Thesis for the Degree of Doctor of Philosophy

Insurance: solvency and valuation

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Chalmers University of Technology
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Göteborg, Sweden 2015

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ISBN 978-91-7597-195-7

Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr 3876
ISSN 0346-718X

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Printed in Göteborg, Sweden 2015

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Abstract

This thesis concerns mathematical and statistical concepts useful to assess an insurer's risk of insolvency. We study company internal claims payment data and publicly available market data with the aim of estimating (the right tail of) the insurer's aggregate loss distribution. To this end, we also develop a framework for market-consistent valuation of insurance liabilities. Moreover, we discuss Solvency II, the risk-based regulatory regime in the European Union, in some detail.

In Paper I, we construct a multidimensional simulation model that could be used to get a better understanding of the stochastic nature of insurance claims payments, and to calculate solvency capital requirements. The assumptions made in the paper are based on an analysis of motor insurance data from the Swedish insurance company Folksam. In Paper II, we investigate risks related to the common industry practice of engaging in interest-rate swaps to increase the duration of assets. Our main focus is on foreign-currency swaps, but the same risks are present in domestic-currency swaps if there is a spread between the swap-zero-rate curve and the zero-rate curve used for discounting insurance liabilities. In Paper III, we study data from the yearly reports the four major Swedish non-life insurers have sent to the Swedish Financial Supervisory Authority (FSA). Our aim is to find the marginal distributions of, and dependence between, losses in the five largest lines of business. In Paper IV, we study the valuation of stochastic cash flows that exhibit dependence on interest rates. We focus on insurance liability cash flows linked to an index, such as a consumer price index or wage index, where changes in the index value can be partially understood in terms of changes in the term structure of interest rates.

Papers I and III are based on data that are difficult to get hold of for people in academia. The FSA reports are publicly available, but actuarial experience is needed to find and interpret them. These two papers contribute to a better understanding of the stochastic nature of insurance claims by providing data-driven models, and analyzing their usefulness and limitations. Paper II contributes by highlighting what may happen when an idea that is theoretically sound (reducing interest-rate risk with swaps) is applied in practice. Paper IV contributes by explicitly showing how the dependence between interest rates and inflation can be modeled, and hence reducing the insurance liability valuation problem to estimation of pure insurance risk.

Keywords: risk aggregation, dependence modeling, solvency capital requirement, market-consistent valuation

Preface

This thesis consists of the following papers.

- ▹ Jonas Alm,
  "*A simulation model for calculating solvency capital requirements for non-life insurance risk*",
  in *Scandinavian Actuarial Journal* **2015:2** (2015), 107–123.
  DOI: 10.1080/03461238.2013.787367
- ▹ Jonas Alm and Filip Lindskog,
  "*Foreign-currency interest-rate swaps in asset-liability management for insurers*",
  in *European Actuarial Journal* **3:1** (2013), 133–158.
  DOI: 10.1007/s13385-013-0069-5
- ▹ Jonas Alm,
  "*Signs of dependence and heavy tails in non-life insurance data*",
  in *Scandinavian Actuarial Journal*. Advance online publication.
  DOI: 10.1080/03461238.2015.1017527
- ▹ Jonas Alm and Filip Lindskog,
  "*Valuation of index-linked cash flows in a Heath-Jarrow-Morton framework*",
  Preprint.

Acknowledgements

As most good things, my time as a graduate student is about to come to an end. I would like to take this opportunity to express my gratitude to a number of people important to me.

To start with I would like to thank my advisors Filip Lindskog, Holger Rootzén and Gunnar Andersson for constant support, encouragement and inspiration during these years. Thank you Filip for a fruitful research cooperation, for helping me formalizing ideas, and for continuously pushing my level of thinking to a higher level. Thank you Holger for convincing me to apply for this position, for introducing me to the subject of mathematical statistics, and for your hospitality. Thank you Gunnar for being a great boss, for sharing your knowledge and experience from both the insurance industry and academia, and for helping me when I needed it. Moreover, I would like to thank my industry advisors Bengt von Bahr, Erik Elvers and Åsa Larson at Finansinspektionen, and Jesper Andersson at Folksam, for valuable input and many interesting discussions. Thanks also to Magnus Lindstedt at Folksam for inspiring asset management discussions.

I would further like to express my warm thanks to Mario Wüthrich and the other members of ETH Risklab for making the stay in Zürich a pleasure for my family and me. A special thank you to Galit Shoham for help finding an apartment.

Many thanks go to two people important to me during my time as a master's student. Thank you Christer Borell for advising my master's thesis and for numerous interesting investment strategy discussions. Thank you Allen Hoffmeyer for guiding me in Atlanta, and through Chung's book.

Thanks to my former office mates at Folksam: Jesper, Micke, Mårten, Erik, Lasse, Tomas, and the other actuaries. Thanks also to my current and former colleagues, senior faculty and fellow graduate students, at Chalmers and KTH: Magnus, Peter, Dawan, Hossein, and others.

Finally, thank you Linnéa, Vilhelm and Ludvig for your patience and support. You mean everything to me. I love you.

Jonas Alm
Göteborg, May 2015

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Part I

INTRODUCTION

Introduction

1. A first overview

Insolvency occurs when a company is unable to meet its financial obligations. A lack of liquidity to pay debts as they fall due is called cash-flow insolvency, and the event that the value of a company's liabilities exceeds the value of its assets is called balance-sheet insolvency or technical insolvency. This thesis concerns mathematical and statistical concepts used to assess the risk of balance-sheet insolvency, and hence could be used as tools for risk management decisions.

Given some valuation method and a pre-defined time period, the loss on an asset is the negative change in asset value over the period. The loss on a liability is the positive change in liability value over the period. The aggregate loss is the sum of losses on all individual assets and liabilities. For a future time period, the aggregate loss and the individual losses may be viewed as random variables. The distribution of the aggregate loss, in particlar the right tail of the distribution, determines the risk of insolvency.

The challenge in solvency modeling is to estimate the right tail of the aggregate loss distribution as well as possible. The first modeling step is to decide on valuation methods to use given some overarching valuation principle. For example, if market-consistent valuation is the principle, then the valuation method for assets (and liabilities) traded in deep and liquid markets is to observe market prices. For non-traded liabilities (and assets) a valuation method based on a subjective choice of state price deflator (stochastic discount factor) may be used. The second step is to decide on a segmentation of asset and liability classes that is optimal in some sense, and the third and final step is to model the marginal distributions and dependence structure of the losses on these classes.

