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Fixed-rate Transmission over Fading Interference Channels Using Point-to-Point Gaussian Codes

Hamed Farhadi, Chao Wang, and Mikael Skoglund

Abstract—**This paper investigates transmission schemes for fixed-rate communications over a Rayleigh block-fading interference channel. There are two source–destination pairs where each source, in the presence of a short-term power constraint, intends to communicate with its dedicated destination at a *fixed* data rate. It encodes its messages using a point-to-point Gaussian codebook. The two users' transmissions can be conducted orthogonally or non-orthogonally. In the latter case, each destination performs either direct decoding by treating the interference as noise, or successive interference cancellation (SIC) to recover its desired message. For each scheme, we seek solutions of a power control problem to efficiently assign power to the sources such that the codewords can be successfully decoded at destinations. However, because of the random nature of fading, the power control problem for some channel realizations may not have any feasible solution and the transmission will be in outage. Thus, for each transmission scheme, we first compute a lower bound and an upper bound on the outage probability. Next, we use these results to find an outer bound and an inner bound on the ϵ-outage achievable rate region, i.e. the rate region in which the outage probability is below a certain value ϵ.**

I. INTRODUCTION

The *fading interference channel* models wireless communication scenarios in which multiple source–destination pairs share the wireless medium and each source's transmission interferes with the reception at non-intended destinations. Wireless communi-

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nications are usually subject to time-varying channels, thus, in order to enable reliable communication users are required to adapt their transmissions according to available channel state information (CSI). In fading interference channel, the optimum transmission of each source–destination pair is interrelated with that of the other pairs and optimizing transmissions is a challenging task. To study transmission in such time-varying environment, an analytically tractable channel model called *block-fading* can be used such that channel gains are constant within the duration of each fading block, but change across different blocks [1]. In each fading block, system parameters (e.g. data transmission rates and transmission powers) can be adapted in order to satisfy certain service requirements. For example, if each user desires maximum data throughput for given transmission powers, a *throughput maximization problem* can be studied to adapt transmission rates. In theses systems the instantaneous mutual information corresponding to each source–destination pair fluctuates over different fading blocks and transmission rates are adapted accordingly to attain the highest throughput while maintaining a reliable communication. In another category of communication systems (e.g. voice communications) in which each user desires a fixed-rate communication, a *power control problem* should be addressed for adapting transmission powers in order to enable reliable communication. Certainly the lowest possible transmission powers are desired. The solution to this problem controls the powers such that the instantaneous mutual information is kept constant over different fading blocks. Consequently, a single fixed-rate codebook is sufficient. The optimal solutions of either the throughput maximization problem or the power control problem in general are unknown.

The throughput maximization problem for a two-user fading interference channel has been studied in [2]–[6] and transmission schemes based on

Han-Kobayashi coding have been devised [7]. The recent development of capacity achieving codes for point-to-point communications, however, makes such codes attractive even for network communications (although they might be sub-optimal in these scenarios). In addition, applying point-to-point code between each source–destination pair has less complexity compared to coding schemes such as Han-Kobayashi. The throughput maximization problem, subject to using Gaussian point-to-point codes, has been addressed in [8] and the capacity region has been characterized. The power control problem, using point-to-point codes, has been studied for single-user communication systems in [9]. In addition, this problem has been partially investigated in the literature only for a class of fading interference channels in which sources simultaneously send their messages and each destination decodes its message by treating the interference as noise (see e.g. [10]–[12]). Certain power control algorithms have been proposed (see e.g. [10], [13]–[15]) that assign power to the sources such that every source–destination pair meets a desired signal-to-interference-plus-noise ratio (SINR). These algorithms require local CSI to be available at terminals. Since in practice available CSI at terminals is subject to errors, power control algorithms in the presence of imperfect CSI have been studied in the literature. For instance, the power control problem when transmitters have only quantized CSI is addressed in [16], and when transmitters have noisy CSI is studied in [17]. A framework for distributed stochastic power control algorithms is proposed in [18], where the transmit powers are updated based on noisy estimation of local CSI. The performance degradation of distributed power control algorithms due to CSI estimation errors is investigated in [19], [20].

Considering a short-term power constraint, a solution to the power control problem may not always exist even if perfect CSI is available at terminals. In fact, due to the random nature of fading, for some channel realizations there is no positive power value that can both satisfy the power constraint and guarantee successful communication at the desired rates. We say, the power control problem is *infeasible* and the system is in *outage*. The probability that an outage event occurs is defined as *outage probability*. In many communication systems a small value of outage probability is tolerable. In these cases an ϵ-*outage achievable rate region* is defined as the set

of the rates for which the outage probability is less than ϵ [21]–[23]. The performance limits of fixed-rate transmission schemes for single-user communication system have been extensively studied (see [9], [21]), but for fading interference channels they are still less known.

In this paper, we consider a two-user Rayleigh block-fading interference channel. Each source utilizes a fixed-rate point-to-point Gaussian code to communicate with its dedicated destination. Perfect CSI is globally available at all terminals. Each source is subject to a short-term power constraint. We consider four different transmission schemes. When the two source–destination pairs are orthogonally activated, inter-user interference can be completely eliminated, with the possible price of spectral inefficiency. When both users non-orthogonally access the channel, inter-user interference must be taken into account at the decoders. This leads to three schemes: 1) both destinations directly decode their desired messages by simply treating interference as noise; 2) both destinations conduct successive interference cancellation (SIC); 3) one destination performs direct decoding and the other destination performs SIC. For each of these schemes, we first find the solution of the power control problem. Next, we compute a lower bound and an upper bound on the outage probability. These results are then used to find an outer bound and an inner bound on the ϵ-outage achievable rate region.

Notations: Lower-case letter x and boldface lower-case letter $\mathbf{x}$ represent scalar and vector, respectively. The transpose of $\mathbf{x}$ is denoted as $\mathbf{x}^T$. The notation $\mathbf{x} \prec \mathbf{y}$ ($\mathbf{x} \preceq \mathbf{y}$) means that each element of vector $\mathbf{x}$ is less than (less than or equal to) the corresponding element of vector $\mathbf{y}$. We denote the set of real numbers as $\mathbb{R}$, the set of positive real numbers as $\mathbb{R}^+$, and the set of complex numbers as $\mathbb{C}$.

The structure of this paper is as follows. In Section II, we describe the system model and define the performance metrics which will be investigated. Section III addresses the orthogonal transmission scheme. The non-orthogonal transmission schemes are discussed in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a single-antenna fading interference channel with two source–destination pairs as shown
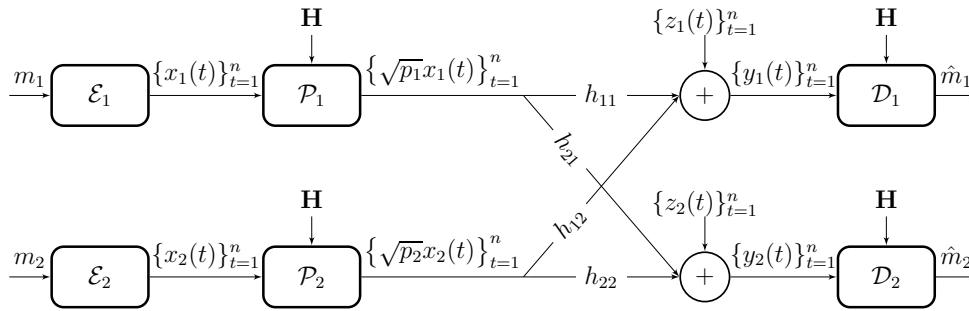
Fig. 1: Two-user fading interference channel.

in Fig. 1. Each source intends to communicate with its dedicated destination. The channels follow a Rayleigh block-fading model, in which the channel gains remain constant within a coherent interval (the time slots in which the considered communications occur and their number is denoted as n). We assume that the channel gains independently change across different fading blocks. We denote the fading coefficient of the link between the kth source ($k \in \{1, 2\}$) and the kth destination (termed direct link) as $h_{kk} \sim \mathcal{CN}(0, \sigma_{\mathsf{S}}^2)$, and that between the kth source and the lth destination ($l \in \{1, 2\}, l \neq k$) (termed interference link) as $h_{lk} \sim \mathcal{CN}(0, \sigma_{\mathsf{I}}^2)$. The parameters σ_{S}^2 and σ_{I}^2 are the variances of the direct and interference link gains, respectively. They generally can have different values and their ratio is denoted as $\alpha = \sigma_{\mathsf{S}}^2 / \sigma_{\mathsf{I}}^2$. The channel gains are mutually independent. We denote the network channel matrix as $\mathbf{H}$, whose element on the ith row and the jth column is h_{ij}. We assume perfect channel knowledge to be globally available, i.e. $\mathbf{H}$ is known at every terminal. During one fading block, each source sends one message to its destination. The message from the kth source, m_k, is encoded using a point-to-point Gaussian codebook with fixed data rate R_k (bits/channel use).

Definition 1 (Messages)**.** *The kth ($k \in \{1, 2\}$) source's message m_k is independently and uniformly chosen from the set $\mathcal{M}_k = \left\{1, 2, ..., 2^{nR_k}\right\}$.*

Definition 2 (Encoders)**.** *The kth source ($k \in \{1, 2\}$) has an encoding function, $\mathcal{E}_k$: $\mathcal{M}_k \rightarrow \mathbb{C}^n$, that maps its message m_k into a length-n codeword $\{x_k(t)\}_{t=1}^n$ with the power constraint*

$$\frac{1}{n} \sum_{t=1}^n |x_k(t)|^2 \leq 1. \tag{1}$$

Each encoder is concatenated by a power controller as shown in Fig. 1.

Definition 3 (Power controllers)**.** *The power controller associated with the kth source ($k \in \{1, 2\}$) applies a function, $\mathcal{P}_k$: $\mathbb{C}^n \rightarrow \mathbb{C}^n$, that according to the channel gains scales the codeword $\{x_k(t)\}_{t=1}^n$ to $\left\{\sqrt{p_k} x_k(t)\right\}_{t=1}^n$, where p_k denotes the average transmission power of the kth source. The assigned power obeys a short-term constraint, i.e. within each fading block it satisfies*

$$p_k \leq p_{\max,k}, \tag{2}$$

where $p_{\max,k}$ is the maximum transmission power of the kth source.

Definition 4 (Decoders)**.** *Each destination has a decoding function, $\mathcal{D}_k$: $\mathbb{C}^n \rightarrow \mathcal{M}_k$, that maps its observed channel outputs $\{y_k(t)\}_{t=1}^n$ to an estimate $\hat{m}_k$ of the transmitted message m_k.*

In the following we provide definitions regarding performance metrics.

