

Cherenkov radiation emitted by solitons in optical fibers

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We demonstrate a simple, fully analytic method of calculating the amount of radiation emitted by optical solitons perturbed by higher-order dispersion effects in fibers and find good agreement with numerical results. It is pointed out that this radiation mechanism is analogous to the well-known Cherenkov radiation processes in nonlinear optics.

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I. INTRODUCTION

Optical solitons in fibers have long been an experimental reality, and they are expected to play an important role as information carriers in future high-speed communication systems. Since the mechanism for forming optical solitons is well known both theoretically and experimentally, current research is devoted to examining how solitons behave when perturbed. The present work is devoted to optical solitons perturbed by higher- (i.e., third- or fourth-) order dispersion.

The governing equation for optical solitons in fibers is the nonlinear Schrödinger (NLS) equation [1]

$$i \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial t^2} + |u|^2 u = \varepsilon \hat{P}(u), \quad (1)$$

where we have included a perturbative operator \hat{P} in the right-hand side. The parameter ε is assumed to be sufficiently small for solitons to exist (see below). In this paper we will consider $\hat{P} = i\partial^3/\partial t^3$ and $\hat{P} = -\partial^4/\partial t^4$, corresponding to third-order dispersion (3OD) and fourth-order dispersion (4OD), respectively. The unperturbed ($\varepsilon=0$) soliton solution of Eq. (1) is

$$u_{\text{sol}}^0(x, t) = A \operatorname{sech}(At) \exp[ik_{\text{sol}}z], \quad (2)$$

where $k_{\text{sol}} = A^2/2$ is the soliton wave number. In the particular case of 3OD, it was shown in Refs. [2–4] that the corrected form of the soliton, to first order in ε , is

$$u_{\text{sol}}^{3OD}(x, t) = A \operatorname{sech}(Ay) \times \exp\{ik_{\text{sol}}z + i\varepsilon[2A^2t - 3\tanh(Ay)]\}, \quad (3)$$

where $y = t - \varepsilon A^2 z$. Thus, the first-order corrections to the soliton (2) only affect the phase and the velocity of the soliton, leaving the amplitude, width, and shape unperturbed. Furthermore, it has been shown numerically [5] that the condition for solitons to exist in the presence of 3OD is $A\varepsilon < 0.04$. The physical interpretation of this is

that a sufficiently large part of the soliton spectrum must lie in the anomalous group velocity dispersion (GVD) regime.

In general, nonlinear Schrödinger solitons (2) are very robust [1], but under certain perturbations they are unstable, e.g., periodic amplification [6] or higher-order dispersion [5]. The instability due to third-order dispersion was first predicted by Wai *et al.* [5], and it manifests itself as radiation at the specific frequency $\omega_0 \approx 1/2\varepsilon$. This lies in the normal GVD regime, and is separated from (but still overlaps) the soliton spectrum. We will show below that it is this overlap that is the main cause of the radiation. In the case of fourth-order dispersion, it was recently shown [7] that the soliton radiates at the two frequencies $\pm 1/\sqrt{2\varepsilon}$, and the relation to the instability discovered by Wai *et al.* was emphasized.

Analytically, the problem of soliton radiation due to higher-order dispersion has been considered previously, both for NLS solitons [3,8–12] and for solitons governed by the Korteweg–de Vries (KdV) equation [12–14]. The primary interest is in two properties of the radiation, viz., its frequency and its intensity. In the case of NLS solitons, the frequency was estimated to first order in ε in the early numerical work [5], but more accurate expressions have been obtained using perturbative inverse scattering methods [8–10]. We will point out below that the same result follows from simple physical arguments. The intensity of the radiation is more difficult to calculate; it has been done in Refs. [3,8,11] by combined numerical and analytical methods. However, these approaches are rather involved, and the physics might in some cases have been obscured by cumbersome mathematics. In the case of the KdV solitons, similar methods have been applied [13,14]. However, recently another somewhat simpler method was proposed by Karpman for both the NLS and the KdV equations [12]. Our aim here is similar—we present a simple, analytical, and physically intuitive derivation of the unstable frequency, and of the intensity of the radiation. Our final expressions are found to be in good agreement with the previous seminumerical results of Refs. [3,8,11]. Moreover, since this radiation is emitted from a wave packet (the soliton) with a phase velocity exceeding the linear phase velocity of the medium, we point out the formal equivalence of this radiation and the well-known Cherenkov radiation processes in nonlinear

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optics [15–18]. The relation to Cherenkov radiation was also briefly mentioned by Karpman [12], and in another context by Cao and Meyerhofer [19]. However, in this work we discuss the underlying physics in some more detail. In particular, we point out that the Cherenkov concept can be generalized to include all processes in which solitonlike pulses (or beams) transfer power to linear waves, e.g., periodic amplification of solitons, transition to stationary states in nonlinear fiber couplers [20], or radiation by solitons in birefringent fibers [19,21].