Governments impose regulations on insurers in order to reduce their probability of insolvency and thereby protect the policyholders. Regulatory frameworks for insurers differs between jurisdictions. For example, the countries in the European Union are about to implement Solvency II, and Switzerland has the Swiss Solvency Test (SST). A short overview of the Solvency II framework is given in the end of this section. The remainder of the introduction is organized as follows. In Section 2, we define the one-period loss of an insurer and introduce the concept of risk measures. We explain how insurance liabilities can be valued using state price deflators in Section 3. Section 4 is devoted to construction and modeling of liability losses, and modeling dependence between asset and liability losses. Summaries of the papers included in this thesis are found in Section 5, and some final thoughts are given in Section 6.

1.1. Solvency II. The regulatory regime Solvency II will harmonize the solvency rules for insurers in the European Union. The Solvency II Directive, which is a recast of several EU directives [**15–17**], will enter into force on January 1, 2016. The Delegated Act of Solvency II [**14**] contains implementing rules that set out more detailed requirements for insurers. The Solvency II Directive replaces 14 existing EU directives commonly known as "Solvency I". While Solvency I focuses on the insurance risks on the liability side of the balance sheet and uses a crude volume-based capital requirement model, Solvency II takes a total balance sheet approach where all risks and their interactions are supposed to be considered. For a historical overview of the steps towards Solvency II, see [**27**].

The framework is divided into three areas, called pillars. Of main concern for this thesis is Pillar 1, which sets out quantitative requirements, including the rules to value assets and liabilities and to calculate capital requirements. Pillar 2 sets out requirements for risk management, governance and supervision, and Pillar 3 addresses transparency and disclosure.

The most well-known capital requirement level is the Solvency Capital Requirement (SCR) which "shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period" [**15**, Article 101]. The "basic own funds" are essentially the difference in value between the insurer's assets and liabilites. The other capital requirement level is the Minimum Capital Requirement (MCR) which is volume based and similar in structure to the capital requirement in Solvency I. There is a standard formula for SCR calculation in Solvency II, but insurers also have the possibility to develop and use their own (full or partial) internal models to calculate SCR. The SCR and MCR may be viewed as soft and hard levels of intervention. If an insurer breaches the SCR, the regulator will intervene to make sure that the insurer takes the appropriate actions to restore SCR. Breaching the MCR will trigger serious regulatory intervention and potential closure of the company.

The fundamental valuation principle in Solvency II is that all assets and liabilities on the insurer's balance sheet should be valued in a market-consistent way. This means that the value of an asset or a liability traded in a deep and liquid market is set to the price paid in the latest market transaction. However, liabilities arising from contractual obligations towards policyholders, known as technical provisions, are in general not traded. The directive states that "the value of technical provisions shall be equal to the sum of a best estimate and a risk margin", where "the best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money" and "the risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations" [**15**, Article 77].

In general, insurance risk cannot (and should not) be hedged. Thus, any model for insurance liability valuation must somehow include the market price of insurance risk. A mathematical interpretation of the above quotes is that the best estimate is calculated under the assumption that the market price of risk is zero.The risk margin is then calculated as the cost of holding the solvency capital until all claims are settled, with the cost-of-capital rate set to 6% (see [**14**, Articles 37–39] for details). The cost-of-capital rate is here interpreted as the expected return in excess of the risk-free rate an investor will require in order to take over the insurance obligations.

2. From ruin theory to risk measures

In this section we start from the classical Cramér-Lundberg model and arrive at a modern setup for solvency modeling based on one-period losses. The line of presentation is inspired by [**32**].

The theoretical foundation of collective risk theory, also known as ruin theory, was laid by Lundberg in his doctoral thesis [**23**]. The pioneering work of Lundberg was later treated with mathematical rigor by Cramér [**6**, **8**]. In the Cramér-Lundberg model, the surplus process $(X_t)_{t\geq 0}$ of the insurer is given by

$$X_t = x_0 + ct - \sum_{i=1}^{N_t} \eta_i, \quad t \geq 0,$$

where $x_0 \geq 0$ is the initial surplus, $c > 0$ is the premium rate, $(\eta_i)_{i\in\mathbb{N}}$ are strictly positive iid claim amounts with finite mean, and $(N_t)_{t\geq 0}$ is a homogeneous Poisson process. The claim amounts $(\eta_i)_{i\in\mathbb{N}}$ and the number of claims $(N_t)_{t\geq 0}$ are assumed to be independent, so $\left(\sum_{i=1}^{N_t} \eta_i\right)_{t\geq 0}$ is a compound Poisson process.

The main quantity of interest in the Cramér-Lundberg model is the probability of ultimate ruin,

$$\mathbb{P}\left(\inf_{t\geq 0} X_t < 0\right),$$

and there are many interesting mathematical results related to this probability. For example, the origin of the theory of large deviations (see [**29**]) is traced back to work by Esscher [**13**] and Cramér [**7**]. Ruin theory for heavy-tailed claim amount distributions is covered in e.g. [**10**, Chapter 1].

In practice, it is not possible to continuously assess the value of an insurer's surplus, so for statistical purposes we consider the discrete version of the ultimate ruin probability,

$$\mathbb{P}\left(\inf_{j\in\mathbb{N}} X_j < 0\right).$$

A modern interpretation is that x_0 represents the insurer's initial equity, i.e. the difference in value between assets and liabilities. Given that new equity may be injected by the owners of the insurance company at each time step it makes sense to study the one-period ruin probability,

$$\mathbb{P}(X_1 < 0).$$

In this modern interpretation, c includes not only the earned premium but also the return on assets over the period. We drop the compound Poisson process assumption, i.e. we allow $\sum_{i=1}^{N_1} \eta_i$ to have any distribution. Moreover, $\sum_{i=1}^{N_1} \eta_i$ is assumed to include not only new claims but also revaluation of existing, not yet completely settled, claims. Putting all this together, we get

$$X_1 = x_0 + c - \sum_{i=1}^{N_1} \eta_i = A_0 - L_0 + A_1 - A_0 - (L_1 - L_0) = A_1 - L_1,$$

where A_i and L_i, $i = 0, 1$, are the values of assets and liabilities, respectively, of the insurer at time i.

