High-SNR Capacity of Multiple-Antenna Phase-Noise Channels with Common/Separate RF Oscillators

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***Abstract*—In multiple antenna systems, phase noise due to instabilities of the radio-frequency (RF) oscillators, acts differently depending on whether the RF circuitries connected to each antenna are driven by separate (independent) local oscillators (SLO) or by a common local oscillator (CLO). In this paper, we investigate the high-SNR capacity of single-input multiple-output (SIMO) and multiple-output single-input (MISO) phase-noise channels for both the CLO and the SLO configurations.**

Our results show that the first-order term in the high-SNR capacity expansion is the same for all scenarios (SIMO/MISO and SLO/CLO), and equal to $0.5\ln(\rho)$, where ρ stands for the SNR. On the contrary, the second-order term, which we refer to as phase-noise number, turns out to be scenario-dependent. For the SIMO case, the SLO configuration provides a diversity gain, resulting in a larger phase-noise number than for the CLO configuration. For the case of Wiener phase noise, a diversity gain of at least $0.5\ln(M)$ can be achieved, where M is the number of receive antennas. For the MISO, the CLO configuration yields a higher phase-noise number than the SLO configuration. This is because with the CLO configuration one can obtain a coherent-combining gain through maximum ratio transmission (a.k.a. conjugate beamforming). This gain is unattainable with the SLO configuration.

I. Introduction

Phase noise due to phase and frequency instability in the local local radio-frequency (RF) oscillators used in wireless communication links results in imperfect synchronization between transmitters and receivers, which degrades the system throughput [1]–[4], especially when high-order modulation schemes are used to support high spectral efficiency.

A fundamental way to assess the impact of phase noise on the throughput of wireless links is to determine the corresponding Shannon capacity. Unfortunately, a closed-form expression for the capacity of wireless channels impaired by phase noise is not available (although it is known that the capacity-achieving distribution has discrete amplitude and uniform independent phase when the phase-noise process is stationary and memoryless, with uniform marginal distribution over $[0, 2\pi)$ [5]). Nevertheless, both asymptotic capacity characterizations for large signal-to-noise ratio (SNR) and nonasymptotic capacity bounds are available in the literature. Specifically, Lapidoth [1] characterized the first two terms in the high-SNR expansion of the capacity of a single-input single-output (SISO) stationary phase-noise channel. Focusing on memoryless phase-noise channels, Katz and Shamai [5] provided upper and lower bounds on the capacity that are tight at high SNR. The results in [1], [5] have been generalized to block-memoryless phase-noise channels in [6], [7]. Numerical methods for the calculation of the information rates achievable with specific modulation formats have been proposed in, e.g., [8]–[10].

In multiple-antenna systems, phase noise acts differently depending on whether the RF circuitries connected to each antenna are driven by *separate* (independent) local oscillators (SLO) or by a *common* local oscillator (CLO). Although the CLO configuration is intuitively more appealing because it results in a single phase-noise process to be tracked, the SLO configuration is unavoidable when the spacing between antennas needed to exploit the available spatial degrees of freedom, and, hence, achieve multiplexing or diversity gains, is large [11], [12]. This occurs for example in multiple-antenna line-of-sight microwave backhaul links operating in the 20–40 GHz frequency band, where the spacing between antennas required to exploit the available spatial degrees of freedom can be as large as few meters [4]. In large-antenna-array systems [13]–[15], cost and packaging considerations may also make the SLO configuration attractive.

For the CLO configuration, a high-SNR capacity expansion together with finite-SNR capacity upper and lower bounds have been recently reported in [16], [4]. For both the CLO and the SLO configurations, the multiplexing gain was partly characterized in [17]. In [18], [15], [19], lower bounds on the sum-rate capacity for the case when multiple single-antenna users communicate with a base station equipped with a large antenna array (uplink channel) have been developed for both CLO and SLO. These bounds suggest that the SLO configuration yields a higher sum-rate capacity than the CLO configuration. However, it is unclear whether these lower bounds are tight.

