (0, 0, \pm 1, \pm 1) \}.$$
(4.6)

With this representation the modulation format can be seen as transmitting one QPSK symbol in either the x- or the y-polarization. Thus, two bits are encoded in the QPSK symbol and one in the selection of polarization and hence the name *polarization switched*. The constellation for PS-QPSK is shown in Fig. 4.5 where the two colors indicate 4D symbols. A different representation of PS-QPSK can be derived by applying a single parity-check (SPC)-code (see section 4.2.4) to the PM-QPSK symbol alphabet. This operation results in two subsets which are given as

$$\mathscr{C}_{\text{PS-QPSK}}' = \{ \pm (1, 1, 1, 1), \mathscr{P}(1, 1, -1, -1) \}$$
(4.7)

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Figure 4.5: Constellation for PS-QPSK. The colors indicate 4D-symbols.

$$\mathscr{C}_{PS-QPSK}^{\prime\prime} = \{\mathscr{P}(-1,1,1,1), \mathscr{P}(1,-1,-1,-1)\}.$$
(4.8)

The two different representations can be achieved by performing a 45° polarization rotation on the constellation $\mathscr{C}_{\text{PS-QPSK}}$ given in Eq. (4.6) [249]. Note that $\mathscr{C}_{\text{PM-QPSK}} = \{\mathscr{C}'_{\text{PS-QPSK}}, \mathscr{C}''_{\text{PS-QPSK}}\}$. The PS-QPSK constellation has SE = 1.5 bit/2D, APE = 1.7609 dB and CFM = 6.0206 dB.

The two representations given here have resulted in two different transmitter setups for PS-QPSK where the transmitter for the polarization-switched representation (Eq. (4.6)) is shown in Fig. 4.6a. Here, a QPSK signal is generated with an I/Q-modulator and this signal is then split into two arms with two MZM driven in a push-pull configuration. The binary signal D then chooses which arm that will allow light through and since the arms are combined with orthogonal polarizations, the signal D switches QPSK signal between the two polarization states. This type of transmitter was used in the first experimental demonstration of PS-QPSK [255] as well as in numerous other experimental studies [256-258]. The second representation (Eq. (4.7) or Eq. (4.8)) has resulted in a transmitter based on the SPC operation as shown in Fig. 4.6b. A conventional PM-QPSK transmitter structure is used with one I/Q-modulator per polarization. The only modification is that three driving signals are given by the information bits while the fourth driving signal is constructed by an exclusive or (XOR) operation of the three information bits. The schematic in Fig. 4.6b results in the even subset given in Eq. (4.7). To obtain the odd subset, an inverter can be placed on the output of the XOR-gate. This type of transmitter has been demonstrated with lower implementation penalty which can be understood by the fact MZMs have a limited extinction ratio and they have higher loss since they are driven with half V_{π} to achieve the on-off behavior. This transmitter structure was used in the experiments in [259, 260] as well as in Paper C. Further, a third type of transmitter was implemented in [261] based on a commercially available polarization-modulator [262], and it should also be mentioned that integrated transmitters optimized for PS-QPSK have been constructed [263].

In experiments, PS-QPSK has been shown to achieve significantly longer transmission distances compared to PM-QPSK. Compared at the same bit rate of 42.9 Gbit/s



Figure 4.6: Two different implementations of a PS-QPSK transmitter showing (a) Polarization-Switched transmitter and (b) SPC-based transmitter.

per channel in WDM transmission, PS-QPSK has been demonstrated with 30 % increased reach compared to PM-QPSK [256]. In Paper C a 50 % increase is seen when the two formats are compared at the same symbol rate and in a dual-carrier setup.

Maybe the most promising application for PS-QPSK is in a flexible transmission scenario where the transmitter and receiver can switch between PM-QPSK and PS-QPSK depending on the quality of the link. Examples of format flexible system are demonstrated in [115, 264], where for instance PM-QPSK or PS-QPSK, among many other formats, can be chosen dependent on the required transmission distance. A similar idea is discussed in [265] where PS-QPSK is proposed as a backup solution for degrading PM-QPSK links.

4.2.3 Binary Pulse Position Modulation QPSK

Pulse position modulation (PPM) encodes data onto a signal by transmitting one pulse in one of K possible timeslots as illustrated for K = 2 in Fig. 4.7. In this section, the two consecutive pulse slots are used to achieve a 4D signal space in the same way as is typically done with two polarizations states. If the PPM-pulses for the binary-PPM (2PPM) shown in Fig. 4.7 are modulated with QPSK data the symbol alphabet can be written as

$$\mathscr{C}_{\text{2PPM-QPSK}} = \{ (\pm 1, \pm 1, 0, 0), (0, 0, \pm 1, \pm 1) \},$$
(4.9)

which is exactly the same as for PS-QPSK in Eq. (4.6) and hence this is another realization of the same modulation format. This format is denoted 2PPM-QPSK and



Figure 4.7: Illustration of a 2PPM pattern. A "1" is encoded as transmitting "10" and a "0" as "01". Solid lines indicates a transmitted pulse and dashed lines the absence of a pulse.



Figure 4.8: Constellation for 2PPM-QPSK. The two color schemes indicate 4D-symbols.

has the same SE, APE and CFM as PS-QPSK. The constellation for 2PPM-QPSK is shown in Fig. 4.8. When this format is implemented, the two polarizations are used as independent channels transmitted 2PPM-QPSK, denoted as PM-2PPM-QPSK. Note that this is not the same format as if the binary PPM is performed over PM symbols, i.e. 2PPM-PM-QPSK. This latter implementation has SE = 1.25 bit/2D, APE = 0.9691 dB and CFM = 6.0206 dB which is worse than 2PPM-QPSK. In [266], a very similar format to 2PPM-QPSK is investigated in simulations for free-space optical communication where BPSK is used instead of QPSK for the modulated pulses.

2PPM-QPSK has been experimentally demonstrated in Paper B where the transmitter was constructed with a MZM for PPM generation followed by an I/Q-modulator for QPSK modulation. It should be noted that it would be possible to implement this format with a single I/Q-modulator using DACs generating 3-level signals. In Paper B, it is shown that 2PPM-QPSK can achieve 40 % increased transmission distance compared to PM-QPSK in single channel transmission at the same bit rate of 42.8 Gbit/s.

4.2.4 Set-Partitioning QAM

As explained in section 4.1, PM-16QAM is typically used in systems where a higher SE than the 2 bit/2D of PM-QPSK is needed. However, the required OSNR to detect 16QAM is much higher, with an CFM which is 3.97 dB worse compared to QPSK. This translates into much shorter transmission distance compared at the same BER. In the search for spectrally efficient modulation formats with better APE than PM-16QAM, Coelho and Hanik [194] introduced a family of 4D modulation formats based on Ungerboeck's set-partitioning scheme [267]. The set-partitioning operation can be explained by considering lattices (section 3.2). By applying the set-partitioning to a QAM-constellation, the lattice that the constellation points are located on is transformed from the Z_4 lattice to the D_4 lattice. This is a result from the process of removing half the constellation points such that the minimum Euclidean distance is increased. If the intrinsic constellation is the 256 points of the PM-16QAM symbol alphabet, two different formats can be found by using the subset of either 128 or 32 points [194, 268]. The most common notation is K-ary set-partitioning MQAM (K-SP-MQAM), such that the format using half the points of PM-16QAM is denoted 128-SP-16QAM and the format choosing 32 points is denoted 32-SP-16QAM.

The set-partitioning operation that chooses half the symbols of a format based on the Z lattice can also be described as a SPC-code where one parity bit per n_b information bits is added. The parity bit is encoded by a modulo-2 addition, denoted \oplus , and is given as

$$b_{\rm SPC} = b_1 \oplus b_2 \oplus b_3 \oplus \dots \oplus b_{n_b-1} \oplus b_{n_b}. \tag{4.10}$$

where b_k denotes the b^{th} of the transmitted symbol. However, for the set-partitioning schemes that chooses less then half the points, such as 32-SP-16QAM, no such simple operation which yields one parity bit exists. However, it is possible to describe this format using parity check codewords that generates three parity bits from five information bits. These 3 parity check bits and 5 information bits are then sent to a PM-16QAM transmitter as described in [269].

128-SP-16QAM

The increased minimum Euclidean distance of the 128-SP-16QAM constellation compared to the PM-16QAM results in APE = -1.549 dB which is an increase of roughly 2.43 dB over 16QAM. The CFM for 128-SP-16QAM is -0.9691 dB which is roughly 3 dB better than PM-16QAM. However, the increase in APE or CFM comes at the cost of a reduced SE by 7/8 since one bit out of eight is used for the SPC, i.e. SE = 3.5 bit/2D.

A possible transmitter structure for 128-SP-16QAM is shown in Fig. 4.9 where a SPC-bit is generated according to Eq. (4.10). As seen, this is a small modification to a conventional PM-16QAM transmitter. In fact, this transmitter architecture can be used for a flexible system that switches from PM-16QAM to 128-SP-16QAM if the link OSNR is not sufficient to receive PM-16QAM.