We define the one-period profit Y_1 as the change in equity over the period, i.e.

$$Y_1 = A_1 - L_1 - e^{r_0}(A_0 - L_0),$$

where r_0 denotes the one-period risk-free rollover at time 0. The discounted one-period loss Z_1 is defined by

$$Z_1 = -e^{-r_0} Y_1 = A_0 - L_0 - e^{-r_0}(A_1 - L_1).$$

Understanding the probability distribution of the insurer's loss Z_1 (or profit Y_1) is the key for both strategic business decisions and risk management.

2.1. Risk measures and capital requirements. A risk measure tries to summarize the risk of the entire probability distribution of the future equity value (or the change in equity value) in a single number. Let $\mathcal{V}$ be a linear vector space of random variables V representing values of the insurer's equity at time 1. A risk measure ρ is then defined as a mapping from $\mathcal{V}$ to $\mathbb{R} \cup \infty$. The quantity $\rho(V)$ is interpreted as the minimum amount of cash that needs to be added to the insurer's equity at time 0 in order to make the position with value V at time 1 acceptable. If no cash is needed, i.e. $\rho(V) \leq 0$, then the position is considered acceptable.

A risk measure ρ is called a monetary risk measure if it is:

(1) translation invariant, i.e. $\rho(V + ae^{r_0}) = \rho(V) - a$ for all $a \in \mathbb{R}$, and
(2) monotone, i.e. $V_2 \leq V_1$ implies that $\rho(V_1) \leq \rho(V_2)$.

It is called a coherent risk measure if it is also:

(3) positively homogenous, i.e. $\rho(aV) = a\rho(V)$ for all $a \geq 0$, and
(4) subadditive, i.e. $\rho(V_1 + V_2) \leq \rho(V_1) + \rho(V_2)$.

The notion of coherent risk measures was introduced in [**1**]. More about risk measures and their properties are found in [**21**, Chapter 6].

Any monetary risk measure ρ determines a capital requirement. Using the translation invariance property, we get

$$\rho(A_1 - L_1) = \rho(Y_1 + e^{r_0}(A_0 - L_0)) = \rho(Y_1) + L_0 - A_0.$$

The insurer's portfolio of assets and liabilities is accepted by the regulator if $\rho(A_1 - L_1) \leq 0$, which is equivalent to

$$A_0 \geq L_0 + \rho(Y_1),$$

i.e. today's value of assets must exceed today's value of liabilities by at least the capital requirement $\rho(Y_1)$.

The two most commonly used risk measures are Value-at-Risk (VaR) and Expected Shortfall (ES). Given a confidence level $p \in (0,1)$ and an equity value V at time 1, the (one-period) Value-at-Risk is defined by

$$\begin{aligned} \mathrm{VaR}_p(V) &= \min\{a : \mathbb{P}(ae^{r_0} + V < 0) \leq p\} \\ &= \min\{a : \mathbb{P}(Z > a) \leq p\} = F_Z^{-1}(1-p) \end{aligned}$$

where $Z = -e^{-r_0}V$ and $F_\cdot^{-1}$ denotes the quantile function. The (one-period) Expected Shortfall is defined by

$$\mathrm{ES}_p(V) = \frac{1}{p}\int_0^p \mathrm{VaR}_u(V)\,du = \frac{1}{p}\int_{1-p}^1 F_Z^{-1}(u)du.$$

Notice that

$$\begin{aligned} \mathbb{P}(X_1 < 0) < p \quad &\Leftrightarrow \quad \mathbb{P}(Z_1 \leq A_0 - L_0) \geq 1 - p \\ &\Leftrightarrow \quad A_0 \geq L_0 + F_{Z_1}^{-1}(1-p), \end{aligned}$$

so setting an upper bound of the one-period ruin probability is equivalent to choosing Value-at-Risk as risk measure.

In Solvency II, the choice of risk measure is VaR at level 0.005 over a one-year horizon (see [**15**, Article 101]). The solvency capital requirement is thus given by

$$\mathrm{SCR} = \mathrm{VaR}_{0.005}(Y_1) = F_{Z_1}^{-1}(0.995),$$

where $Z_1 = -e^{-r_0} Y_1$ is the (discounted) one-year loss. Since VaR at level 0.005 is the 0.995-quantile of the one-year loss distribution, the SCR in Solvency II is often referred to as "the Value-at-Risk at level 99.5%".

The risk measure used in the Swiss Solvency Test is ES at level 0.01 over a one-year horizon (see [**28**]), so the SCR is given by

$$\mathrm{SCR} = \mathrm{ES}_{0.01}(Y_1) = 100 \int_{0.99}^{1} F_{Z_1}^{-1}(u) du.$$

On the one hand, ES should be a better measure of risk than VaR since it takes the entire right tail of the one-period loss distribution into account. Moreover, ES is a coherent risk measure while VaR is not. On the other hand, there are very few data available on the extreme levels specified by the regulators, so in practice the statistical problem (more or less) boils down to determining the shape of the right tail, and the variance, of the loss distribution given a total number of, say, 20 observations.

3. The valuation framework

In this section we introduce the concept of state price deflators (stochastic discount factors), as presented in [**31**], to create valuation functionals for insurance liability cash flows. Moreover, we show how a state price deflator defines a risk-neutral probability measure. The existence of a risk-neutral measure is equivalent to absence of arbitrage opportunities in the market.

We consider a discrete-time setting given by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, where $\mathbb{F} = (\mathcal{F}_t)_{t=0,\dots,T}$, with $\mathcal{F}_t$ denoting the information available at time t. Here, $\mathbb{P}$ is the real-world probability measure under which all cash flows and price processes are observed, and the expectation operator with respect to $\mathbb{P}$ is denoted by $\mathbb{E}$. We let $P(t,u)$ denote the price at time t of a non-defaultable zero-coupon bond maturing at time $u \geq t$, with $P(u,u) = 1$ by convention, and $r_t = -\log P(t, t+1)$ denote the one-period risk-free rollover at time t.

Each asset or liability has a corresponding $\mathbb{F}$-adapted price process $(\xi_t)_{t=0,\dots,T}$, where ξ_t denotes the price at time t. For assets (or liabilities) traded in deep and liquid markets, we equate ξ_t with the market price at time t.