Definition 5 (Achievable rate region)**.** *Within each fading block, for a given transmission power vector $\mathbf{p}$ ($\mathbf{p} = [p_1 \ p_2]^T$), applying a transmission scheme 'A', the average probability of error is defined as*

$$P_e^{(n)} = \Pr\left\{(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\right\}, \tag{3}$$

where M_k and $\hat{M}_k$ denote a randomly transmitted message and the corresponding decoded message, respectively. If there exist channel encoding and decoding functions such that $\lim_{n\to\infty} P_e^{(n)} = 0$, then we say that a rate pair (R_1, R_2) is achievable. We denote the achievable rate region as $\mathcal{C}^A(\mathbf{p}, \mathbf{H})$.

Although in general one can use time-sharing with point-to-point codes, we assume that there is no

time-sharing as it requires additional coordination between the terminals.

Definition 6 (Feasible transmission scheme)**.** *Given a globally known channel matrix* $\mathbf{H}$*, if the achievable rate region corresponding to a power vector* $\mathbf{p}$ $\left(\mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}_{\max}, \ \mathbf{p}_{\max} = [p_{\max,1} \ p_{\max,2}]^T\right)$ *includes the transmission rates, i.e.* $(R_1, R_2) \in \mathcal{C}^A(\mathbf{p}, \mathbf{H})$*, then the power vector is a solution of the power control problem. In the case that the power control problem for a transmission scheme 'A' has a solution, we say that the transmission scheme 'A' is feasible. We define the set of feasible solutions of the power control problem for the transmission scheme 'A' as*

$$\mathcal{P}_{\mathbf{H}}^{A}((R_1, R_2), \mathbf{p}_{\max}) \triangleq$$
$$\left\{ \mathbf{p} : \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}_{\max}, (R_1, R_2) \in \mathcal{C}^A(\mathbf{p}, \mathbf{H}) \right\}. \quad (4)$$

Using $\emptyset$ *to denote an empty set,* $\mathcal{P}_{\mathbf{H}}^{A} \neq \emptyset$ *means that the transmission scheme 'A' is feasible.*

In fact, due to the random nature of fading, for some channel realizations, a transmission scheme 'A' may not be feasible. If $\mathcal{P}_{\mathbf{H}}^{A}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \emptyset$, then the network is said to be in outage. This implies that if there exists $\mathbf{p}$ $(\mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}_{\max})$ such that the desired rate pair (R_1, R_2) is in the rate region $\mathcal{C}^A(\mathbf{p}, \mathbf{H})$, then the transmission scheme is feasible and no outage occurs. According to this definition, if for any power $\mathbf{p}$ $(\mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}_{\max})$ at least one of the rates (either R_1 or R_2) does not fall in the achievable rate region, then an outage event occurs. We define the *outage probability* as follows.

Definition 7 (Outage probability)**.** *The outage probability of a transmission scheme 'A' is*

$$P_{\text{out}}^{A}((R_1, R_2), \mathbf{p}_{\max}) \triangleq$$
$$\int_{\hat{\mathbf{H}} \in \mathcal{H}} \Pr\left\{ \mathcal{P}_{\hat{\mathbf{H}}}^{A}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \emptyset \right\} f_{\mathbf{H}}\left(\hat{\mathbf{H}}\right) d\hat{\mathbf{H}}, \quad (5)$$

where $\mathcal{H}$ *denotes the set of all possible channel matrices and* $f_{\mathbf{H}}$ *is the joint pdf of* $\mathbf{H}$*.*

Wireless communication systems depending on their underlying applications can tolerate a certain amount of outage probability. Thus, it is desirable to characterize the rate region for which an outage probability less than the maximum tolerable outage probability can be attained. In the following we provide the definition of a related performance measure which will be investigated in this paper [23].

Definition 8 (ϵ-outage achievable rate region)**.** *An* ϵ*-outage achievable rate region of a transmission scheme 'A' is*

$$\mathcal{C}_{\epsilon}^{A}(\mathbf{p}_{\max}) \triangleq \left\{ (R_1, R_2) : P_{\text{out}}^{A}((R_1, R_2), \mathbf{p}_{\max}) \leq \epsilon \right\}, \quad (6)$$

where ϵ *is the maximum outage probability that a specific application can tolerate.*

In the following sections, we study the performance of different transmission schemes. Our analysis starts from orthogonal transmission.

III. FIXED-RATE ORTHOGONAL TRANSMISSION SCHEME

In wireless networks, inter-user interference may significantly degrade the communication system's performance. One intuitive solution to deal with interference is to orthogonalize different users' operations. Since each user has access to only a fraction of the available channel, this can provide interference-free communication for each user. This transmission scheme is referred as *time-division multiple access (TDMA)* in the literature, and we term it *orthogonal transmission* (OT) throughout the paper to be consistent with the other schemes that we study in this paper. Using δ $(0 < \delta < 1)$ to denote the *channel-sharing factor*, the fractions of the total channel used by the first and the second sources are δ and $(1 - \delta)$, respectively. In a fading block, the achievable rate region $\mathcal{C}^{OT}(\mathbf{p}, \mathbf{H})$ includes the rate pairs (R_1, R_2) which satisfy

$$R_1 \ \leq \ \delta \log_2\left(1 + \frac{|h_{11}|^2 p_1}{\delta N_0}\right) \quad (7)$$

$$R_2 \ \leq \ (1 - \delta) \log_2\left(1 + \frac{|h_{22}|^2 p_2}{(1 - \delta) N_0}\right), \quad (8)$$

where N_0 is the noise power. The set of feasible solutions of the power control problem is

$$\mathcal{P}_{\mathbf{H}}^{OT}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \{\mathbf{p} : \ \mathbf{n}_T \preceq \mathbf{p} \preceq \mathbf{p}_{\max}\}, \quad (9)$$

in which

$$\mathbf{n}_T = \left[\left(2^{R_1/\delta} - 1\right)\frac{\delta N_0}{|h_{11}|^2} \ \ \left(2^{R_2/(1-\delta)} - 1\right)\frac{(1-\delta) N_0}{|h_{22}|^2}\right]^T. (10)$$

Clearly, the minimum required transmission powers of the OT scheme are

$$p_1^{OT} \ = \ \left(2^{R_1/\delta} - 1\right)\frac{\delta N_0}{|h_{11}|^2} \quad (11)$$

$$p_2^{OT} \ = \ \left(2^{R_2/(1-\delta)} - 1\right)\frac{(1-\delta) N_0}{|h_{22}|^2}. \quad (12)$$

This is similar to the *channel inversion* method proposed for single-user point-to-point communications [24]. The solutions in (11) and (12), however, may violate the maximum power constraints in (2). Thus, with a certain probability, the scheme is infeasible and outage events occur. The following proposition characterizes the outage probability.

Proposition 1. *The outage probability of the OT scheme is*

$$P_{\text{out}}^{\text{OT}}\left((R_1, R_2), \mathbf{p}_{\max}\right) =$$
$$1 - e^{-\frac{N_0}{\sigma_S^2}\left(\left(2^{R_1/\delta}-1\right)\frac{\delta}{p_{\max,1}}+\left(2^{R_2/(1-\delta)}-1\right)\frac{(1-\delta)}{p_{\max,2}}\right)}. \quad (13)$$

The channel sharing factor δ can be carefully chosen to minimize the outage probability. It can be shown that $P_{\text{out}}^{\text{OT}}\left((R_1, R_2), \mathbf{p}_{\max}\right)$ is a convex function of δ. The optimum choice of δ, denoted as δ_{opt}, can be found by solving the following equation:

$$\left(2^{R_1/\delta_{\text{opt}}}\left(1 - R_1 \ln 2/\delta_{\text{opt}}\right) - 1\right) =$$
$$\frac{p_{\max,1}}{p_{\max,2}}\left(2^{R_2/(1-\delta_{\text{opt}})}\left(1 - R_2 \ln 2/(1 - \delta_{\text{opt}})\right) - 1\right). (14)$$

For instance, when $p_{\max,1} = p_{\max,2}$ and $R_1 = R_2$, we can see that $\delta_{\text{opt}} = 0.5$ is the solution. The result in **Proposition 1** can be used to obtain the boundary of the ϵ-outage achievable rate region $\mathcal{C}_\epsilon^{\text{OT}}\left(\mathbf{p}_{\max}\right)$ by solving $P_{\text{out}}^{\text{OT}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \epsilon$ for (R_1, R_2). The solutions are

$$\begin{cases} R_1 = t \\ R_2 = (1-\delta)\log_2\left(1 - \frac{\delta p_{\max,2}\left(2^{t/\delta}-1\right)}{(1-\delta)p_{\max,1}} - \frac{p_{\max,2}\sigma_S^2\ln(1-\epsilon)}{(1-\delta)N_0}\right) \end{cases} (15)$$

where $0 \leq t \leq \delta \log_2\left(1 - \frac{p_{\max,1}}{\delta} \times \frac{\sigma_S^2}{N_0}\ln(1-\epsilon)\right)$. As mentioned, the OT scheme eliminates interuser interference with the possible price of spectral inefficiency. In fact, depending on the value of σ_S^2, σ_I^2 and $\mathbf{p}_{\max}$, permitting both sources to send messages non-orthogonally may outperform the OT scheme. In the next section, we will focus on non-orthogonal transmission schemes.

IV. FIXED-RATE NON-ORTHOGONAL TRANSMISSION SCHEMES

We permit the two sources to transmit non-orthogonally. The kth source ($k \in \{1, 2\}$) sends a codeword x_k as described in Section II. The channel outputs at time t ($t \in \{1, ..., n\}$) are

$$y_1(t) = \sqrt{p_1}h_{11}x_1(t) + \sqrt{p_2}h_{12}x_2(t) + z_1(t)$$
$$y_2(t) = \sqrt{p_1}h_{21}x_1(t) + \sqrt{p_2}h_{22}x_2(t) + z_2(t), (16)$$

where $z_k(t)$ is the additive white Gaussian noise with power N_0. Each destination may either directly decode its intended message by treating interference as noise, or it may first decode the message of the unintended source and next decode its intended message after removing the interference. Thus, depending on the decoding strategy, we have three different transmission schemes. We study their power control strategies and their performance limits in what follows.

A. Direct Decoding at Both Destinations

We require each destination to directly decode its desired message by treating the interference as noise. We term this scheme NOT_1. For a fading block, the achievable rate region $\mathcal{C}^{\text{NOT}_1}(\mathbf{p}, \mathbf{H})$ includes the rate pairs (R_1, R_2) which satisfy

$$R_1 \leq \log_2\left(1 + \frac{|h_{11}|^2 p_1}{|h_{12}|^2 p_2 + N_0}\right) \quad (17)$$

$$R_2 \leq \log_2\left(1 + \frac{|h_{22}|^2 p_2}{|h_{21}|^2 p_1 + N_0}\right). \quad (18)$$

After some manipulations, we can present these conditions as the following power constraint

$$\mathbf{p} \succeq \mathbf{D}_S\mathbf{F}_S\mathbf{p} + \mathbf{n}_S, \quad (19)$$

where

$$\mathbf{D}_S = \begin{bmatrix} 2^{R_1}-1 & 0 \\ 0 & 2^{R_2}-1 \end{bmatrix}, \mathbf{F}_S = \begin{bmatrix} 0 & \frac{|h_{12}|^2}{|h_{11}|^2} \\ \frac{|h_{21}|^2}{|h_{22}|^2} & 0 \end{bmatrix},$$

$$\mathbf{n}_S = \left[\left(2^{R_1}-1\right)\frac{N_0}{|h_{11}|^2} \quad \left(2^{R_2}-1\right)\frac{N_0}{|h_{22}|^2}\right]^T. \quad (20)$$

The matrix $\mathbf{D}_S$ is related to the transmission rates, while $\mathbf{F}_S$ depends only on the channel gains.