128-SP-16QAM has been studied in numerical simulations in [269] where the format was compared to PM-QPSK and PM-8QAM where parts of the increased



Figure 4.9: Transmitter structure for generating 128-SP-16QAM using a PM-16QAM transmitter.

spectral efficiency of 128-SP-16QAM over these formats was used for extra overhead for LDPC codes. In [270], 128-SP-16QAM is compared to PM-16QAM in numerical simulations targeting WDM transmission systems.

The first experimental demonstrations of 128-SP-16QAM were done for singlechannel transmission in [23] (on which Paper A is an extension), and for WDM transmission in [271]. In Paper A, 128-SP-16QAM is experimentally compared to PM-16QAM in long-haul transmission for both single-channel and WDM transmission at the same bit rate and symbol rate. It is shown that in WDM transmission at the same symbol rate of 10.5 Gbaud, the achievable transmission distance is increased by 69 % for 128-SP-16QAM over PM-16QAM. At the same bit rate the corresponding reach increase is 54 % which is also what is seen in the simulations in [270]. The achievable transmission distance for 128-SP-16QAM compared to 2D-formats as well as to other set-partitioning formats and POL-QAM has been evaluated using the Gaussian noise model [46]. The performance of 128-SP-16QAM in combination with LDPC or TPC-based FEC is evaluated in [272]. 128-SP-16QAM is experimentally compared together with other set-partitioning formats based on MQAM formats up to PM-64QAM in [273]. In [274], 128-SP-16QAM is proposed for few-mode transmission systems where the four dimensions are realized by two consecutive timeslots instead of over the two polarizations.

Other 4D Set-Partitioning Formats

32-SP-16QAM is derived from the PM-16QAM symbol alphabet where 32 symbols are obtained by two consecutive set-partitioning operations [194, 268]. This modulation format has an SE of 2.5 bit/2D which is closer to the 2 bit/2D of QPSK than to 4 bit/2D of PM-16QAM, and thus it is more comparable to QPSK. 32-SP-16QAM has APE = 0 dB which is the same as for QPSK. In other words 32-SP-16QAM can increase the SE over QPSK without any loss in APE. The CFM for 32-SP-16QAM is 2.0412 dB. In the simulations in [269], it is shown that when compared at the same information rate after decoding of the used LDPC code, the sensitivity of 32-SP-16QAM is only 0.1 dB lower than that of PM-QPSK. In the numerical simulations

in [275], 32-SP-16QAM is compared to a modulation scheme interleaving QPSK and 8QAM symbols with the same effective SE as 32-SP-16QAM and it is shown that 32-SP-16QAM has slightly higher back-to-back (BtB) sensitivity and better nonlinear transmission performance. The WDM transmission experiments in [271] show that the transmission reach for 32-SP-16QAM is in between those of QPSK and 8QAM, when the formats are compared at the same symbol rate. The same result is in [46], where the Gaussian noise model is used to evaluate different 4D formats.

64-SP-16QAM is of little interest since it has APE = -2.2185 dB and CFM = -0.9691 dB which is slightly worse compared to circular 8QAM (Fig. 4.1d) and the formats have the same SE of 3 bit/2D. This is also seen in the numerical simulations in [269] where it is shown that the uncoded sensitivity at BER = 10^{-3} and the sensitivity after LDPC-decoding is very similar compared to 8QAM. The similar performance can be understood from the fact that the points of 64-SP-16QAM lies on the Z₄ lattice opposed to the D₄ of the 128- and 32-SP-16QAM.

512-SP-32QAM applies the set-partitioning operation to PM-32QAM and has SE = 4.5 bit/2D, APE = -3.4679 dB, CFM = -3.9794 dB. Note that this is the modulation format implemented in [273, 276] where a single set-partitioning operation is applied to the 32QAM constellation. This is a different format compared to 512-SP-64QAM where two consecutive set-partitioning operations on the 4096 points of the PM-64QAM constellation yield the 512 points, which is the format that is given in [268]. 512-SP-64QAM has SE = 4.5 bit/2D, APE = -3.6798 dB and CFM = -4.1913 dB. The 512-SP-32QAM format can also be expressed as a SPC-code, where nine information bits used to generate one SPC bit according to Eq. (4.10)and the bits are then mapped on the 32QAM constellation. 512-SP-32QAM was experimentally realized in [276], where the performance is compared to 32QAM and 16QAM. The main findings is that the achievable transmission distance is intermediate to that of PM-16QAM and PM-32QAM offering a simple solution to move between 32QAM and the slightly less spectrally efficient 512-SP-32QAM if longer transmission reach is needed. This format was also studied and compared to other 2D and set-partitioning formats in [46, 273].

2048-SP-64QAM is studied in [46, 273], this format applies the set-partitioning operation, or equivalent the SPC code over 11 information bits, to a PM-64QAM constellation. In these experiments it is shown that 2048-SP-64QAM offers a well-needed option in terms of trade-off between throughput and transmission reach which is intermediate to that of PM-32QAM and PM-64QAM.

4.2.5 256-D₄

Using the D_4 lattice as a base from which constellation points are chosen, modulation formats with densely packed structures can be found by sphere cutting [277]. In Paper E, a modulation format with 256 points from the D_4 lattice is studied, which will be denoted 256- D_4 . The 256 points are cut out from the D_4 lattice using a hypersphere and the choice of 256 constellation points gives an SE of 4 bit/2D which



Figure 4.10: Constellation diagrams for $256-D_4$. Showing (a) the constellation given in Eq. (4.11) and (b) the constellation after a polarization rotation is applied.

is the same as for PM-16QAM. The symbol alphabet of this format can be given as

$$\mathscr{C}_{256-D_4} = \{\mathscr{P}(\pm 1, 0, 0, 0), \mathscr{P}(\pm 1, \pm 1, \pm 1, 0), \mathscr{P}(\pm 1, \pm 2, 0, 0), \\ \mathscr{P}(\pm 1, \pm 1, \pm 1, \pm 2), \mathscr{P}(\pm 1, \pm 2, \pm 2, 0), \mathscr{P}(\pm 3, 0, 0, 0)\}.$$
(4.11)

This format has APE = -2.2724 dB and CFM = -2.2724 dB and it is the most power efficient modulation format in four dimensions with 256 points that is currently known. The constellation for 256-D₄ is shown in Fig. 4.10a. In Paper E it was found that by applying a polarization rotation, the required resolution of the ADCs could be relaxed since the number of discrete level in each quadrature could be reduced from 7 (as seen in the constellation in Fig. 4.10a) to 6. The constellation after this rotation is shown in Fig. 4.10b.

This format has been studied for bit-interleaved coding [278]. Also, a very similar format was studied in [279]. In Paper E, the 256-D₄ format is experimentally realized for the first time. It is shown that for systems operating with pre-FEC BER targets of lower than BER = 10^{-3} , the 256-D₄ is an interesting format that can achieve longer transmission reach compared to PM-16QAM at the same SE. However, after decoding of an applied turbo product code (TPC) with 21.3 % overhead, PM-16QAM outperforms 256-D₄.

4.2.6 Optimized 16-point 4D Formats

An interesting question is how to optimally pack M constellation points in four dimensions, where M is a power of two to achieve an integer number of bits per symbol. Note that PM-QPSK has M = 16 points in four dimensions. The best known packing of 16 points in 4D space is called $\mathcal{C}_{opt,16}$ [280]. This format is given by the points

$$\mathscr{C}_{\text{opt},16} = \{ (a + \sqrt{2}, 0, 0, 0), (a, \pm\sqrt{2}, 0, 0), (a, 0, \pm\sqrt{2}, 0), (a, 0, 0, \pm\sqrt{2}), (a - c, \pm 1, \pm 1, \pm 1), (a - c - 1, 0, 0, 0) \},$$
(4.12)

where $c = \sqrt{2\sqrt{2} - 1}$ and $a = (1 - \sqrt{2} + 9c)/16$. The constellation for $\mathscr{C}_{\text{opt,16}}$ is shown in Fig. 4.11. As seen, for this format there is no longer any symmetry between the



Figure 4.11: Constellation for $\mathscr{C}_{opt,16}$.



Figure 4.12: Constellation for SO-PM-QPSK. The two colors indicates 4D symbols. (a) shows the representation given in Eq.(4.13) and (b) shows the constellation with a 45° polarization rotation.

signal in the two polarizations. Due to the irregularity, this format will require high resolutions DACs. The $\mathscr{C}_{opt,16}$ format has SE = 2 bit/2D, APE = 1.1137 dB and CFM = 4.1240 dB. This modulation format has been experimentally implemented and compared in a BtB setup for orthogonal frequency division multiplexing (OFDM) signals [191]. The $\mathscr{C}_{opt,16}$ format has also been realized in transmission using dataaided training sequences to enable polarization demultiplexing and equalization [253]. Further, it has also been studied in terms of mutual information (MI) and generalized mutual information (GMI) [218]. It was found that $\mathscr{C}_{opt,16}$ has a higher MI compared to PM-QPSK. However, for the more practical case of GMI, $\mathscr{C}_{opt,16}$ has a much worse sensitivity compared to PM-QPSK.

