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Abstract: We present an optimized CD compensation filter with improved performance and increased resilience to quantization errors. The filter is given in closed form based on a constrained least-squares problem.
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1. Introduction

In coherent optical communication systems, the pulse broadening from chromatic dispersion (CD) can be compensated using digital signal processing (DSP) [1]. In systems with a large amount of accumulated dispersion at the receiver, this compensation is performed in the frequency domain. However, in short systems, such as metro systems, or when CD is compensated in the optical domain, one alternative is to perform CD compensation in the time domain. An FIR filter for CD compensation has been calculated by performing a direct sampling of the CD compensation impulse response [2], and we will refer to this as the *DS filter*. This approach is applicable only with large accumulated CD and the maximum number of filter taps is limited by aliasing [2]. The bandwidth limitations of the signal spectrum are not considered in this method and as will be shown, the method shows poor performance for higher-order modulations. Based on a least-squares (LS) optimization approach, an FIR filter that compensates the CD only over the signal band has been proposed [3]. With this method, CD can be compensated using a smaller number of taps but a drawback is that it may lead to a filter with large out-of-band gain. This gain leads to noise enhancement and can also increase channel crosstalk, which degrades system performance. Moreover, the filter tap coefficients depend on an adjustment parameter, ε, for which there is no analytic selection method. Therefore, an exhaustive search is required to find the optimal ε, which depends on the system parameters, including the resolution used in the fixed-point implementation.

In this paper, we propose a novel filter design method which introduces a constraint on the out-of-band gain of the CD compensation filter. This LS filter design method with constrained optimization (LS-CO) provides a closed-form solution for the filter tap coefficients. We show that the proposed filter is more resilient to quantization errors and has improved performance compared with other methods previously proposed in the literature. Moreover, as can be seen in a companion paper [4], a power-efficient ASIC implementation of the proposed filter can lead to a significant power dissipation reduction compared to CD compensation in the frequency domain.

2. CD compensation FIR filter design

CD can be modeled as an all-pass filter with a frequency response given by [2, Eq. (4)]

$$H_{\mathrm{CD}}(e^{j\omega T}) = e^{-jM(\omega T)^2}, \quad M = \frac{D\lambda^2 L}{4\pi c T^2}, \tag{1}$$

where D is the dispersion parameter, T is the sampling interval, L is the fiber length, λ is the operating wavelength, and c denotes the speed of light. CD is compensated by a static filter with the inverse frequency response, which in the ideal case is given by

$$H(e^{j\omega T}) = H_{\mathrm{CD}}^{-1}(e^{j\omega T}) = e^{jM(\omega T)^2}. \tag{2}$$

Assuming that the spectrum of the pulse-shaping filter at the transmitter is known, CD needs to be compensated only within the signal band. Denoting the frequency response of the FIR CD compensation filter to be designed by $\hat{H}(e^{j\omega T})$, the normalized value of the filter response error within the signal band is defined as

$$\xi_s = \frac{\int_{\Omega_s} |\hat{H}(e^{j\omega T}) - H(e^{j\omega T})|^2 d(\omega T)}{\int_{\Omega_s} d(\omega T)} = \frac{1}{\Omega_2 - \Omega_1}\int_{\Omega_1}^{\Omega_2} |\hat{H}(e^{j\omega T}) - H(e^{j\omega T})|^2 d(\omega T), \tag{3}$$

where Ω_1 and Ω_2 denote the boundary frequencies of the signal band Ω_s. The average out-of-band gain of the filter is

$$\xi_o = \frac{\int_{\Omega_o} |\hat{H}(e^{j\omega T})|^2 d(\omega T)}{\int_{\Omega_o} d(\omega T)} = \frac{1}{2\pi + \Omega_1 - \Omega_2} \left[\int_{-\pi}^{\Omega_1} |\hat{H}(e^{j\omega T})|^2 d(\omega T) + \int_{\Omega_2}^{\pi} |\hat{H}(e^{j\omega T})|^2 d(\omega T) \right], \quad (4)$$

where Ω_o denotes the spectral bands with no signal content. For the DS filter, the maximum number of filter tap coefficients is, due to aliasing, given by $N_{c,\max} = 2\lfloor 2\pi|M| \rfloor + 1$ [2, Eq. (9)], where $\lfloor\cdot\rfloor$ indicates the floor function. Using (3) and (4), the optimization problem for calculating the filter tap coefficients $\hat{\mathbf{h}} = [\hat{h}_{-(N_c-1)/2}, \ldots, \hat{h}_0, \ldots, \hat{h}_{(N_c-1)/2}]^{\mathrm{T}}$, where N_c is the number of filter taps, can be formulated as