1) Power Control Solution: The positive element-wise minimum transmission power vector among those satisfying (19) - if there is any such vector - is $\mathbf{p}^{\text{NOT}_1} = (\mathbf{I} - \mathbf{D}_S\mathbf{F}_S)^{-1}\mathbf{n}_S$ [10], where $\mathbf{I}$ is the 2×2 identity matrix. Therefore, the minimum required powers are

$$p_1^{\text{NOT}_1} = \frac{N_0\left(\left(2^{R_1}-1\right)\frac{|h_{21}|^2}{|h_{11}|^2}+l\right)}{|h_{21}|^2(1-l)} \quad (21)$$

$$p_2^{\text{NOT}_1} = \frac{N_0\left(\left(2^{R_2}-1\right)\frac{|h_{12}|^2}{|h_{22}|^2}+l\right)}{|h_{12}|^2(1-l)}, \quad (22)$$

where $l = \left(2^{R_1}-1\right)\left(2^{R_2}-1\right)\frac{|h_{12}|^2|h_{21}|^2}{|h_{11}|^2|h_{22}|^2}$. The powers are positive only if $l < 1$. For some channel
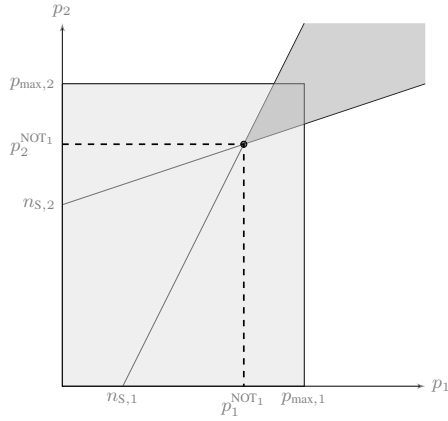
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Fig. 2: The set of feasible solutions of the power control problem for the NOT$_1$ scheme.

realizations, there is no positive power vector that can satisfy the constraints in (2) and (19). In these cases, the power control problem does not have any feasible solution, and transmissions cause outage events. In the following, we investigate the outage probability.

2) Outage Probability Analysis: For a given channel $\mathbf{H}$, the set of feasible solutions of the power control problem is

$$\mathcal{P}_{\mathbf{H}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \{\mathbf{p} : \mathbf{p} \succeq \mathbf{D}_{\mathrm{S}}\mathbf{F}_{\mathrm{S}}\mathbf{p} + \mathbf{n}_{\mathrm{S}}, \ \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{p}_{\max}\}. \quad (23)$$

Fig. 2 illustrates this set - when there is at least one vector $\mathbf{p}$ that satisfies the constraints in (2) and (19). The light gray region in the figure indicates the powers which satisfy (2), and the dark gray region illustrates the powers which satisfy (19). The intersection of the two regions indicates the powers within $\mathcal{P}_{\mathbf{H}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right)$. For some channel realizations, there is no positive power vector which satisfies (19), or such a positive power vector exists but does not satisfy the maximum power constraints in (2). In this case, $\mathcal{P}_{\mathbf{H}}^{\mathrm{NOT_1}} = \emptyset$, i.e. the power control problem is *infeasible*, and an outage event occurs. We aim to characterize the outage probability. For this purpose, we first present the following lemma to provide a necessary and sufficient condition for the existence of a positive vector that satisfies (19).

Lemma 1. *There exists at least one positive vector* $\mathbf{p}$ ($\mathbf{p} \succ \mathbf{0}$) *that satisfies the inequality in (19), if and only if* $\lambda_{\max}\left(\mathbf{D}_{\mathrm{S}}\mathbf{F}_{\mathrm{S}}\right) < 1$, *where* $\lambda_{\max}\left(\mathbf{D}_{\mathrm{S}}\mathbf{F}_{\mathrm{S}}\right)$ *denotes the largest magnitude of the eigenvalues of matrix* $\mathbf{D}_{\mathrm{S}}\mathbf{F}_{\mathrm{S}}$ *given in (20).*

Proof: The proof is similar to that of Theorem 5 in [25]. ∎

Now, we utilize the above result to characterize a lower bound and an upper bound on the outage probability of the NOT$_1$ scheme.

Proposition 2. *The outage probability of the* NOT$_1$ *scheme is bounded as follows*

$$P_{\mathrm{out,l}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) \leq P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right), \quad (24)$$

$$P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) \leq P_{\mathrm{out,u}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right), \quad (25)$$

where

$$P_{\mathrm{out,l}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - \min\{P_{\mathrm{F},1}^{\mathrm{NOT_1}}, P_{\mathrm{F},2}^{\mathrm{NOT_1}}\} \quad (26)$$

$$P_{\mathrm{out,u}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \min\{1, 2 - P_{\mathrm{F},1}^{\mathrm{NOT_1}} - P_{\mathrm{F},2}^{\mathrm{NOT_1}}\} \quad (27)$$

In these equations ($k \in \{1, 2\}$)

$$P_{\mathrm{F},k}^{\mathrm{NOT_1}} =$$

$$-\frac{\gamma \beta_k \sigma_{\mathrm{S}}^4}{\alpha(1-\gamma)} e^{-(b_k + \alpha a_k)} + \frac{\beta_k a_k \sigma_{\mathrm{S}}^4}{1-\gamma} E_1\left(\frac{a_k \alpha}{\gamma}\right) e^{\frac{a_k \alpha(1-\gamma) - \gamma b_k}{\gamma}}$$

$$+ \frac{\beta_k \gamma \sigma_{\mathrm{S}}^4 e^{-b_k}}{\alpha(1-\gamma)^2} \left(E_1\left(a_k \alpha\right) - E_1\left(\frac{a_k \alpha}{\gamma}\right) e^{\frac{a_k \alpha(1-\gamma)}{\gamma}}\right), \quad (28)$$

where $\beta_k = \frac{\alpha}{\sigma_{\mathrm{S}}^4} e^{(a_k \alpha - b_k)}$, $a_k = N_0/(\sigma_{\mathrm{S}}^2 p_{\max,k})$, $b_k = N_0\left(2^{R_k} - 1\right)/(\sigma_{\mathrm{S}}^2 p_{\max,k})$, $\alpha = \sigma_{\mathrm{S}}^2/\sigma_{\mathrm{I}}^2$, $\gamma = \alpha^2/\left(\left(2^{R_1} - 1\right)\left(2^{R_2} - 1\right)\right)$. *The function* $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt$ *denotes the* exponential integral [26].

Proof: See Appendix A. ∎

In general, computing a closed-form expression for the outage probability is involved. However, in the following two special cases, the exact value of $P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right)$ can be found. The first case is when one source has sufficiently large power constraint. Specifically, if $p_{\max,1} \to \infty$, then condition $p_1^{\mathrm{NOT_1}} < p_{\max,1}$ almost surely holds and comparing (60) and (62) in Appendix A we can see that $P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - P_{\mathrm{F},2}^{\mathrm{NOT_1}}$. Similarly, if $p_{\max,2} \to \infty$, then $P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - P_{\mathrm{F},1}^{\mathrm{NOT_1}}$. The next case is when one of the transmission rates is sufficiently small. Specifically, if $R_1 \to 0$, then according to (21) $p_1^{\mathrm{NOT_1}} \to 0$, and condition $p_1^{\mathrm{NOT_1}} < p_{\max,1}$ almost surely holds. Comparing (60) and (62) in Appendix A it can be concluded that $P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - P_{\mathrm{F},2}^{\mathrm{NOT_1}}$. Also, it can be shown that if $R_2 \to 0$, then $P_{\mathrm{out}}^{\mathrm{NOT_1}}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - P_{\mathrm{F},1}^{\mathrm{NOT_1}}$. We can also

compute the closed-form of outage probability at asymptotically high $p_{\max,1}$ and $p_{\max,2}$.

Corollary 1. *The outage probability of the* NOT$_1$ *scheme at asymptotically high* $p_{\max,1}$ *and* $p_{\max,2}$ *is*

$$P_{\text{out}}^{\text{NOT}_1}((R_1, R_2), \infty) = \begin{cases} 1 - \frac{\gamma}{\gamma-1} + \frac{\gamma \ln(\gamma)}{(\gamma-1)^2} & \gamma \neq 1 \\ 0.5 & \gamma = 1 \end{cases},$$

(29)

where $\gamma = \alpha^2 / \left(\left(2^{R_1} - 1 \right) \left(2^{R_2} - 1 \right) \right)$ *and* $\alpha = \sigma_{\text{S}}^2 / \sigma_{\text{I}}^2$.
    Proof: See Appendix B. ∎

It is worth mentioning that, although **Proposition 2** provides a lower bound on the outage probability, **Corollary 1** gives the exact value of the outage probability at high $p_{\max,1}$ and $p_{\max,2}$. This result shows that at this regime, the outage probability only depends on parameter γ, which is a function of R_1, R_2, and the channel parameter α.

3) ϵ-outage Achievable Rate Region: We can use the lower bound in **Proposition 2** to obtain an outer bound on $\mathcal{C}_\epsilon^{\text{NOT}_1}(\mathbf{p}_{\max})$ denoted as $\mathcal{C}_{\epsilon,\text{out}}^{\text{NOT}_1}(\mathbf{p}_{\max})$ by solving $P_{\text{out},l}^{\text{NOT}_1}((R_1, R_2), \mathbf{p}_{\max}) = \epsilon$ for (R_1, R_2). This equation has different solutions for (R_1, R_2): each of them denotes one point on the boundary of $\mathcal{C}_{\epsilon,\text{out}}^{\text{NOT}_1}(\mathbf{p}_{\max})$. Similarly, we can characterize an inner bound on $\mathcal{C}_\epsilon^{\text{NOT}_1}(\mathbf{p}_{\max})$ denoted as $\mathcal{C}_{\epsilon,\text{in}}^{\text{NOT}_1}(\mathbf{p}_{\max})$ by solving $P_{\text{out},u}^{\text{NOT}_1}((R_1, R_2), \mathbf{p}_{\max}) = \epsilon$ for (R_1, R_2).

