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Global spiral modes in stellar disks containing gas

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Summary. Much of the current discussion on the comparison between expected values of the velocity dispersion of stars (i.e. the Q parameter) in galaxy disks and the relevant observational data relies on theoretical work on the stability of infinitesimally thin one-component models. On the other hand a given model should be supplemented with studies of many physical effects that are often not directly included for simplicity, such as those associated with resonances and other stellar dynamical properties, finite thickness and disk-halo interaction. Here we contribute to this theoretical effort by a linear study of the role of *gas* on *global* spiral modes in stellar disks using a simple two-fluid model. The *local* destabilizing role of a cold component has been known for many years and calculated by various authors. We confirm the effectiveness of a small amount of gas in fueling spiral instabilities with a response which is primarily at short wavelengths by constructing a synthetic diagram which summarizes the local stability analysis. Then we proceed to consider *global* spiral modes in two-component systems with applications to regimes of astrophysical interest that are expected to be associated with normal spiral structure. Moderate growth spiral modes are found that are very close in structure to those of one-component systems, but for substantially higher values of the stability parameter Q. A simple model of self-regulation is also presented.

Key words: galaxies: kinematics and dynamics

1. Introduction

Recent progress in the study of the dynamics of disk galaxies is mostly based on the theory of infinitesimally thin one-component disks (see, e.g., Lin and Lau, 1979; Bertin, 1980; Toomre, 1981). These simplified basic states are to be considered as representative (equivalent) *models* of the real systems. Any resulting prediction should be carefully interpreted, since many physical effects, such as those associated with resonances and other stellar dynamical properties, finite thickness and disk-halo interaction, even though studied separately by various authors, are often not explicitly included for simplicity. This approach is quite natural and even a one-component fluid model, if properly modified and carefully interpreted, has proved to be very useful (see Lin and Bertin, 1985).

Following certain observational developments, renewed attention has been paid recently to one important feature of galaxy disks, i.e. the fact that (except for the rare case of smooth armed spirals) spiral galaxies are associated with the presence of cold interstellar gas, mostly detected as atomic hydrogen. *Local* analyses have long recognized the *dual* dynamical role of such a cold gas component. On the one hand cold material, even in small amounts, can significantly destabilize a stellar system (Lin and Shu, 1966; Lynden-Bell, 1967; Miller et al., 1970; Quirk, 1971; Jog and Solomon, 1984a, b; Sellwood and Carlberg, 1984). On the other hand cold gas can be shocked and can dissipate energy in such a way as to saturate otherwise exponentially growing spiral instabilities (Roberts, 1969; Kalnajs, 1972; Roberts and Shu, 1972). In this sense the gas has a damping role which can counteract and balance, at finite amplitudes, the excitation mechanisms of spiral structure. This latter effect is typically nonlinear (see remark by Shu, 1985). Thus the interstellar medium provides a welcome source of *self-regulation* for spiral instabilities, by inhibiting excessive heating in the stellar disk (cf. Ostriker, 1985; Lin and Bertin, 1985).

A complete understanding of the dynamics of stellar disks with gas would require a self-consistent global non-linear analysis of a two-component system, which is presently not available. While certain authors (Lubow et. al., 1986) have recently focused their efforts on the non-linear aspects of the problem by developing a full self-consistent *local* theory, in this paper we restrict our attention to the *linear* analysis but are able to offer a self-consistent theory of *global* spiral modes in two-component systems. It is interesting to note that so far both approaches rely on approximations that apply to tightly wound spirals and, therefore, they should be taken only as indicative of possible trends for late spirals (that are richer in gas but not consistent with the tightly wound limit). These studies might be generalized in the near future to the regime of more open spiral structure (see Bertin et al., 1984), but we now limit our conclusions to the case of early spirals.

Before starting our analysis, we briefly recall some observational studies that have attracted new interest in the dynamics of two-component systems. Relatively recent radio data have shown that a substantial fraction of the interstellar gas could be in the form of molecular hydrogen (see, e.g., Blitz and Shu, 1980). It is generally agreed that in the Milky Way, and also in some external galaxies, the distribution of molecular hydrogen exhibits a rather sharp peak, in a ring-like fashion, in contrast to the essentially flat distributions which characterize atomic hydrogen. However the exact values of the mass density of the molecular component are somewhat controversial (Blitz and Shu, 1980; Solomon, 1981;

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Young and Scoville, 1982a, b; Solomon et al., 1983; Sanders et al., 1984; Bhat et al., 1985; Young, 1985). The equivalent acoustic speed associated with the molecular component is known to be smaller than that relevant to the atomic hydrogen (which is in the range 8–10 km s^{-1}), but its value is also under debate. Proposed values for the effective velocity dispersion range from 4 km s^{-1} (Liszt and Burton, 1981) to 8 km s^{-1} (Stark, 1979). One theoretical problem motivated by the above mentioned observations is the formation of giant molecular clouds and complexes (Jog and Solomon, 1984a, b; Balbus and Cowie, 1985; see also Mouschovias et al., 1974; Blitz and Shu, 1980), but we will not pursue this line of research in the present paper.

Another source of interest in two-component systems derived from the recent progress in measuring the velocity dispersion of stars in galaxy disks. The level of random motions in the stellar component is expected to be determined by a process of self-regulation. Values of the dimensionless parameter Q larger than unity, as reported for NGC 488 by Kormendy (1985), are not at all surprising for a number of reasons (see Lin and Bertin, 1985). One important factor to keep in mind is that the presence of some cold material, even in small amounts, can indeed destabilize a stellar system so that higher values of the stellar velocity dispersion can be allowed for a given degree of stability with respect to spiral modes. One purpose of the present paper is to substantiate this point further, at the level of *global* spiral modes. However, even in gas-poor galaxies such as NGC 1553 and NGC 936 (Kormendy, 1984a, b), other factors can justify relatively high values of the observed Q. One such a factor, that we will not discuss in the present paper, is the different regime of spiral instabilities that occurs when the disk density is sufficiently high ($J \gtrsim 0.5$, see Bertin et al., 1984, Fig. 3).

2. Local properties of stellar disks with gas

In order to proceed with the global analysis of two-component systems, in this section we recover a number of results for the local analysis. Even if many of these are essentially known (Lin and Shu, 1966; Lynden-Bell, 1967; Graham, 1967; Kato, 1972; Jog and Solomon, 1984a, b), our contribution here is to clarify certain points by constructing a handy phase diagram (Fig. 4) and by discussing the relative response of the two components. For simplicity we adopt a two-fluid model, which we compare in a set of marginal stability curves with the more appropriate fluid-kinetic model of Lin and Shu (1966).