II. RADIATION FREQUENCY

The main reason for the robustness of solitons is that the wave numbers of the solitons lie in a range that is forbidden for linear dispersive waves. Therefore, linear waves cannot be in resonance with the soliton, and energy cannot be transferred from the soliton to the linear waves. On the other hand, this fact makes solitons sensitive to perturbations having the soliton wavenumber, since the soliton then is in resonance with the perturbation, and can transfer power to it. More specifically, the soliton of Eq. (2) has a wave number $k_{\text{sol}} = A^2/2$, and the linear dispersion relation corresponding to Eq. (1) is obtained by substituting $\exp[i(k_{\text{lin}}z + \omega t)]$ into Eq. (1) and neglecting the nonlinear term

$$k_{\text{lin}}(\omega) = -\frac{1}{2}\omega^2 - \varepsilon\omega^3, \quad (4a)$$

$$k_{\text{lin}}(\omega) = -\frac{1}{2}\omega^2 + \varepsilon\omega^4. \quad (4b)$$

The indexing (a) and (b) of this and all subsequent equations refers to the cases of 3OD and 4OD, respectively. We see that in the absence of the perturbation, $\varepsilon = 0$, $k_{\text{lin}} < 0$, and since $k_{\text{sol}} > 0$, the soliton is stable [22]. In the presence of the perturbation, however, the soliton is in resonance with the dispersive waves at $k_{\text{sol}} = k_{\text{lin}}$, i.e. at the frequency ω_0 , defined by

$$\frac{A^2}{2} = -\frac{1}{2}\omega_0^2 - \varepsilon\omega_0^3, \quad (5a)$$

$$\frac{A^2}{2} = -\frac{1}{2}\omega_0^2 + \varepsilon\omega_0^4, \quad (5b)$$

which is the phase-matching condition for the instability. To lowest orders in ε , this is

$$\omega_0 = -\frac{1}{\varepsilon} \left[\frac{1}{2} + 2(\varepsilon A)^2 + O(\varepsilon A)^4 \right], \quad (6a)$$

$$\omega_0 = \frac{1}{\sqrt{2\varepsilon}} \left[1 + (A\sqrt{\varepsilon})^2 + O((A\sqrt{\varepsilon})^4) \right], \quad (6b)$$

as found numerically [5,8]. In the case of 3OD, there is only one unstable frequency, but for 4OD the problem is spectrally symmetric and the unstable frequencies are $+\omega_0$ and $-\omega_0$. The group velocities at the unstable frequencies are easily calculated to be

$$V_g^{-1} = \left. \frac{\partial k_{\text{lin}}}{\partial \omega} \right|_{\omega=\omega_0} \approx -\frac{1}{4\varepsilon}, \quad (7a)$$

$$V_g^{-1} = \left. \frac{\partial k_{\text{lin}}}{\partial \omega} \right|_{\omega=\pm\omega_0} \approx \pm \frac{1}{\sqrt{2\varepsilon}}. \quad (7b)$$

Since we are in the frame of reference in which the soliton is stationary, these are the velocities at which the linear radiation leaves the soliton. It is evident that 3OD causes radiation behind (in front of) the soliton when $\varepsilon > 0$ ($\varepsilon < 0$). The radiation from 4OD will be in both temporal directions, due to the symmetry of Eq. (1). We also see that the resonant frequency ω_0 depends on the soliton amplitude A , as pointed out by Elgin [9]. However, this dependence is rather weak due to the condition that $A\varepsilon$ needs to be sufficiently small for soliton creation (see Sec. I). Nevertheless, the dependence must be taken into account when integrating the total radiated energy (see Sec. IV).