If no (deep and liquid) market exists, we view a liability (or an asset) as a stochastic cash flow $\boldsymbol{X} = (X_0, \dots, X_T)$, where X_t is the payment due at time t. We decide on a valuation functional Q_t that maps the cash flow to a market-consistent (liability) value at time t, and set $\xi_t = -\mathrm{Q}_t(\boldsymbol{X})$. In general, the market of insurance liabilities is incomplete which implies that there are infinitely many valuation functionals that allow for arbitrage-free pricing. One should aim for a valuation functional that correctly captures the risk appetite of the market participants, and yields a simple change of measure between the risk-neutral and the real-world probability measures.

3.1. State price deflators. A cash flow $\boldsymbol{X}$ is an $\mathbb{F}$-adapted random vector with integrable components, and we write $\boldsymbol{X} \in L^1(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$. A state price deflator $\boldsymbol{\varphi} = (\varphi_0, \dots, \varphi_T) \in L^1(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is a strictly positive random vector with normalization $\varphi_0 \equiv 1$. The component φ_t transports a random cash amount X_t at time t to a value at time 0.

The set of cash flows that can be valued relative to a given state price deflator $\boldsymbol{\varphi}$ is

$$\mathcal{L}_{\boldsymbol{\varphi}} = \left\{ \boldsymbol{X} \in L^1(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}) : \mathbb{E}\left(\sum_{t=0}^{T} \varphi_t |X_t| \middle| \mathcal{F}_0 \right) < \infty \right\},$$

and the value at time t of a cash flow $\boldsymbol{X} \in \mathcal{L}_{\boldsymbol{\varphi}}$ is defined by

$$\mathrm{Q}_t(\boldsymbol{X}) = \frac{1}{\varphi_t} \mathbb{E}\left(\sum_{u=0}^{T} \varphi_u X_u \middle| \mathcal{F}_t \right), \quad t = 0, \dots, T.$$

By the tower property of conditional expectation,

$$\begin{aligned} \mathbb{E}\left(\varphi_{t+1} \mathrm{Q}_{t+1}(\boldsymbol{X}) | \mathcal{F}_t\right) &= \mathbb{E}\left(\mathbb{E}\left(\sum_{u=0}^{T} \varphi_u X_u \middle| \mathcal{F}_{t+1} \right) \middle| \mathcal{F}_t \right) \\ &= \mathbb{E}\left(\sum_{u=0}^{T} \varphi_u X_u \middle| \mathcal{F}_t \right) = \varphi_t \mathrm{Q}_t(\boldsymbol{X}). \end{aligned}$$

So, the deflated price process $(\varphi_t \mathrm{Q}_t(\boldsymbol{X}))_{t=0,\dots,T}$ is a $(\mathbb{P}, \mathbb{F})$-martingale.

The cash flow corresponding to a zero-coupon bond maturing at time u consists of one single deterministic payment of size 1 at time u. Thus, for any state price deflator $\boldsymbol{\varphi}$, the condition

$$P(t, u) = \frac{1}{\varphi_t} \mathbb{E}(\varphi_u | \mathcal{F}_t), \quad t \le u,$$

must be fulfilled.

3.2. Equivalent martingale measure. The value B_t of one unit of the bank account at time t is given by

$$B_0 = 1 \quad \text{and} \quad B_t = \exp\left\{\sum_{s=1}^{t} r_{s-1}\right\}, \quad t \geq 1.$$

We define a probability measure $\mathbb{P}^*$, equivalent to $\mathbb{P}$, via the Radon-Nikodym derivative

$$\frac{d\mathbb{P}^*}{d\mathbb{P}}\Big|\mathcal{F}_t = \varphi_t B_t > 0.$$

We have

$$\mathbb{E}^*\left(B_{t+1}^{-1}\,\mathrm{Q}_{t+1}(\boldsymbol{X})\,|\mathcal{F}_t\right) = B_t^{-1}\varphi_t^{-1}\mathbb{E}(\varphi_{t+1}\,\mathrm{Q}_{t+1}(\boldsymbol{X})\,|\mathcal{F}_t) = B_t^{-1}\,\mathrm{Q}_t(\boldsymbol{X}),$$

(see e.g. [**31**, Lemma 11.3.]), i.e. $(B_t^{-1}\,\mathrm{Q}_t(\boldsymbol{X}))_{t=0,\dots,T}$ is a $(\mathbb{P}^*, \mathbb{F})$-martingale. The martingale measure $\mathbb{P}^*$ is often called a risk-neutral measure. According to the Fundamental Theorem of Asset Pricing, the existence of a risk-neutral measure $\mathbb{P}^*$ is equivalent to that the market is free of arbitrage (see e.g. [**9**, Section 1.6.] or [**19**, Theorem 5.16.]). In general, there exist more than one risk-neutral measure which implies that the market is incomplete.

A natural way to model state price deflators is to set up a model for the interest-rate dynamics under $\mathbb{P}^*$ such that it via a convenient change of measure yields interest-rate dynamics under $\mathbb{P}$ that are in line with historical observations of interest-rate changes. Typically, the change of measure corresponds to a Girsanov transformation, where the kernel can be interpreted as the market price of risk. In this case, the Radon-Nikodym derivative is completely determined by the market-price-of-risk function, which may be estimated from historical interest-rate changes. For an introduction to interest-rate modeling, see [**4**], [**18**] (continuous time) or [**31**] (discrete time).

3.3. Non-life insurance liabilities. Consider a fixed non-life insurance liability class (or line of business), and let $X_{i,j}$ denote the (incremental) claims payment for accident period i and development period j, i.e. the amount paid in accounting period $i + j$ for claims in accident period i. Moreover, let J denote the ultimate development period, i.e. $X_{i,j} = 0$ if $j > J$.

Accident period	Development period 0	1	$\cdots$	J
$-J+1$				$X_{-J+1,J}$
$-J+2$			$\iddots$	$X_{-J+2,J}$
$\vdots$		$\iddots$	$\iddots$	$\vdots$
1	$X_{1,0}$	$X_{1,1}$	$\cdots$	$X_{1,J}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
K	$X_{K,0}$	$X_{K,1}$	$\cdots$	$X_{K,J}$

TABLE 1. Future claims payments at time 0.