2.1. *Local dispersion relation*

We consider an infinitesimally thin, self-gravitating two-component disk system in differential rotation $\Omega(r)$. The two components are denoted by different labels, H and C, in order to recall that they are characterized by different equivalent acoustic speeds. Having in mind cases of astrophysical interest, we will refer to them as the stars (H) and the cold interstellar gas (C). However we could also consider the case where gas is absent but two stellar populations with different velocity dispersions can be identified. Each component is described by the standard Euler-continuity equations supplemented by a polytropic equation of state. The only interaction between the two components is taken to occur via the gravitational field (Poisson equation).

Linear perturbations of the spiral form

$$f_1 = \hat{f}_1 \exp\left[i\left(\int^r k\,dr' + m\vartheta - \omega t\right)\right],$$

with $m > 0$ (in this notation *trailing* disturbances are characterized by $k > 0$), on the axisymmetric basic state are studied under the ordering

$$\frac{m}{|k|\,r} \sim \varepsilon = \frac{c_{\rm H}}{r\kappa} \ll 1\,, \tag{1}$$

where $c_{\rm H}$ represents the radial velocity dispersion of the stars and κ is the epicyclic frequency. This is the approximation of tightly wound spiral structure which is expected to be applicable to early normal spirals (see discussion of regimes of spiral structure by Lin and Bertin, 1985; Bertin et al., 1984). To lowest order the density response for each component is given by

$$\sigma_{1j} = \sigma_j \frac{(k^2 \Phi_1/\kappa^2)}{\nu^2 - 1 - (k^2 c_j^2/\kappa^2)}, \tag{2}$$

where $j = {\rm H}, {\rm C}$, σ is the unperturbed disk density, c the equivalent acoustic speed, and $\nu = (\omega - m\Omega)/\kappa$ is the relative dimensionless frequency of the spiral perturbation. The perturbed potential Φ_1 obeys the (asymptotic) Poisson equation

$$-|k|\,\Phi_1 = 2\pi G\,(\sigma_{1\rm H} + \sigma_{1\rm C})\,. \tag{3}$$

Since we adopt an infinitesimally thin disk model, we should keep in mind, when dealing with observational implications, that the present discussion is restricted to relatively long waves, in the sense that when $|k\langle z\rangle| \gtrsim 1$ finite thickness ($\langle z\rangle$) effects should be considered. The *local dispersion relation* is derived by combining Eq. (2) with Eq. (3) (see, e.g., Jog and Solomon, 1984a). Here we write it in *dimensionless form:*

$$\hat{\nu}^4 + \hat{\nu}^2\,|\tilde{k}|\,[|\tilde{k}|\,(1+\beta) - (1+\alpha)] + |\tilde{k}|^3\,[\beta\,|\tilde{k}| - (\alpha+\beta)] = 0\,, \tag{4}$$

where $\tilde{k}$ and $\hat{\nu}$ are convenient dimensionless forms of the local radial wavenumber k and frequency, respectively:

$$\tilde{k} \equiv \frac{k}{\Sigma_{\rm H}}, \tag{5}$$

$$\hat{\nu}^2 \equiv \frac{Q_{\rm H}^2}{4}(1-\nu^2)\,, \tag{6}$$

$$\Sigma_{\rm H} \equiv \frac{2\pi G \sigma_{\rm H}}{c_{\rm H}^2}, \tag{7}$$

$$Q_{\rm H} \equiv \frac{c_{\rm H}\kappa}{\pi G \sigma_{\rm H}}. \tag{8}$$

Marginal stability is characterized by $\hat{\nu}^2 = Q_{\rm H}^2/4$. Thus the local dispersion relation depends on the two dimensionless parameters α and β which are defined as:

$$\alpha = \frac{\sigma_{\rm C}}{\sigma_{\rm H}} \tag{9}$$

$$\beta = \frac{c_{\rm C}^2}{c_{\rm H}^2}. \tag{10}$$

Roughly speaking α represents how much gas is locally present in the disk and β how cold it is. In the following we will take α, β to vary in the ranges $0 < \alpha < \infty$, $0 < \beta < 1$. The cases $\alpha = 0$ and $\beta = 1$ represent the limit of a one-component system. The parameter $Q_{\rm H}$ related to the hotter component is analogous to the Q-parameter for one-component systems, and $\Sigma_{\rm H}/2$ is analogous to $\Sigma/2$ which defines a typical wavenumber for tightly wound spiral modes. Finally we note that with a proper transformation of variables ($\hat{\nu}^2 \to -\tilde{k}^2\hat{\omega}^2$; $|\tilde{k}| \to \tilde{k}^2$) the discussion of this dispersion relation can be given in complete analogy with the case of the Jeans stability for a two-component three-dimensional homogeneous system in the absence of rotation.

2.2. Wave branches

For any fixed $|\tilde{k}|$ the local dispersion relation (4) is a quadratic algebraic equation in the variable $\hat{\nu}^2$. The two solutions, which are always real, will be labeled by the symbols (+) and (−) according to the property $\nu_-^2 > \nu_+^2$. Note that $\nu_-^2 > 1$. These two branches of the local dispersion relation merge only for $|\tilde{k}| \to 0$, where $\nu_-^2 \sim \nu_+^2 \to 1$. From a physical point of view this means that these wave branches are independent and do not interact with each other except, possibly, at the Lindblad resonances (where the present model is inadequate).

In order to study the relative response of the two components for a given perturbation, it is convenient to introduce the response function χ defined as

$$\chi = \chi(|\tilde{k}|) = \frac{\sigma_{1C}}{\sigma_{1H}} = \alpha \frac{\hat{\nu}^2 + \tilde{k}^2}{\hat{\nu}^2 + \beta \tilde{k}^2} \equiv \alpha \hat{\chi}. \tag{11}$$

In analogy with simple mechanical systems of two coupled pendula it is found that the two wave branches ν_-^2 and ν_+^2 have opposite parity. In particular, the branch ν_-^2 is *odd*, in the sense that $\chi_- < 0$, and ν_+^2 is *even*, as $\chi_+ > 0$. In addition, the odd branch is entirely dominated by the hotter component, because for any value of α, β we have $|\chi_-| \leqq 1$. When the colder component is heavier ($\alpha > 1$), it dominates entirely the even branch ($\chi_+ > 1$). Otherwise, for $\alpha < 1$, the colder component dominates the even branch only at short wavelengths $|\tilde{k}| \geqq \tilde{k}_0$, with $\tilde{k}_0 = 2(1-\alpha)/(1-\beta)$. (The decoupling of the two components can be noted directly by inspection of the dispersion relation). In Fig. 1 we illustrate these properties.

In Eq. (11) we have also introduced the reduced response function $\hat{\chi}$, which is independent of the values of the unperturbed densities. Thus the function $\hat{\chi}$ measures the relative response referred to the initial concentrations: $\hat{\chi} = (\sigma_{1C}/\sigma_C)/(\sigma_{1H}/\sigma_H)$, which we may call relative compressibility. In particular for the even branch ν_+^2 the colder component is more compressible than the hotter one at *any* wavelength and for *any* value of α, β, since $\hat{\chi}_+ > 1$.