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \xi_s \quad \text{subject to} \quad \xi_o \leq \xi_{o,\max}, \quad (5)$$

where $\xi_{o,\max}$ is a selected threshold on the out-of-band gain. This optimization problem is a constrained LS problem and can easily be shown to be convex. This problem can be solved by minimizing its corresponding Lagrangian function [5]. The optimum filter tap coefficients can be calculated as $\mathbf{h} = \mathbf{Q}^{-1}\mathbf{v}$, where $\mathbf{Q}$ is a $N_c \times N_c$ matrix and $\mathbf{v}$ is a $N_c \times 1$ vector. By setting the first derivative of the corresponding Lagrangian function to zero, the elements of $\mathbf{Q}$ are calculated according to

$$Q_{m,n} = \begin{cases} \frac{2\pi(\lambda+1)+(\lambda+1)\Omega_1-(\lambda+1)\Omega_2}{2\pi+\Omega_1-\Omega_2} & \text{if } m = n \\ \frac{\lambda}{j(m-n)(2\pi+\Omega_1-\Omega_2)}\left[e^{j(m-n)\Omega_1} - e^{j(m-n)\Omega_2}\right] & \\ \qquad + \frac{1}{j(m-n)(\Omega_1-\Omega_2)}\left[e^{j(m-n)\Omega_1} - e^{j(m-n)\Omega_2}\right] & \text{if } m \neq n \end{cases} \quad (6)$$

and the elements for $\mathbf{v} = [v_{-(N_c-1)/2}, \ldots, v_0, \ldots, v_{(N_c-1)/2}]^{\mathrm{T}}$ according to

$$v_m = \frac{e^{-j\left(\frac{m^2}{4M}+\frac{3\pi}{4}\right)}}{2(\Omega_2-\Omega_1)}\sqrt{\frac{\pi}{M}}\left\{\operatorname{erf}\left[\frac{e^{j\frac{3\pi}{4}}(2M\Omega_2 - m)}{2\sqrt{M}}\right] + \operatorname{erf}\left[\frac{e^{j\frac{3\pi}{4}}(2M\Omega_2 + m)}{2\sqrt{M}}\right]\right\}, \quad (7)$$

where $\operatorname{erf}(\cdot)$ is the error function. By satisfying the complementary slackness condition [5], it can be shown that the Lagrangian parameter λ can be computed by solving

$$\frac{2\pi\hat{\mathbf{h}}^{\mathrm{H}}\hat{\mathbf{h}} + \Omega_1 - \Omega_2}{2\pi + \Omega_1 - \Omega_2} = \xi_{o,\max}, \quad (8)$$

where $^{\mathrm{H}}$ denotes Hermitian conjugation.

3. Simulation results and discussion

The performance of the proposed *LS-CO filter* has been compared with the DS filter [2] and the LS approach [3] for a fixed-point implementation. This introduces quantization errors due to the finite-precision calculations. The performance of the different filters was evaluated using a selected word length for the CD compensation part of the receiver. The CD was compensated using the fixed-point version of the FIR CD compensation filters and we investigated the LS approach for a band-limited pulse spectrum and refer to this as the *LS-BL filter*. In the simulations, $\lambda = 1550$ nm and a fiber length $L = 250$ km with $D = 16$ ps/(nm km) was simulated. The symbol rate was 28 GBd and 2 samples per symbol were assumed for the receiver. Square-root raised cosine pulses with roll-off factor $\rho = 0.25$ were used and since CD compensation is performed only within the signal band, $\Omega_2 = -\Omega_1 = (1+\rho)\pi/2$. By increasing the threshold on the out-of-band gain, the constraint in (4) is less stringent and the LS-CO problem provides a better approximation of the ideal CD compensation filter. On the other hand, a large threshold leads to noise enhancement and quantization errors. The best threshold is found numerically. Then, for a given threshold, the filter tap coefficients are found in closed-form. We remark that the sensitivity to the threshold is rather low, therefore the numerical optimization is fast. To reduce complexity, a reduction of the number of filter taps is advocated, at the expense of a performance loss [2]. In our simulations, we fixed the filter length to $0.6N_{c,\max}$ and evaluate the effect of other parameters on the performance of the different filtering methods.