At time 0, the insurer's liability cash flow is $\tilde{\boldsymbol{X}} = (0, \tilde{X}_1, \ldots, \tilde{X}_T)$, where $T = J + K$, and

$$\tilde{X}_t = \sum_{i=-J+t}^{\min(t,K)} X_{i,t-i}, \quad t = 1, \ldots, T,$$

with K denoting the minimum integer greater than or equal to the maximum remaining lifetime of contracts written at time 0. Since most non-life insurance contracts have a lifetime of one year, K is often the number of periods in one year. The claims payments included in the cash flow $\tilde{\boldsymbol{X}}$ are shown in Table 1.

For a given state price deflator $\boldsymbol{\varphi}$, we have

$$\mathrm{Q}_s\left(\tilde{\boldsymbol{X}}\right) = \frac{1}{\varphi_s} \sum_{t=1}^{T} \mathbb{E}\left(\varphi_t \tilde{X}_t | \mathcal{F}_s\right), \quad s = 0, 1.$$

In the most general case we need a joint model for $\boldsymbol{\varphi}$ and $\tilde{\boldsymbol{X}}$ to calculate $\mathrm{Q}_s(\tilde{\boldsymbol{X}})$. Here, we consider the case $\tilde{X}_t = I_t Y_t$, $t = 1, \ldots, T$, where I_t is the value of an index at time t that may depend of $\boldsymbol{\varphi}$, and Y_t is a pure insurance risk independent of both the state price deflator and the index. The index could be, e.g., a consumer price index, a wage level index, or a claims inflation index.

We have

$$\begin{aligned}\frac{1}{\varphi_s} \mathbb{E}\left(\varphi_t \tilde{X}_t | \mathcal{F}_s\right) &= \frac{1}{\varphi_s} \mathbb{E}\left(\varphi_t I_t | \mathcal{F}_s\right) \mathbb{E}\left(Y_t | \mathcal{F}_s\right) \\ &= B_s \mathbb{E}^*\left(B_t^{-1} I_t | \mathcal{F}_s\right) \mathbb{E}\left(Y_t | \mathcal{F}_s\right),\end{aligned}$$

and hence,

$$\mathrm{Q}_s\left(\tilde{\boldsymbol{X}}\right) = B_s \sum_{t=1}^{T} \mathbb{E}^*\left(B_t^{-1} I_t | \mathcal{F}_s\right) \mathbb{E}\left(Y_t | \mathcal{F}_s\right), \quad s = 0, 1. \tag{1}$$

Thus, to value the cash flow we need both a method to calculate $\mathbb{E}\left(Y_t | \mathcal{F}_s\right)$, and a joint model for the bank account and the index under $\mathbb{P}^*$.

We set

$$\mathbb{E}\left(Y_t | \mathcal{F}_s\right) = \hat{Y}_t^{(s)}, \quad \text{with} \quad \hat{Y}_t^{(s)} := \sum_{i=-J+t}^{\min(t,K)} \hat{Y}_{i,t-i}^{(s)},$$

where $\hat{Y}_{i,j}^{(s)}$ is a prediction of the index-adjusted payment $Y_{i,j} = X_{i,j}/I_{i+j}$ at time s given some actuarial method, e.g. the chain-ladder method (see [**24**]) or the Bornhuetter-Ferguson method (see [**3**]) with some additional assumptions regarding future accident periods where no payments yet are made.

Notice that if there is a bond linked to the index, then

$$\frac{1}{B_s^{-1} I_s} \mathbb{E}^*\left(B_t^{-1} I_t | \mathcal{F}_s\right) = P_r(s,t),$$

where $P_r(s,t)$ denotes the price at time s of an index-linked zero-coupon bond maturing at time t. In this case,

$$\mathrm{Q}_s\left(\tilde{\boldsymbol{X}}\right) = I_s \sum_{t=1}^{T} P_r(s,t) \mathbb{E}\left(Y_t | \mathcal{F}_s\right), \quad s = 0, 1,$$

so there is no need for a joint model for the bank account and the index.

In Papers I and III, we assume independence between $\tilde{\boldsymbol{X}}$ and $\boldsymbol{\varphi}$, i.e. we set $I_t = 1$ for all t. Moreover, we assume a low-interest-rate environment and use the approximation

$$\mathrm{Q}_s\left(\tilde{\boldsymbol{X}}\right) \approx \sum_{t=1}^{T} \mathbb{E}\left(Y_t | \mathcal{F}_s\right), \quad s = 0, 1. \tag{2}$$

In Paper IV we model state price deflators and the market price of risk in a Heath-Jarrow-Morton (HJM) framework. Moreover, we give a suggestion of how to model the dependence between interest rates and the index under $\mathbb{P}^*$.

4. Dependence modeling

In this section we formulate the risk aggregation problem as the problem of statistically estimate (the tail of) the distribution of a sum of dependent random variables. Moreover, we explain how to construct time series of insurance liability losses from data, and formulate a stochastic model for one-period losses for the case when an index can be linked to the liability cash flow.

Assume that all assets but no liabilities of the insurer are traded in deep and liquid markets. Let d_A and d_L be the insurer's number of classes of assets and liabilities, respectively. Moreover, let A_t^ℓ denote the total market value of assets of class ℓ at time t, and let $\tilde{\boldsymbol{X}}^\ell$ denote the cash flow corresponding to liability class ℓ at time 0.

Then, the (discounted one-period) loss on asset class ℓ is

$$Z_{A,1}^\ell = A_0^\ell - e^{-r_0} A_1^\ell, \quad \ell = 1, \dots, d_A,$$

and the loss on liability class ℓ is

$$Z_{L,1}^\ell = e^{-r_0}\, \mathrm{Q}_1\left(\tilde{\boldsymbol{X}}^\ell\right) - \mathrm{Q}_0\left(\tilde{\boldsymbol{X}}^\ell\right), \quad \ell = 1, \dots, d_L.$$

The total loss Z_1 of the insurer may now be written

$$Z_1 = \sum_{\ell=1}^{d} Z_1^\ell,$$

where $d = d_A + d_L$ and

$$Z_1^\ell = \begin{cases} Z_{A,1}^\ell, & \text{if } 1 \le \ell \le d_A, \\ Z_{L,1}^{\ell - d_A}, & \text{if } d_A < \ell \le d. \end{cases}$$

The statistical challenge is to get the best estimate possible of the joint distribution of $Z_1^1, \dots, Z_1^d$. Given this joint distribution, the distribution of Z_1 follows directly. We must make sure that d_A and d_L are large enough to capture the essential parts of the insurance business, but not larger. Ideally, there should be independence between some disjoint sets of Z_1^ℓs that simplifies the dependence modeling.

