Antibunched Photons from Inelastic Cooper-Pair Tunneling

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We demonstrate theoretically that charge transport across a Josephson junction, voltage-biased through a resistive environment, produces antibunched photons. We develop a continuous-mode description of the emitted radiation field in a semi-infinite transmission line terminated by the Josephson junction. Within a perturbative treatment in powers of the tunneling coupling across the Josephson junction, we capture effects originating in charging dynamics of consecutively tunneling Cooper pairs. We find that within a feasible experimental setup the Coulomb blockade provided by high zero-frequency impedance can be used to create antibunched photons at a very high rate and in a very versatile frequency window ranging from a few GHz to a THz.

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Introduction.—Photons from ordinary thermal sources have a tendency to bunch together, and the first controlled generation of single photons was only performed in 1974 [1]. Since then, single photons have been used to explore fundamental aspects of quantum mechanics, such as the interference fringes of single particles [2] and Bell's inequalities [3]. A good single-photon source can be characterized by the vanishing probability of detecting two photons at the same time from its output. This property is called antibunching [4], and is also a sign that the field is nonclassical [5]. In addition to being of fundamental interest, the capability to create single propagating photons is an indispensable tool for many quantum information applications, including quantum key distribution [6].

In the optical domain, the workhorse for many single-photon experiments has been parametric down-conversion [5]. Here, photon pairs are generated at random times, and one of the two photons can be used to herald the other one [7]. Other type of sources for photons in the visible spectrum are based on photoluminescence in systems such as single ions, single molecules, semiconductor quantum dots, and diamond color centers [8,9].

In the microwave domain, superconducting quantum circuits [10–12] have been used to build single photon sources [13–18]. In most of these realizations, a two-level system is first excited and, consequently, spontaneously emits a microwave photon into a transmission line. The emission rate can often be controlled *in situ*, allowing for waveform shaping [17–19]. The use of the field reflected from a single artificial atom, which is indeed perfectly antibunched, has equally been demonstrated [20–22]. In addition to fundamental quantum optics experiments, single photon sources in the microwave domain might be useful for metrological purposes due to their well-defined output power and, in combination with single photon detectors, for quantum nondemolition measurements due to their ideal amplitude squeezing.

In this Letter, we study theoretically an alternative type of single-photon source, based on the Coulomb blockade of charge transport across a Josephson junction in series with an electromagnetic environment [23–25]. Our proposed setup has a very simple operating principle and promises to be a bright, robust, and versatile "photon gun."

Antibunched photons are created from the applied voltage, making use of inelastic (photon-assisted) Cooper-pair tunneling [25–28] and the long charging time of the junction. When biased below the superconducting gap, the applied voltage defines the frequency spectrum of the emitted photons through the Josephson frequency, $\omega_{\mathrm{J}}/2\pi = 2eV/h$. This frequency is not limited to the plasma frequency of the Josephson junction and the energy of the emitted photons is, in principle, only limited by the gap of the superconductor ($\hbar\omega_{\mathrm{J}} < 4\Delta$). Using appropriate materials, it should thus be possible to reach frequencies from a few GHz to 1 THz, making our source compatible with the energy scales of several other quantum systems including semiconductor quantum dots. The high frequencies would also allow for single photon envelopes on the cm length scale, facilitating experiments on quantum nonlocality within a cryostat.

The single-photon source we consider is shown in Fig. 1. It consists of a Josephson junction biased at voltage V and embedded in an electromagnetic environment characterized by the impedance $Z(\omega)$ as seen by the junction. The impedance is engineered to be high at the desired frequency ω_0, opening a window for photon