Fig. 3 shows the inner bounds and the outer bounds on the ϵ-outage achievable rate regions of the NOT$_1$ scheme for different values of α. We also plot the ϵ-outage achievable rate regions of the OT scheme for comparison. In this example, we set $\epsilon = 0.001$ and $p_{\max,1}/N_0 = p_{\max,2}/N_0 = 50$ dB. This figure shows that for these parameters, if α is sufficiently large, then the ϵ-outage achievable rate region of the NOT$_1$ scheme is larger than that of the OT scheme.

Although for the single-user fading Gaussian channel with perfect CSI it has been shown that a fixed codebook that does not depend on the CSI concatenated by a separate power-control based on CSI can achieve the rates without loss of generality [27], for the interference channel, it is not known whether fixed codebook with concatenated power control can achieve any rate within the capacity region. Therefore, there might be some points which are not achievable with the considered schemes when time-sharing is not allowed. This justifies non-convex achievable rate regions in this figure.

Although the outage probability of the NOT$_1$ is small when the interference links are relatively weak compared to the direct links (i.e. large α), it is large when the interference links become relatively stronger than the direct links (i.e. small α). Therefore, for such channels, decoding the desired message by treating interference as noise may not be the best decoding strategy to apply. In the next section, we consider another scheme which is suitable for such scenarios.

B. Successive Interference Cancellation at Both Destinations

If each destination knows the codebooks of both sources, it can perform SIC by decoding the message of the interfering source, removing the interference, and then decoding the message of the desired source. We term this scheme NOT$_2$. The destinations would be able to decode the messages of the interfering source in a fading block, if the following conditions are satisfied:

$$R_1 \leq \log_2 \left(1 + \frac{|h_{21}|^2 p_1}{|h_{22}|^2 p_2 + N_0} \right)$$ (30)

$$R_2 \leq \log_2 \left(1 + \frac{|h_{12}|^2 p_2}{|h_{11}|^2 p_1 + N_0} \right).$$ (31)

These are equivalent to the following power constraint in matrix format

$$\mathbf{p} \succeq \mathbf{D}_\text{I} \mathbf{F}_\text{I} \mathbf{p} + \mathbf{n}_\text{I},$$ (32)

where

$$\mathbf{D}_\text{I} = \begin{bmatrix} 2^{R_1} - 1 & 0 \\ 0 & 2^{R_2} - 1 \end{bmatrix}, \quad \mathbf{F}_\text{I} = \begin{bmatrix} 0 & \frac{|h_{22}|^2}{|h_{21}|^2} \\ \frac{|h_{11}|^2}{|h_{12}|^2} & 0 \end{bmatrix},$$

$$\mathbf{n}_\text{I} = \left[\left(2^{R_1} - 1 \right) \frac{N_0}{|h_{21}|^2} \quad \left(2^{R_2} - 1 \right) \frac{N_0}{|h_{12}|^2} \right]^T.$$ (33)

The matrix $\mathbf{D}_\text{I}$ depends on the rates and the matrix $\mathbf{F}_\text{I}$ is a function of only the channel gains. After interference cancellation, each destination can successfully decode its desired message if the following conditions are satisfied:

$$R_1 \leq \log_2 \left(1 + \frac{|h_{11}|^2 p_1}{N_0} \right)$$ (34)

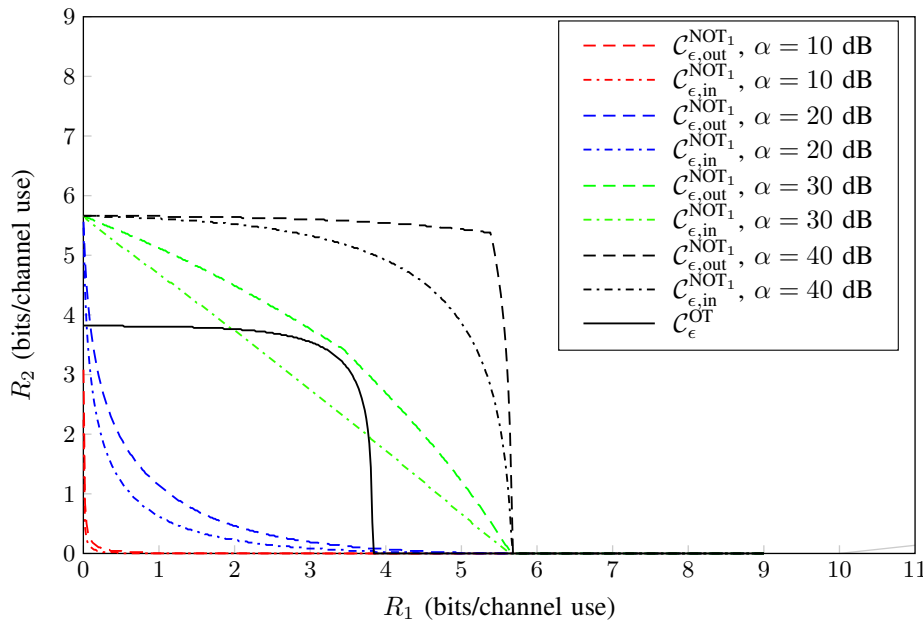$$R_2 \leq \log_2 \left(1 + \frac{|h_{22}|^2 p_2}{N_0} \right).$$ (35)

Fig. 3: The ϵ-outage achievable rate region of the OT and NOT_1 schemes.

Therefore, the transmission powers should also satisfy the following condition:

$$\mathbf{p} \succeq \mathbf{n}_\text{F}, \tag{36}$$

where

$$\mathbf{n}_\text{F} = [n_{\text{F},1}\ n_{\text{F},2}]^T = \left[\left(2^{R_1}-1\right)\frac{N_0}{|h_{11}|^2}\ \ \left(2^{R_2}-1\right)\frac{N_0}{|h_{22}|^2}\right]^T.$$

1) Power Control Solution: The positive element-wise minimum transmission power vector among those satisfying the constraints in (32) and (36) - if there is any such vector - can be found as described in the following. Define $\mathbf{p}_\text{I} \triangleq [p_{\text{I},1}\ p_{\text{I},2}]^T = (\mathbf{I} - \mathbf{D}_\text{I}\mathbf{F}_\text{I})^{-1}\mathbf{n}_\text{I}$. Then, depending on the channel gains and transmission rates, the minimum required powers can be found according to one of the four cases that will be mentioned in the following. Fig. 4 shows four plots each corresponding to one possible case. The light gray region denotes powers which satisfy the power constraint in (2); the dark gray shows the powers which satisfy (32); the gray region illustrates the powers which satisfy the constraint in (36). The intersection of these regions (the darkest region) shows powers which can provide successful transmission. There are four different cases where this happens. In each case, we find the minimum required powers as follows.

a) Case 1: If $\mathbf{n}_\text{F} \preceq \mathbf{p}_\text{I} \preceq \mathbf{p}_\text{max}$, as shown in Fig. 4 (a), the minimum power solution of the power control problem is

$$\mathbf{p}^{\text{NOT}_2} = \mathbf{p}_\text{I}. \tag{37}$$

This solution is marked by a small circle in the figure.

b) Case 2: If $p_{\text{I},1} \leq n_{\text{F},1} \leq p_{\text{max},1}$ and $n_{\text{F},2} \leq 2^{R_1}\left(2^{R_2}-1\right)\frac{N_0}{|h_{12}|^2} \leq p_{\text{max},2}$, as shown in Fig. 4 (b), the minimum required powers are

$$p_1^{\text{NOT}_2} = n_{\text{F},1} = \left(2^{R_1}-1\right)\frac{N_0}{|h_{11}|^2}, \tag{38}$$

$$p_2^{\text{NOT}_2} = 2^{R_1}\left(2^{R_2}-1\right)\frac{N_0}{|h_{12}|^2}. \tag{39}$$

The solution is shown by a small circle in the figure.

c) Case 3: If $n_{\text{F},1} \leq 2^{R_2}\left(2^{R_1}-1\right)\frac{N_0}{|h_{21}|^2} \leq p_{\text{max},1}$ and $p_{\text{I},2} \leq n_{\text{F},2} \leq p_{\text{max},2}$, as shown in Fig. 4 (c), the minimum required powers are

$$p_1^{\text{NOT}_2} = 2^{R_2}\left(2^{R_1}-1\right)\frac{N_0}{|h_{21}|^2}, \tag{40}$$

$$p_2^{\text{NOT}_2} = n_{\text{F},2} = \left(2^{R_2}-1\right)\frac{N_0}{|h_{22}|^2}. \tag{41}$$

This solution is shown in the figure by a small circle.

d) Case 4: If $p_{\text{I},1} \leq n_{\text{F},1} \leq p_{\text{max},1}$ and $\left(2^{R_1}-1\right)\frac{N_0}{|h_{12}|^2}\left(\frac{|h_{11}|^2}{|h_{22}|^2}\left(2^{R_2}-1\right)+1\right) \leq n_{\text{F},2} \leq$

(a) Case 1

(b) Case 2
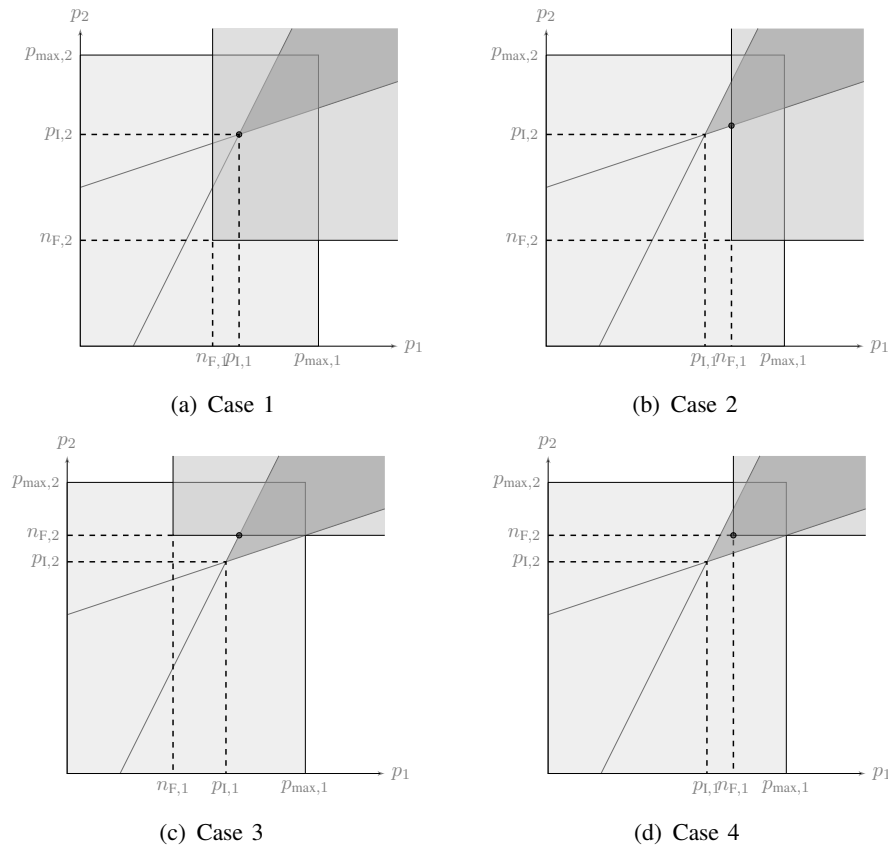

(c) Case 3

(d) Case 4

Fig. 4: Possible solutions of the power control problem for the NOT$_2$ scheme. The small circle in each plot indicates the minimum required powers in each case.