Another 4D format with 16 points is the subset-optimized PM-QPSK (SO-PM-QPSK) [281] which optimizes the amplitude ratio between the even and odd subsets of PM-QPSK given in Eqs. (4.7) and Eq. (4.8), respectively. The constellation for

SO-PM-QPSK is given as

$$\mathscr{C}_{\text{SO-PM-QPSK}} = \{ \pm (1, 1, 1, 1), \mathscr{P}(1, 1, -1, -1), \\ \frac{1}{A_r} \mathscr{P}(-1, 1, 1, 1), \frac{1}{A_r} \mathscr{P}(1, -1, -1, -1) \},$$
(4.13)

where A_r is the amplitude ratio between the two subsets and it is found that $A_r = (\sqrt{5} + 1)/2$ is optimal in terms of APE [281]. The constellation for SO-PM-QPSK is plotted in Fig. 4.12 for two different polarization rotations. SO-PM-QPSK has SE = 2 bit/2D, APE = 0.4359 dB and CFM = 3.4462 dB. Thus, at asymptotically high SNR this format is more sensitive compared to PM-QPSK. However, when compared in terms of MI and GMI, it was found that SO-PM-QPSK has a lower achievable information rate [218].

4.3 Eight-Dimensional Modulation Formats

As seen in the previous sections, increasing the dimensionality of the modulation space allows for higher degree of freedom for optimization, and has given rise to many interesting formats. This opens up for research on systems that increases the dimensionality further and the next natural choice is to use eight dimensions. The single-mode optical signal is inherently four dimensional and to increase the dimensionality to eight, the signal has to be constructed using other dimensions such as considering modulation over two WDM channels or two consecutive timeslots as in Fig. 4.13a-b. An alternative way is to consider the single-polarization field of



Figure 4.13: Different options of achieving eight dimensions showing: (a) Two polarization-multiplexed WDM-channels, (b) two consecutive polarization-multiplexed timeslots, (c) four consecutive single-polarization timeslots, (d) two modes of a multimode fiber, and (e) two cores of a multicore fiber.

four consecutive timeslots as in Fig. 4.13c. For an SDM system, two modes of a multimode fiber (MMF), as shown in Fig. 4.13d, or two cores of a MCF, as shown in Fig. 4.13e, can be used to achieve eight dimensions. In Paper C, two wavelength channels in a dual-carrier setup is used. This technique has also been used in [282–284] to increase the dimensionality of the modulation space. In Paper D two timeslots are used. This concept is also used in several studies of multidimensional modulation formats [277, 285–287]. Further, Papers H and I study, among other, 8D modulation formats over two cores of a MCF. Using the cores of MCF to increase the number of dimensions has also been studied in [288–290]. The concept of modulation over several modes was discussed in [284, 291–294].

4.3.1 8D Biorthogonal Modulation

Biorthogonal modulation transmits energy in only one dimension per symbol with an amplitude that is ± 1 [295, section 3.2-4]. This can also be described as transmitting a BPSK symbol in the selection of one of the N possible dimensions per symbol. This can be expressed as

$$\mathscr{C}_{\text{ND-biorth.}} = \{\mathscr{P}(\pm 1, 0^{N-1})\},\tag{4.14}$$

where 0^{N-1} is used to denote that the number of dimensions containing a zero is N-1. As an example, with N = 1, Eq. (4.14) gives $\mathscr{C}_{1D\text{-biorth.}} = \mathscr{P}(\pm 1)$ which is the same as $\mathscr{C}_{\text{BPSK}}$ in Eq. (4.1). With N = 2, $\mathscr{C}_{2D\text{-biorth.}} = \mathscr{P}(\pm 1, 0)$ which corresponds to QPSK, given a $\pi/4$ phase rotation and scaling of the constellation given in Eq. (4.2). Further, N = 4 gives $\mathscr{C}_{4D\text{-biorth.}} = \mathscr{P}(\pm 1, 0, 0, 0)$ which corresponds to PS-QPSK. Given the popularity of these formats, it is a logical next step to investigate biorthogonal modulation in higher dimensions. The next dimensionality of $\mathscr{C}_{ND\text{-biorth.}}$ that gives an integer number of bits per symbol is eight. From a geometrical point of view, N-dimensional biorthogonal modulation corresponds to the N-dimensional cross-polytope which has been studied in [296], where an exact sym-

N	Modulation Format	SE [bit/2D]	APE [dB]	CFM [dB]	$\mid M^{\ddagger}$
1	BPSK	2	0	3.0103	2
2	QPSK	2	0	3.0103	4
4	PS-QPSK (Paper C), 2PPM-QPSK (Paper B),	1.5	1.7609	6.0206	8
8	4FPS-QPSK, 2PPM-PS- QPSK (Papers C and D)	1	3.0103	9.0309	16
16	-	0.6250	3.9794	12.0412	32
32	-	0.3750	4.7712	15.0515	64
64	-	0.2188	5.4407	18.0618	128
128	-	0.1250	6.0206	21.0721	256

Table 4.1: Biorthogonal modulation in N dimensions and, when applicable, more commonly used designation of the formats.

^{\ddagger} number of N-dimensional symbols in the constellation.



(b) 2PPM-PS-QPSK

Figure 4.14: Illustration of two different implementations of 8D-biorthogonal modulation where the eight dimensions are realized by (a) two wavelengths in 4FPS-QPSK and (b) two consecutive timeslots in 2PPM-PS-QPSK. Solid lines indicate a transmitted QPSK symbol and dashed lines that no power is transmitted.

bol error probability is given. In Table 4.1, the SE, APE and CFM for biorthogonal modulation with different dimensionalities are given.

8D-biorthogonal modulation was experimentally investigated in Paper C and D. In Paper C, the eight dimensions were realized by considering two wavelength channels in a dual-carrier setup and due to this configuration the format was designated 4-ary frequency and polarization switched QPSK (4FPS-QPSK). An illustration of the transmitted symbols of this implementation is shown in Fig. 4.14a. In Paper D, 8D-biorthogonal modulation was instead implemented using two consecutive timeslots to achieve a dimensionality of eight. This implementation can be seen as a combination of binary-PPM and PS-QPSK, i.e. for each 8D-symbol, one QPSK signal is transmitted in one of the four positions given by two timeslots and two polarizations. These formats belong to the family of multidimensional position modulation (MDPM) which is introduced in section 4.5.

In Paper C, 8D-biorthogonal modulation implemented as 4FPS-QPSK is compared to dual-carrier PM-QPSK at the same symbol rate of 10 Gbaud (i.e. the same bandwidth), which means that the bit rate is half for 4FPS-QPSK. Transmission of up to 14,000 km is demonstrated, which corresponds to an increase in transmission reach by 84 % compared to dual-carrier PM-QPSK. In Paper D, 8D-biorthogonal modulation implemented as 2PPM-PS-QPSK is compared to PM-QPSK at the same bit rate of 85.6 Gbit/s. Transmission of 2PPM-PS-QPSK with a reach of up to 12,300 km is demonstrated, showing a reach increase over PM-QPSK of 84 %.

A format with the same properties as 8D-biorthogonal modulation can be derived from the (8,4) extended Hamming code [198, section 10.2]. The parity check matrix



(b) 2PPM-PS-QPSK

Figure 4.15: The transmitter structure used to generate (a) 4FPS-QPSK in Paper C and (b) 2PPM-PS-QPSK in Paper D.

for this code can be given as

$$H_{(8,4)} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$
 (4.15)

The symbol alphabet for this format is then given by

$$\mathscr{C}_{H(8,4)} = \{ 2\boldsymbol{b}H_{(8,4)} - 1 : \boldsymbol{b} = (0,0,0,0), (0,0,0,1), \dots, (1,1,1,1) \}.$$
(4.16)

This implementation requires only binary driving signals as the generated constellation only contains 1 or -1.

Yet another implementation of this format is given in [285] where the eight dimensions are constructed by two consecutive timeslots. The implementation is very similar to the constellation generated by the (8,4) extended Hamming code. However, the format in [285] is designed for increased nonlinear tolerance in systems with inline dispersion compensation. The constellation is a rotated version of $\mathscr{C}_{8D-biorth.}$ (Eq. (4.14)) where the rotation is performed such that all symbol slots have constant power and more importantly that the sub-symbols in the two timeslots considered as one symbol have opposite Stokes vectors. This ensures that the polarization is rapidly changing, which has been shown to reduce the intra-channel nonlinear effects [297]. This modulation format is now commercially available [235].



Figure 4.16: The constellation in four IQ-planes for \mathcal{C}_{8D-ASK} .

4.3.2 Other 8D Modulation Formats

In [286], three different 8D modulation formats are investigated in simulations where two consecutive timeslots are used to span the eight dimensions. Two of the formats are based on sphere cutting of 128 and 256 points of the E_8 lattice which is the most densely packed lattice in eight dimensions [186]. The second format uses the 8D hypercube with a SPC bit. These formats are shown to have increased APE when compared at the same SE as PS-QPSK or PM-QPSK. In [277] more 8D formats are introduced, including 16 points from the E_8 lattice and the (8,4) extended Hamming code.