III. RADIATION INTENSITY

In order to calculate the exact intensity of the dispersive radiation, one may refer to complicated analytical-numerical analyses, similar to those of Refs. [3,8,11]. However, an accurate estimate can easily be found by using the following method. Guided by the numerical results [3,5,7], we know that the solution of Eq. (1) is given by $u(z,t) = u_{\text{sol}}(z,t) + f(z,t)$, where the radiation part is much less than the soliton part, i.e., $|f(z,t)| \ll |u_{\text{sol}}(z,t)|$, over the body of the soliton. Moreover, if $\varepsilon \ll 1$, the radiation amplitude appears to be proportional to $\exp[-1/\varepsilon]$, i.e., it is “beyond all orders” [8] of corrections for the shape and the phase of the soliton itself. This means that we can calculate the shape and the phase of the soliton up to all orders in ε and after that we can calculate the amplitude of radiation using the expression for the corrected soliton. Thus, for 3OD we take $u_{\text{sol}}(z,t)$ from Eq. (3) and for 4OD we take $u_{\text{sol}}(z,t)$ from Eq. (2). To simplify the calculations, we restrict ourselves to the first-order corrections for the soliton given by Eq. (3). Higher-order terms in ε would give additional corrections, which we ignore here. Now, linearizing Eq. (1) in f ,

$$i \frac{\partial f}{\partial z} + \frac{1}{2} \frac{\partial^2 f}{\partial t^2} - \varepsilon \hat{P}(f) + 2|u_{\text{sol}}|^2 f + (u_{\text{sol}})^2 f^* = \varepsilon \hat{P}(u_{\text{sol}}), \quad (8)$$

where according to the discussion above, u_{sol} can be given by either Eq. (2) or Eq. (3), depending on the accuracy we want. It may be noted that an exact way of solving this equation has been devised by Elgin [9], but this does not give any simple explicit expressions for the radiation f , which we are interested in.

Away from the soliton, $k_{\text{sol}} \rightarrow 0$ and Eq. (8) becomes homogeneous and thus has solutions in terms of linear dispersive waves. We can interpret the term on the right-hand side as a source term for these waves. The terms $2|u_{\text{sol}}|^2 f$ and $u_{\text{sol}}^2 f^*$ are responsible for frequency variations along t inside the range where $u_{\text{sol}} = 0$, i.e., inside the soliton. The radiated energy is mostly governed by the source term, and the two above-mentioned terms give corrections to it. Hence, in principle, in order to estimate the radiated energy, we can omit these two terms in favor of the source term. Our numerical simulations show that the corrections related to these two terms are

less than 10% of the full amplitude of radiation. This is certainly acceptable for the rough estimates we are doing here. Obviously, the linear waves in our approximation have to have the same dependence on z as the source term, viz., $f \propto \exp[ik_{\text{sol}}z]$, as was explained in the preceding section. After canceling the common factors $\exp[ik_{\text{sol}}z]$, we rewrite Eq. (8) in terms of its Fourier components

$$[-k_{\text{sol}} + k_{\text{lin}}(\omega)]F(\omega) + \frac{1}{\pi} \int_{-\infty}^{+\infty} U(\omega - \omega')F(\omega')d\omega' + \frac{1}{2\pi} \int_{-\infty}^{+\infty} U(\omega - \omega')\bar{F}(\omega')d\omega' = \varepsilon P(\omega), \quad (9)$$

where

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{+\infty} f(t)\exp[-i\omega t]dt, \\ \bar{F}(\omega) &= \int_{-\infty}^{+\infty} f(t)^*\exp[-i\omega t]dt, \\ U(\omega) &= \int_{-\infty}^{+\infty} u_{\text{sol}}^2(t)\exp[-i\omega t]dt, \\ P(\omega) &= \int_{-\infty}^{+\infty} \hat{P}(u_{\text{sol}}(t))\exp[-i\omega t]dt. \end{aligned}$$

According to the discussion above, we can omit the convolution integrals, and the function F may then be written as

$$F(\omega) = \frac{\varepsilon P(\omega)}{-k_{\text{sol}} + k_{\text{lin}}(\omega)}. \quad (10)$$