Since the odd wavebranch is available only outside the Lindblad resonances ($\nu_-^2 > 1$), where a completely different analysis is required for a stellar system with gas, in the following we will focus our attention only on the even branch. This is characterized by $\nu_+^2 < 1$ in the wavenumber range $|\tilde{k}| < \tilde{k}_J = 1 + (\alpha/\beta)$. It is in this range that we can find, for a given value of α, β, conditions of local instability ($\nu_+^2 < 0$).

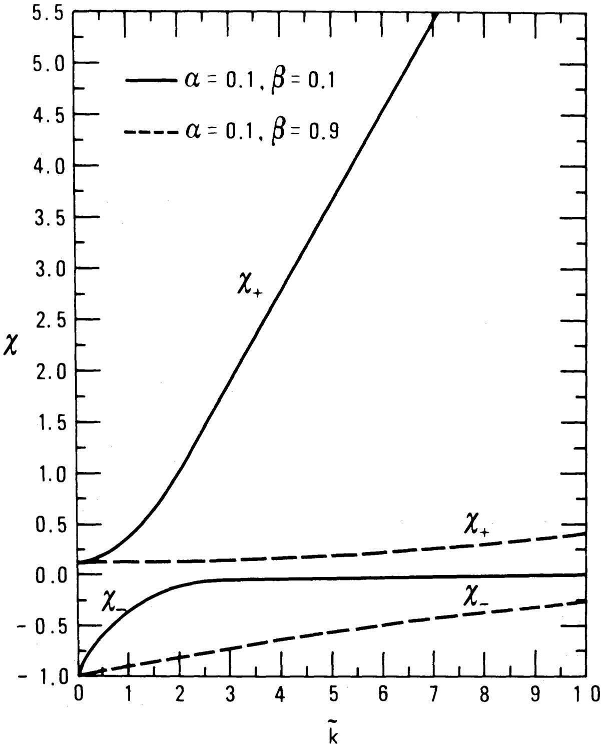

Fig. 1. Plots of the response function $\chi = \sigma_{1C}/\sigma_{1H}$, as a function of $\tilde{k}$, relative to the odd wave-branch (χ_-) and to the even wave-branch (χ_+) for two values of the local parameter β with fixed $\alpha = 0.1$. The wavenumber $\tilde{k}_0 \equiv 2(1-\alpha)/(1-\beta)$, beyond which the response of the colder component dominates the even branch ($\chi_+ > 1$), is $\tilde{k}_0 = 2$ in the case $\alpha = 0.1$ and $\beta = 0.1$, $\tilde{k}_0 = 18$ in the case $\alpha = 0.1$ and $\beta = 0.9$

2.3. Local stability

Setting $\hat{\nu}^2 = Q_H^2/4$, i.e. $\nu^2 = 0$, in the local dispersion relation (4), we obtain the conditions for marginal stability of our perturbations. Note that this can be seen as determining a value for $Q_H^2/4$, when α, β and $\tilde{k}$ have been fixed. This value is positive in the range $|\tilde{k}| < \tilde{k}_J$. From this relation a *local stability criterion* can be stated in analogy with the case of one-component systems. A function $\bar{Q}^2 = \bar{Q}^2(\alpha, \beta)$ can be defined in such a way that when $Q_H^2 \geqq \bar{Q}^2$ the system is locally stable at *all* wavelengths. The function $\bar{Q}^2$, which reduces to unity when $\alpha = 0$, will be discussed in this subsection. Here we note that for $Q_H^2 < \bar{Q}^2$ we expect the system to be locally unstable in ranges of wavelengths defined by the marginal stability condition.

In order to compare more easily the two-fluid model with the well-known one-fluid marginal stability curve, and with the fluid-kinetic results reported by Lin and Shu (1966) (see our Fig. 2), it is convenient to use the following, more standard scaling. We define

$$\bar{\lambda} = \frac{k_H}{|k|}, \tag{12}$$

where

$$k_H \equiv \frac{\kappa^2}{2\pi G \sigma_H}. \tag{13}$$

Then the *marginal stability curve* in the $(\bar{\lambda}, Q_H^2)$ plane is defined by the following relation:

$$Q_H^2 = \left(\frac{2\bar{\lambda}}{\beta}\right) \cdot [(\alpha+\beta) - \bar{\lambda}(1+\beta) + \sqrt{\bar{\lambda}^2(1-\beta)^2 - 2\bar{\lambda}(1-\beta)(\alpha-\beta) + (\alpha+\beta)^2}] \tag{14}$$

which we consider in the range $0 \leqq \bar{\lambda} \leqq 1 + \alpha$.

In Fig. 3 we show situations characterized by different values of the parameters α, β. Note that in certain cases the marginal stability curve exhibits two maxima. We shall refer to the maximum which occurs at short wavelengths as the *gaseous peak*, and to the maximum which occurs at intermediate wavelengths as the *stellar peak*. The gaseous peak can be very high at low values of β. This fact shows how effective a small amount of gas can be in destabilizing the disk, provided the gas is sufficiently cold.

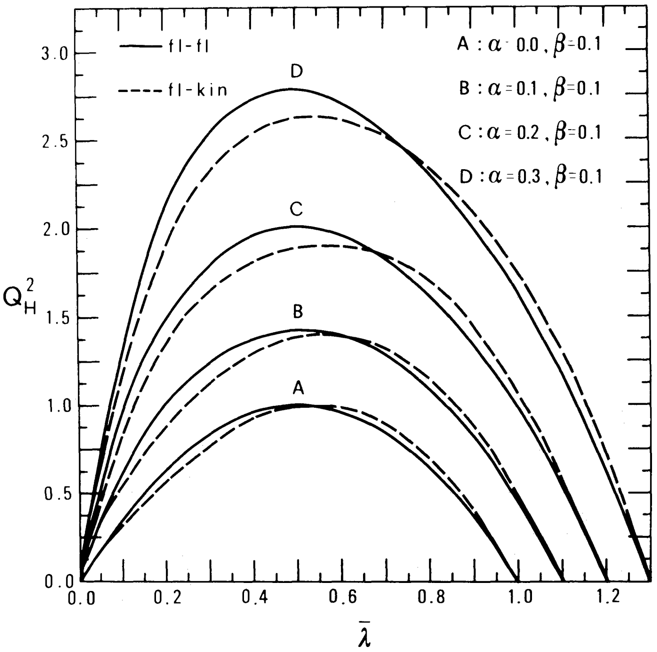

Fig. 2. Fluid-kinetic (readapted from Lin and Shu, 1966) and two-fluid marginal stability curves in the $(\bar{\lambda}, Q_H^2)$ plane for some values of the local parameter α, with fixed $\beta = 0.1$. Note that the peaks of the two-fluid curves are always higher and the corresponding wavelengths are always smaller than in the fluid-kinetic case