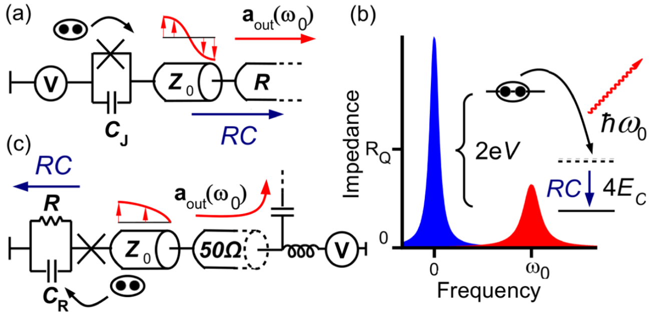

FIG. 1 (color online). (a) The single-photon source we consider consists of a voltage-biased Josephson junction (JJ) and a parallel junction capacitor C_J, in series with a semi-infinite transmission line (TL) with an impedance step, $Z_0 < R$. The TL provides a $\lambda/2$-type standing wave between the step and the junction, and results in similar impedance as in (b)–(c). (b) The desired impedance as seen by the JJ, presents a peak at zero frequency described by a zero frequency resistance $R > h/4e^2 = R_Q$ and a capacitance C defining its bandwidth, as well as a peak at finite frequency ω_0 opening a window for photon emission. A photon can be emitted by Cooper-pair (CP) tunneling when the voltage V is chosen so that $2eV = 4e^2/2C + \hbar\omega_0$. Further CP tunneling events are momentarily blocked because more energy would be needed to add a second CP to the effective capacitor C. After a time RC the capacitor discharges and the next photon-assisted CP tunneling can occur. (c) An experimentally feasible realization allowing for large zero-frequency impedance $R \gg R_Q$, while maintaining compatiblity with standard 50 Ω TLs. Here the voltage bias is applied to the junction via a $Z = 50\ \Omega$ TL and a $\lambda/4$ segment with $Z_0 > 50\ \Omega$. The same TL is used to collect the high-frequency response by splitting the signal in high- and low-frequency components. In this case the large resistance R can be realized on chip as a thin-film resistor.

emission at this frequency, as well as high at zero frequency $[Z(0) > h/4e^2 = R_Q]$. This 0-frequency peak is further characterized by a capacitance C, in the simplest case the junction capacitance C_J. When a Cooper-pair tunnels across the junction, it gains the energy $2eV$ from the voltage source. However, the electrostatic energy of the capacitor (C) also increases by the charging energy $4E_C = 2e^2/C$. For operation, the released energy is made to match the frequency of the mode ω_0, i.e., $2eV - 4E_C = \hbar\omega_0$. After each photon-assisted tunneling event, the high zero-frequency impedance ($R > R_Q$) and capacitance lead to a long charging time RC of the junction, which temporarily blocks further tunneling [24,29–32]. This results in photon antibunching in the field emitted into the transmission line. In this picture, the slow recharging dissipates the rest of the energy supplied by the voltage source ($2eV - \hbar\omega_0 = 4E_C$) in the form of a large number of emitted low frequency photons, that in an experimental setup are absorbed by the physical resistor.

Methods.—To quantitatively test if such an interplay between the low- and high-frequency parts of the same electromagnetic environment is possible, we take a specific implementation of the environment response function $Z(\omega)$, as described in Fig. 1(a), and study the solution of the propagating continuous-mode flux field $\hat{\Phi}(x,t)$ in the neighborhood of the Josephson junction ($x = 0$) and in the transmission line. In the semi-infinite transmission line the quantized field can be presented as a sum of incoming and outgoing waves [33–35],

$$\hat{\Phi}(x,t) = \sqrt{\frac{\hbar R}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \times [\hat{a}_{\rm in}(\omega) e^{-i(k_\omega x + \omega t)} + \hat{a}_{\rm out}(\omega) e^{-i(-k_\omega x + \omega t)} + \text{H.c.}]. \quad (1)$$

Here, $R = \sqrt{L'/C'}$ is the characteristic impedance, expressed via inductance L' and capacitance C' per unit length, and $k_\omega = \omega\sqrt{C'L'}$ is the wave number. The incoming (outgoing) wave at frequency ω is created by the operator $\hat{a}^\dagger_{\rm in}(\omega)$ $[\hat{a}^\dagger_{\rm out}(\omega)]$ and annihilated by $\hat{a}_{\rm in}(\omega)$ $[\hat{a}_{\rm out}(\omega)]$. They fulfill the standard commutation relation $[\hat{a}_{\rm in}(\omega), \hat{a}^\dagger_{\rm in}(\omega')] = \delta(\omega - \omega')$. A similar solution exists also in the cavity region (impedance $Z_0 < R$).