$N_0 \left(\frac{1}{|h_{11}|^2} - \frac{1}{|h_{21}|^2} \right) \frac{|h_{21}|^2}{|h_{22}|^2}$, as shown in Fig. 4 (d), the minimum required powers are

$$p_1^{\text{NOT}_2} = \left(2^{R_1} - 1 \right) \frac{N_0}{|h_{11}|^2}, \qquad (42)$$

$$p_2^{\text{NOT}_2} = \left(2^{R_2} - 1 \right) \frac{N_0}{|h_{22}|^2}. \qquad (43)$$

This solution is marked in the figure by a small circle.

2) Outage Probability Analysis: The set of the feasible solutions of the power control problem for the NOT$_2$ scheme is

$$\mathcal{P}_{\mathbf{H}}^{\text{NOT}_2} \left((R_1, R_2), \mathbf{p}_{\max} \right) =$$
$$\{ \mathbf{p} : \ \mathbf{p} \succeq \mathbf{D}_{\text{I}} \mathbf{F}_{\text{I}} \mathbf{p} + \mathbf{n}_{\text{I}}, \ \mathbf{n}_{\text{F}} \preceq \mathbf{p} \preceq \mathbf{p}_{\max} \}. \ (44)$$

This set is illustrated in Fig. 4 as the intersection of the green region, the red region, and the blue region. The red region in this figure denotes powers which satisfy the constraint in (2); the green region shows the powers which satisfy (32); the blue region illustrates the powers which satisfy the constraint in (36). Therefore, if these three regions have no common intersection region, then $\mathcal{P}_{\mathbf{H}}^{\text{NOT}_2} \left((R_1, R_2), \mathbf{p}_{\max} \right) = \emptyset$ and an outage event occurs. In the following proposition, we provide a lower bound on the outage probability of the NOT$_2$ scheme.

Proposition 3. *The outage probability of the* NOT$_2$ *scheme is bounded as follows:*

$$P_{\text{out,l}}^{\text{NOT}_2} \left((R_1, R_2), \mathbf{p}_{\max} \right) \leq P_{\text{out}}^{\text{NOT}_2} \left((R_1, R_2), \mathbf{p}_{\max} \right), (45)$$

where

$$P_{\text{out,l}}^{\text{NOT}_2}((R_1, R_2), \mathbf{p}_{\max}) = 1 - \min \left\{ P_{\text{F,D,1}}^{\text{NOT}_2}, P_{\text{F,D,2}}^{\text{NOT}_2}, P_{\text{F,I}}^{\text{NOT}_2} \right\}$$
$$(46)$$

and

$$P_{\text{F,I}}^{\text{NOT}_2} = e^{-\frac{N_0}{\sigma_S^2}\left(\frac{(2^{R_1}-1)}{p_{\max,1}} + \frac{(2^{R_2}-1)}{p_{\max,2}}\right)} \qquad (47)$$

$$P_{\text{F,D,k}}^{\text{NOT}_2} = -\frac{\gamma'\alpha\beta_k\sigma_S^4}{1-\gamma'}e^{-\frac{\alpha b_k + a_k}{\alpha}}$$
$$+ \frac{\beta_k a_k \sigma_S^4}{1-\gamma'}E_1\left(\frac{a_k}{\alpha\gamma'}\right)e^{\frac{a_k(1-\gamma')-\gamma'\alpha b_k}{\alpha\gamma'}}$$
$$+ \frac{\beta_k\alpha\gamma'\sigma_S^4 e^{-b_k}}{(1-\gamma')^2}\left(E_1\left(\frac{a_k}{\alpha}\right) - E_1\left(\frac{a_k}{\alpha\gamma'}\right)e^{\frac{a_k(1-\gamma')}{\alpha\gamma'}}\right)$$
$$k \in \{1,2\}, \qquad (48)$$

in which $\gamma' = 1/\left(\alpha^2\left(2^{R_1}-1\right)\left(2^{R_2}-1\right)\right)$, $a_k = N_0/(p_{\max,k}\sigma_S^2)$, $b_k = N_0\left(2^{R_k}-1\right)/p_{\max,k}\sigma_S^2$, $\alpha = \sigma_S^2/\sigma_I^2$, *and* $\beta_k' = \frac{1}{\alpha\sigma_S^4}e^{(a_k-\alpha b_k)/\alpha}$.

Proof: See Appendix C. ∎

Even at high $p_{\max,1}$ and $p_{\max,2}$, the outage event may occur due to the fact that no positive power vector may exist to satisfy (32). The following corollary characterizes the value of the outage probability of NOT$_2$ scheme at asymptotically high $p_{\max,1}$ and $p_{\max,2}$.

Corollary 2. *The outage probability of the* NOT$_2$ *scheme at asymptotically high $p_{\max,1}$ and $p_{\max,2}$ is*

$$P_{\text{out}}^{\text{NOT}_2}((R_1,R_2),\infty) = \begin{cases} 1 - \frac{\gamma'}{\gamma'-1} + \frac{\gamma'\ln(\gamma')}{(\gamma'-1)^2} & \gamma' \neq 1 \\ 0.5 & \gamma' = 1 \end{cases}, \qquad (49)$$

where $\gamma' = 1/\left(\alpha^2\left(2^{R_1}-1\right)\left(2^{R_2}-1\right)\right)$ *and* $\alpha = \sigma_S^2/\sigma_I^2$.

Proof: The proof is similar to that of **Corollary 1**. ∎

3) The ϵ-outage Achievable Rate Region: We can characterize an outer bound on $\mathcal{C}_\epsilon^{\text{NOT}_2}(\mathbf{p}_{\max})$ denoted as $\mathcal{C}_{\epsilon,\text{out}}^{\text{NOT}_2}(\mathbf{p}_{\max})$ using the lower bound given in **Proposition 3** by solving $P_{\text{out,l}}^{\text{NOT}_2}((R_1,R_2),\mathbf{p}_{\max}) = \epsilon$ for (R_1,R_2). Each solution of this equation for (R_1,R_2) denotes one point on the boundary of $\mathcal{C}_{\epsilon,\text{out}}^{\text{NOT}_2}(\mathbf{p}_{\max})$. The outer bound on $\mathcal{C}_\epsilon^{\text{NOT}_2}(\mathbf{p}_{\max})$, for different values of α, and the ϵ-outage achievable rate region of the OT scheme are shown in Fig. 5. In this particular example, we set $\epsilon = 0.001$ and $p_{\max,1}/N_0 = p_{\max,2}/N_0 = 50$ dB.

If only one of the destinations observes strong interference, SIC at both destinations may not be the best decoding strategy. Instead, SIC can be employed at the destination which observes strong interference and direct decoding by treating interference as noise at the other destination to achieve a better performance. We investigate this scheme in more detail in the next subsection.

C. Successive Interference Cancellation at One Destination

The destinations may implement different decoding strategies. For instance, the first destination can perform SIC, while the second destination decodes its message directly by treating the interference as noise. We refer to this scheme as NOT$_3$. The following conditions should be satisfied for a successful transmission in a fading block:

$$R_2 \leq \log\left(1 + \frac{|h_{12}|^2 p_2}{|h_{11}|^2 p_1 + N_0}\right) \qquad (50)$$

$$R_1 \leq \log\left(1 + \frac{|h_{11}|^2 p_1}{N_0}\right) \qquad (51)$$

$$R_2 \leq \log\left(1 + \frac{|h_{22}|^2 p_2}{|h_{21}|^2 p_1 + N_0}\right). \qquad (52)$$

Therefore, the powers should satisfy

$$\mathbf{p} \succeq \mathbf{n}_{\text{R}} \qquad (53)$$

where

$$\mathbf{n}_{\text{R}} = \left[\left(2^{R_1}-1\right)\frac{N_0}{|h_{11}|^2}\right.$$
$$\left.\left(2^{R_2}-1\right)\max\left\{\frac{N_0 2^{R_1}}{|h_{12}|^2}, \frac{N_0\left(|h_{11}|^2+|h_{21}|^2\left(2^{R_1}-1\right)\right)}{|h_{22}|^2|h_{11}|^2}\right\}\right]^T. \qquad (54)$$

1) Power Control Solution: The minimum powers which satisfy the constraint in (53) are

$$p_1^{\text{NOT}_3} = \left(2^{R_1}-1\right)\frac{N_0}{|h_{11}|^2} \qquad (55)$$

$$p_2^{\text{NOT}_3} = N_0\left(2^{R_2}-1\right) \times$$
$$\max\left\{\frac{2^{R_1}}{|h_{12}|^2}, \frac{|h_{11}|^2+|h_{21}|^2\left(2^{R_1}-1\right)}{|h_{22}|^2|h_{11}|^2}\right\}. \qquad (56)$$

However, for some channel realizations, the calculated powers in (55) and (56) for NOT$_3$ may violate the power constraint in (2); thus, an outage event may occur. In the next part, we investigate the outage probability in more details.
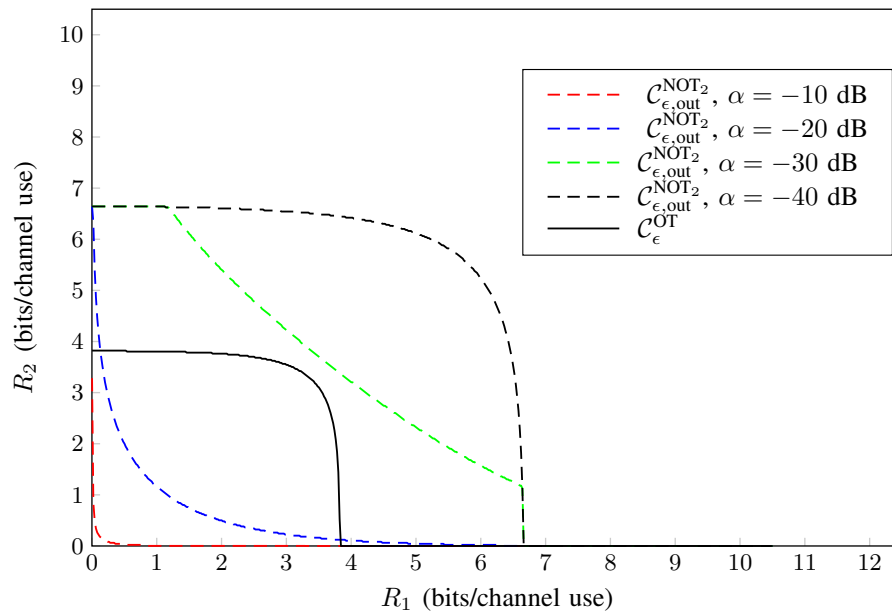
Fig. 5: Outer and inner bounds on the ϵ-outage achievable rate region of the OT and NOT$_2$ scheme.
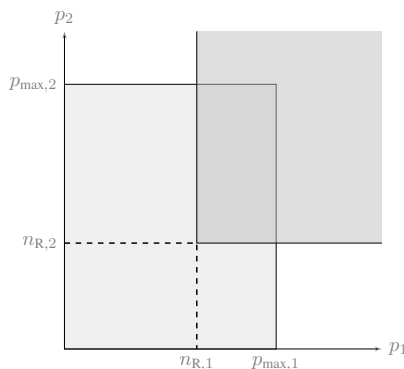

Fig. 6: The set of the feasible solutions of the power control problem for the NOT$_3$ scheme.