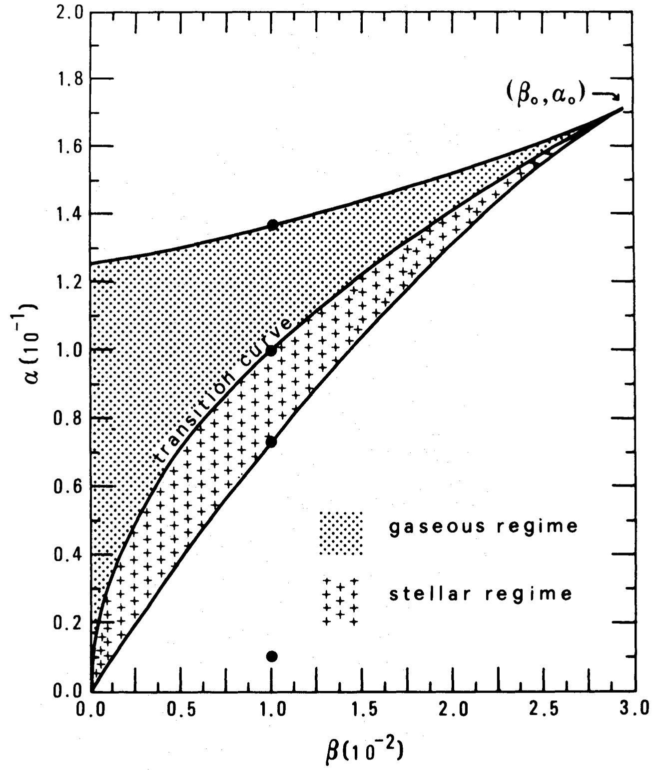

Fig. 4. Inside the two-phase region in the (β, α) plane, which is the triangular region with vertex (β_0, α_0), the marginal stability curve exhibits two maxima. In the *stellar (gaseous) regime* the stellar (gaseous) peak is dominant; the curve $\alpha = \sqrt{\beta}$ corresponds to the *transition* between these two regimes. A cusp formed by the intersection between the upper and lower boundaries of the two-phase region and the transition curve occurs at the "*triple point*" (β_0, α_0). Four points are marked corresponding to the cases illustrated in Fig. 3

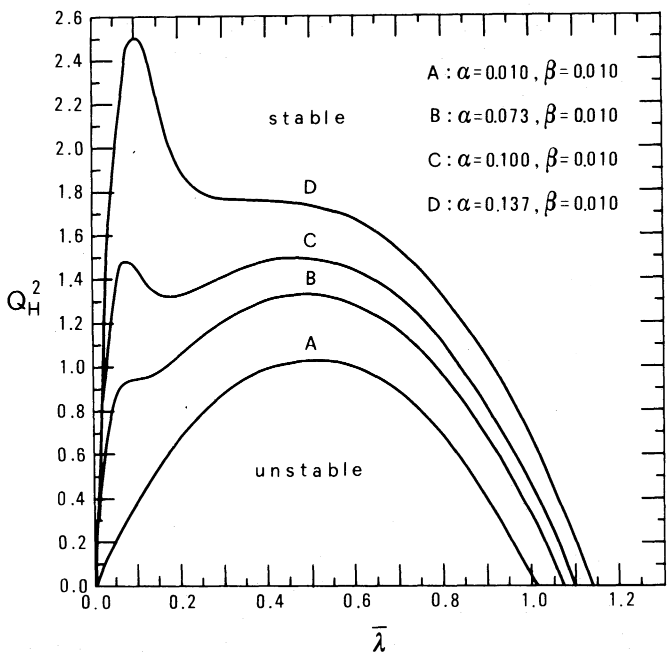

Fig. 3. Two-fluid marginal stability curves in the $(\bar{\lambda}, Q_H^2)$ plane for some values of the local parameter α and fixed $\beta = 0.01$. Note the quick appearance of the gaseous peak which occurs at short wavelengths $\bar{\lambda} \approx \frac{1}{2}\alpha$ and the smoother behavior of the stellar peak occurring at intermediate wavelengths $\bar{\lambda} \approx \frac{1}{2}$

In Fig. 4 we illustrate the *phase diagram* which summarizes the properties of the marginal stability curve (14). Inside a triangular region of the (β, α) plane that we will call *two-phase region* the marginal stability curve is characterized by the presence of two peaks, so that local stability properties are qualitatively different from those of single-component systems. Outside the two-phase region the marginal stability curve exhibits a single maximum.

The upper and the lower boundaries of the two-phase region join at a cusp (β_0, α_0) that we will call *triple point*:

$$\alpha_0 = 3 - 2\sqrt{2} \approx 0.172$$

$$\beta_0 = 17 - 12\sqrt{2} \approx 0.0294\,.$$

Correspondingly, the marginal stability curve exhibits a single flat maximum at $\bar{\lambda}_0 = 1 - (\sqrt{2}/2) \approx 0.293$ with $\bar{Q}^2 = 2$. Certain points that define the cases shown in Fig. 3 are marked in the phase diagram. The two-phase region is divided in two parts by the curve $\alpha = \sqrt{\beta}$ that defines the situation where the two peaks have the same height. Thus we will call the case $\alpha^2 > \beta$ the *gaseous regime*, since the value of $\bar{Q}^2$ is there determined by the gaseous peak. In contrast, in the *stellar regime* $(\alpha^2 < \beta)$ the stellar peak is higher and determines the marginal value $\bar{Q}^2$ for stability at all wavelengths. Therefore we expect a discontinuity in the behavior of the $\bar{Q}^2$-contours on this curve (see Fig. 5). This distinction of regimes may be extended to the part of parameter space outside the two-phase region, especially for $\beta < \beta_0$.