The interaction between the radiation field and Cooper-pair tunneling across the Josephson junction is described by the boundary condition ($x = 0$ corresponds to the junction location)

$$C_J \ddot{\hat{\Phi}}(0,t) - \frac{1}{L_0'} \frac{\partial \hat{\Phi}(x,t)}{\partial x}\bigg|_{x=0} = I_c \sin[\omega_J t - \hat{\phi}(t)]. \quad (2)$$

Here, C_J is the junction capacitance, L_0' the inductance per unit length in the region between the junction and the impedance step. The Josephson current is limited by the critical current I_c and controlled by the phase $\hat{\phi}(t) \equiv 2\pi\hat{\Phi}(0,t)/\Phi_0$. Equation (2) corresponds to current conservation at the junction. The boundary condition at the impedance step is linear and can be solved by Fourier transformation [35]. These two conditions can now be used to solve the free-space out-field $\hat{a}_{\rm out}(\omega)$ as a function of the equilibrium in-field $\hat{a}_{\rm in}(\omega)$. The general solution for the out-field can be written formally as

$$\hat{a}_{\rm out}(\omega) = r(\omega)\hat{a}_{\rm in}(\omega) + iI_c A(\omega)\sqrt{\frac{Z_0}{\hbar\omega\pi}} \int_{-\infty}^\infty dt e^{i\omega t} \times \hat{U}^\dagger(t,-\infty) \sin[\omega_J t - \hat{\phi}_0(t)] \hat{U}(t,-\infty). \quad (3)$$

Here $A(\omega)$ (Supplemental Material [36]) is related to the impedance as seen by the Josephson junction, $\text{Re}[Z(\omega)] \equiv Z_0|A(\omega)|^2$, and $r(\omega) = A(\omega)/A(\omega)^*$ corresponds to the phase shift in the out-field when $I_c = 0$. The solution is expressed via time evolution (operator) of the current across the Josephson junction [37–39],

$$\hat{U}(t,t_0) = \mathcal{T} \exp\left\{\frac{i}{\hbar}\int_{t_0}^t dt' E_J \cos[\omega_J t' - \hat{\phi}_0(t')]\right\}. \quad (4)$$

Here, $E_\mathrm{J} = (\hbar/2e)I_c$ is the Josephson coupling and $\mathcal{T}$ stands for time ordering. This is an expansion in terms of the phase at the Josephson junction in the absence of the tunneling current,

$$\hat{\phi}_0(t) = \frac{\sqrt{4\pi\hbar Z_0}}{\Phi_0}\int_0^\infty \frac{d\omega}{\sqrt{\omega}} A(\omega)\hat{a}_\mathrm{in}(\omega)e^{-i\omega t} + \mathrm{H.c.} \tag{5}$$

From here on, we make the natural assumption that temperature is low compared to the mode frequency, $k_\mathrm{B}T \ll \hbar\omega_0$. The flux density of photons due to Cooper-pair tunneling can be evaluated to the leading order in the critical current I_c [28,34,35],

$$\begin{aligned} f(\omega) &= \int_0^\infty d\omega' \frac{1}{2\pi}\langle \hat{a}^\dagger_\mathrm{out}(\omega)\hat{a}_\mathrm{out}(\omega')\rangle \\ &= \frac{I_c^2\mathrm{Re}[Z(\omega)]}{2\omega}P(2eV - \hbar\omega). \end{aligned} \tag{6}$$

The photon flux is a function of the well-known probability density [25],

$$P(E) = \int_{-\infty}^\infty dt \frac{1}{2\pi\hbar}e^{J(t)}e^{i(E/\hbar)t}, \tag{7}$$

where $J(t) = \langle[\hat{\phi}_0(t) - \hat{\phi}_0(0)]\hat{\phi}_0(0)\rangle$ is a measure of equilibrium phase fluctuations and is a function of input impedance and temperature [25]. The function $P(E)$ describes the ability of the transmission line to absorb an energy E when a Cooper-pair tunnels. In the setup we consider, the $P(E)$ function has a simple analytical form

$$P(E) \approx (1-p)P_\mathrm{CB}(E) + pP_\mathrm{CB}(E - \hbar\omega_0), \tag{8}$$

where $p \ll 1$ (Supplemental Material [36]) and $P_\mathrm{CB}(E)$ is the probability distribution for a high-Ohmic impedance, $R = Z_0 \gg R_Q$, with cutoff $1/RC$,

$$P_\mathrm{CB}(E) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(1/2\sigma^2)(E-4E_C)^2}.$$