Contributions: We consider the scenario where a multiple-antenna base station communicates with a single-antenna user over an AWGN channel impaired by phase noise and study the first two terms in the high-SNR capacity expansion, for both the uplink (SIMO) and the downlink (MISO) channel, and for both CLO and SLO. We characterize the first term and provide bounds on the second term that are tight for some phase-noise models of practical interest. Our findings are as follows. The first-order term in the high-SNR capacity expansion turns out to be the same in all four scenarios, and equal to $0.5\ln(\rho)$, where ρ stands for the SNR. In contrast, the second-order term, which we

This work was partly supported by the Swedish Foundation for Strategic Research under grant SM13-0028.

denote as *phase-noise number*, takes different values in the four cases. For the uplink channel, the SLO phase-noise number is larger than the CLO one. Intuitively, this holds because the SLO configuration provides a diversity gain. For the specific case of Wiener phase noise [20], we show that a diversity gain of at least $0.5 \ln M$, where M is the number of receive antennas, can be achieved. This result provides a theoretical justification of the observation reported in [18], [15], [19] that SLO yields a higher sum-rate capacity than CLO for the uplink channel.

For the downlink channel, the ordering is reversed: the CLO configuration results in a higher phase-noise number than the SLO configuration. Coarsely speaking, this holds because CLO allows for maximum-ratio transmission (also known as conjugate beamforming), which yields a coherent-combing gain, whereas this gain is lost in the SLO case. For the case of Wiener phase noise, we determine numerically the extent to which the quality of the local oscillators in the SLO configuration must be improved to overcome the loss of coherent-combing gain.

Our results are derived under the assumption that the continuous-time phase noise process remains constant over the duration of the symbol time. This assumption allows us to obtain a discrete-time equivalent channel model by sampling at Nyquist rate. As shown recently in [21]–[23], by dropping this assumption one may obtain drastically different high-SNR behaviors. In the Wiener phase-noise case, for example, the first-order term in the high-SNR capacity expression was shown in [21] to be at least as large as $0.5 \ln(\rho)$. However, it is unclear whether this lower bound is tight.

Notation: Boldface letters such as $\mathbf{a}$ and $\mathbf{A}$ denote vectors and matrices, respectively. The operator $\operatorname{diag}(\cdot)$, applied to a vector $\mathbf{a}$, generates a square diagonal matrix having the entries of $\mathbf{a}$ on its main diagonal. With $\mathcal{N}(0, \sigma^2)$ and $\mathcal{CN}(0, \sigma^2)$, we denote the probability distribution of a real Gaussian random variable and of a circularly symmetric complex Gaussian random variable with zero mean and variance σ^2. Furthermore, $\mathcal{U}[0, 2\pi)$ stands for the uniform distribution over the interval $[0, 2\pi)$. Throughout the paper, all sums between angles (both random and deterministic) are performed modulus 2π. For a given discrete-time vector-valued random process $\{\boldsymbol{\theta}_k\}$, we denote the sequence $\{\boldsymbol{\theta}_m, \dots, \boldsymbol{\theta}_n\}$, $m < n$ as $\boldsymbol{\theta}_m^n$. When $m = 1$, we omit the subscript. For two functions $f(\cdot)$ and $g(\cdot)$, the notation $f(x) = o(g(x))$, $x \to \infty$, means that $\lim_{x\to\infty} |f(x)/g(x)| = 0$. Finally, $\ln(\cdot)$ denotes the natural logarithm.

II. Review of the SISO Case

Consider the discrete-time SISO phase-noise channel

$$y_k = e^{j\theta_k} h x_k + w_k, \quad k = 1, \dots, n. \tag{1}$$

Here, x_k denotes the input symbol at time k. The constant h is the path-loss coefficient, which is assumed deterministic, time-invariant, and known to the transmitter and the receiver; $\{w_k\}$ are the additive Gaussian noise samples, drawn independently from a $\mathcal{CN}(0, 2)$ distribution.[1] Finally, the phase-noise process $\{\theta_k\}$ is assumed stationary, ergodic, independent of the noise process $\{w_k\}$, and with finite differential-entropy rate[2]

$$h(\{\theta_k\}) > -\infty. \tag{2}$$

Under these assumptions, the capacity of the SISO phase-noise channel (1) is given by

$$C(\rho) = \lim_{n\to\infty} \frac{1}{n} \sup I(y^n; x^n) \tag{3}$$

where the supremum is over all probability distributions on $x^n = (x_1, \dots, x_n)$ that satisfy the average-power constraint

$$\frac{1}{n} \sum_{k=1}^{n} \mathbb{E}\left[|x_k|^2\right] \le 2\rho. \tag{4}$$

Here, ρ can be thought of as the SNR (recall that we set the noise variance to 2; hence, the SNR is equal to half the signal power 2ρ). A closed-form expression for the capacity of the phase-noise channel is not available. Lapidoth [1] proved the following asymptotic characterization of $C(\rho)$.