The function $F(\omega)$ can be transformed back to the time domain by contour integration in the complex plane. The contour consists of the real axis, a small semicircle around the pole at ω_0 , and a large semicircle in the upper (lower) half-plane for $t > 0$ ($t < 0$). The main contribution to the integral comes from the pole at $\omega = \omega_0$ ($\omega = \pm\omega_0$ for 4OD). All other poles in the upper half-plane correspond to evanescent waves that vanish for large t . We therefore neglect their contribution, and obtain

$$f(t) = a \exp[i\omega_0 t] = \frac{i}{2} \frac{\varepsilon P(\omega_0)}{V_g^{-1}} H(t\varepsilon) \exp[i\omega_0 t], \quad (11a)$$

$$\begin{aligned} f(t) &= a_+ \exp[i\omega_0 t] + a_- \exp[-i\omega_0 t] \\ &= \frac{i\varepsilon}{2} \left[\frac{P(\omega_0)}{V_g^{-1}(\omega_0)} H(-t) \exp[i\omega_0 t] \right. \\ &\quad \left. + P \frac{P(-\omega_0)}{V_g^{-1}(-\omega_0)} H(t) \exp[-i\omega_0 t] \right], \quad (11b) \end{aligned}$$

where $H(x)$ is the Heaviside step function. In the 3OD case, it shows that the radiation only exists on one side of the soliton. The amplitude in front of the exponential term $\exp[i\omega_0 t]$ is what we need for further calculations. Taking into account additional poles and the two terms omitted in Eqs. (8) and (9) will change the step function into a smooth transition function, and it will also alter ω_0 inside this transition region. We now treat 3OD and 4OD separately. Following the discussion given around Eq. (8), we use the first-order expression (3) to calculate $P(\omega_0)$ in the case of 3OD. In the 4OD case we use the unperturbed soliton (2). We find

$$|P(\omega_0)| = \frac{\pi}{4\varepsilon^3} \left[\frac{\varepsilon}{2} - \pi A \varepsilon + O((A\varepsilon)^2) \right] \exp \left[-\frac{\pi}{4A\varepsilon} \right], \quad (12a)$$

$$|P(\omega_0)| = \frac{\pi}{2\varepsilon^2} [1 + 4A^2\varepsilon + O(A^4\varepsilon^2)] \exp \left[-\frac{\pi\omega_0}{2A} \right]. \quad (12b)$$

In deriving Eqs. (12a) we have used the first-order expressions for ω_0 from Eqs. (6). In the exponential in (12b), however, we retain the full dependence of A in ω_0 . This is important when integrating to get the total energy (see Sec. IV). Thus, the moduli of the Fourier amplitudes of the radiation become

$$|a| = \frac{5\pi}{4\varepsilon} \left[1 - \frac{2\pi}{5} A \varepsilon \right] H(t) \exp \left[-\frac{\pi}{4\varepsilon A} \right], \quad (13a)$$

$$|a_{\pm}| = \frac{\pi}{2\sqrt{2}\varepsilon} H(\mp t) \exp \left[-\frac{\pi\omega_0}{2A} \right]. \quad (13b)$$

We can see that the dominant factor in these expressions is the spectral value of the source term at ω_0 . This is satisfying from a physical point of view, as the radiation amplitude is proportional to the spectral amplitude of the soliton at the radiation frequency. It is thus the spectral tail of the soliton in the normal GVD regime that boosts the radiation.

The only difference between our expression and the seminumerical one obtained by Wai *et al.* [8] lies in the function of $A\varepsilon$ in front of the exponential in Eq. (13a). Karpman [12] obtained similar results, although an undetermined constant prevented it from being a fully analytical one. All previous expressions for the radiation amplitude [as well as our Eq. (13a)] have the same exponential factor, but the factor in front of the exponential differs in the different papers. A comparison between those factors the present Eq. (13a), that of Wai *et al.* [8], and that of Karpman [12], yields

$$\begin{aligned} &\frac{5\pi}{4} \left[1 - \frac{2\pi}{5} A \varepsilon \right] \quad (\text{present}), \\ &|\text{Im}\Gamma(A\varepsilon)| \approx 13.2 - 36A\varepsilon \quad (\text{Wai } et \text{ al.}), \quad (14) \\ &\frac{B}{2} \quad (\text{Karpman}), \end{aligned}$$