In [287], an 8D modulation format is constructed from joining 4D QPSK symbols with 2PPM-QPSK symbols where these QPSK symbols are rotated $\pi/2$ compared to that of the 4D-QPSK symbols. This format can be given as

$$\mathscr{C}_{4D-QPSK} = \{ (\pm 1, \pm 1, \pm 1, \pm 1) \}, \tag{4.17}$$

$$\mathscr{C}'_{\rm 4D-QPSK} = \{\mathscr{P}(\pm\sqrt{2}, 0, 0, 0)\},\tag{4.18}$$

$$\mathscr{C}_{\text{8D-ASK}} = (\mathscr{C}_{\text{4D-QPSK}} \times \mathscr{C}'_{\text{4D-QPSK}}) \cup (\mathscr{C}'_{\text{4D-QPSK}} \times \mathscr{C}_{\text{4D-QPSK}}), \tag{4.19}$$

where \cup denotes the union. This format has SE = 2 bit/2D, APE = 1.2494 dB and CFM = 4.2597 dB. In other words the APE and CFM can be increased over QPSK, without a reduction in SE. The constellation in all four IQ-planes for this modulation format is shown in Fig. 4.16. It is interesting to note that \mathscr{C}_{8D-ASK} has the same SE, APE and CFM as 4iMDPM-QPSK which was introduced in Paper F. These types of formats are discussed in section 4.5. The biggest difference, from a geometric point of view is that \mathscr{C}_{8D-ASK} has 10 nearest neighboring constellation points while 4iMDPM-QPSK has 18.

4.4 Time-Domain Hybrid Formats

An alternative method of achieving higher dimensional formats is to combine constellations from conventional 2D formats transmitting for instance two different constellations in two orthogonal polarizations or in two consecutive timeslots. Examples of such formats include a combination of QPSK and 8QAM in alternating polarizations and timeslots [298–301], a combination of two 8QAM symbols and one 16QAM symbol [301, 302], a combination of 32QAM and 64QAM [303] and combination of QPSK and 16QAM [301]. The latter has also been studied for systems where the ratio between the number of slots with QPSK and the number of slots with 16QAM is varied [304].

For the analysis in this section, these formats can be treated as higher dimensional formats. A time-domain hybrid of QPSK and 8QAM can be described as [298–301]

$$\mathscr{C}_{\text{Hybrid-QPSK/8QAM}} = \mathscr{C}_{\text{QPSK}} \times \mathscr{C}_{\text{8QAM}}, \qquad (4.20)$$

which yields a 4D format with 32 constellation points. This format has SE = 2.5 bit/2D, i.e. the average of QPSK and 8QAM. Further, this format has APE = -1.2918 dB and CFM = 0.7494 dB. Compared to a modulation format optimized in 4D, i.e. 32-SP-16QAM, which has the same SE, this hybrid format has roughly 1.29 dB decrease in APE and CFM.

In the same fashion as in Eq. (4.20), the time-domain hybrid of 8QAM and 16QAM can be constructed [301, 302]. This format has SE = 3.5 bit/2D, APE = -3.2317 dB and CFM = -2.6517 dB. The hybrid between 16QAM and 32QAM [303] has SE = 4.5 bit/2D, APE = -5.2288 dB, and CFM = -5.7403 dB. Further, the hybrid transmission with 32QAM and 64QAM has SE = 5.5 bit/2D, APE = -7.5100 dB and CFM = -8.8930 dB.

The system combining QPSK and 16QAM [301] in a time domain hybrid has SE = 3 bit/2D, APE = -3.0103 dB and CFM = -1.7609 dB. Better performance can be achieved if the ratio between the number of slots carrying the two formats are optimized [304]. An extension of this format combines 16QAM and QPSK with 45° rotation and an increased symbol energy by $\sqrt{2}$ compared to the points on the inner circle of 16QAM. The 16QAM and the rotated and scaled QPSK symbol can be selected in each timeslot, making the SE of this format to be 3.5 bit/2D. The APE is -3.0103 dB and the CFM is -2.4304 dB, showing that at the same APE, this format can achieve an extra 0.5 bit/2D in SE compared to the pure 16QAM-QPSK hybrid. However, it is important to note that both these formats are optimized in an ad hoc fashion and a format optimized with the full four-dimensions taken into consideration such as 256-D₄ and 128-SP-16QAM can achieve better APE/CFM at the same or higher SE.

4.4.1 Single-Parity Check Formats

The SPC code given in Eq. (4.10) can be applied over several symbols to generate higher dimensional SPC codes. The main benefit is that the SPC-bit is shared among more bits, reducing the penalty in SE associated with the SPC bit. In [126], an LDPC code is interleaved over pairs of WDM channels, where the channel with lower



Figure 4.17: Illustration of SPC over N number of channels with PM-16QAM transmitted over (a) different cores or (b) in different timeslots.

performance is encoded by two concatenated SPC codes. This coded modulation scheme allowed for a new record throughput-distance product at that time. The same coding scheme was applied in [234]. A similar technique was also used in [125], using time-interleaved symbols which are partly encoded by a SPC.

In Papers H and I, SPC codes are applied over several cores in MCFs transmission experiments. Since the concept of spatial superchannels requires signals in the same wavelength over the different cores to be routed to the same node and that they are synchronized in the receiver to resolve propagation effects such as deterministic crosstalk this opens up for coding over the full spatial channel. However, this is not limited to signals over several cores, any method of increasing the dimensionality of the signal space shown in Fig. 4.13 could be used, for instance by applying the SPC over several timeslots. An example of SPC-encoding of PM-16QAM signals over Nchannels with PM-16QAM is shown in Fig. 4.17 where the output dual-polarization signals in this example are mapped to either different cores of a MCF (Fig. 4.17a) or consecutive timeslots (Fig. 4.17b). However, the analysis in this section will be kept general by not considering any specific implementation.

The APE for a SPC-format applying the SPC over N_{4D} sets of 4D signals is given as

$$APE_{SPC} = 2 \cdot APE_{2D\text{-format}} \left(1 - \frac{1}{2N_{4D}\log_2(M_{2D\text{-format}})} \right), \quad (4.21)$$

where $\log_2(M_{2D-\text{format}})$ is the number bits of the 2D base modulation format used, i.e. 2 bits for QPSK and 4 bits for 16QAM. APE_{2D-format} is the APE of the base format and is calculated according to Eq. (3.7). Note that N_{4D} could for instance be the number of cores or the number of timeslots. The CFM for the SPC formats



Figure 4.18: SPC over increasing number of 4D-symbols for (a) QPSK (b) 16QAM (c) 32QAM (d) 64QAM.

is given as

$$CFM_{SPC} = \frac{2d_{\min}^2}{E_{s,2D}},$$
(4.22)

where $E_{s,2D}$ denotes the average symbol energy of the base 2D format used and is calculated according to Eq. (3.4). As seen, the CFM for the SPC-formats is independent of the number of 4D symbols the parity bit is shared among. Finally, the SE is given as

$$SE_{SPC} = \log_2(M_{2D-format}) - \frac{1}{2N_{4D}}.$$
 (4.23)

As seen, the SE is increased when the parity bit is shared over more 4D symbols. An interesting limit is when $N_{4D} \rightarrow \infty$ which makes the SE_{SPC} approach that of the base 2D modulation format while the gain in APE approaches 3 dB. The CFM is independent of N_{4D} .

The SE and APE for SPC in combination with QPSK, 16QAM, 32QAM and 64QAM are shown in Fig. 4.18 for increasing N_{4D} . As seen for all formats, when N_{4D} is very large the SE approaches that of the base 2D format used and the APE is approaching a 3 dB increase over the base format. Note that the CFM is fixed with 6.0103 dB for SPC-QPSK, -0.9691 dB for SPC-16QAM, -3.9794 dB for SPC-32QAM, and -7.2016 dB for SPC-64QAM. Interesting from a practical point of view is the fact that at low N_{4D} , such as 2 or 3, the gain in SE over $N_{4D} = 1$ is significant. For instance for QPSK, the SE is increased from 1.5 for $N_{4D} = 1$ (or PS-QPSK) to 1.8333 bit/2D with $N_{4D} = 3$ while having a fixed CFM gain over QPSK of 3.0103 dB or with an APE gain of 2.6324 dB. Note that the SPC-formats have to be detected using the soft information over the full $N \times N_{4D}$ dimensional space, i.e. if the 2D-constellations are first decoded into bits the information about the most likely erroneous bit is lost.

4.5 Multidimensional Position Modulation

In 2011, Liu *et al.* demonstrated a coherent optical transmission system with a record sensitivity of only 3.5 photons per bit [294]. This sensitivity was achieved by combining 16-ary PPM in combination with PM-QPSK. The concept of combining PPM with coherent modulation formats was generalized in [305] where PPM is studied in combination with QPSK, PM-QPSK and PS-QPSK in terms of SE and APE.