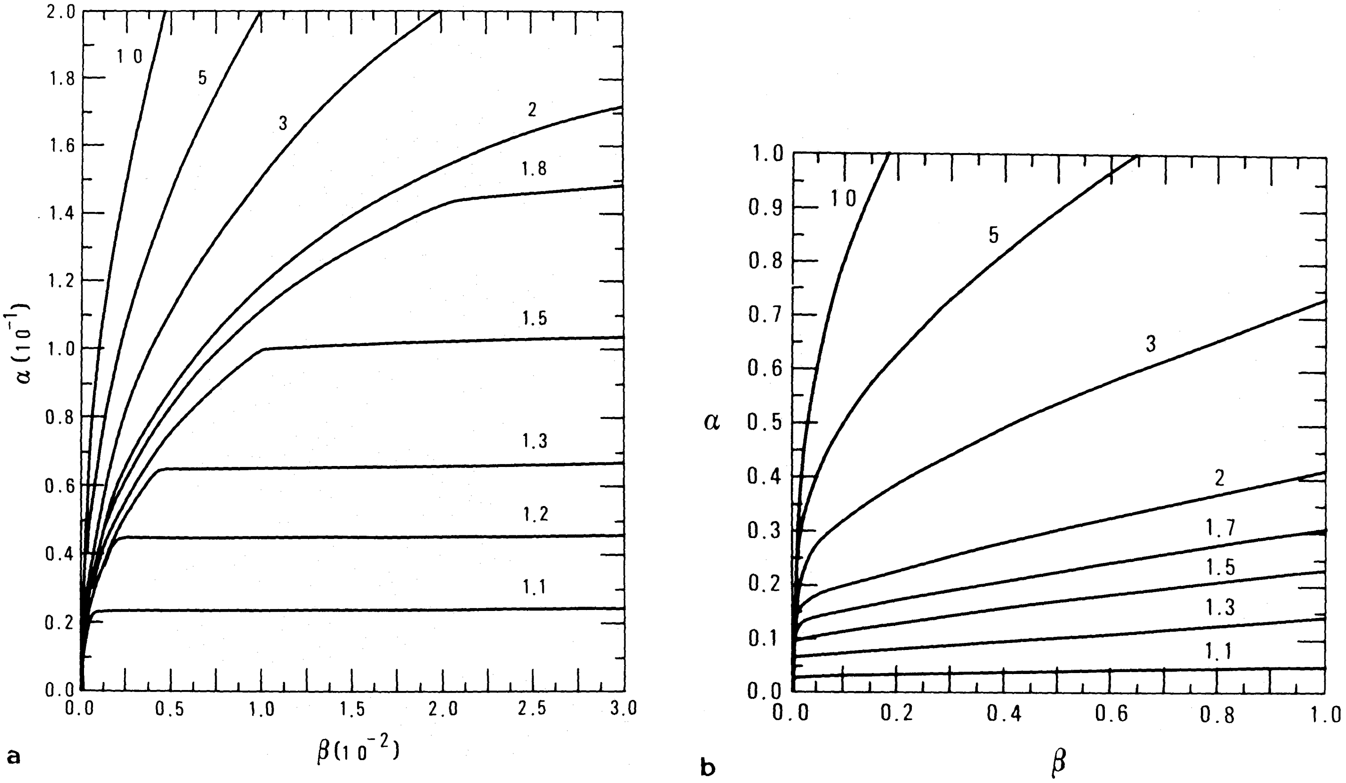

Fig. 5. a $\bar{Q}^2$-contours in the (β, α) plane, showing the transition inside the two-phase region and **b** the large-scale behavior in the ranges $0 < \alpha < 1$ and $0 < \beta < 1$. Each contour is labeled with the appropriate value of $\bar{Q}^2$. Note the flat behavior in the stellar regime

In the two-phase region we can apply the ordering $\alpha \ll 1$, $\beta = O(\alpha^2) \ll 1$ and derive approximate expressions for some interesting quantities. For example, the gaseous peak is located approximately at $\bar{\lambda} \sim \alpha/2$ with height $Q_H^2 \sim (\alpha^2/\beta) + 4\alpha$. Note the arbitrarily large values that are obtained for fixed values of α when $\beta \to 0$. The stellar peak occurs at $\bar{\lambda} = (1/2) + O(\alpha^2)$ with $Q_H^2 \sim 1 + 4\alpha$. The last expression for the height of the stellar peak holds even outside the two-phase region provided $\beta \sim \alpha \ll 1$ (see Lin and Bertin, 1985).

In Fig. 5 we illustrate the behavior of the function $\bar{Q}^2(\alpha, \beta)$ by providing contours of constant $\bar{Q}^2$ in the (β, α) plane. This function plays a crucial role in the discussion of global modes that we will give in the next section. We re-iterate that the condition $Q_H^2 > \bar{Q}^2$ replaces the well-known (see Toomre, 1964) condition $Q^2 > 1$ for one-component systems. These and other dispersion properties are discussed in detail by Romeo (1985), where some of the conclusions are generalized to the case of systems with many (more than two) components.

3. Discrete global spiral modes

In this section we will calculate some examples of discrete global spiral modes in stellar disks with gas, by applying asymptotic methods that have been used widely in the study of one-component systems (see Lin and Lau, 1979; Bertin, 1980). Our aim is to investigate this new case of two-component disks and to compare modes with moderate growth rates and the equilibria that support them. In particular we will show (i) to what extent the basic states must be changed, when some gas is added, in order to be subject to modes with a growth rate comparable to that of the no-gas case and (ii) whether modes with comparable growth rates change their morphology when the presence of gas is taken into account.

3.1. *Choice of the equilibrium models*

We have decided to base our investigations on the family of equilibrium disks called E3 models (Lowe, 1988). The reason for this choice is that these models were designed so as to include the key features of disk galaxies and their stability properties were accurately computed in an extensive numerical survey (Bertin et al., 1988). For the present purposes a detailed definition of the E3 models is not needed. The main properties of these equilibria are: (i) the mobile part of the basic state is an infinitesimally thin one-component disk with mass density $\sigma(r)$ which, outside a bulge region where it is cut, becomes exponential with a scalelength h. (ii) the rotation curve rises for $r < h$ and becomes asymptotically flat for $r \gtrsim h$. (iii) the additional mass that is required so support the rotation curve is taken to be distributed in a bulge-halo spheroid which does not participate in the spiral perturbations.

Within the family of E3 models we refer to one case which is representative of the regime of normal spiral structure (i.e. $J < 0.5$, see Bertin et al., 1984, Fig. 3). The particular model we refer to is illustrated in Fig. 6, where we also show the Q-profile which is consistent with the presence of a bulge in the inner regions and with a mechanism of self-regulation (see Lin and Bertin, 1985) in the active disk. We will call this model $E3_0$.

Now we modify $E3_0$ into a two-component model E3g with the same total mass distribution $\sigma(r)$ and with gas mass distribution $\sigma_C(r)$ so that:

$$\sigma_C(r) + \sigma_H(r) = \sigma_H(r)\,(1 + \alpha(r)) = \sigma(r)\,. \tag{15}$$