Here, the broadening $\sigma = \sqrt{8E_C k_\mathrm{B}T}$ (assuming $1/RC < k_\mathrm{B}T/\hbar$) describes thermal fluctuations of the junction voltage induced by the series resistor. These fluctuations are accounted for as interaction with (thermal) photons in the transmission line, treated to all orders. The total $P(E)$ function has not only a peak at $E = 4E_C$, but also at $E = 4E_C + \hbar\omega_0$, which corresponds to the possibility to emit the energy $\hbar\omega_0$ to the lowest cavity mode and to use the rest to charge the junction capacitor.

Results.—We now investigate the probability to observe two photons with a time separation τ. This is usually quantified in terms of the second order coherence function $g^{(2)}(\tau)$ [40]. To calculate this, we need the first order coherence function, defined via the photon flux as

$$G^{(1)}(\tau) = \frac{R\hbar}{2}\int_0^\infty d\omega e^{i\omega\tau}\omega|F(\omega)|^2 f(\omega).$$

Here, $F(\omega)$ describes the measurement bandpass filter [41], centered at the chosen measurement frequency, $\omega \sim \omega_\mathrm{J} - 4E_C/\hbar$. The inverse of its bandwidth W determines the temporal resolution of the photon detection. The expression for the unnormalized second-order correlation function of photon detection reads

$$\begin{aligned} G^{(2)}(\tau) \equiv& \left(\frac{\hbar R}{4\pi}\right)^2 \\ &\times \int_\mathrm{BW} e^{i\tau(\omega_2-\omega_3)}\sqrt{\omega_1\omega_2\omega_3\omega_4}\langle \hat{a}^\dagger_{\omega_1}\hat{a}^\dagger_{\omega_2}\hat{a}_{\omega_3}\hat{a}_{\omega_4}\rangle, \end{aligned} \tag{9}$$

using the shorthand notation $\int_\mathrm{BW} \equiv \Pi_i \int_0^\infty d\omega_i F(\omega_i)$ and $a_{\omega_i} \equiv a_\mathrm{out}(\omega_i)$. The dominating contribution comes from the fourth order in I_c (Supplemental Material [36]),

$$\begin{aligned} G^{(2)}_\mathrm{4th}(\tau) =& \left(\frac{I_\mathrm{c}^2 R}{4\pi\hbar}\right)^2 \int_\mathrm{BW} e^{i\tau(\omega_2-\omega_3)}A_\omega \\ &\times \int_\mathrm{times}\langle \mathcal{T}^\dagger\{\hat{I}^\dagger_{\omega_1,t_1}\hat{I}^\dagger_{\omega_2,t_2}\}\mathcal{T}\{\hat{I}_{\omega_3,t_3}\hat{I}_{\omega_4,t_4}\}\rangle, \end{aligned} \tag{10}$$

where $A_\omega \equiv A^*(\omega_1)A^*(\omega_2)A(\omega_3)A(\omega_4)\delta(\omega_1 + \omega_2 - \omega_3 - \omega_4)$ and $\int_\mathrm{times} \equiv \Pi_i \int_{-\infty}^\infty dt_i$. The operator $\hat{I}_{\omega,t} = e^{i\omega t}\exp\{i[\hat{\phi}_0(t) - \omega_\mathrm{J}t]\}$ can be interpreted as a creation of a photon with frequency ω via Cooper-pair tunneling at time t. The final expression for the second order coherence can be presented as

$$g^{(2)}(\tau) \equiv \frac{G^{(2)}_\mathrm{4th}(\tau)}{|G^{(1)}_\mathrm{2nd}(0)|^2} = 1 + \left[\frac{G^{(1)}_\mathrm{2nd}(\tau)}{G^{(1)}_\mathrm{2nd}(0)}\right]^2 + \mathcal{G}(\tau), \tag{11}$$