Theorem 1 ([1]): The capacity of the SISO phase-noise channel (1) is given by

$$C(\rho) = \eta \ln(\rho) + \chi + o(1), \quad \rho \to \infty \tag{5}$$

where $\eta = 1/2$ and

$$\chi = (1/2) \ln\left(|h|^2/2\right) + \ln(2\pi) - h(\{\theta_k\}). \tag{6}$$

The factor $\eta = 1/2$ in (5) is the so-called capacity *prelog*, defined as the asymptotic ratio between capacity and the logarithm of SNR as SNR grows to infinity: $\eta = \lim_{\rho\to\infty} C(\rho)/\ln(\rho)$. The capacity prelog can be interpreted as the fraction of complex dimensions available for communications in the limiting regime of high signal power, or equivalently vanishing noise variance [26]. For the phase-noise channel (1), only the amplitude $|x_k|$ of the transmitted signal x_k can be perfectly recovered in the absence of additive noise, whereas the phase $\angle x_k$ is lost. Hence, the fraction of complex dimensions available for communication is $\eta = 1/2$.

We denote the second term in the high-SNR expansion (5) of $C(\rho)$ as the *phase-noise number* χ

$$\chi = \lim_{\rho\to\infty} \{C(\rho) - \eta \ln(\rho)\}. \tag{7}$$

We can see from (6) that the phase-noise number depends only on the statistics of the phase-noise process and on the path-loss coefficient h. It is worth mentioning that the approximation $C(\rho) \approx \eta \ln(\rho) + \chi$, although based on a high-SNR capacity expansion, is often accurate already for low SNR values [5], [7], [4]. Next, we characterize χ for some phase-noise models commonly used in the wireless literature.

[1] Normalizing the noise variance to 2 turns out to be convenient [1], [24].

[2] Note that the differential-entropy rate of the complex random process $\{e^{j\theta_k}\}$ is equal to $-\infty$. This means that the results obtained in [25] in the context of fading channel are not applicable to (1).

Noncoherent System: Consider the case where the phase-noise process $\{\theta_k\}$ is stationary and memoryless with uniform marginal distribution over $[0, 2\pi)$. This scenario models accurately a noncoherent communication system where the phase of x_k is not used to transmit information (see [5]). The phase-noise number for this case can be readily obtained from (6) by using that $h(\{\theta_k\}) = \ln(2\pi)$.

Partially Coherent System: When a phase tracker such as a phase-locked loop (PLL) is employed at the receiver, the output signal after phase tracking is impaired only by the residual phase error. Systems employing phase trackers are sometimes referred to as partially coherent [5]. It is often accurate to assume that the residual phase-error process $\{\theta_k\}$ is stationary and memoryless. Under this assumption, the phase-noise number for the partially-coherent case simplifies to

$$\chi = (1/2)\ln\Big(|h|^2/2\Big) + \ln(2\pi) - h(\theta) \tag{8}$$

where θ is the random variable modeling the residual phase error. When a PLL is used, the statistics of θ are accurately described by a Tikhonov distribution.

The Wiener Process: The case of phase-noise process with memory is relevant when a free-running oscillator is used or when the phase tracker is not able to completely remove the memory of the phase-noise process [20], [27]. The samples $\{\theta_k\}$ of a free-running oscillator are typically modeled using a Wiener process [20], [28], according to which

$$\theta_{k+1} = (\theta_k + \Delta_k) \bmod (2\pi) \tag{9}$$

where $\{\Delta_k\}$ are Gaussian random samples, independently drawn from a $\mathcal{N}(0, \sigma_\Delta^2)$ distribution. Hence, the sequence $\{\theta_k\}$ is a Markov process, i.e.,

$$f_{\theta_k \,|\, \theta_{k-1},\dots,\theta_0} = f_{\theta_k \,|\, \theta_{k-1}} = f_\Delta \tag{10}$$

where the wrapped Gaussian distribution f_Δ is the pdf of the innovation Δ modulus 2π. Under the assumption that the initial phase-noise sample θ_0 is uniformly distributed over $[0, 2\pi)$, the process $\{\theta_k\}$ is stationary. Hence, its differential-entropy rate is given by the differential entropy of the innovation process

$$h(\{\theta_k\}) = h(\Delta). \tag{11}$$

The differential entropy $h(\Delta)$ can be well-approximated by that of an unwrapped $\mathcal{N}(0, \sigma_\Delta^2)$ random variable:

$$h(\Delta) \approx (1/2)\ln(2\pi e \sigma_\Delta^2) \tag{12}$$

whenever the standard deviation σ_Δ is below $55°$ (see [29]). The oscillators commonly used in wireless transceivers result in a phase-noise standard variation that is well below $55°$ [24].