For conventional PPM, data is modulated by transmitting one pulse per PPM frame with K possible timeslots. In other words, the modulation is performed in the selection of timeslot. Note that PPM itself is not new, and has been studied for direct detection systems since the 1980's [306–308]. An example of KPPM with K=4 is shown in Fig. 4.19a which gives $\log_2(K) = 2$ bits per PPM-frame. For conventional KPPM the symbol alphabet is given as

$$\mathscr{C}_{KPPM} = \{(1, 1, 0, 0, \dots, 0, 0), (0, 0, 1, 1, \dots, 0, 0), \dots, (0, 0, 0, 0, \dots, 1, 1)\}$$
(4.24)

where the length of each symbol vector is 2K. Typically K is a power of two to assure an integer number of bits per symbol. The polarizations can be used as separate channels transmitting individual PPM in each polarization as indicated in Fig. 4.19b which will be denoted PM-KPPM. As explained in section 4.2.3, this is not the same format as if the PPM would be applied over both polarizations. PPM is a hardware-efficient method of increasing the APE. However, this comes at the cost of a drastically reduced SE, especially when K is large. Due to the high sensitivity and low complexity, KPPM in combination with FEC has been proposed for optical communication links between Earth and Mars [309]. Successful laser communication between Earth and a spacecraft located at the moon using 4PPM and 16PPM has been demonstrated [310].

An alternative to KPPM is to increase the number of pulses per frame from 1 to L which is typically called multi-pulse position modulation (MPPM) and has been studied extensively for intensity modulated systems [311–313]. MPPM is also



Figure 4.19: Illustration of (a) KPPM and (b)PM-KPPM.

suggested for visible light communication using indoor lightning, where the number of pulses per frame is set by the desired dimming of the light in the room [314].

For coherent systems, the transmitted pulses can be modulated with an I/Qmodulation format such as QPSK which will increase the SE of the system as the I/Q-modulation allows for more bits to be encoded onto the pulses. This family of modulation formats includes, e.g., 16PPM-PM-QPSK [294], 2PPM-QPSK [Paper B], and 2PPM-PS-QPSK [Paper D]. In analogy with the previous examples of the 8D modulation formats (section 4.3) and the higher dimensional SPC formats (section 4.4.1), the PPM is not limited to timeslots but other methods of increasing the dimensionality can be used such as modes, cores or wavelength channels as shown in Fig. 4.13. Although the name PPM does not inherently indicates timeslots, it is traditionally associated with this and therefore the designation multidimensional position modulation (MDPM) is introduced to differentiate between these cases. As such, PPM is one implementation of MDPM. With this designation, PS-QPSK can be described as 2MDPM-QPSK, for which PS-QPSK and 2PPM-QPSK are two different realizations of this format as illustrated in Fig. 4.20. Note that in this section, only memoryless AWGN channels are considered and for such channels there is no difference between different realizations of the MDPM formats. However, the transmitter structure and the design of the DSP algorithms would be different for PS-QPSK and 2PPM-QPSK. Further, with nonlinear effects from the transmission link taken into considerations, there could possibly be different penalties depending on the realization of the MDPM format.

For coherent systems utilizing the in-phase and quadrature components of the signal, only a handful of studies exist for MPPM formats. One such study is [315], where MPPM for different number of pulses per slot is studied in combination with PS-QPSK and it is shown that this format can increase both the SE and APE over conventional PS-QPSK. In [316], MPPM in combination with BPSK is studied for both coherent and direct detection systems. Paper F studies MDPM with multiple pulses per frame in combination with QPSK, PM-QPSK and PS-QPSK.



Figure 4.20: Two different realizations of 2MDPM-QPSK showing (a) PS-QPSK and (b) 2PPM-QPSK.

4.5.1 Inverse Pulse Position Modulation

An interesting subset of the formats studied in Paper F is when the number of pulses per frame is L - 1 where L is the number of slots in one frame. This can be seen as the inverse of conventional PPM, as data is encoded in the position of an empty slot. This family of modulation formats is denoted K-ary inverse-MDPM (KiMDPM). This type of modulation has been investigated for visible light communication for indoor lightning systems [317] and for vehicular communication [318]. For coherent communication systems, this means that the number of bits per frame can be increased since all slots except one can have an overlaid I/Q-modulation such as QPSK. The constellation for KiMDPM-QPSK is given as

$$\mathscr{C}_{KiMDPM-QPSK} = \{(0, 0, \pm 1, \pm 1, \dots, \pm 1, \pm 1), \\ (\pm 1, \pm 1, 0, 0, \dots, \pm 1, \pm 1), \dots, (\pm 1, \pm 1, \pm 1, \pm 1, \dots, 0, 0)\}.$$
(4.25)

This modulation format has an integer number of bits per symbol when K is a power of two and of particular interest is KiMDPM-QPSK with K = 4 and 8. The format 4iMDPM-QPSK is an 8D format with 256 points which gives the same SE as QPSK of 2 bit/2D. This can be understood from the fact that 3 out of the four pulse slot contain a QPSK symbol, which carry two bits each, and the 4iMDPM modulation also carries two bits. 4iMDPM-QPSK has APE = 1.2494 dB and CFM = 4.2597 dB which both are increased by roughly 1.25 dB over conventional QPSK. As shown in Fig. 4.21, the four pulse slots can be achieved by for instance two consecutive timeslots or two timeslots over two polarizations. Asymptotically, this format has the same performance in terms of SE, APE and CFM as $\mathscr{C}_{\text{8D-ASK}}$ which was explained in section 4.3.2. The extra bit in eight dimensions for \mathscr{C}_{8D-ASK} comes from the selection of sequence of the two subsets of QPSK and rotated QPSK and the extra bit for 8iMDPM-QPSK comes from the fact that sacrificing the two bits that one timeslot of QPSK carries, $\log_2 8 = 3$ bits can be modulated in the inverse-PPM. 4iMPDM-QPSK is less complex to generate in the sense that it only requires 3 levels per dimension. However, \mathscr{C}_{8D-ASK} has the advantage of having 10 nearest neighboring constellation points compared to 4iMPDM-QPSK which has 18.

Going to 8iMDPM-QPSK, it is possible to simultaneously increase the APE, CFM and the SE over QPSK. This format has 2^{17} constellation points in 16 dimensions and has SE = 2.125 bit/2D which is larger than that of QPSK. This format has


Figure 4.21: Two different implementations of 4iMDPM-QPSK showing (a) the four slots are realized over two timeslots and the two polarizations and (b) the four slots are realized by four consecutive timeslots and the polarizations are used for multiplexing.



Figure 4.22: Two different implementations of 8iMDPM-QPSK showing (a) the eight slots are realized over four timeslots and the two polarizations and (b) the eight slots are realized by eight consecutive timeslots and the polarizations are used for multiplexing.

APE = 0.8432 dB, which is a slight reduction compared to 4iMDPM-QPSK but still a gain over QPSK. Finally the CFM is 3.5902 dB which is also reduced compared to 4iMPDM-QPSK but still larger than that of QPSK. Two different implementations of 8iMDPM-QPSK are shown in Fig. 4.22, following the same reasoning as for 4iMPDM-QPSK. For K = 16, the same SE, APE and CFM as 8iMDPM-QPSK is achieved and for higher values of K the SE, CFM and APE are all reduced compared to 8iMDPM-QPSK.

Chapter 5

Comparison of Multidimensional Modulation Formats

In this section, different modulation formats introduced in Chapter 4 will be compared in terms of SE, APE, CFM and MI.

5.1 Modulation Formats Comparable to QPSK

In Fig. 5.1 the SE as a function of APE is plotted for various modulation formats with performance comparable to that of QPSK. The different colors indicate the dimensionality of the format. In four dimensions, PS-QPSK is the most power efficient modulation format [22]. In two dimensions, the most power efficient format is 3PSK which however does not carry an integer number of bits. At the same APE, 32-SP-16QAM has 0.5 bit/2D higher SE compared to QPSK. Further, it can be seen that for the same SE of 2 bit/2D as QPSK a variety of formats with higher APE exist. The 4D format SO-PM-QPSK has roughly 0.44 dB higher APE and a second 4D format, $\mathscr{C}_{opt,16}$, has roughly 1.11 dB higher APE compared to QPSK. By increasing the dimensionality to 8, the APE gain over QPSK at SE = 2 bit/2D can be increased by 1.25 dB using 4iMDPM-QPSK or $\mathcal{C}_{\text{SD-ASK}}$. POL-QAM can simultaneously increase both the SE and the APE over QPSK, even when bits are mapped to two consecutive symbol slots with some constellation points removed. The same is true for 8-iMDPM-QPSK, which has a higher APE than POL-QAM but slightly lower SE. The most power efficient modulation format in this plot is 8D-biorthogonal modulation, which can be implemented as for instance 2PPM-PS-QPSK or 4FPS-QPSK. This format has a roughly 3 dB increased APE over QPSK at the cost of a reduction in SE by one half.