where $\mathcal{G}(\tau)$ captures correlations between consecutive Cooper-pair tunneling events and the other terms describe the emission from uncorrelated Cooper-pair tunneling. When the temporal resolution of the detection cannot resolve individual tunnel events, i.e., for $W \to 0$, we get $\mathcal{G}(\tau) \to 0$ and $g^{(2)}(0) \to 2$, which is characteristic for chaotic (thermal) light, having no phase coherence between different photons [40]. For times much longer than the temporal resolution $\tau \gg W^{-1}$ and recovery time $\tau \gg RC$, only the first term is finite leaving $g^{(2)}(\tau) = 1$, a characteristic of uncorrelated (Poissonian) photon emission. Outside these limits, we see clear effects of $\mathcal{G}(\tau)$, as shown in Fig. 2, where we plot the second-order coherence calculated numerically using Eqs. (10)–(11). We consider a cavity length such that a resonance mode appears at $\omega_0/2\pi = 5$ GHz, for $R = 4R_\mathrm{Q}$ and $Z_0/R = 1/10$. The detection filter is centered at the mode frequency with a bandwidth of 1 GHz. The $P(E)$ function (inset) is close to the analytical form in Eq. (8) and the simple interpretation presented in Fig. 1 implies clear antibunching [$g^{(2)}(0) < 1$]

of the outgoing radiation. Indeed, when biased optimally at $2eV = 4E_C + \hbar\omega_0$ (black arrow in the inset), we observe a clear antibunching over a time scale given by the RC time. The depth of the antibunching dip is limited by temperature, and increasing it beyond the charging energy, $k_B T \gtrsim 3E_C$, the output radiation instead becomes bunched $[g^{(2)}(0) > 1]$.

Classically, at zero temperature, a sudden $2e$ reduction of the equilibrium capacitor charge ($Q = CV$) induces recharging dynamics $v(t) = V - (2e/C)e^{-t/RC}$. A first guess would be that (in the full quantum treatment) the probability to detect a second photon after a time τ is given by the photon emission rate at voltage $v(\tau)$. Following this simple idea, we write down a semianalytical formula for the expected second order coherence,

$$g^{(2)}(\tau) = \frac{\int_{\mathrm{BW}} \mathrm{Re}[Z(\omega)]P[2ev(\tau) - \hbar\omega]}{\int_{\mathrm{BW}} \mathrm{Re}[Z(\omega)]P(2eV - \hbar\omega)}. \qquad (12)$$

In Fig. 2, we see that Eq. (12) reproduces the long-time behavior, whereas can fail when the separation between the tunneling events becomes short, i.e., when the tunneling processes start to overlap. This can also be understood more mathematically, as detailed in the Supplemental Material [36]. The form implies that to suppress $g^{(2)}(0)$ at the optimal bias point, we need the width of the $P(E)$ to be smaller than the bias jump $4E_C$, which leads to the conditions $k_B T \ll E_C$ and $R \gg R_Q$.

For further illustration of recharging effects in this system, we study the dependence of bunching on the bias voltage, see Fig. 3. For high bias voltages the second-order coherence shows superbunching $[g^{(2)}(0) > 2]$, whereas for lower bias voltages both antibunching and bunching can coexist for a single curve. These properties are qualitatively reproduced by Eq. (12), and occurs here since the first emission rate is decreased compared to the optimal point ($2eV = 4E_C + \hbar\omega_0$), while the rate for the secondary emission is always higher, or sweeps through the maximum, during the recharging. We also observe that $g^{(2)}(0)$ can be suppressed below the value of the optimal point, by a bias voltage $2eV < 4E_C + \hbar\omega_0$, with the cost of a reduced photon flux. This is a result of a reduction in thermal-fluctuation triggered two-photon emission processes.