2) Outage Probability Analysis: The set of the feasible solutions of the power control problem for the NOT$_3$ scheme is

$$\mathcal{P}_{\mathbf{H}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \left\{\mathbf{p} : \mathbf{n}_{\text{R}} \preceq \mathbf{p} \preceq \mathbf{p}_{\max}\right\}. \quad (57)$$

This set is illustrated in Fig. 6 as the intersection region of the light gray and the dark gray regions. The light gray region shows powers which satisfy the power constraint in (2) and the dark gray region denotes the powers which satisfy the constraint in (53). If the light gray and the dark gray regions do not overlap, then an outage event occurs. The outage probability is characterized as follows.

Proposition 4. *The outage probability of the* NOT$_3$

scheme is bounded as follows

$$P_{\text{out,l}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) \leq P_{\text{out}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) \quad (58)$$
$$P_{\text{out}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) \leq P_{\text{out,u}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right), \quad (59)$$

where

$$P_{\text{out,l}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) = 1 - \min\left\{P_{\text{F,1}}^{\text{NOT}_3}, P_{\text{F,2}}^{\text{NOT}_3}\right\}$$
$$P_{\text{out,u}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \min\left\{1, 2 - P_{\text{F,1}}^{\text{NOT}_3} - P_{\text{F,2}}^{\text{NOT}_3}\right\},$$

and

$$P_{\text{F,1}}^{\text{NOT}_3} = e^{-\left(2^{R_1}-1\right)N_0/\left(p_{\max,1}\sigma_{\text{S}}^2\right)}$$
$$P_{\text{F,2}}^{\text{NOT}_3} = \left(e^{-b_2} - \frac{c_1 b_2}{\alpha}e^{\frac{c_1 b_2}{\alpha}}E_1\left(\frac{c_1 b_2}{\alpha}\right)\right)e^{-(c_1+1)b_2\alpha}.$$

In these equations $c_1 = 2^{R_1} - 1$ *and* $b_2 = N_0\left(2^{R_2} - 1\right)/p_{\max,2}\sigma_{\text{S}}^2$.

Proof: See Appendix D. ∎

3) The ϵ-outage Achievable Rate Region: The lower bound on the outage probability in **Proposition 4** can be used to obtain an outer bound on $\mathcal{C}_{\epsilon}^{\text{NOT}_3}\left(\mathbf{p}_{\max}\right)$, denoted as $\mathcal{C}_{\epsilon,\text{out}}^{\text{NOT}_3}\left(\mathbf{p}_{\max}\right)$, by solving $P_{\text{out,l}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \epsilon$ for (R_1, R_2). This equation may have many solutions for (R_1, R_2); each of them denotes one point on the boundary of the outer bound region. Similarly, an inner bound on $\mathcal{C}_{\epsilon}^{\text{NOT}_3}\left(\mathbf{p}_{\max}\right)$, denoted as $\mathcal{C}_{\epsilon,\text{in}}^{\text{NOT}_3}\left(\mathbf{p}_{\max}\right)$, can be found by solving $P_{\text{out,u}}^{\text{NOT}_3}\left((R_1, R_2), \mathbf{p}_{\max}\right) = \epsilon$ for (R_1, R_2).

Fig. 7 shows the inner bound and the outer bound on the ϵ-outage achievable rate regions of the NOT$_3$

scheme and the ϵ-outage achievable rate region of the OT scheme when $\alpha = 0$ dB. In this example, we set $\epsilon = 0.001$ and $p_{\max,1}/N_0 = p_{\max,2}/N_0 = 50$ dB. The figure shows that, the achievable rate region of none of these schemes is strictly larger that the other. In fact, each of them can achieve certain rates that may not be achievable by the other scheme. For instance, NOT$_3$ can achieve certain asymmetric rate pairs which can not be achieved by OT.

We can similarly characterize the performance limits of a transmission scheme where the first destination decodes its message by treating interference as noise and the second destination performs SIC.

Remark 1. *For each specific set of network parameters, one of the transmission schemes outperforms the others; specifically, for each value of α and $\mathrm{SNR}_{\max}$, one of the considered transmission schemes achieves the minimum outage probability for transmission at the desired rates. Since these parameters are a priori known at terminals, an adaptive transmission strategy can be deployed. This transmission strategy based on the values of α and $\mathrm{SNR}_{\max}$ selects the transmission scheme which has the minimum outage probability for transmission at the desired rates.*

V. Conclusions

In this paper, we have studied fixed-rate transmission for a two-user Rayleigh block-fading interference channel. The studied schemes use a point-to-point encoder at each source in concatenation with a power controller. The power controllers adjust the transmission power during each fading block, considering a short-term individual power constraint, to successfully transmit at desired rates. We have considered orthogonal and non-orthogonal transmission schemes. In the latter case, we have investigated different decoding strategies in which each destination either directly decodes its desired message by treating the interference from the undesired source as noise, or it performs successive interference cancellation. For each of these schemes, we have found the solution of the power control problem to assign the minimum required power to each source. Since the power control problem for some channel realizations may not have any feasible solution, for each transmission scheme we have characterized the probability of such events, based

on which the ϵ-outage achievable rate regions have been evaluated.

APPENDIX A
PROOF OF **PROPOSITION 2**

The probability $P_{\mathrm{out}}^{\mathrm{NOT}_1}\left(\left(R_1, R_2\right), \mathbf{p}_{\max}\right)$ is

$$
\begin{aligned}
&P_{\mathrm{out}}^{\mathrm{NOT}_1}\left(\left(R_1, R_2\right), \mathbf{p}_{\max}\right) \\
&= 1 - \Pr\left\{\mathcal{P}_{\mathbf{H}}^{\mathrm{NOT}_1}\left(\left(R_1, R_2\right), \mathbf{p}_{\max}\right) \neq \emptyset\right\} \\
&\overset{(a)}{=} 1 - \Pr\left\{\lambda_{\max}(\mathbf{D}_{\mathbf{S}}\mathbf{F}_{\mathbf{S}}) < 1, p_k^{\mathrm{NOT}_1} < p_{\max,k}, k \in \{1,2\}\right\},
\end{aligned}
$$
(60)

where $p_1^{\mathrm{NOT}_1}$ and $p_2^{\mathrm{NOT}_1}$ are the minimum required transmission powers for the NOT$_1$ scheme calculated in (21) and (22). The equality (a) follows the definition in (23) and the feasibility condition given in **Lemma 1**. Defining the probabilities $P_{\mathrm{F},1}^{\mathrm{NOT}_1}$ and $P_{\mathrm{F},1}^{\mathrm{NOT}_2}$ as

$$
P_{\mathrm{F},1}^{\mathrm{NOT}_1} \triangleq \Pr\left\{\lambda_{\max}(\mathbf{D}_{\mathbf{S}}\mathbf{F}_{\mathbf{S}}) < 1, p_1^{\mathrm{NOT}_1} < p_{\max,1}\right\}
$$
(61)
$$
P_{\mathrm{F},2}^{\mathrm{NOT}_1} \triangleq \Pr\left\{\lambda_{\max}(\mathbf{D}_{\mathbf{S}}\mathbf{F}_{\mathbf{S}}) < 1, p_2^{\mathrm{NOT}_1} < p_{\max,2}\right\}
$$
(62)

we can bound the feasibility probability as follows

$$
\begin{aligned}
&P_{\mathrm{F},1}^{\mathrm{NOT}_1} + P_{\mathrm{F},2}^{\mathrm{NOT}_1} - 1 \\
&\overset{(a)}{\leq} \Pr\left\{\lambda_{\max}(\mathbf{D}_{\mathbf{S}}\mathbf{F}_{\mathbf{S}}) < 1, p_1^{\mathrm{NOT}_1} < p_{\max,1}, p_2^{\mathrm{NOT}_1} < p_{\max,2}\right\} \\
&\overset{(b)}{\leq} \min\left\{P_{\mathrm{F},1}^{\mathrm{NOT}_1}, P_{\mathrm{F},2}^{\mathrm{NOT}_1}\right\},
\end{aligned}
$$

where the equality (a) follows the fact that $\Pr\{\mathcal{A} \bigcap \mathcal{B}\} = 1 - \Pr\left\{\overline{\mathcal{A} \bigcap \mathcal{B}}\right\} = 1 - \Pr\left\{\overline{\mathcal{A}} \bigcup \overline{\mathcal{B}}\right\} \geq 1 - \Pr\left\{\overline{\mathcal{A}}\right\} - \Pr\left\{\overline{\mathcal{B}}\right\} = \Pr\{\mathcal{A}\} + \Pr\{\mathcal{B}\} - 1$, where $\mathcal{A}$ and $\mathcal{B}$ are random events; and the equality (b) follows $\Pr\{\mathcal{A} \bigcap \mathcal{B}\} \leq \Pr\{\mathcal{A}\}$. The probability $P_{\mathrm{F},1}^{\mathrm{NOT}_1}$ can be further simplified as follows

$$
\begin{aligned}
P_{\mathrm{F},1}^{\mathrm{NOT}_1} = \Pr\Bigg\{ &\frac{|h_{12}|^2 |h_{21}|^2}{|h_{11}|^2 |h_{22}|^2} < \frac{\gamma}{\alpha^2}, \\
&\frac{N_0\left(\left(2^{R_1} - 1\right)\frac{|h_{21}|^2}{|h_{11}|^2} + l\right)}{|h_{21}|^2(1-l)} < p_{\max,1}\Bigg\},
\end{aligned}
$$
(63)