III. Uplink Channel

Building on the results reviewed in Section II, we next analyze the uplink channel of a wireless communication system where a single-antenna terminal communicates with a base station equipped with M antennas over an AWGN channel impaired by phase noise. This yields the following $1 \times M$ single-input multiple-output (SIMO) phase-noise channel:

$$\mathbf{y}_k = \Theta_k \mathbf{h} x_k + \mathbf{w}_k, \quad k = 1, \dots, n. \tag{13}$$

Here, the matrix $\Theta_k = \operatorname{diag}([e^{j\theta_{1,k}}, \dots, e^{j\theta_{M,k}}])$ contains the phase-noise samples. We assume that, for each $m = 1, \dots, M$, the phase-noise process $\{\theta_{m,k}\}$ is stationary, ergodic, independent of the additive-noise process $\{\mathbf{w}_k\}$, and has finite differential-entropy rate. Note that we do not necessarily assume that the phase-noise processes $\{\theta_{m,k}\}$, $m = 1, \dots, M$ are independent. It will turn out convenient to define also the phase-noise vector-valued process $\{\boldsymbol{\theta}_k\}$ where $\boldsymbol{\theta}_k = [\theta_{1,k}, \dots, \theta_{M,k}]^T$. The vector $\mathbf{h} = [h_1, \dots, h_M]^T$ contains the path-loss coefficients. Finally, the vector $\mathbf{w}_k = [w_{1,k}, \dots, w_{M,k}]^T$ contains the AWGN samples, which are drawn independently from a $\mathcal{CN}(0, 2)$ distribution. The capacity of the SIMO phase-noise channel (13) is

$$C(\rho) = \lim_{n\to\infty} \frac{1}{n} \sup I(\mathbf{y}^n; x^n) \tag{14}$$

where the supremum is over all probability distributions on x^n that satisfy the average-power constraint (4).

A. Uplink, Common Local Oscillator (UL-CLO)

In the CLO configuration, we have that $\theta_{1,k} = \cdots = \theta_{M,k} = \theta_k$ for all k. Hence, the input-output relation (13) simplifies to

$$\mathbf{y}_k = e^{j\theta_k} \mathbf{h} x_k + \mathbf{w}_k. \tag{15}$$

By projecting $\mathbf{y}_k$ on $\mathbf{h}/\|\mathbf{h}\|$, i.e., by performing coherent/maximal-ratio combining, we obtain a sufficient statistics for the detection of x_k from $\mathbf{y}_k$. Through this projection, the SIMO phase-noise channel (15) is transformed into an equivalent SISO phase-noise channel with channel gain $\|\mathbf{h}\|$. Therefore, using Theorem 1, we conclude that the prelog for the UL-CLO case is $\eta_{\text{ul-clo}} = 1/2$ and that the phase-noise number is

$$\chi_{\text{ul-clo}} = (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - h(\{\theta_k\}). \tag{16}$$

B. Uplink, Separate Local Oscillators (UL-SLO)

In the SLO case, the M phase-noise processes $\{\theta_{m,k}\}$, are independent and identically distributed (i.i.d.) across the receive antennas. Hence, coherent combining does not yield a sufficient statistics. In Theorem 2 below, we provide a characterization of the high-SNR capacity of $C(\rho)$, which holds irrespectively of the dependency between the M phase-noise processes $\{\theta_{m,k}\}$, $m = 1, \dots, M$.