In Fig. 5.2, the SE and CFM are plotted for the same formats as in Fig. 5.1. It is interesting to note that for a scenario when CFM is the appropriate measure, the

5. COMPARISON OF MULTIDIMENSIONAL MODULATION FORMATS



Figure 5.1: APE and SE for different modulation formats which are comparable to QPSK.



Figure 5.2: CFM and SE for different modulation formats which are comparable to QPSK.

formats compare differently to QPSK than for the APE case. For instance, POL-QAM is now the same CFM as QPSK but with an increased SE by 0.25 bit/2D. For 32-SP-16QAM, the increased SE by 0.5 bit/2D over QPSK comes at a cost of reduced CFM by roughly 0.97 dB. Of the formats compared here, 8iMDPM-QPSK is the only format which can simultaneously increase both the SE and the CFM over QPSK with 0.125 bit/2D higher SE and 0.58 dB higher CFM. Compared at the same SE as QPSK, there are several formats with higher CFM, listed with increasing CFM is SO-PM-QPSK, $\mathcal{C}_{opt,16}$ and the highest CFM at this SE is achieved by 4iMPDM-QPSK and \mathcal{C}_{8D-ASK} . The format 3PSK, which is the most power efficient format in 2D, is included for comparison and as it has marginally higher SE than PS-QPSK but with much lower CFM. PS-QPSK has roughly 3 dB higher CFM compared to QPSK and by using SPC over more 2D symbols the SE can be increased over PS-QPSK with a maintained CFM. The highest CFM of the compared formats is achieved by 8D-biorthogonal modulation with roughly 6 dB higher CFM compared to QPSK at the cost of a reduction in SE by one half.

5.1.1 Mutual Information for Formats Comparable to QPSK

The APE and CFM give the uncoded sensitivity for the different modulation formats. When modulation is combined with FEC, these measures are no longer valid. In this section the formats are compared in terms of MI for an AWGN channel which can be interpreted as a measure of the performance in presence of strong coding. Fig. 5.3 shows the achievable rate as a function of SNR for formats with SE comparable to QPSK. For the two formats with 2 bit/4D, i.e. BPSK and 8D-biorthogonal modulation (2PPM-PS-QPSK or 4FPS-QPSK), it can be seen that in terms of MI the 8D-biorthogonal modulation has a better sensitivity. For instance at an achievable information rate of 1.5 bit/4D, 8D-biorthogonal requires 1.19 dB lower SNR. PS-QPSK is often considered as a more sensitive format compared to QPSK. However, in the presence of a strong FEC this is no longer true and if the two formats are compared in terms if MI at an achievable rate of 2.75 bit/4D, QPSK has a 1.36 dBlower required SNR. This can be understood from the fact that PS-QPSK is a subset of the PM-QPSK constellation on which it can be described as a SPC code that is a very simple code with low performance. It should be noted though that if the overhead is limited to, say 7 %, PS-QPSK has a better sensitivity at the cost of lower SE. It is also interesting to compare PS-QPSK, which is the most power efficient modulation format in 4D, to 3PSK, which is the most power efficient format in 2D. When these two curves are compared, it can be seen that for moderate overheads 3PSK has lower required SNR at the same R. However, at around 2.5 bit/4D the two curves overlap and there is only a marginal sensitivity difference. 2-SPC-QPSK is a subset of QPSK in eight dimensions and thus has a lower achievable rate for



Figure 5.3: Achievable rate as a function of SNR for different modulation formats which are comparable to QPSK.

any SNR value compared to QPSK. However, compared with a fixed small overhead, 2-SPC-QPSK offers a tradeoff between sensitivity and achievable rate in-between PS-QPSK and QPSK.

Also plotted in Fig. 5.3 are several formats with SE = 4 bit/4D, namely QPSK, SO-PM-QPSK, $\mathcal{C}_{opt,16}$, and \mathcal{C}_{8D-ASK} . First, note that in terms of MI, SO-PM-QPSK has the worst performance in the region were a distinguishable difference can be seen. Further, $\mathcal{C}_{opt,16}$ has a significantly higher achievable rate at most SNR values compared to QPSK. For instance, at 3.5 bit/4D, $\mathcal{C}_{opt,16}$ has 0.30 dB lower required SNR. Finally, best performing, in terms of achievable rate, of these four formats is \mathcal{C}_{8D-ASK} which at an achievable information rate of 3.5 bit/4D has 0.55 dB increased sensitivity over QPSK. The format 4iMDPM-QPSK is omitted from the figure for clarity as the curve for this format overlaps with $\mathcal{C}_{opt,16}$ with just a fractional difference.

POL-QAM is more sensitive compared to QPSK at any SNR, which again can be explained by the fact that QPSK is a subset of the POL-QAM symbol alphabet. Finally, 32-SP-16QAM achieves the highest R at low overheads and it is more sensitive compared to QPSK. For instance again at R = 3.5 bit/4D, it has 0.92 dB increased sensitivity over QPSK. At this R, 32-SP-16QAM is marginally more sensitive compared to POL-QAM with 0.04 dB better sensitivity.

5.2 Modulation Formats Comparable to 16QAM

For systems requiring a higher SE than QPSK, the most commonly used modulation format is 16QAM. In this section, alternative formats with an SE, APE and CFM comparable to that of 16QAM are compared. Fig. 5.4 shows the SE and APE for such formats. At the same SE of 4 bit/2D, the format optimizing 16 points from the A₂ lattice can achieve roughly 0.54 dB increase in APE. However, optimization of 256 points in 4D on the D_4 lattice, i.e. 256- D_4 yields an increase of roughly 1.71 dB in APE at the same SE as 16QAM. By increasing the QAM order to 32QAM, the SE is increased by 1 bit/2D over 16QAM at the cost of roughly 2 dB lower APE. The time-domain hybrid of 16QAM and 32QAM can increase the APE over 32QAM at the cost of 0.5 bit/2D in SE. However, at the same SE as this hybrid format, formats with the same dimensionality but significantly higher APE can be found. Such formats are 512-SP-64QAM and 512-SP-32QAM, which both simultaneously increase both the APE and the SE over 16QAM. As can be noted, 512-SP-32QAM has higher APE than 512-SP-64QAM and should be less complex to implement since the base format has a lower number of levels. With 512-SP-32QAM, a gain in SE of 0.5 bit/2D and a gain in APE of 0.51 dB can be achieved compared to 16QAM. Applying SPC over several symbol slots can simultaneously increase both the APE and SE over 512-SP-32QAM, where 2-SPC-32QAM has roughly 0.23 dB higher APE and 0.25 bit/2D higher SE. Increasing the number of slots for the SPC, diminishing returns are seen. When applying SPC, or equivalently set-partitioning, to the 16QAM symbol alphabet, 128-SP-16QAM is obtained which has 2.43 dB higher APE compared to 16QAM at the cost of 0.5 bit/2D lower SE. This format requires the same resolution of the DACs as 16QAM, and as shown in Paper A, it is more resilient to small transmitter errors. In the same fashion as for N_{4D} -



Figure 5.4: APE and SE for different modulation formats which are comparable to 16QAM.



Figure 5.5: CFM and SE for different modulation formats which are comparable to 16QAM.

SPC-32QAM, applying the SPC over more symbols of 16QAM, increases the SE and APE over 128-SP-16QAM (which is the same as 1-SPC-16QAM). Most notably, 2-SPC-16QAM has an increase of roughly 0.30 dB in APE and 0.25 bit/2D over 128-SP-16QAM. The time-domain hybrid between 8QAM and 16QAM does increase the APE over 16QAM at the cost of lower SE, but the gain is much smaller compared to 128-SP-16QAM which has the same SE. Also shown in Fig. 5.4 are two different implementations of 8QAM, where it can be seen that the circular implementation has roughly a 1 dB increased APE over the rectangular constellation. Noteworthy is the fact that 64-SP-16QAM has worse APE compared to circular 8QAM at the same SE. Finally 7-A₂, has more than 1 dB increase over 8QAM in APE at the cost of a small reduction in SE. However, this format is not practical since it does not carry an integer number of bits per constellation and it is included in the plot for comparison.

In Fig. 5.5, the same formats are compared in terms of CFM and SE. At the same SE of 4 bit/2D as 16QAM, a small gain in CFM can be seen for $16-A_2$ and

while a much larger gain of 0.71 dB over 16QAM is seen for $256\text{-}D_4$. For the timedomain hybrid between 8QAM and 16QAM, an increase in CFM over 16QAM is seen at the cost of reduced SE. However, $256-D_4$ has a higher CFM compared to this format at no reduction of SE compared to 16QAM. For 128-SP-16QAM a large increase in CFM of 3.01 dB over 16QAM is seen at the cost of 0.5 bit/2D lower SE. If SPC is applied over 2 or more PM-16QAM symbols, the same CFM as 128-SP-16QAM can be achieved while the SE is increased with the number of symbols the SPC is applied over. The circular 8QAM has a marginally higher CFM compared to 128-SP-16QAM but with lower SE and 64-SP-16QAM and rectangular 8QAM both have lower CFM than the circular 8QAM, as well as compared to 128-SP-16QAM. Compared at the same CFM as 16QAM, 512-SP-32QAM can increase the SE by 0.5 bit/2D. In analogy with SPC over 16QAM symbols, applying SPC over more than one PM-32QAM symbol can increase the SE further while the same CFM as 16QAM is retained. Again 512-SP-64QAM is inferior to 512-SP-32QAM, as it is both requires a higher resolution of the DACs and has lower CFM. The time-domain hybrid between 16QAM and 32QAM has the same SE 512-SP-32QAM but 1.76 dB lower CFM while requiring the same resolution in the DACs since both formats rely on 32QAM. Finally, 32QAM has 1 bit/2D higher SE compared to 16QAM at the cost of roughly 3 dB lower CFM.