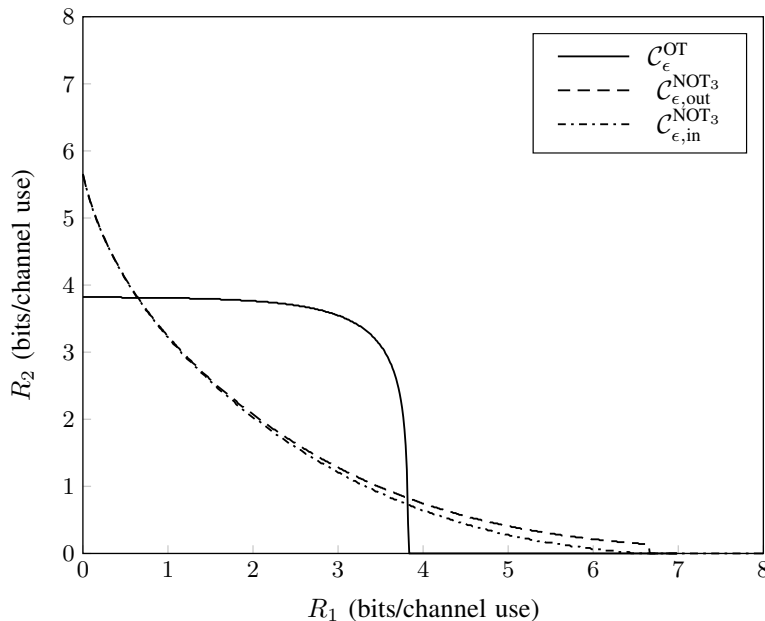Fig. 7: The ϵ-outage achievable rate region of the NOT$_3$ scheme.

where $\gamma = \frac{\alpha^2}{(2^{R_1}-1)(2^{R_2}-1)}$, and $l = \frac{|h_{12}|^2|h_{21}|^2}{|h_{11}|^2|h_{22}|^2} \times \frac{\alpha^2}{\gamma}$.
It can be shown that

$$
\begin{aligned}
P_{\text{F},1}^{\text{NOT}_1} &= \Pr\left\{\frac{|h_{22}|^2}{|h_{21}|^2}\left(|h_{11}|^2 - b_1\sigma_{\mathsf{S}}^2\right) > \left(|h_{12}|^2 + a_1\sigma_{\mathsf{S}}^2\right)\frac{\alpha^2}{\gamma}\right\} \\
&= \Pr\left\{b_1\sigma_{\mathsf{S}}^2 < |h_{11}|^2\right\} \times \\
&\quad \Pr\left\{0 < \frac{|h_{21}|^2\left(|h_{12}|^2 + a_1\sigma_{\mathsf{S}}^2\right)}{|h_{22}|^2\left(|h_{11}|^2 - b_1\sigma_{\mathsf{S}}^2\right)} < \frac{\gamma}{\alpha^2}\right\} \\
&= e^{-b_1} \times Q
\end{aligned}
\tag{64}
$$

where $Q = \Pr\left\{0 < \frac{|h_{21}|^2\left(|h_{12}|^2+a_1\sigma_{\mathsf{S}}^2\right)}{|h_{22}|^2\left(|h_{11}|^2-b_1\sigma_{\mathsf{S}}^2\right)} < \frac{\gamma}{\alpha^2}\right\}$,
$a_1 = \frac{N_0}{\sigma_{\mathsf{S}}^2 p_{\max,1}}$, and $b_1 = \frac{N_0}{\sigma_{\mathsf{S}}^2 p_{\max,1}}\left(2^{R_1}-1\right)$. To calculate Q, we derive the probability density function
(pdf) of the random variable $M \triangleq \frac{|h_{21}|^2\left(|h_{12}|^2+a_1\sigma_{\mathsf{S}}^2\right)}{|h_{22}|^2\left(|h_{11}|^2-b_1\sigma_{\mathsf{S}}^2\right)}$.
Let $X_{kl} \triangleq |h_{kl}|^2$ and $Y_{kk} \triangleq |h_{kk}|^2$ ($\forall k,l \in \{1,2\}$, $k \neq l$), we have:

$$
f_{X_{kl}}(x) = \frac{1}{\sigma_{\mathsf{I}}^2}e^{-x/\sigma_{\mathsf{I}}^2} \quad x \geq 0 \tag{65}
$$

$$
f_{Y_{kk}}(y) = \frac{1}{\sigma_{\mathsf{S}}^2}e^{-y/\sigma_{\mathsf{S}}^2} \quad y \geq 0. \tag{66}
$$

Let $Z_{12} \triangleq \frac{|h_{12}|^2+a_1\sigma_{\mathsf{S}}^2}{|h_{11}|^2-b_1\sigma_{\mathsf{S}}^2} = \frac{X_{12}+a_1\sigma_{\mathsf{S}}^2}{Y_{11}-b_1\sigma_{\mathsf{S}}^2}$, and
$Z_{21} \triangleq \frac{|h_{21}|^2}{|h_{22}|^2} = \frac{X_{21}}{Y_{22}}$, using the fact that the pdf of
$Z = \frac{X}{Y}$ is $f_Z(z) = \int_{-\infty}^{+\infty}|y|f_{XY}(zy,y)dy$, we can
show that the pdf of Z_{21} and Z_{12} are

$$
f_{Z_{21}}(z) = \frac{\alpha}{(1+\alpha z)^2}, \quad z \in \mathbb{R}^+ \tag{67}
$$

$$
f_{Z_{12}}(z) = \left(\frac{\beta_1 a_1\sigma_{\mathsf{S}}^4}{(1+\alpha z)z} + \frac{\beta_1\sigma_{\mathsf{S}}^4}{(1+\alpha z)^2}\right)e^{-a_1\left(\frac{1+\alpha z}{z}\right)} \tag{68}
$$

where $\alpha = \frac{\sigma_{\mathsf{S}}^2}{\sigma_{\mathsf{I}}^2}$ and $\beta_1 = \frac{\alpha}{\sigma_{\mathsf{S}}^4}e^{(a_1\alpha - b_1)}$. The pdf
of $M = \frac{|h_{21}|^2\left(|h_{12}|^2+a_1\sigma_{\mathsf{S}}^2\right)}{|h_{22}|^2\left(|h_{11}|^2-b_1\sigma_{\mathsf{S}}^2\right)} = Z_{12}Z_{21}$ can be derived as $f_M(m) = \int_{-\infty}^{+\infty}\frac{1}{|t|}f_{Z_{12}Z_{21}}(t,\frac{m}{t})dt$. Since
the cumulative distribution function (cdf) of M is
$F_M(m) = \int_{x=-\infty}^{m}f_M(x)dx$, by using the fact that
the random variables Z_{21} and Z_{12} are independent
we have

$$
\begin{aligned}
F_M(m) = &-\frac{\beta_1\sigma_{\mathsf{S}}^4}{1-\alpha^2 m}\left(\frac{1}{\alpha}e^{-\alpha a_1} - a_1 E_1\left(a_1\alpha\right)\right) \\
&- \left(\left(\frac{\beta_1 a_1\sigma_{\mathsf{S}}^4}{1-\alpha^2 m} - \frac{\beta_1\alpha m\sigma_{\mathsf{S}}^4}{(1-\alpha^2 m)^2}\right) \times \right. \\
&\left. \left(E_1\left(a_1\alpha\right) - E_1\left(\frac{a_1}{\alpha m}\right)e^{\frac{a_1(1-\alpha^2 m)}{\alpha m}}\right)\right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
Q &= F_M(\gamma/\alpha^2) - F_M(0) \\
&= -\frac{\gamma\beta_1\sigma_{\mathsf{S}}^4}{\alpha(1-\gamma)}e^{-\alpha a_1} + \frac{\beta_1 a_1\sigma_{\mathsf{S}}^4}{1-\gamma}E_1\left(\frac{a_1\alpha}{\gamma}\right)e^{\frac{a_1\alpha(1-\gamma)}{\gamma}} \\
&\quad + \frac{\beta_1\gamma\sigma_{\mathsf{S}}^4}{\alpha(1-\gamma)^2}\left(E_1(a_1\alpha) - E_1\left(\frac{a_1\alpha}{\gamma}\right)e^{\frac{a_1\alpha(1-\gamma)}{\gamma}}\right)
\end{aligned}
\tag{69}
$$

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By substituting (69) in (64), we can find $P_{\mathrm{F,1}}^{\mathrm{NOT_1}}$. It is possible to find $P_{\mathrm{F,2}}^{\mathrm{NOT_1}}$ with similar approach. By plugging these in (63), the bounds on $P_{\mathrm{out}}^{\mathrm{NOT_1}}((R_1,R_2),\mathbf{p}_{\max})$ can be obtained.

APPENDIX B
PROOF OF **COROLLARY 1**

We have

$$
\begin{aligned}
\lim_{p_{\max,1},p_{\max,2}\to\infty} P_{\mathrm{out}}^{\mathrm{NOT_1}} &\overset{(a)}{=} 1 - \lim_{p_{\max,2}\to\infty} P_{\mathrm{F,2}}^{\mathrm{NOT_1}} \\
&\overset{(b)}{=} 1 - \lim_{p_{\max,1}\to\infty} P_{\mathrm{F,1}}^{\mathrm{NOT_1}} \\
&= 1 - \lim_{a_1,b_1\to 0} P_{\mathrm{F,1}}^{\mathrm{NOT_1}} \\
&= 1 - A - B - C, \quad (70)
\end{aligned}
$$

where (a) and (b) follow from the equality in (60), and the definitions of $P_{\mathrm{F,1}}^{\mathrm{NOT_1}}$ and $P_{\mathrm{F,2}}^{\mathrm{NOT_1}}$ in (61) and (62), respectively. The values A, B and C can be calculated as follows

$$
\begin{aligned}
A &= \lim_{a_k,b_k\to 0} -\frac{\gamma\beta_k\sigma_{\mathrm{S}}^4}{\alpha(1-\gamma)} e^{-(b_k+\alpha a_k)} = -\frac{\gamma}{1-\gamma} \\
B &= \lim_{a_k,b_k\to 0} \frac{\beta_k a_k\sigma_{\mathrm{S}}^4}{1-\gamma} E_1\left(\frac{a_k\alpha}{\gamma}\right) e^{\frac{a_k\alpha(1-\gamma)-\gamma b_k}{\gamma}} = 0 \\
C &= \lim_{a_1,b_1\to 0} \frac{\beta_k\gamma\sigma_{\mathrm{S}}^4 e^{-b_k}}{\alpha(1-\gamma)^2}\Bigg(E_1\left(a_1\alpha\right) - \\
&\qquad\qquad E_1\left(\frac{a_1\alpha}{\gamma}\right) e^{\frac{a_1\alpha(1-\gamma)}{\gamma}} \Bigg) \\
&= -\frac{\gamma}{(1-\gamma)^2}\ln\gamma. \quad (71)
\end{aligned}
$$

APPENDIX C
PROOF OF PROPOSITION 3

We have

$$
\begin{aligned}
&P_{\mathrm{out}}^{\mathrm{NOT_2}}((R_1,R_2),\mathbf{p}_{\max}) \\
&\geq 1 - \Pr\left\{\lambda_{\max}(\mathbf{D_I F_I}) < 1, \mathbf{p_I}\preceq\mathbf{p}_{\max}, \mathbf{n_F}\preceq\mathbf{p}_{\max}\right\}.
\end{aligned}
\quad (72)
$$