Theorem 2: The prelog of the SIMO phase-noise channel (13) is given by $\eta_{\text{ul}} = 1/2$. Furthermore, the phase-noise number is bounded by

$$\begin{aligned}\chi_{\text{ul}} \geq{}& (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) \\ & - h\big(\phi_0 \,|\, \boldsymbol{\theta}_0 + \phi_0, \boldsymbol{\theta}_{-\infty}^{-1}\big) \end{aligned}\tag{17a}$$

$$\begin{aligned}\chi_{\text{ul}} \leq{}& (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) \\ & - h(\phi_0 \,|\, \boldsymbol{\theta}_0 + \phi_0) + I\big(\boldsymbol{\theta}_0; \boldsymbol{\theta}_{-\infty}^{-1}\big) \end{aligned}\tag{17b}$$

where $\{\phi_k\}$ is a stationary memoryless process, with marginal distribution uniform over $[0, 2\pi)$.

Proof: See [24, App. I]. ∎

Remark 1: The lower and upper bounds in (17) match when the phase noise processes are memoryless. Indeed, under this assumption,

$$\chi_{\text{ul}} = (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - h(\phi_0 \,|\, \boldsymbol{\theta}_0 + \phi_0)\,. \quad (18)$$

Remark 2: For the CLO case where $\theta_{1,k} = \cdots = \theta_{M,k} = \theta_k$ for all k, the bounds in (17) match and reduce to (16) (see [24]).

C. Discussion

The fact that $\eta_{\text{ul-clo}} = \eta_{\text{ul-slo}} = \eta_{\text{siso}}$ comes perhaps as no surprise because adding multiple antennas at the receiver only (SIMO channel) does not yield spacial multiplexing gains. We next compare the phase-noise number of the CLO and the SLO configurations. We see from (16) and (17) that the term $0.5\ln(\|\mathbf{h}\|^2/2)$ appears in the phase-noise number of both the CLO and SLO configuration. As already pointed out, in the CLO case this term comes from coherently combining the signals received at the M antennas.

In the SLO case, coherent combining is not possible because the received signals at the different antennas are subject to independent random phase shifts. It turns out (see [24, App. I-A, App. I-B]) that a coherent-combining gain can be harvested regardless by separately decoding the amplitude and the phase of the transmitted signal, and by adding the square of the received signals when decoding the amplitude.

The CLO and SLO phase-noise numbers coincide in the noncoherent case (stationary, memoryless phase noise, with uniform marginal distribution over $[0,2\pi)$):

$$\chi_{\text{ul-clo}} = \chi_{\text{ul-slo}} = (1/2)\ln\big(\|\mathbf{h}\|^2/2\big)\,. \quad (19)$$

For phase-noise processes with memory, the CLO configuration results in a smaller phase-noise number than the SLO configuration. Indeed by rewriting the second and the third term on the RHS of (16) as follows

$$\ln(2\pi) - h(\{\theta_k\}) = I(\theta_0+\phi_0;\phi_0 \,|\, \theta_{-\infty}^{-1}) \quad (20)$$
$$= \ln(2\pi) - h(\phi_0 \,|\, \phi_0+\theta_0, \theta_{-\infty}^{-1}) \quad (21)$$

where $\phi_0 \sim \mathcal{U}(0,2\pi]$ is independent of $\{\theta_k\}$, we see that the differential entropy on the RHS of (21) is larger than the differential entropy in the SLO phase-noise lower bound in (17a). To shed further light on the difference between the CLO and the SLO configuration, we now consider the special case of Wiener phase noise. For the CLO configuration, by substituting (12) in (11), and then (11) in (16) we obtain

$$\chi_{\text{ul-clo}} \approx (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - (1/2)\ln(2\pi e\sigma_\Delta^2). \quad (22)$$

For the SLO configuration, we manipulate the lower-bound in (17a) as follows (see [24] for details):

$$\chi_{\text{ul-slo}} \geq (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - h\bigg(\phi_0\bigg|\phi_0 + \frac{1}{M}\sum_{m=1}^{M}\Delta_{m,-1}\bigg) \quad (23)$$
$$= \frac{1}{2}\ln\bigg(\frac{\|\mathbf{h}\|^2}{2}\bigg) + \ln(2\pi) - h\Bigg(\frac{1}{M}\sum_{m=1}^{M}\Delta_{m,-1}\Bigg) \quad (24)$$
$$\approx (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - (1/2)\ln\big(2\pi e\sigma_\Delta^2/M\big)\,. \quad (25)$$

By comparing (22) and (25) we see that—as expected—$\chi_{\text{ul-slo}} \geq \chi_{\text{ul-clo}}$. This gain can be explained as follows: in the SLO case, we have M independent noisy observations of the phase of the transmitted signal. These independent noisy observations can be used to improve the estimation of the transmitted phase. In order to obtain equal phase-noise numbers in the CLO and SLO configurations, the phase-noise variance σ_Δ^2 in the CLO case must be at least M times lower than in the SLO case. It is perhaps also worth mentioning that the SLO gains cannot be achieved in the CLO case simply by independently phase-shifting the signal received at each antenna. In fact, this strategy does not even achieve the CLO phase-noise number (16).