In conclusion, by extending the well-known $P(E)$ theory to two-Cooper-pair processes (i.e., to 4th order in E_J) we have shown that the Coulomb blockade of Cooper-pair tunneling allows for the creation of strongly antibunched microwave photons from a simple dc bias under experimentally realistic conditions. The validity of our theoretical approach is limited to tunneling rates lower than $1/RC$,

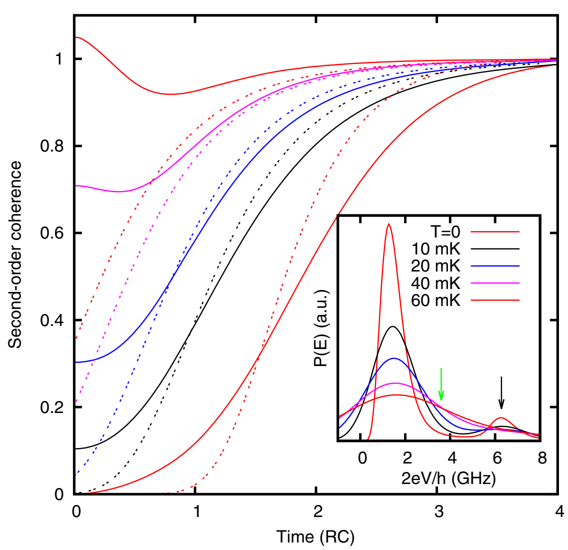

FIG. 2 (color online). Second-order coherence $g^{(2)}(\tau)$ based on equations (10)–(11) (solid lines) and on semianalytical approximation (12) (dashed lines), at different temperatures. When biased optimally (the black arrow) we observe antibunching within the RC time scale and limited by temperature. The expected voltage immediately after the first tunneling is pointed to by the green arrow. The long-time behavior is reproduced by the semi-analytical formula, based on calculating the leading-order emission flux with a slowly evolving voltage. At short times, thermal fluctuations increase the bunching and wash out antibunching approximately when $T = 60\ \mathrm{mK} \approx 3E_C/k_B$. (Other parameters are given in the Supplemental Material [36].)

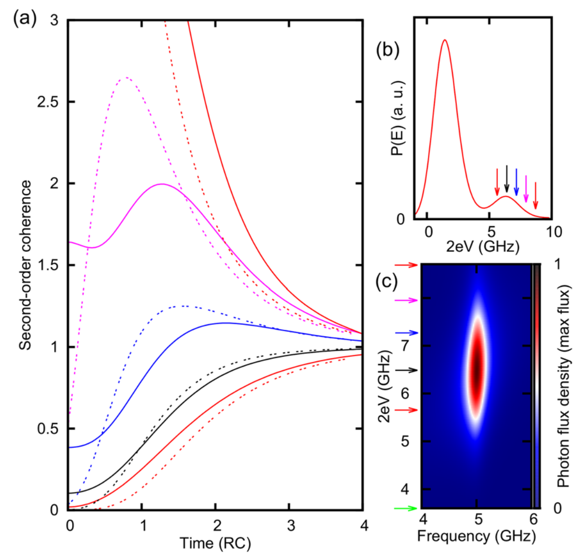

FIG. 3 (color online). (a) Second-order coherence $g^{(2)}(\tau)$ for different bias voltages, based on Eqs. (10)–(11) (solid lines) and on approximation (12) (dashed lines), for the system as in Fig. 2 with $T = 10$ mK. (b) The $P(E)$ function and the positions of the studied voltage-bias points, marked by the arrows of the corresponding colors. (c) The photon-flux density as a function of the bias voltage in the neighborhood of the studied mode at $\omega_0/2\pi = 5$ GHz. Depending on the bias point, the second-order coherence can have a deep antibunching dip, a local maximum, or show a steep decay, as a function of the measurement separation-time τ. The steep decay or local maximum occurs since above the optimal bias (black arrow) the first emission rate is decreased, while the rate for the secondary emission can be always higher, or sweep through the maximum, during the recharging.

but we expect antibunching to survive also at higher tunneling rates. This regime would be characterized by modified junction voltage dynamics, and is indeed an interesting regime to explore, both experimentally and theoretically. The proposed source of antibunched photons can be transformed into a photon gun by replacing the Josephson junction by a dc SQUID. Cooper-pair tunneling can then be efficiently suppressed by threading a half flux quantum through the SQUID and a flux pulse short compared to RC will then release a single photon on demand with high probability for large E_J. Therefore, this system allows for an on-demand photon emission at a very high rate and in a wide range of frequencies. Such a bright microwave single photon source would be useful in various microwave quantum measurement setups because of its perfect amplitude squeezing and well-defined power.

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