For an introduction to the basic concepts of multivariate modeling, e.g. spherical and elliptical distributions, extreme value methods, and copulas, we refer to [**21**] and [**25**]. More about extreme value modeling is found in [**5**]. For a comprehensive view on regular variation and statistical inference for heavy tails, see [**26**]. Financial time series analysis and related stochastic processes are covered in [**10**, Chapter 7] and [**25**, Chapter 4]. Some recent interesting results on risk aggregation with dependence uncertainty are found in [2], [**11**] and [22].

4.1. Observing and modeling liability losses. It is a straightforward task to create time series of asset losses to analyze from historical market prices. The dependence between losses on a bond portfolio, a stock portfolio and a foreign interest-rate swap is studied in Paper II. However, to "construct" losses on the different liability classes (often chosen as the lines of business) we need both company internal data and a valuation functional. In Papers I and III, we construct normalized losses U^ℓ via

$$U^\ell = \frac{\mathrm{Q}_1\left(\tilde{\boldsymbol{X}}^\ell\right) - \mathrm{Q}_0\left(\tilde{\boldsymbol{X}}^\ell\right)}{\mathrm{Q}_0\left(\tilde{\boldsymbol{X}}^\ell\right)},$$

where Q_0 and Q_1 are given by the approximation in (2).

Now, suppose that $\tilde{X}_t^\ell = I_t^\ell Y_t^\ell$, where I_t^ℓ is the value of index ℓ at time t that may depend of $\boldsymbol{\varphi}$, and Y_t^ℓ is a pure insurance risk independent of both the state price deflator and the index. From (1) we get

$$\mathrm{Q}_s\left(\tilde{\boldsymbol{X}}^\ell\right) = B_s \sum_{t=1}^{T} \mathbb{E}^*\left(B_t^{-1} I_t^\ell | \mathcal{F}_s\right) \mathbb{E}\left(Y_t^\ell | \mathcal{F}_s\right), \quad s = 0, 1.$$

Thus,

$$Z_{L,1}^\ell = \sum_{t=1}^{T} \left(V_{1,t}^\ell W_{1,t}^\ell - V_{0,t}^\ell W_{0,t}^\ell\right)$$

where $V_{s,t}^\ell = \mathbb{E}^*\left(B_t^{-1} I_t^\ell | \mathcal{F}_s\right)$ and $W_{s,t}^\ell = \mathbb{E}\left(Y_t^\ell | \mathcal{F}_s\right)$.

The distribution of $Z_{L,1}^\ell | \mathcal{F}_0$ is determined by the distributions of $\boldsymbol{V}_1^\ell | \mathcal{F}_0$ and $\boldsymbol{W}_1^\ell | \mathcal{F}_0$, where

$$\boldsymbol{V}_1^\ell = (V_{1,1}^\ell, \dots, V_{1,T}^\ell) \quad \text{and} \quad \boldsymbol{W}_1^\ell = (W_{1,1}^\ell, \dots, W_{1,T}^\ell).$$

A joint model for the bank account and index ℓ yields the distribution of $\boldsymbol{V}_1^\ell | \mathcal{F}_0$. We give an example of such a model in Paper IV. The distribution of $\boldsymbol{W}_1^\ell | \mathcal{F}_0$ is given by the choice of stochastic claims reserving model (see e.g. [**12**] or [**30**]).

Losses on two liability classes, ℓ and m, may be dependent due to index or interest rate dependence, i.e. dependence between $\boldsymbol{V}_1^{\ell}|\mathcal{F}_0$ and $\boldsymbol{V}_1^{m}|\mathcal{F}_0$, or due to pure insurance risk dependence, i.e. dependence between $\boldsymbol{W}_1^{\ell}|\mathcal{F}_0$ and $\boldsymbol{W}_1^{m}|\mathcal{F}_0$. Moreover, for any ℓ, there may exist dependence between the index factor $\boldsymbol{V}_1^{\ell}|\mathcal{F}_0$ and losses on interest-rate sensitive assets, e.g. bonds and interest-rate swaps.

If the general price inflation in a region is not close to zero, we expect to see positive index dependence between many non-life liability classes. This is why the standard formula in Solvency II assumes positive correlations between the non-life modules (see [**14**, Annex IV]). Since the subjective part in any solvency calculation is the assumption about future claims or price inflation (see e.g. Papers I and IV), it is a good idea to make this assumption explicit in the model and separate it from the modeling of pure insurance risk dependence. A natural approach is to create two independent models: a joint model for the bank account and all indices that yields the distributions of $\boldsymbol{V}_1^{\ell}|\mathcal{F}_0$s, and a multivariate stochastic claims reserving model (see e.g. [**30**, Chapter 8]), with some additional assumptions regarding future accident periods, that yields the distributions of $\boldsymbol{W}_1^{\ell}|\mathcal{F}_0$s.

The findings in Paper III suggest that there exists pure insurance risk dependence between the Swedish LoBs Home and Motor Other, but there are no clear signs of dependence between other lines of business. Moreover, Paper I suggests that there is dependence between subclasses of the LoBs Motor Liability and Motor Other.

4.2. A simple example. In this example, we construct a simple model for the stochastic behavior of a non-life insurer's assets and liabilities. The aim is to show how model assumptions may induce interest-rate, index, and pure insurance risk dependence between losses for different lines of business.

We consider a yearly grid with time points $t = 0, 1, 2, 3$, where $t = 0$ is the current time. The non-life insurer has two short-tailed lines of business ($J = 1$ in both LoBs) and an asset portfolio that only consists of n zero-coupon bonds maturing at time 2. All insurance contracts have a lifetime of one year, so we set $K = 1$.