5.2.1 Mutual Information for Formats Comparable to 16QAM

In Fig. 5.6, the achievable rate as a function of SNR is shown for several constellations with an SE comparable to that of 16QAM. If the three formats with a maximum achievable rate of 6 bit/4D are compared, it can be shown that the rectangular version of 8QAM has significantly lower sensitivity when compared with coding. However, the difference between 64-SP-16QAM and the circular 8QAM is small. Interesting to note is that at very low SNR, the curve for $7-A_2$ constellation starts to crosse the curves for 64-SP-16QAM and the two 8QAM constellations and below 6.4 dBSNR (outside the figure) 7- A_2 has the highest achievable rate. Hybrid-8/16QAM has poor performance compared to 128-SP-16QAM and below 6.8 dB SNR (also outside the figure) it has lower achievable rate compared to 64-SP-16QAM, circular 8QAM, and 7-A₂. For all but very low SNR, 128-SP-16QAM has a significantly lower achievable rate compared to 16QAM which can be understood from the fact that 128-SP-16QAM is a subset of PM-16QAM. The same goes for 512-SP-32QAM and 32QAM. However, if for practical reason the overhead of the coding scheme used is limited, it can be seen that the set-portioning formats can increase the sensitivity significantly over the QAM format at the expense of reduced achievable rate. Further, it can be seen that the difference between 16QAM and 16-A₂ is small. However, at the same maximum achievable rate $256-D_4$ sees a significantly higher achievable rate at any SNR. In fact, at very low SNR this is the format with the highest achievable rate of all the formats plotted in this figure. Hybrid-16/32QAM has a bad performance and crosses many of the other constellations at lower SNR. It can also be seen in Fig. 5.6, that 512-SP-32QAM has slightly higher achievable rate compared to 512-SP-64QAM. For most SNR values 32QAM has the highest achievable rate, only beaten by $256-D_4$ at very low SNR.



Figure 5.6: Achievable rate as a function of SNR for different modulation formats which are comparable to 16QAM.

Chapter 6

Future Outlook

This chapter reviews three interesting future research areas closely connected to the topics addressed in this thesis.

Coded Modulation for the Nonlinear Channel

For today's systems, modulation and coding are usually designed under the assumption that the channel is memoryless with AWGN. For long-haul systems without inline dispersion compensation it has been shown that the total distortions can be modeled to have circularly Gaussian statistics [41–45] which has also shown to predict the performance for various modulation formats in experiments [46]. However, it has been shown that taking the memory from the deterministic nonlinear effects into consideration can increase the performance of the system [47–49]. Interesting topics for future research include modulation and coding schemes that are aware of the deterministic nonlinear effects. Examples of such schemes includes the phase-conjugated twin-waves scheme which mitigates nonlinear effects by optimizing dispersion maps and transmitting signals only on the real part of the signal [319]. The 8D format in [285] is designed to reduce the nonlinear distortions in inline dispersion compensated links by ensuring rapid variation in the SOP in the 8D symbols. Another interesting transmission scheme is the nonlinear Fourier transform [320, 321] which includes the nonlinear effects into the signal description. The results from Paper J and K shows that for highly nonlinear channels, adopting the decoder to four-dimensional channel distributions gives significant gains in the achievable information rate. However, in these papers a memoryless channel is assumed. Future research on memory-aware decoders for nonlinear channels would thus have a lot of potential.

Photonic Bandgap Fibers

In section 2.6, spatial division multiplexing systems are discussed as one solution to overcome the "capacity crunch" in single-mode fibers [154]. An intriguing technology

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is the photonic bandgap fiber where the light is mainly guided in air with a potential of lower loss, latency and nonlinear interference compared to conventional SMF [185]. Multimode transmission over such fibers are thus interesting since two of the effects that are limiting the transmission distance, namely loss and nonlinear distortions, can be reduced compared to conventional fibers. Several challenges exists though such as fabricating long enough fibers and reaching the theoretically predicted loss. Further, these fibers have the lowest loss around a wavelength of 2 μ m requiring all the system architecture to migrate to this wavelength, requiring a lot of research attention to sub-components for these systems.

Self-Homodyne Superchannels

In coherent optical systems there is tendency to use higher order modulation formats such as 32QAM and higher. One reason for this was discussed in section 2.5 where it was shown that with advanced FEC higher order modulation formats are capable of a higher achievable information rate at the same SNR (see Fig. 2.9). However, the higher order of the modulation format, the harder it is to perform phase tracking which put constraints on the linewidth of the lasers [113]. One possible solution to overcome the linewidth constraint for these formats is to use self-homodyne detection where a pilot tone of the transmitter laser is co-propagated with the signal and used as a local oscillator [322]. For a conventional SMF systems however, this usually means a sacrifice by 50 % in SE since the pilot is transmitted on the orthogonal polarization. For multicore systems the pilot symbols for every WDM channel can be transmitted over one core reducing the penalty in SE significantly [173]. For conventional SMF system, recent work has demonstrated regeneration of a frequency comb from two frequency and phase-locked carriers at low OSNR [323]. This is a promising technique for self-homodyne detection of superchannels where the LO tones are regenerated from two unmodulated carriers transmitted in a frame with several WDM channels carrying data [323]. Interesting future research includes systems implementing this technique in transmission and studying the impact of the regeneration on higher order formats and the impact of nonlinear crosstalk to the pilot carriers.

Chapter 7

Summary of Papers

Paper A

"Comparison of 128-SP-QAM and PM-16QAM in Long-haul WDM Transmission", *Optics Express*, vol. 21, no. 16, pp. 19269-19279, 2013.

In this paper we experimentally investigate 128-ary set-partitioning quadrature amplitude modulation (128-SP-QAM) which is a subset of PM-16QAM. We investigate the back-to-back sensitivity gain of 128-SP-QAM and compare the results to PM-16QAM both at the same symbol rate and bit rate. The two formats are compared in both single channel and wavelength-division multiplexed transmission. We show that the maximum transmission reach can be increased by 50% by going to 128-SP-QAM to PM-16QAM, compared at the same bit rate.

My Contribution: I planned the experiment, built the experimental setup, extended the existing signal processing code to fit this experiment, and performed the measurement. I wrote the paper.

Paper B

"Long-Haul Transmission of PM-2PPM-QPSK at 42.8 Gbit/s", Optical Fiber Communication Conference (OFC), Anaheim, CA, USA, 2013, paper OTu2B.7.

Polarization-switched QPSK (PS-QPSK) is the most power efficient modulation format in 4 dimensions and has attracted a lot of research interest for coherent fiber optical systems. In this paper we experimentally investigate binary pulse position modulation QPSK (2PPM-QPSK) which is a modulation format with the same spectral efficiency and asymptotic power efficiency as PS-QPSK. Instead of transmitting a QPSK symbol in either the x- or y-polarization as for PS-QPSK, 2PPM-QPSK transmits one QPSK in either of two neighboring timeslots. In this paper we show the first experimental realization of PM-2PPM-QPSK and compare the performance to PM-QPSK at the same bit rate. We show that the transmission distance can be increased by approximately 40 % by using PM-2PPM-QPSK over PM-QPSK compared at the same bit rate.

My Contribution: M. Sjödin and myself jointly built the experimental setup and conducted the experiments.

Paper C

"Frequency and Polarization Switched QPSK", European Conference and Exhibition on Optical Communication (ECOC), London, England, 2013, paper Th.2.D.4.

Much research, both theoretical and experimental, has been devoted to 4-dimensional modulation formats. In this paper we explore the possibilities of going to 8 dimensions by considering two neighboring wavelength channels as one signal. We experimentally implement biorthogonal modulation in 8 dimensions which for each symbol transmits one QPSK signal in one of the four possible slots given by the two polarization states of the two wavelengths. Due to this implementation we designate the format 4-ary frequency and polarization switched QPSK (4FPS-QPSK). This format has 3 dB increased asymptotic power efficiency over QPSK at the cost of half spectral efficiency. We compare this format to dual-carrier (DC) PM-QPSK and DC polarization-switched QPSK at the same symbol rate and we detect the two wavelength channels using a single coherent receiver. We show a 4.2 dB increased back-to-back sensitivity for 4FPS-QPSK over DC-PM-QPSK and transmission of 4FPS-QPSK of up to 14,000 km is an increase of 84 % over DC-PM-QPSK.

My Contribution: I planned and carried out the experiment and wrote the signal processing code for dual-carrier detection, polarization demultiplexing and equalization as well as for 8D constellation detection. I wrote and presented the paper.