where $\text{Im}\Gamma$ is some function of $A\varepsilon$ that was numerically computed in Refs. [3,8,11] and it is fairly well approximated with a linear function of $A\varepsilon$ in the region of interest ($A\varepsilon < 0.04$). Karpman's result is a constant B that has to be numerically obtained. Our function is approximately a factor of 3 lower than the numerically computed function in the interval $0 < A\varepsilon < 0.05$, but it has the similar functional dependence on $A\varepsilon$. It should be observed that if perturbed soliton, Eq. (2), had been used to calculate the radiation in Eq. (12a), then the terms $(\frac{\varepsilon}{2} - \pi A \varepsilon)$ would have been replaced with a factor of 1. This shows that the first-order correction of the soliton gives a better description than the unperturbed soliton. If expressions

for the soliton to higher order (in ϵ) had been used in the calculation of Eq. (12a), the result would have been even better. In the limit of using the exact, infinite-order correction to the soliton, the exact amplitude found numerically by Wai *et al.* is reached. Finally, it should also be observed that Ref. [3] predicts approximately a factor of 3 *higher* radiation amplitude than Refs. [8,11]. In view of the mutual disagreement between the seminumerical works, and the fact that our calculations are much simplified and fully analytic, we believe that our result is quite reasonable. It gives us at least an analytical expression rather than the numerical ones obtained in previous works.

IV. RADIATED ENERGY

Using the formulas for the radiation given above, we may now calculate the energy loss of the solitons. The first invariant of Eq. (1) can be expressed as a ‘‘continuity equation’’ in integral form:

$$\frac{\partial}{\partial z} \int_{-t_0}^{t_0} |u|^2 dt = -[Q_{\text{rad}}]_{t=-t_0}^{t=t_0}, \quad (15)$$

where

$$Q_{\text{rad}} = \frac{i}{2} (uu_t^* - u_t u^*) - \epsilon (uu_{tt}^* + u_{tt} u^* - |u_t|^2), \quad (16a)$$

$$Q_{\text{rad}} = \frac{1}{2} (uu_t^* - u_t u^*) - i\epsilon (u^* u_{ttt} - u_{ttt}^* u + u_{tt}^* u_t - u_{tt} u_t^*) \quad (16b)$$

define the energy flux at any t , and $\pm t_0$ are arbitrary boundaries of integration. Equation (15) tells us that the decrease of energy in the range $[-t_0, +t_0]$ is equal to the energy flow out of this region. It is reasonable to set $\pm t_0$ beyond the main body of the soliton, where only small-amplitude radiation exists. Then the integral in Eq. (15) is, to a good approximation, the soliton energy $2A$, and the energy flow is determined by the dispersive waves, i.e.,

$$\begin{aligned} [Q_{\text{rad}}]_{t=-t_0}^{t=t_0} &= |a|^2 (\omega_0 + 3\epsilon\omega_0^2) \\ &= \frac{25\pi^2}{64\epsilon^3} \left[1 - \frac{4\pi}{5} A\epsilon \right] \exp \left[-\frac{\pi}{2\epsilon A} \right], \end{aligned} \quad (17a)$$

$$\begin{aligned} [Q_{\text{rad}}]_{t=-t_0}^{t=t_0} &= 2|a|^2 (\epsilon 4\omega_0^3 - \omega_0) \\ &= \frac{\pi^2}{8\epsilon\sqrt{2\epsilon}} \exp \left[-\frac{\pi\omega_0}{A} \right]. \end{aligned} \quad (17b)$$

Physically, this means that the energy radiates away with the group velocity, i.e., $Q_{\text{rad}} = |a|^2 v_g^{-1}$. It is now a straightforward task to find $A(z)$ numerically from Eq. (15) for every case of interest. This works well in the case of 4OD, but for 3OD complications arise due to the temporal (and spectral) asymmetry. These complications, due to the ‘‘spectral recoil’’ effect, are briefly outlined below.