Defining

$$
P_{\mathrm{F,I}} \triangleq \Pr\{\mathbf{n_F}\preceq\mathbf{p}_{\max}\} \quad (73)
$$
$$
P_{\mathrm{F,D,k}} \triangleq \Pr\{\lambda_{\max}(\mathbf{D_I F_I}) < 1, p_{\mathrm{I},k}\leq p_{\max,k}\}, k\in\{1,2\}
\quad (74)
$$

we can show that

$$
\begin{aligned}
&P_{\mathrm{F,I}} + P_{\mathrm{F,D,1}} + P_{\mathrm{F,D,2}} - 2 \\
&\overset{(a)}{\leq} \Pr\{\lambda_{\max}(\mathbf{D_I F_I}) < 1, \mathbf{p_I}\preceq\mathbf{p}_{\max}, \mathbf{n_F}\preceq\mathbf{p}_{\max}\} (75)
\end{aligned}
$$

The inequality (a) follows the fact that $\Pr\{\mathcal{A}\bigcap\mathcal{B}\bigcap\mathcal{C}\} = 1 - \Pr\{\overline{\mathcal{A}\bigcap\mathcal{B}\bigcap\mathcal{C}}\} = 1 - \Pr\{\overline{\mathcal{A}}\bigcup\overline{\mathcal{B}}\bigcup\overline{\mathcal{C}}\} \geq 1 - \Pr\{\overline{\mathcal{A}}\} - \Pr\{\overline{\mathcal{B}}\} - \Pr\{\overline{\mathcal{C}}\} = \Pr\{\mathcal{A}\} + \Pr\{\mathcal{B}\} + \Pr\{\mathcal{C}\} - 2$, where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are random events. We have

$$
P_{\mathrm{F,I}} = e^{-\frac{N_0}{\sigma_{\mathrm{S}}^2}\left(\frac{(2^{R_1}-1)}{p_{\max,1}}+\frac{(2^{R_2}-1)}{p_{\max,2}}\right)} \quad (76)
$$

and

$$
\begin{aligned}
P_{\mathrm{F,D,1}} &= \Pr\left\{\lambda_{\max}(\mathbf{D_I F_I}) < 1, p_{\mathrm{I},1}\leq p_{\max,1}\right\} \\
&= \Pr\Bigg\{(2^{R_1}-1)(2^{R_2}-1)\frac{|h_{11}|^2|h_{22}|^2}{|h_{12}|^2|h_{21}|^2} < 1, \\
&\qquad \frac{\frac{N_0}{|h_{21}|^2}(2^{R_1}-1)\left(1+(2^{R_2}-1)\frac{|h_{22}|^2}{|h_{12}|^2}\right)}{1-(2^{R_1}-1)(2^{R_2}-1)\frac{|h_{11}|^2|h_{22}|^2}{|h_{12}|^2|h_{21}|^2}} < p_{\max,1}\Bigg\} \\
&= \Pr\Big\{(2^{R_1}-1)(2^{R_2}-1)(a_1\sigma_{\mathrm{S}}^2+|h_{11}|^2)|h_{22}|^2 < \\
&\qquad (|h_{21}|^2 - b_1\sigma_{\mathrm{S}}^2)|h_{12}|^2\Big\} \\
&= \Pr\left\{b_1 < |h_{21}|^2\right\}\times \\
&\qquad \Pr\left\{0 < \frac{|h_{22}|^2(|h_{11}|^2+a_1\sigma_{\mathrm{S}}^2)}{|h_{12}|^2(|h_{21}|^2-b_1\sigma_{\mathrm{S}}^2)} < \alpha^2\gamma'\right\} (77)
\end{aligned}
$$

where $a_1 = N_0/(\sigma_{\mathrm{S}}^2 p_{\max,1})$, and $b_1 = N_0(2^{R_1}-1)/\sigma_{\mathrm{S}}^2 p_{\max,1}$. Comparing (77) and (64), we can obtain (48) by substituting σ_{S}^2, $1/\alpha$, and σ_{I}^2, instead of σ_{I}^2, α, and σ_{S}^2 in (28), respectively.

APPENDIX D
PROOF OF **PROPOSITION 4**

The probability $P_{\mathrm{out}}^{\mathrm{NOT_3}}((R_1,R_2),\mathbf{p}_{\max})$ can be simplified as

$$
\begin{aligned}
&P_{\mathrm{out}}^{\mathrm{NOT_3}}((R_1,R_2),\mathbf{p}_{\max}) \\
&= 1 - \Pr\left\{\mathcal{P}_{\mathbf{H}}^{\mathrm{NOT_3}}((R_1,R_2),\mathbf{p}_{\max})\neq\emptyset\right\} \\
&= 1 - \Pr\left\{p_1^{\mathrm{NOT_3}} < p_{\max,1}, p_2^{\mathrm{NOT_3}} < p_{\max,2}\right\} (78)
\end{aligned}
$$

Defining the probabilities $P_{\mathrm{F,1}}^{\mathrm{NOT_3}}$ and $P_{\mathrm{F,2}}^{\mathrm{NOT_3}}$ as follows

$$
\begin{aligned}
P_{\mathrm{F,1}}^{\mathrm{NOT_3}} &\triangleq \Pr\left\{p_1^{\mathrm{NOT_3}} < p_{\max,1}\right\} \quad (79) \\
P_{\mathrm{F,2}}^{\mathrm{NOT_3}} &\triangleq \Pr\left\{p_2^{\mathrm{NOT_3}} < p_{\max,2}\right\}, \quad (80)
\end{aligned}
$$

we have

$$
\begin{aligned}
2 - P_{\mathrm{F,1}}^{\mathrm{NOT_3}} - P_{\mathrm{F,2}}^{\mathrm{NOT_3}} &\overset{(a)}{\leq} \Pr\left\{p_1^{\mathrm{NOT_3}} < p_{\max,1}, p_2^{\mathrm{NOT_3}} < p_{\max,2}\right\} \\
&\overset{(b)}{\leq} \min\left\{P_{\mathrm{F,1}}^{\mathrm{NOT_3}}, P_{\mathrm{F,2}}^{\mathrm{NOT_3}}\right\}, \quad (81)
\end{aligned}
$$

where the equality (a) follows the fact that $\Pr\{\mathcal{A}\bigcap\mathcal{B}\} = 1-\Pr\{\overline{\mathcal{A}\bigcap\mathcal{B}}\} = 1-\Pr\{\overline{\mathcal{A}}\bigcup\overline{\mathcal{B}}\} \geq 1-\Pr\{\overline{\mathcal{A}}\} - \Pr\{\overline{\mathcal{B}}\} = \Pr\{\mathcal{A}\} + \Pr\{\mathcal{B}\} - 1$, where $\mathcal{A}$ and $\mathcal{B}$ are random events; and the equality (b) follows $\Pr\{\mathcal{A}\bigcap\mathcal{B}\} \leq \Pr\{\mathcal{A}\}$. We have

$$
\begin{aligned}
P_{\mathrm{F,1}}^{\mathrm{NOT_3}} &= \Pr\left\{p_1^{\mathrm{NOT_3}} < p_{\max,1}\right\} \\
&= \Pr\left\{|h_{11}|^2 > \frac{N_0}{p_{\max,1}}\left(2^{R_1}-1\right)\right\} \\
&= e^{-\frac{N_0}{p_{\max,1}\sigma_{\mathrm{S}}^2}\left(2^{R_1}-1\right)},
\end{aligned} \tag{82}
$$

and

$$
P_{\mathrm{F,2}}^{\mathrm{NOT_3}} = \Pr\left\{0 < M < 1/\left(b\sigma_{\mathrm{S}}^2\right)\right\} \tag{83}
$$

where $M \triangleq \max\left\{\frac{2^{R_1}}{|h_{12}|^2}, \frac{1}{|h_{22}|^2}\left(\frac{|h_{21}|^2}{|h_{11}|^2}\left(2^{R_1}-1\right)+1\right)\right\}$, and $b = \left(2^{R_2}-1\right)\frac{N_0}{p_{\max,2}\sigma_{\mathrm{S}}^2}$. Therefore, we require to find the cdf of M. We define $X \triangleq \left(\frac{|h_{21}|^2}{|h_{11}|^2}c+1\right)$, and $Y \triangleq \frac{1}{|h_{22}|^2}$, where $c = \left(2^{R_1}-1\right)$. It can be shown that

$$
f_X(x) = \frac{\alpha c}{(c-a\alpha+\alpha x)^2} \quad x \geq 1 \tag{84}
$$

$$
f_Y(y) = \frac{1}{y^2\sigma_{\mathrm{S}}^2}e^{-\frac{1}{y\sigma_{\mathrm{S}}^2}} \quad y \geq 0. \tag{85}
$$

We define $Z \triangleq XY$, and we have

$$
\begin{aligned}
f_Z(z) &= \int_{-\infty}^{\infty} \frac{1}{|x|} f_X(x) f_Y\left(\frac{z}{x}\right) dx \\
&= \int_1^{\infty} \frac{\alpha c x}{\sigma_{\mathrm{S}^2}(c-\alpha+\alpha x)^2 z^2} e^{-\frac{x}{\sigma_{\mathrm{S}}^2 z}} dx.
\end{aligned} \tag{86}
$$

The cdf of Z is

$$
F_Z(z) = e^{-\frac{1}{\sigma_{\mathrm{S}}^2 z}} - \frac{c}{\sigma_{\mathrm{S}}^2 z\alpha} e^{\frac{c}{\alpha\sigma_{\mathrm{S}}^2 z}} E_1\left(\frac{c}{\alpha\sigma_{\mathrm{S}}^2 z}\right). \tag{87}
$$

Now, defining $W \triangleq \frac{c+1}{|h_{12}|^2}$, we have

$$
F_W(w) = e^{-\frac{c+1}{\sigma_{\mathrm{I}}^2 w}} \quad w > 0. \tag{88}
$$

Therefore, for the random variable $M = \max\{Z, W\}$, since Z and W are independent, we have

$$
F_M(m) = F_Z(m) F_W(m). \tag{89}
$$

Then,

$$
\begin{aligned}
P_{\mathrm{F,2}}^{\mathrm{NOT_3}} &= \Pr\left\{0 < M < 1/\left(b\sigma_{\mathrm{S}}^2\right)\right\} \\
&= F_M\left(1/\left(b\sigma_{\mathrm{S}}^2\right)\right) - F_M(0) \\
&= \left(e^{-b} - \frac{bc}{\alpha}e^{\frac{bc}{\alpha}}E_1\left(\frac{bc}{\alpha}\right)\right)e^{-(c+1)b\alpha}.
\end{aligned} \tag{90}
$$

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