We assume a Ho-Lee framework (see [**20**]) with interest-rate dynamics given by

$$r_t = r_{t-1} + \theta_t + \sigma_r \epsilon_t^*, \quad t = 1, 2,$$

where $\epsilon_1^*|\mathcal{F}_0$ and $\epsilon_2^*|\mathcal{F}_1$ are independent standard normal random variables under $\mathbb{P}^*$, and θ_1 and θ_2 are known constants. We further assume that $\theta_1 = \theta_2 = -\frac{1}{2}\sigma_r^2$. With these dynamics, the one-year loss on the asset portfolio is given by

$$Z_1^1 = n(P(0,2) - e^{-r_0}P(1,2)) = ne^{-r_0}(e^{-r_0+\sigma_r^2} - e^{-r_1}).$$

For LoB ℓ, we have the cash flow $\tilde{\boldsymbol{X}}^\ell = (0, \tilde{X}_1^\ell, \tilde{X}_2^\ell, 0)$, where

$$\tilde{X}_1^\ell = I_1^\ell Y_1^\ell = I_1^\ell (Y_{0,1}^\ell + Y_{1,0}^\ell) \quad \text{and} \quad \tilde{X}_2^\ell = I_2^\ell Y_2^\ell = I_2^\ell Y_{1,1}^\ell.$$

We let

$$I_0^\ell = 1 \quad \text{and} \quad I_t^\ell = e^{q_0^\ell + \ldots + q_{t-1}^\ell}, \quad t \geq 1,$$

and assume that, for $t = 1, 2$,

$$q_{t-1}^\ell = r_t + d_t^\ell, \quad \text{with} \quad d_t^\ell = -\frac{1}{2}\sigma_d^2 + \sigma_d \delta_t^{\ell *},$$

where $(\delta_1^{1*}, \delta_1^{2*})|\mathcal{F}_0$ and $(\delta_2^{1*}, \delta_2^{2*})|\mathcal{F}_1$ are independent bivariate standard normal random vectors with correlation ρ_d under $\mathbb{P}^*$. If we further, still under $\mathbb{P}^*$, assume independence between the ϵ^* and the δ^*, we get

$$\begin{aligned}
\mathbb{E}^*\left(B_1^{-1} I_1^\ell | \mathcal{F}_0\right) &= 1, \qquad & \mathbb{E}^*\left(B_1^{-1} I_1^\ell | \mathcal{F}_1\right) &= e^{-r_0 + r_1 + d_1^\ell}, \\
\mathbb{E}^*\left(B_2^{-1} I_2^\ell | \mathcal{F}_0\right) &= 1, \qquad & \mathbb{E}^*\left(B_2^{-1} I_2^\ell | \mathcal{F}_1\right) &= e^{-r_0 + r_1 + d_1^\ell}.
\end{aligned}$$

Now, we consider the pure insurance risk. For $i = 0, 1$, let

$$Y_{i,1}^\ell = Y_{i,0}^\ell (f_i^\ell - 1), \quad \text{with} \quad f_i^\ell = f + \sigma_f \beta_i^\ell,$$

where $\beta_0^1|\mathcal{F}_0, \beta_0^2|\mathcal{F}_0, \beta_1^1|\mathcal{F}_1, \beta_1^2|\mathcal{F}_1$ are independent standard normal random variables under $\mathbb{P}$. Notice that the chain-ladder factors f_i^ℓ are independent under $\mathbb{P}$. Moreover, let

$$Y_{1,0}^\ell = \mu_Y + \sigma_Y \gamma^\ell,$$

where $\gamma^1|\mathcal{F}_0$ and $\gamma^2|\mathcal{F}_0$ are standard normal random variables with correlation ρ_Y under $\mathbb{P}$. Assuming independence between the β and the γ under $\mathbb{P}$, we get

$$\begin{aligned}
\mathbb{E}\left(Y_{0,1}^\ell | \mathcal{F}_0\right) &= Y_{0,0}^\ell (f - 1), \qquad & \mathbb{E}\left(Y_{0,1}^\ell | \mathcal{F}_1\right) &= Y_{0,0}^\ell (f_0^\ell - 1), \\
\mathbb{E}\left(Y_{1,0}^\ell | \mathcal{F}_0\right) &= \mu_Y, \qquad & \mathbb{E}\left(Y_{1,0}^\ell | \mathcal{F}_1\right) &= Y_{1,0}^\ell, \\
\mathbb{E}\left(Y_{1,1}^\ell | \mathcal{F}_0\right) &= \mu_Y (f - 1), \qquad & \mathbb{E}\left(Y_{1,1}^\ell | \mathcal{F}_1\right) &= Y_{1,0}^\ell (f - 1).
\end{aligned}$$

Thus,

$$\begin{aligned} Q_0\left(\tilde{X}^\ell\right) &= Y^\ell_{0,0}(f-1) + \mu_Y f, \\ e^{-r_0} Q_1\left(\tilde{X}^\ell\right) &= e^{-r_0+r_1+d^\ell_1}(Y^\ell_{0,0}(f^\ell_0 - 1) + Y^\ell_{1,0} f), \end{aligned}$$

and we get the losses

$$\begin{aligned} Z^2_1 &= e^{-r_0+r_1+d^1_1}(Y^1_{0,0}(f^1_0 - 1) + Y^1_{1,0} f) - Y^1_{0,0}(f-1) - \mu_Y f, \\ Z^3_1 &= e^{-r_0+r_1+d^2_1}(Y^2_{0,0}(f^2_0 - 1) + Y^2_{1,0} f) - Y^2_{0,0}(f-1) - \mu_Y f, \end{aligned}$$

for LoBs 1 and 2, respectively. We have index dependence due to the correlation between $d^1_1|\mathcal{F}_0$ and $d^2_1|\mathcal{F}_0$, and pure insurance risk dependence due to the correlation between $Y^1_{1,0}|\mathcal{F}_0$ and $Y^2_{1,0}|\mathcal{F}_0$. Moreover, we have interest rate dependence between all losses since r_1 is in the expressions for Z^1_1, Z^2_1 and Z^3_1.

All insurers in a market are subject to the same interest rates, and similiar levels of claims inflation in each common line of business. Thus, it is reasonable that all insurers use the same model for interest rates and indices, and their dependence structure. We discuss this further in Section 6.

5. Summary of papers

Here we give brief summaries of the contents of the papers in this thesis.