The second invariant of Eq. (1) is the conservation of momentum, i.e.,

$$\frac{\partial}{\partial z} \int_{-\infty}^{+\infty} (u^* u_t - u_t^* u) dt = 0. \quad (18)$$

Physically, this means that the spectral center of mass is invariant. Thus, if a soliton loses energy by emitting linear waves in the normal dispersion regime, it will ‘‘recoil’’ into the anomalous regime of the spectrum [23,24]. This is the physical reason for the frequency shift (or temporal velocity shift) of order ϵ that was given in Eq. (3). Moreover, this spectral recoil sets a lower limit on the proximity to the zero-dispersion frequency at which solitons can be launched. As mentioned in Sec. I, this criterion can be expressed as $A\epsilon < 0.04$. If this condition is not fulfilled, the soliton radiates at ω_0 and recoils further into the anomalous regime. Since the recoil pushes the soliton spectrum away from the frequency of radiation ω_0 , the amplitude of the radiation decreases. Finally, a quasistationary state is reached, where the radiation rate is so small that the spectral recoil of the soliton is negligible [23,24]. This can be expected to occur at $A\epsilon \approx 0.04$. The soliton therefore stabilizes itself through the radiative losses, which is a rather unexpected example of soliton robustness. Using $A=1$ and $\epsilon=0.04$ in Eq. (17a) we find $Q_{\text{rad}} \approx 10^{-12}$, in good agreement with the numerics of Ref. [8]. In comparison with the effects of fiber loss, this effect is negligible, and we can therefore make the important conclusion that radiative losses due to third-order dispersion are negligible in communication systems utilizing stationary solitons. It should be noted that the spectral recoil effect prevents us from integrating the energy loss of the soliton using Eq. (16), since the above analysis (as well as Refs. [3,8,11]) neglects the spectral shift of the soliton.

In the case of 4OD, the radiation is spectrally symmetric, and the spectral recoil from each sideband cancels out. Therefore, the perturbative character of $A\epsilon$ is less important than it is in the 3OD case, and we may use Eq. (16) to calculate the radiative losses. The most significant contribution to Q_{rad} comes from the exponential, and it is therefore important to retain the full functional dependence of A in ω_0 :

$$\omega_0 = \left[\frac{1 + \sqrt{1 + 8\epsilon A^2}}{4\epsilon} \right]^{1/2}. \quad (19)$$

In Fig. 1, we show the peak intensity $A^2(z)$ for some different values of ϵ . The specific case $\epsilon=0.08$ was solved numerically in Fig. 1 of Ref. [7] to yield $A^2(z=16) \approx 0.7$, which is in excellent agreement with our value $A^2(z=16) \approx 0.74$. It is clearly seen in Fig. 1 that the asymptotic decay at high z is very slow. As the soliton loses energy, its spectral width decreases. The intensity of the radiation, which is proportional to the spectral intensity of the soliton at the unstable frequency ω_0 , will therefore decrease. Thus the soliton will radiate less as its total energy loss becomes greater, in analogy with the 3OD scenario outlined above. Similarly, a quasistationary state will be reached with negligible radiation loss. Due to the absence of spectral recoil, however, a longer propagation distance is required to reach this state than for the 3OD soliton.

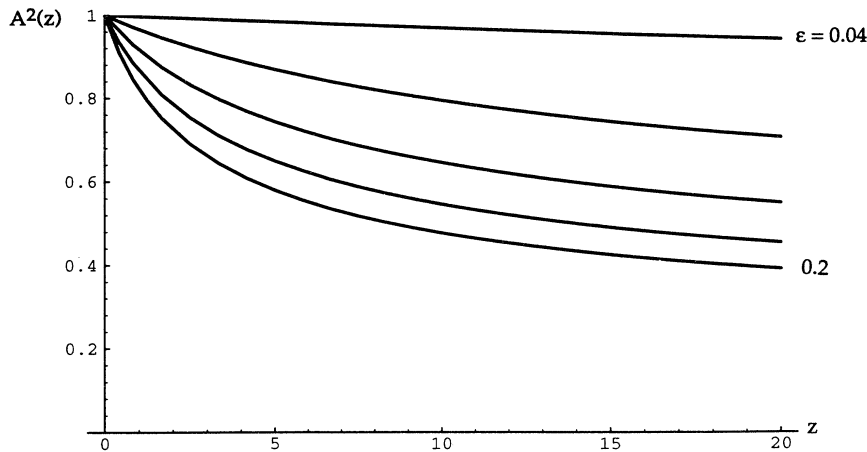


FIG. 1. Decaying intensity $A^2(z)$ of a soliton perturbed by fourth-order dispersion, for $\epsilon = \{0.04, 0.08, 0.12, 0.16, 0.2\}$, and $A(z=0)=1$.