Paper D

"Biorthogonal Modulation in 8 Dimensions Experimentally Implemented as 2PPM-PS-QPSK", Proc. Optical Fiber Communication Conference (OFC), San Francisco, CA, USA, 2014, paper W1A.5.

In this paper we implement biorthogonal modulation in eight dimensions by considering modulation over two consecutive timeslots. This format can be seen as transmitting polarization-switched QPSK in combination with binary pulse position modulation and is hence designated 2PPM-PS-QPSK. This is an alternative experimental realization of the format implemented in paper C with slightly less complex transmitter and receiver structures. The 2PPM-PS-QPSK format is compared to PM-QPSK at the same bit rate of 85.6 Gbit/s in long-haul transmission experiments. We demonstrate an increased back-to-back sensitivity gain of 1.4 dB for 2PPM-PS-QPSK over PM-QPSK at a bit error rate of 10^{-3} . We also show transmission of this format of up to 12.300 km which corresponds to an increase over PM-QPSK by 84 %.

My Contribution: I planned and carried out the experiment, extended the code from paper C to fit pulse-position modulated signals. I wrote and presented the paper.

Paper E

"Experimental Investigation of a Four-Dimensional 256-ary Lattice-Based Modulation Format", Proc. Optical Fiber Communication Conference (OFC), Los Angeles, CA, USA, 2015, paper W4K.3.

For coherent optical systems which requires high spectral-efficiency, polarizationmultiplexed 16-ary quadrature amplitude modulation (PM-16QAM) is well studied modulation format. An alternative to PM-16QAM is the 4D format 128-SP-16QAM which is studied in Paper A. This format however, sacrifices spectral efficiency to gain in transmission reach. In this paper we study a 4D modulation format with 256 constellation points derived from the D₄ lattice, designated 256-D₄. The choice of 256 points gives the same spectral efficiency as PM-16QAM. This format has an increased asymptotic power efficiency by 1.71 dB over PM-16QAM. We show in experiments that with an FEC operating in the region of BER = 10^{-2} , PM-16QAM has a longer transmission reach in WDM transmission. However, if the target is a system with an FEC operating at BER $\leq 10^{-3}$, 256-D₄ shows an increased transmission reach over PM-16QAM.

My Contribution: I performed the experiments together with S. Alreesh, and wrote the paper. The DSP used was developed at HHI. E. Agrell found the bit-to-symbol mapping and S. Alreesh implemented the encoder and decoder for the FEC coding. I presented the paper at the conference.

Paper F

"K-over-L Multidimensional Position Modulation", Journal of Lightwave Technology, vol. 32, no. 12, pp. 2254–2262, Jun. 2014.

In this theoretical paper we analyze a family of modulation formats based on multidimensional position modulation (MDPM) which is a generalization of pulse position modulation (PPM). For MDPM, the different slots can be realized by different timeslots, polarization states, wavelength channels, different modes of a multimode fiber, different cores of a multicore fiber or any combination of these. We show that by using multiple pulses per frame, K-over-L-MDPM, in combination with QPSK it is possible to simultaneously increase both the spectral efficiency and the asymptotic power efficiency over conventional QPSK. We identify K-ary inverse-MDPM (KiMDPM), the special case of K-over-(K-1)-MDPM, as practically interesting modulation formats since they carry an integer number of bits per frame when K is a power of two. We show that 4iMDPM-QPSK has a 1.25 dB increased asymptotic power efficiency over QPSK with a maintained spectral efficiency. We also investigate 4iMDPM-QPSK and 8iMDPM-QPSK in the low SNR regime by Monte Carlo simulations with additive white Gaussian noise as the only impairment and show that the sensitivity of 4iMDPM-QPSK is better compared to QPSK for bit-error probabilities lower than 8.3×10^{-3} .

My Contribution: I came up with the idea of multidimensional position modulation, performed the theoretical and simulation analysis and wrote the paper.

Paper G

"Experimental Investigation of Crosstalk Penalties in Multicore Fiber Transmission Systems", *IEEE Photonics Journal*, vol. 7, no. 1, paper 7200507, Feb. 2015.

Spatial division multiplexed optical communication systems has received a lot of research attention in recent years as a solution to increase the data throughput of a single fiber. In this paper we consider multicore fibers where one of the biggest difference compared to single-core fibers is that the signal integrity is affected by the in-band crosstalk between different cores which arises due to power coupling during fiber propagation. We study the impact of inter-core crosstalk in long-haul multicore fiber transmission with varying amounts of crosstalk per span. We show for instance that a crosstalk level of -25.7 dBm per span of 28.6 km, reduced the achievable transmission distance of PM-QPSK by 25 %.

My Contribution: I came up with the idea of the experimental setup to vary the amount of crosstalk per transmission span, I built the setup and carried out the experiments. I wrote the paper.

Paper H

"Modulation Formats for Multi-Core Fiber Transmission", *Optics Express*, vol. 22, no. 26, pp. 32457–32469, Dec. 2014.

With multicore fiber transmission systems, a new dimension for modulation is available, i.e., space. For systems with strong coupling between the cores, spatial superchannels has to be routed to the same receiver where digital signal processing is used to undo the effects of coupling. This opens up for modulation formats spanning the spatial superchannel. In this paper we investigate several different options of modulation over several cores including pulse position modulation and single parity check (SPC) coding over the spatial superchannel. With SPC, the parity bit can be split among all cores, reducing the penalty in spectral efficiency associated with the parity bit. We show that with SPC over three cores, the achievable transmission distance can be increased by 20 % over PM-QPSK at BER = 10^{-3} .

My Contribution: I performed parts of the experiments regarding single-parity check coded QPSK and performed parts of the theoretical analysis. I wrote parts of the paper.

Paper I

"Single Parity Check-Coded 16QAM over Spatial Superchannels in Multicore Fiber Transmission", *Optics Express*, vol. 23, no. 1, pp. 14569–14582, May 2015.

In Paper H, single-parity check (SPC) coded PM-QPSK superchannels were identified as a practical modulation scheme for multicore fiber systems. For PM-16QAM, the achievable transmission reach is much more limited compared to PM-QPSK, especially in the presence of crosstalk between the cores. In this paper we experimentally study SPC coded spatial superchannels where the SPC code is applied over up to the full seven cores of the multicore fiber that is used. We show that SPC over one core can more then double the transmission distance compared to PM-16QAM at the cost of 0.91 bit/s/Hz/core in spectral efficiency. Sharing the parity bit over seven cores, the loss in spectral efficiency is only 0.13 bit/s/Hz/core while the achievable transmission distance is 44 % longer than that of PM-16QAM.

My Contribution: I planned and performed the experiment. I wrote code to perform the offline joint demodulation over the spatial super channels and loading of single-parity check coded signals in the transmitter. I wrote the paper.

Paper J

"Four-Dimensional Estimates of Mutual Information in Coherent Optical Communication Experiments", Proc. European Conference on Optical Communication (ECOC), Valencia, Spain, 2015, paper We.4.6.5.

In recent fiber optical communication experiments, soft-decision coding schemes are an essential part. For these systems, it can be shown that the mutual information is a better figure of merit compared to the pre-FEC bit-error-rate. In this paper, we investigate mutual information fiber optical experiments for different assumptions of the channel distribution. We show that for WDM transmission systems of PM-QPSK or PS-QPSK without inline dispersion compensation, assuming a 2D independent and identically distributed Gaussian noise distribution is a good approximation. This

7. SUMMARY OF PAPERS

is a common assumption in the literature which is validated for experiments in this paper. However, we also show that this is not true for highly nonlinear channels such as the system in this paper that transmits single channel PM-16QAM over a link with inline dispersion compensation. In this case, 4D distributions can achieve significantly higher achievable rates.

My Contribution: This contribution is in close collaboration with T. Fehenberger. T. Fehenberger wrote the code for mutual information estimates, I extended it to four dimensions. I constructed the experimental setups and performed the measurements. I wrote the paper with major input from T. Fehenberger and E. Agrell. I presented the paper.

Paper J

"Impact of 4D Channel Distribution on the Achievable Rates in Coherent Optical Communication Experiments", Submitted to *Journal of Lightwave Technology*, 16 Nov. 2015.

In this paper we investigate the impact of channel distribution assumption on the achievable information rate calculated using mutual information and generalized mutual information for different optical transmission channels with PM-16QAM modulation. We show that for WDM transmission systems without inline dispersion compensation, the circularly symmetric complex Gaussian distribution is a good approximation. However, for systems with inline dispersion compensation, we show that 4D distributions can increase the achievable rate. Part of this gains comes from estimating individual distributions for each received constellation point. We also show that a significant part of the gain comes from estimating the mean values of the received constellations. The distribution with the highest achievable rate in our experiments is the 4D correlated Gaussian distribution.

My Contribution: This contribution is in close collaboration with T. Fehenberger and is a continuation of Paper J. T. Fehenberger extended the estimates to also include GMI. I built the experimental setup and performed the experiments. I wrote the paper together with T. Fehenberger.

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