A configuration that is perhaps more relevant from a practical point of view is the one where the M phase-noise processes $\{\theta_{m,k}\},\ m = 1,\ldots,M$ result from the sum of the phase-noise contribution $\theta_k^{(\text{tx})}$ at the transmitter and of M independent phase-noise contributions $\{\theta_{m,k}^{(\text{rx})}\},\ m=1,\ldots,M$ at the receivers. Assuming that both $\theta_k^{(\text{tx})}$ and $\{\theta_{m,k}^{(\text{rx})}\}$ evolve according to independent Wiener processes with iid innovations $\Delta_k^{(\text{tx})} \sim \mathcal{N}(0,\sigma_{\Delta,\text{tx}}^2)$ and $\Delta_{m,k}^{(\text{rx})} \sim \mathcal{N}(0,\sigma_{\Delta,\text{rx}}^2)$, we obtain [24]

$$\chi_{\text{ul-slo}} \geq (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - h\Bigg(\Delta_{-1}^{(\text{tx})} + \frac{1}{M}\sum_{m=1}^{M}\Delta_{m,-1}^{(\text{rx})}\Bigg) \quad (26)$$
$$\approx (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - (1/2)\ln\big(2\pi e\left(\sigma_{\Delta,\text{tx}}^2 + \sigma_{\Delta,\text{rx}}^2/M\right)\big)\,. \quad (27)$$

The case where a single oscillator is used at the receiver can be obtained from (27) by setting $M=1$. Also for this setup, using independent oscillators at the receiver is advantageous, although the gain is smaller than what suggested by (25).

IV. Downlink Channel

We next analyze the downlink channel, i.e., the scenario where a base station equipped with M antennas communicates with a single-antenna terminal. This yields the following $M \times 1$ multiple-input single-output (MISO) phase-noise channel

$$y_k = \mathbf{h}^T \Theta_k \mathbf{x}_k + w_k. \quad (28)$$

Here, the phase-noise process $\{\Theta_k\}$ and the path-loss vector $\mathbf{h}$ are defined as in Section III; $\mathbf{x}_k = [x_{1,k},\ldots,x_{M,k}]^T$, where $x_{m,k}$ denotes the symbol transmitted from antenna m at time instant k; finally, $\{w_k\}$ is the additive noise process, with samples drawn independently from a $\mathcal{CN}(0,2)$ distribution. The capacity of the MISO phase-noise channel (28) is

$$C(\rho) = \lim_{n\to\infty}\frac{1}{n}\sup I(y^n;\mathbf{x}^n) \quad (29)$$

where the supremum is over all probability distributions on $\mathbf{x}^n = (\mathbf{x}_1,\ldots,\mathbf{x}_n)$ that satisfy the average-power constraint

$$\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}\big[\|\mathbf{x}_k\|^2\big] \leq 2\rho. \quad (30)$$

A. Downlink, Common Local Oscillator (DL-CLO)

In the CLO case, we have that $\theta_{1,k} = \cdots = \theta_{M,k} = \theta_k$ for all k. Hence, the input-output relation (28) simplifies to

$$y_k = e^{j\theta_k}\mathbf{h}^T\mathbf{x}_k + w_k. \quad (31)$$

Maximum ratio transmission, i.e., setting $\mathbf{x}_k = s_k\mathbf{h}^*/\|\mathbf{h}\|$, with $\{s_k\}$ chosen so that (30) holds, is capacity achieving. With maximum ratio transmission, the MISO channel is transformed into a SISO channel. Hence, Theorem 1 allows us to conclude that, for the DL-CLO case, $\eta_{\text{dl-clo}} = 1/2$ and

$$\chi_{\text{dl-clo}} = (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - h(\{\theta_k\}). \quad (32)$$

B. Downlink, Separate Local Oscillator (DL-SLO)

In Theorem 3 below, we characterize the prelog and provide bounds on the phase-noise number of the MISO phase-noise channel (28). Afterwards, we shall discuss specific phase-noise models for which the bounds are tight.