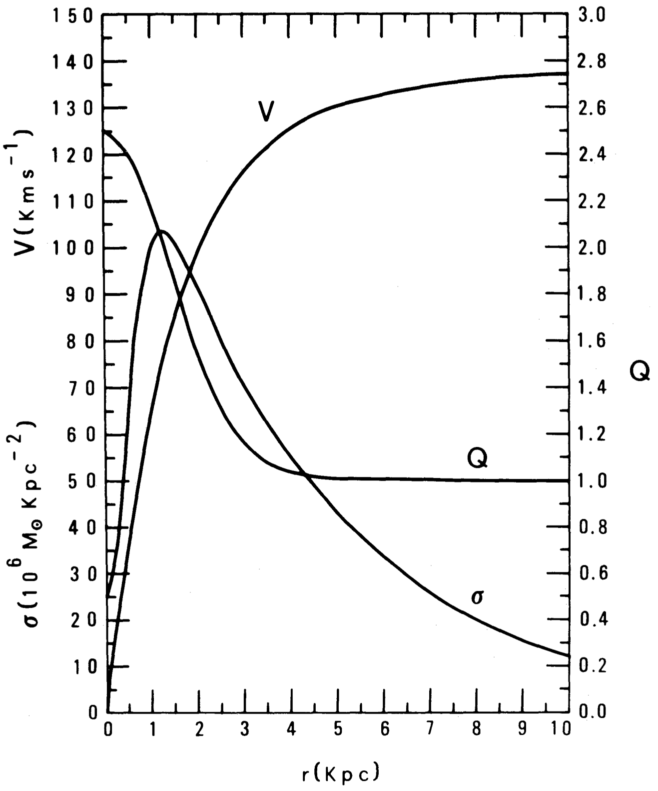

Fig. 6. Rotation curve, disk density distribution, and Q profile for the one-component equilibrium model $E3_0$. The rotation curve is supported by the combined effect of an active disk and of a spheroidal bulge-halo (the density distribution of the latter component is not shown here)

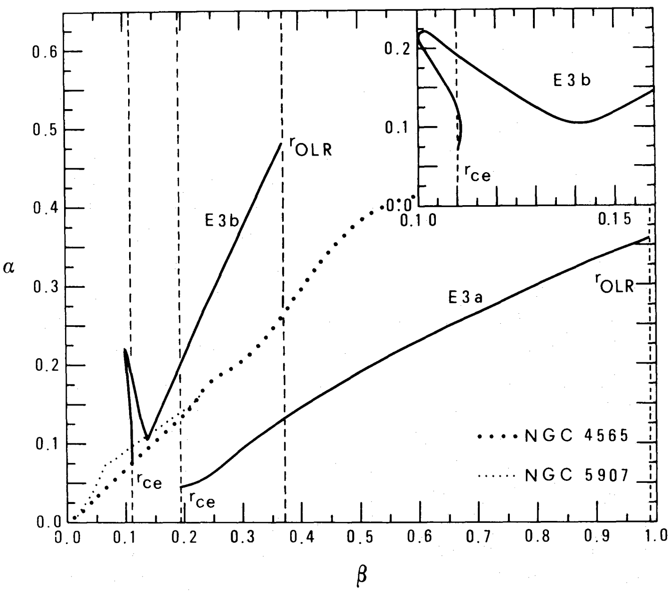

Fig. 7. (β, α) tracks for the galaxies NGC 4565, NGC 5907 and for the two-component equilibrium models E 3a and E 3b. *E3a* is characterized by a flat gas mass-distribution $\sigma_C = 3\ 10^6 M_\odot\ \mathrm{kpc}^{-2}$, simulating the atomic neutral hydrogen distribution, and by a constant equivalent acoustic speed $c_C = 8\ \mathrm{km\,s}^{-1}$. *E3b* is characterized by a gaussian ring overlapped to a flat background $\sigma_C = \{4 + 6 \exp[-4(\hat{r}-4)^2]\}\ 10^6 M_\odot\ \mathrm{kpc}^{-2}$ with $\hat{r} = r/1$ kpc, simulating the presence of a ring of molecular hydrogen, and by a constant equivalent acoustic speed $c_C = 6\ \mathrm{km\,s}^{-1}$

As for the velocity dispersion distributions, we refer for simplicity to a case where the *cold* component has an approximately *constant* equivalent acoustic speed c_C. In order to be consistent with processes of self-regulation, i.e. to keep a comparable level of stability with respect to spiral perturbations, we impose the following constraint:

$$Q_H(r) = Q(r)\,\bar{Q}(\alpha, \beta) \tag{16}$$

since for $\alpha, \beta \neq 0$ the stellar dispersion required for marginal stability is higher and governed by $\bar{Q}$. In Eq. (16) the profile of $\alpha = \alpha(r)$ is determined by our choice of $\sigma_C(r)$ and by Eq. (15). Then, for a given choice of the equivalent dispersion speed c_C, Eq. (16) determines the profile of the parameter $\beta = \beta(r)$. In fact, we can rewrite (16) as

$$\sqrt{\beta} = \frac{c_C \kappa}{\pi G \sigma} \frac{(1+\alpha)}{Q \bar{Q}(\alpha, \beta)}, \tag{17}$$

where κ, σ, and Q are given by the model $E3_0$. At each location r, once c_C, α are specified, we can determine the solution β by iteration, starting from

$$\hat{\beta} = c_C^2 \frac{\kappa^2}{(\pi G \sigma)^2} \frac{1}{Q^2}. \tag{18}$$

As a result of this discussion we see that *the model* E3g *is completely determined once the gas distributions* σ_C, c_C *are specified.* In order to make a realistic choice for these distributions we have considered profiles of the kind reported by studies of external galaxies, such as NGC 5907 (see Casertano, 1983) and NGC 4565 (Sancisi and Casertano, 1987), which suggest a relatively flat H I distribution. Then we have taken (i) the case of a flat σ_C (E 3a) and (ii) the case of a distribution with a (molecular) ring (E 3b; see discussion of H_2 distributions in the Sect. 1). In the first case a flat distribution of gas with $c_C = 8\ \mathrm{km\,s}^{-1}$ is taken, in the other a (gaussian) ring is added with $c_C = 6\ \mathrm{km\,s}^{-1}$. These being taken as representative plausible models, we did not try to model any specific galaxy. In Fig. 7 we show the (β, α) tracks for models of NGC 5907 and NGC 4565 provided for us by Casertano (private communication), where for simplicity we assume that the cold component is given by the H I distribution only (i.e. no molecular hydrogen is considered) with $c_C = 8\ \mathrm{km\,s}^{-1}$. On the same plot we consider the theoretical models E 3a, E 3b.