In Fig. 1a, bit error rate (BER) simulations are shown in order to visualize the power penalty of the different filters. Results are shown for QPSK, 16-QAM, and 64-QAM using 5-bit word length. At a target BER of 10^{-3}, the

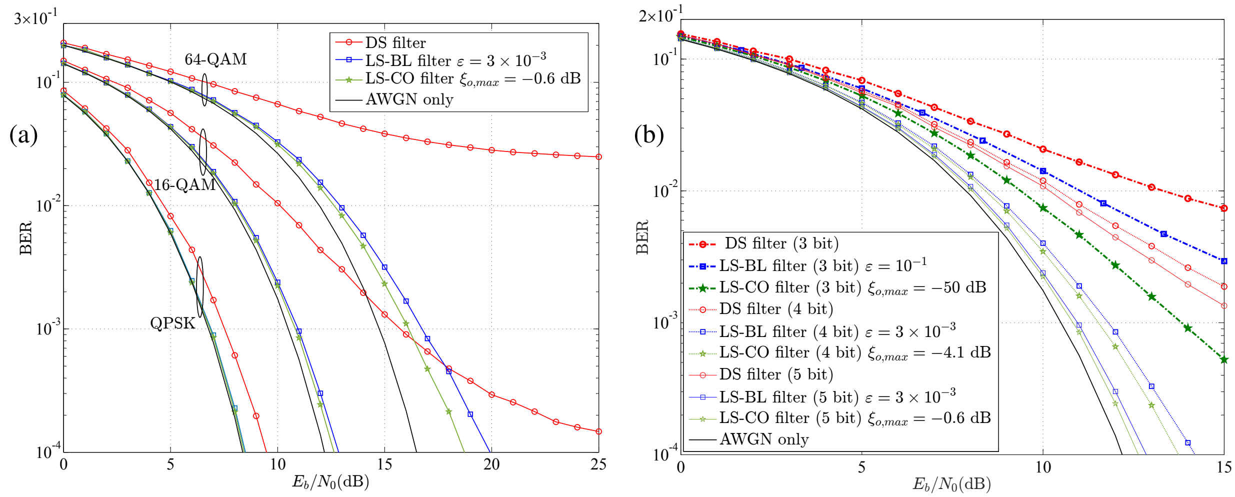

Fig. 1. (a) BER comparison of the DS, LS-BL ($\varepsilon = 3 \times 10^{-3}$), and LS-CO ($\xi_{o,\max} = -0.6$ dB) filters using QPSK, 16-QAM, and 64-QAM modulation, respectively. The word length is 5-bits and $0.6N_{c,\max}$ filter taps are used in all cases. (b) BER comparison of different CD compensation filters for 3, 4, and 5-bit word lengths using 16-QAM modulation and $0.6N_{c,\max}$ FIR filter taps.

SNR penalties incurred by reducing the number of taps to $0.6N_{c,\max}$ for the LS-CO filter for QPSK, 16-QAM, and 64-QAM are 0.045 dB, 0.145 dB and 0.685 dB, respectively. The SNR penalty can be decreased by increasing the filter length or the word length. As mentioned earlier, the LS-BL filtering method requires an adjustment parameter (ε) and it was previously found that the best performance is achieved for $\varepsilon = 10^{-14}$ [3]. This value yields the best performance in floating-point, but in a fixed-point implementation it leads to a performance degradation. Therefore, for a fair comparison, an exhaustive search has been performed to find the best ε for the LS-BL filter for the fixed-point implementation. According to our simulations, the best ε depends on the system parameters such as fiber length, dispersion parameter, operating wavelength, sampling interval, number of filter taps, and word length. From Fig. 1a, it can be seen that the LS-CO filter has improved performance compared to the DS and the LS-BL filters and when increasing the modulation order, the performance gain increases. For example, using 16-QAM, 5-bit word length, and a target BER of 10^{-3}, the SNR penalties of the LS-CO filter, the LS-BL filter, and the DS filter compared to the limit set by the additive white Gaussian noise (AWGN) are 0.33 dB, 0.46 dB and 5.18 dB, respectively.

In Fig. 1b, BER results are presented for 16-QAM and different word lengths. When decreasing the word length, which corresponds to increasing the quantization errors, the performance gain of the proposed filter increases; therefore, the proposed filter is more resilient to quantization errors. Furthermore, a trade-off between quantization errors and out-of-band gain of the LS-CO filter can be seen where with decreasing the word length, while keeping the other parameters fixed, the out-of-band gain of the filter should be reduced.

4. Conclusion

An optimized FIR CD compensation filter based on a constrained LS optimization problem has been proposed. A closed-form expression for the CD compensation filter tap coefficients has been derived and the performance of the resulting filter has been investigated using numerical simulations. The proposed filter is more resilient to quantization errors and yields better performance compared to other filtering methods proposed in the literature. The improvement in performance is more significant for higher-order modulation formats and decreasing word length.

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