5.1. Paper I. In the first paper, *A simulation model for calculating solvency capital requirements for non-life insurance risk*, we construct a multidimensional simulation model that could be used to get a better understanding of the stochastic nature of insurance claims payments, and to calculate solvency capital requirements, best estimates, risk margins and technical provisions. The only model input is assumptions about distributions of payment patterns, i.e. how fast claims are handled and closed, and ultimate claim amounts, i.e. the total amount paid to policyholders for accidents occuring in a specified time period. This kind of modeling works well on lines of business where claims are handled rather quickly, say in a few years. The assumptions made in the paper are based on an analysis of motor insurance data from the Swedish insurance company Folksam. Motor insurance is divided into the three subgroups *collision*, *major first party* and *third party property insurance*. The data analysis is interesting in itself and presented in detail in Chapter 3 of the paper.

Some of the findings of Paper I are that: the multivariate normal distribution fitted the motor insurance data rather well; modeling data for each subgroup individually, and the dependencies between the subgroups, yielded more or less the same SCR as modeling aggregated motor insurance data; uncertainty in prediction of trends in ultimate claim amounts affects the SCR substantially.

5.2. Paper II. In the second paper, *Foreign-currency interest-rate swaps in asset-liability management for insurers*, co-authored with Filip Lindskog, we investigate risks related to the common industry practice of engaging in interest-rate swaps to increase the duration of assets. Our main focus is on foreign-currency swaps, but the same risks are present in domestic-currency swaps if there is a spread between the swap-zero-rate curve and the zero-rate curve used for discounting insurance liabilities.

We set up a stylized insurance company, where the size of the swap position can be varied, and conduct peaks-over-threshold analyses of the distribution of monthly changes in net asset value given historical changes in market values of bonds, swaps, stocks and the exchange rate. Moreover, we consider a 4-dimensional sample of risk-factor changes (domestic yield change, foreign-domestic yield-spread change, exchange-rate log return, and stock-index log return) and develop a structured approach to identifying sets of equally likely extreme scenarios using the assumption that the risk-factor changes are elliptically distributed. We define the *worst area* which is interpreted as the subset of a set of equally extreme scenarios that leads to the worst outcomes for the insurer.

The fundamental result of Paper II is that engaging in swap contracts may reduce the standard deviation of changes in net asset value, but it may at the same time significantly increase the exposure to tail risk; and tail risk is what matters for the solvency of the insurer.

5.3. Paper III. In the third paper, *Signs of dependence and heavy tails in non-life insurance data*, we study data from the yearly reports the four major Swedish non-life insurers (*Folksam*, *If*, *Länsförsäkringar* and *Trygg-Hansa*) have sent to the Swedish Financial Supervisory Authority (FSA).

The aim is to find the marginal distributions of, and dependence between, losses in the five largest lines of business. These findings are then used to create models for SCR calculation. We try to use data in an optimal way by defining an accounting year loss in terms of actuarial liability predictions, and by pooling observations from several companies when possible, to decrease the uncertainty about the underlying distributions and their parameters.

We find that dependence between lines of business is weaker in the FSA data than what is assumed in the Solvency II standard formula. We also find dependence between companies that may affect financial stability, and must be taken into account when estimating loss distribution parameters.

5.4. Paper IV. In the fourth paper, *Valuation of index-linked cash flows in a Heath-Jarrow-Morton framework*, co-authored with Filip Lindskog, we study valuation of index-linked cash flows under the assumption that the index return and the changes in nominal interest rates have significant dependence. The cash flows we consider are such that each payment is a product of two independent random variables: one is the index value and the other may represent pure insurance risk or simply a constant. Typically, the index is a consumer price index or a wage index, but the index returns could also be interpreted as claims inflation, i.e. increase in claims cost per sold insurance contract. Given a deep and liquid market of bonds linked to the same index as the cash flow, the natural market-consistent value of the cash flow would be a best estimate of the non-index factor times the market-implied price of an index-linked zero-coupon bond. Here we focus mainly on market-consistent valuation of index-linked cash flows when market-implied prices of index zero-coupon bonds are absent or unreliable.

We apply the valuation principles in [**31**] with the aim of setting up a credible valuation machinery for index-linked cash flows. The valuation formulas we derive allow us to understand how the volatility structure of the calibrated Heath-Jarrow-Morton model, the market-price-of-risk vector, the forecasts of trends in index values and interest rates, and the necessary modeling assumptions affect the value of an index-linked cash flow. The index we consider in the empirical analysis is the Swedish Consumer Price Index (CPI) which, for example, illness and accident insurance contracts often are linked to. Market prices of CPI-linked bonds offer possibilities to investigate how the market's anticipation of future price inflation can be understood in terms of our valuation machinery.

Our main contributions in this paper can be summarized as follows. Firstly, we present an approach, for assigning a monetary value to a stochastic cash flow, that does not require full knowledge of the joint dynamics of the cash flow and the term structure of interest rates. Secondly, we investigate in detail model selection, estimation and validation in an HJM framework. Finally, we analyze the effects of model uncertainty on the valuation of the cash flows, and also how forecasts of cash flows and interest rates translate into model parameters and affect the valuation.

6. Some thoughts about solvency modeling and regulation

We have seen in Section 4 that if claims inflation indices are available, then the modeling of interest rates and indices can be separated from the modeling of (index-adjusted) claims payments. In this case, the regulator could decide on a joint model for interest rates and indices with parameters specified so that $\mathrm{Q}_s(\tilde{\boldsymbol{X}})$ becomes the amount the fictive investor will require at time s in order to take over the liability cash flow $\tilde{\boldsymbol{X}}$. The only dependence left to model then is the dependence between insurance events, and this modeling could (and probably should) be left to the actuaries in the different companies. A qualified guess is that the linear correlation between most lines of business will be weak when payments are adjusted for claims inflation. However, there may exist extremal dependence due to catastrophes that somehow must be modeled.

One interesting future research problem is how data from different companies best could be used to construct claims inflation indices for the major lines of business. Another interesting problem is how to choose parameters so that the valuation functional corresponds to realistic assumptions regarding the investor's risk profile.

As a final note, I would like to emphasize that we should be careful so that the development of new valuation models does not lead to more securitization of insurance risk. It is of great importance for policyholder protection that the insurance risk stays in the insurance company where it can be (somewhat) regulated.

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