3.2. *Reference spiral modes in the one-component case*

For a given model the *local* dispersion relation can be used to calculate the *global* (discrete) spiral modes. This is obtained by imposing the following quantum condition (see, e.g., Lin and Lau, 1979; Bertin, 1980):

$$\oint k(r, \omega_R)\, dr = (2n+1)\pi \tag{19}$$

taken between the inner turning point r_{ce}, where the bulge terminates, and the corotation circle r_{co}. This equation fixes the pattern frequency of the n-th mode. The growth rate of the mode is inversely proportional to the propagation time τ taken along the relevant wave-cycle. Our $E3_0$ model supports a normal tight two-armed spiral mode with moderate growth rate, since the relevant parameter regime is that of domain A in Fig. 3 of Bertin et al. (1984). The propagation diagram $k = k(r)$ of the $(n=0,\ m=2)$ mode, which identifies the relevant wave-cycle and excitation mechanism, is essentially the same as that shown in Fig. 8a. A discussion of the mechanisms of excitation and maintenance of normal spiral modes of this kind has been given on several

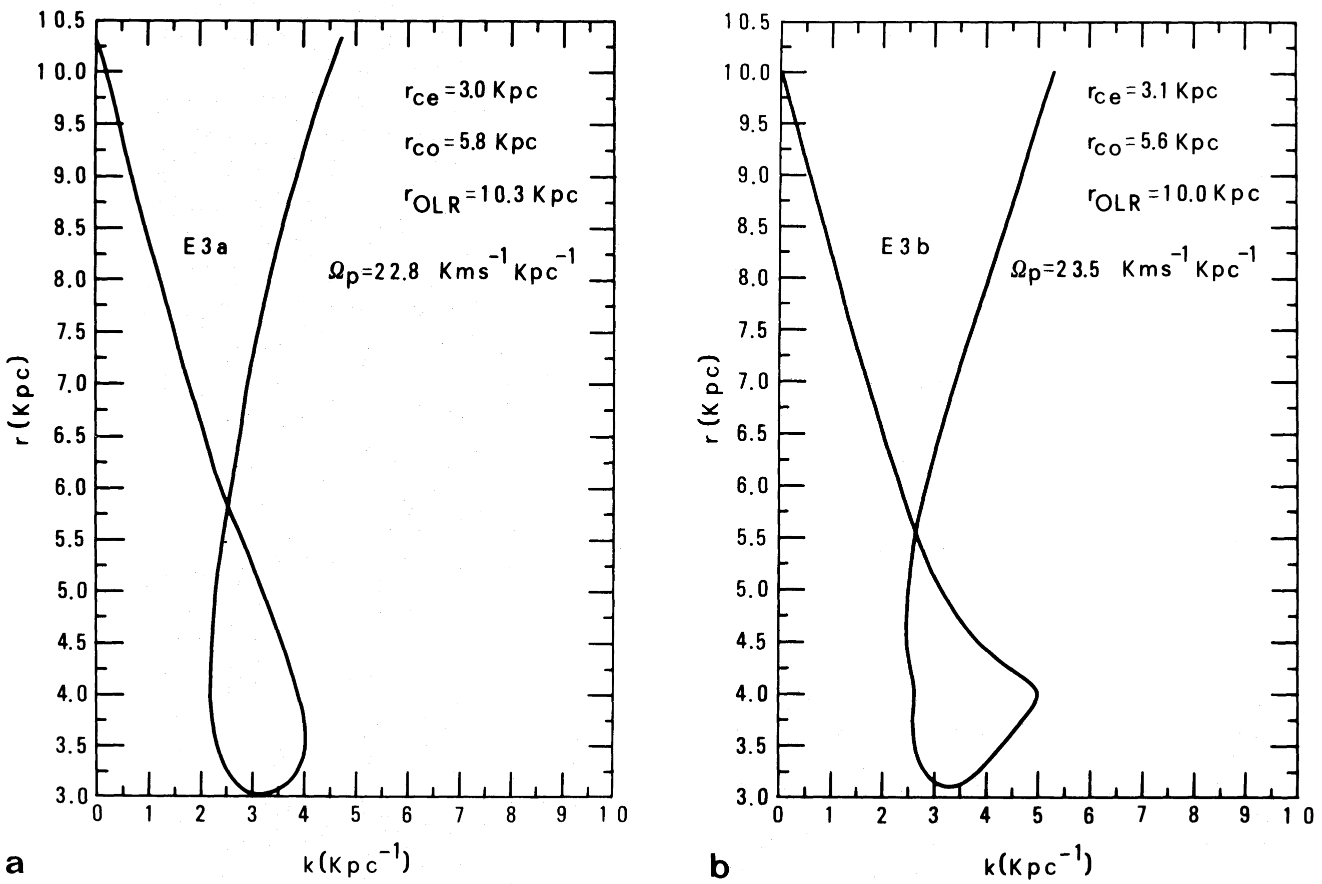

Fig. 8. **a** Propagation diagrams relative to the mode ($n=0$, $m=2$) for the two-component equilibrium models E3a and **b** E3b, with the indication of the corresponding bulge radius r_{ce}, corotation circle r_{co}, location of the outer Lindblad resonance r_{OLR} and pattern frequency Ω_p. The propagation diagram of **a** essentially coincides with that obtained for the one-component equilibrium model E3_0. The distortion in **b** which affects appreciably the short-wave branch is due to the presence of the molecular ring. Thus, when gas is included, there is a shift towards shorter wavelengths and the size of the pattern, as measured by the radial range of the loop, shrinks

occasions and we refer the reader to the literature cited. We only note that in this regime of low disk mass (i.e. of low J) the approximations we made in Sect. 2 in deriving the local dispersion relation are reasonably justified.

3.3. Global modes in stellar disks with gas

At this stage it is very easy, at least for parameter regimes well outside the two-phase region of Fig. 4 (see further discussion at the end of this paragraph), to proceed to calculate global spiral modes in our two-component models. We have computed the *new* eigenvalues of the E3a and E3b models for the $n=0$ mode by calculating the propagation diagram $k=k(r)$ following the local dispersion relation (4), and by adjusting slightly the value of the pattern frequency in order to satisfy the quantum condition (19). The resulting propagation diagrams for the two cases are shown in Fig. 8a and b.

It is easily recognized that the eigenvalue and the structure of the mode are qualitatively unchanged. For the case with a cold ring, some modifications are present and affect mostly the short wave branch. But in general we expect only a slight shift of the mode to a tighter spiral with a smaller corotation radius. The reason for this modest qualitative change is obviously related to two facts: (i) the self-regulation constraint (16) helps the propagation diagrams to stay close to each other because a local condition of marginal stability is imposed. (ii) the (β, α) tracks of the basic states stay away from the two-phase triangular region of Fig. 4; since the basic models lie well outside, within the stellar regime, only modest quantitative and no significant qualitative changes are expected and found. In fact, the present simple analysis would not hold for parameter regimes closer to or inside the two-phase region of Fig. 4, since more complicated wave channels and wave cycles would be available in this latter case. More comments and details are reported by Romeo (1985).

The present analysis provides an important message. The basic states E3a and E3b that support these modes are characterized by relatively high stellar velocity dispersions. The profiles of the "stellar Q-parameter" are shown in Fig. 9. One easily notes that Q_H lies above unity everywhere even for this regime of low disk density. Still the models allow for spiral modes with moderate growth rates, comparable to that of E3_0. Obviously this is made possible by the presence of the small amount of gas. Interestingly enough, the qualitative behaviour of Q_H for E3a resembles that inferred by Casertano (private communication) for cases like NGC 4565 and NGC 5907 in his dynamical models of these galaxies.