V. RELATION TO CHERENKOV RADIATION

Classical Cherenkov radiation appears when a small object (particle) moves in a medium with a velocity exceeding the phase velocity of the waves in the given medium [15–18]. It is assumed that the source has dimensions much smaller than the wavelength. The source of radiation does not necessarily have to be a real particle, but can be, for example, waves of polarization induced in a nonlinear medium by external fields. If the size of the source is finite and comparable with the wavelength at least in one direction, then the radiation is defined in this direction by the “Cherenkov conditions” rather than full phase-matching conditions. Usually in nonlinear optics, the range of the induced polarization is large in comparison with the wavelength, so that satisfying full phase-matching conditions is necessary for the emission of radiation. In the latter case, the direction of the radiation is defined by these phase-matching conditions or the “resonance between the acting force connected with the presence of the source and the oscillators of the acting fields” [15].

In nonlinear optics, the concept of Cherenkov radiation was introduced by Tien, Ulrich, and Martin in 1970 [17]. Their experiment shows that a thin-film pump beam can generate a second-harmonic (SH) beam in the substrate below the film. The phase velocity of the induced nonlinear polarization is higher than the phase velocity of the waves in the substrate, or equivalently, the effective wave number of the polarization in the film must be shorter than the wave number of the waves in the substrate at the SH frequency. The emerging SH beam is therefore tilted with respect to the longitudinal direction of the pump, due to the longitudinal phase matching of the wave vectors. This is therefore an example of Cherenkov radiation with longitudinal phase matching, but without transverse phase matching.

The same considerations apply to the radiation emitted by solitons in fibers. In this case, the radiation can be considered as phase matched along the z direction and not phase matched along the “transverse” t axis [see Fig. (2)]. Moreover, the “phase velocity” of the soliton in the z direction is higher than the phase velocities of linear

waves outside the soliton. The physical consequence of this fact is that radiation is emitted in the “direction” that is inclined to the z direction at the angle ϕ , defined by $\sin\phi = \omega_0 / \sqrt{(k_{\text{sol}}^2 + \omega_0^2)}$. The intensity of this radiation is proportional to the square of the spectral component of the source. It decreases when the width of the source increases, but becomes higher for small soliton widths as would be expected for Cherenkov radiation without phase matching. It should be observed that, in this sense, all perturbations, that match linear waves to solitons in at least one direction can be considered as Cherenkov radiation. One more example of Cherenkov radiation by solitons was given in a recently published paper [19], in which two colliding solitons in a birefringent fiber transformed into a Cherenkov-emitting breatherlike state. Similarly, the radiation emitted when an initial condition evolves towards a stationary “solitonlike” state in other coupled nonlinear Schrödinger systems [20,21] is

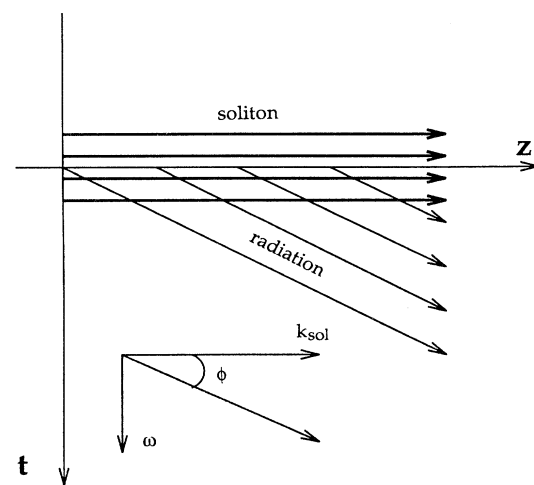


FIG. 2. Interpreting the retarded time t as a transverse coordinate, we can view the radiation process as Cherenkov radiation under the longitudinal phase-matching conditions $k_{\text{sol}} = k_{\text{lin}}$. The radiation is emitted at the “Cherenkov angle” ϕ defined by $\sin\phi = \omega_0 / \sqrt{(k_{\text{sol}}^2 + \omega_0^2)}$.

Cherenkov radiation.

In conclusion, we have demonstrated a straightforward analytic method for investigating the radiation emitted by solitons due to higher-order dispersion. We have found good agreement with previous numerical results. Emphasizing the physics of the problems we have identified the radiation as Cherenkov radiation in the t - z plane.

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