3.4. A simple model for self-regulation

In this paper we have often referred to a mechanism of self-regulation as an important feature of galactic disks. In particular,

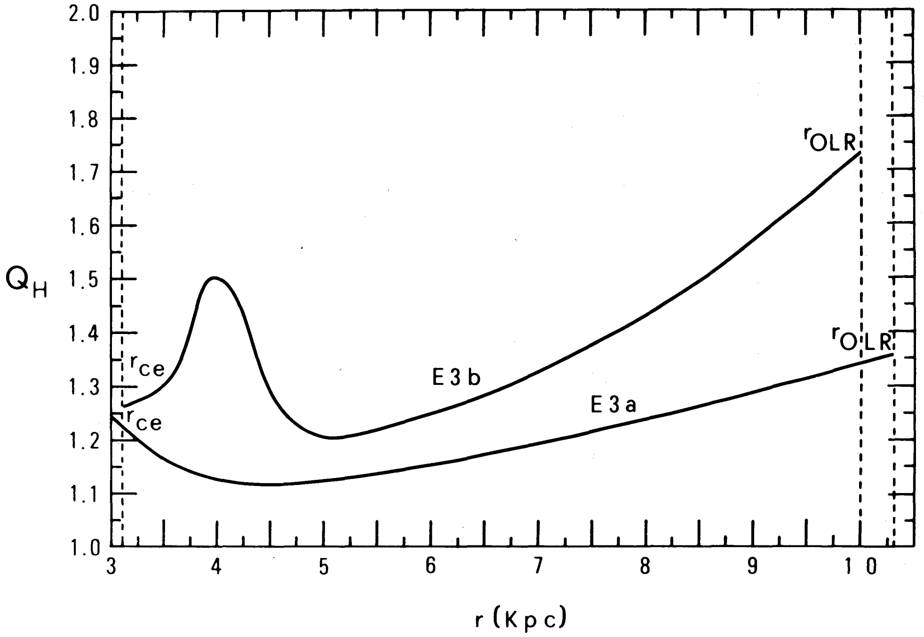

Fig. 9. The stability parameter Q_H for the two-component equilibrium models E3a and E3b. These profiles have been derived by imposing the condition $Q_H(r) = Q(r)\,\bar{Q}(\alpha, \beta)$, $Q(r)$ being specified in the basic one-component model $E3_0$

we have imposed it [Eq. (16)] in order to justify our choice of equilibrium models for the global mode calculation. In closing this article we would like to be more specific about self-regulation. This short discussion will also show how crucial is the role of gas in the physical picture of spiral galaxies.

We would like to construct a simple mathematical model to describe the following facts: (i) In the disk the *stellar component* tends to heat up via gravitational instabilities and possibly because of interaction with giant molecular clouds. The collective heating mechanisms are quite sensitive to the value of the *effective Q*-parameter, i.e. to the ratio $Q_H/\bar{Q}(\alpha, \beta)$. No obvious cooling mechanisms are at hand. (ii) The *gas component* tends to cool off on a short time-scale because of turbulent dissipation (inelastic cloud-cloud collisions)[1]. It also suffers heating via the *same* gravitational instabilities mentioned in (i), but at a faster rate because of the stronger reaction of the (thinner) gas component. (iii) Cooling is a source of dynamical instabilities, thus it generates heating, ensuring self-regulation. Therefore a schematic simple model can be formulated:

$$\frac{d \ln Q_H}{dt} = f_H \gtrsim 0\,, \tag{20}$$

$$\frac{d \ln c_C}{dt} = -g + f_C\,, \tag{21}$$

where $f_C > f_H$ represent heating rates which are rapid functions of $(1-Q)$, so that for $Q > 1$, $f_H \sim f_C \sim 0$ and for $Q < 1$, f_C and f_H are very large. The g-term is large and essentially Q-independent: it represents turbulent dissipation, which is expected to act on a *short* time-scale. On this short time scale, c_C reaches a quasi-steady state determined by the near balance of g anf f_C; the small difference between these terms induces a slow evolution described below. Given this physical picture we should check what are the implications on the quantities β and $\bar{Q}(\alpha, \beta)$ which describe the stability properties of the system. In this context α can be taken to be a constant. We have:

$$\frac{d \ln Q}{dt} = \frac{d \ln Q_H}{dt} - \frac{d \ln \bar{Q}}{dt}\,, \tag{22}$$

$$= \frac{d \ln Q_H}{dt} + \frac{|h|}{2}\frac{d \ln \beta}{dt}\,, \tag{23}$$

$$= (1-|h|)\, f_H + |h|\,(-g + f_C)\,, \tag{24}$$

where we have defined

$$h = 2\left(\frac{\partial \ln \bar{Q}}{\partial \ln \beta}\right)_\alpha < 0\,. \tag{25}$$

We notice that a qualitative idea of this gradient is provided by Fig. 5. In general for interesting values of α, β, $|h| < 1$, but it is not small.

From Eq. (24) the mechanism of self-regulation is apparent, due to the strong dependence of the heating functions on the value of $(1-Q)$ which implies $d \ln Q/dt \sim 0$. Thus

$$\frac{d \ln \beta}{dt} \simeq -\frac{2}{|h|}\frac{d \ln Q_H}{dt} < 0\,. \tag{26}$$

Therefore, the stellar component continually heats. In turn β decreases in the process and the system moves on a horizontal line to the left of the (β, α) plane. Depending on the initial value of α the system may meet the sudden experience of a "phase transition" by encountering the two-phase region of Fig. 4, or for $\alpha > \alpha_0$ it can move towards a gas-dominated regime more smoothly directly from the original star-dominated case. We note that these processes of β-evolution are expected to occur secularly on a long time-scale. A slow decrease of α could also be included in order to describe the possible evolution of the Population I disk.

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[1] In reality a fraction of the cold component, which is here referred to as "gas component", is stellar and thus not subject to cooling. This fraction slowly increases in time as a result of star formation processes. The following presents just a simplified discussion of the mechanism of self-regulation

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Note added in proof. Regarding the schematic model of self-regulation described in Sect. 3.4, we should stress that the focus of this paper is on the destabilizing role of the gas and on the choice of the basic state as determined by the *long-term* evolution of galactic parameters. On the other hand, as we mentioned in Sect. 1, the gas also has a damping role on the mode amplitude, because of phase shifts induced by density-wave shocks. The two roles are not unrelated in the overall self-regulation of the system. In fact, the source terms appearing in Eqs. (20) and (21) depend on the mode amplitude. Thus Eqs. (20) and (21) should be supplemented by an equation for the mode amplitude A of the form $dA/dt = \gamma A - S$, which determines the amplitude A on the *intermediate* time scale of wave-cycles, where S is a non-linear function of A representing the damping of the shocks. Note that even γ would be A-dependent, so as to take into account intrinsic non-linearities. Of course, a more complete analysis of self-regulation should also include spatial variations (see Shu, 1985) that are not considered here for simplicity. We thank Frank Shu for reminding us of the importance of these points