Theorem 3: The prelog of the MISO phase-noise channel (28) is given by $\eta_{\text{dl}} = 1/2$. Furthermore, the phase-noise number is bounded by

$$\chi_{\text{dl}} \geq \ln(2\pi) + \max_{m=1,\dots,M}\left\{\frac{1}{2}\ln\left(\frac{|h_m|^2}{2}\right) - h(\{\theta_{m,k}\})\right\} \quad (33a)$$

$$\chi_{\text{dl}} \leq \ln(2\pi) + \sup_{\|\hat{\mathbf{x}}\|=1}\left\{\frac{1}{2}\ln\left(\frac{1}{2}\mathbb{E}\left[\left|\mathbf{h}^T\Theta_0\hat{\mathbf{x}}\right|^2\right]\right)\right\} - \inf_{\|\hat{\mathbf{x}}\|=1}\left\{h\big(\underline{/\mathbf{h}^T\Theta_0\hat{\mathbf{x}}}\,|\,\boldsymbol{\theta}_{-\infty}^{-1}\big)\right\} \quad (33b)$$

where $\hat{\mathbf{x}}$ is a unit-norm vector in $\mathbb{C}^M$.

Proof: See [24, App. B]. ■

Remark 3: The lower bound on the phase-noise number in (33a) is achieved by antenna selection, i.e., by activating only the transmit antenna that leads to the largest SISO phase-noise number. The other $M-1$ transmit antennas are switched off.

Remark 4: In the CLO case where $\theta_{1,k} = \cdots = \theta_{1,k} = \theta_k$ for all k, the upper bound in (33b) is tight. On the contrary, the lower bound in (33a) is not tight because antenna selection is not optimal for the CLO case.

C. Discussion

The lower bound and the upper bounds in (33) match when the phase-noise processes are independent across antennas, and have uniform marginal distributions over $[0, 2\pi)$. This occurs in noncoherent systems and for the Wiener model. We formalize this result in Theorem 4 below.

Theorem 4: The phase-noise number of the MISO phase-noise channel (28) under the additional assumptions that the M phase-noise processes $\{\theta_{m,k}\}$, $m = 1,\dots,M$ i) are independent and identically distributed (i.i.d.) across antennas (SLO configuration), ii) have uniform marginal distributions over $[0, 2\pi)$, is given by

$$\chi_{\text{dl-slo}} = \frac{1}{2}\max_{m=1,\dots,M}\ln\left(\frac{|h_m|^2}{2}\right) + \ln(2\pi) - h(\{\theta_k\}) \quad (34)$$

where $h(\{\theta_k\})$ is the differential-entropy rate of one of the i.i.d. phase-noise processes.

Proof: See [24, App. C] ■

We next compare $\chi_{\text{dl-clo}}$ and $\chi_{\text{dl-slo}}$. For the noncoherent case, we have that

$$\chi_{\text{dl-clo}} = (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) \quad (35)$$

$$\chi_{\text{dl-slo}} = (1/2)\max_{m=1,\dots,M}\ln\Big(|h_m|^2/2\Big). \quad (36)$$

For the Wiener case, by substituting (12) in (11) and then (11) in (32) and in (34), we obtain

$$\chi_{\text{dl-clo}} \approx (1/2)\ln\big(\|\mathbf{h}\|^2/2\big) + \ln(2\pi) - (1/2)\ln(2\pi e\sigma_\Delta^2) \quad (37)$$

$$\chi_{\text{dl-slo}} \approx (1/2)\max_{m=1,\dots,M}\ln\Big(|h_m|^2/2\Big) + \ln(2\pi) - (1/2)\ln(2\pi e\sigma_\Delta^2). \quad (38)$$

In both the noncoherent and the Wiener case, we see that the SLO configuration results in no coherent-combining gain: $\|\mathbf{h}\|^2$ is replaced by $\max_{m=1,\dots,M}|h_m|^2$. The resulting throughput loss is most pronounced when the entries of $\mathbf{h}$ have all the same magnitude.

To shed further light on this loss, we depart from the model we considered so far, where the $\{h_m\}$, $m = 1,\dots,M$ are deterministic, and move to a quasi-static fading model [30, p. 2631] where the $\{h_m\}$ are independently drawn from a $\mathcal{CN}(0,1)$ distribution and stay constant over the duration of a codeword. We also assume that the $\{h_m\}$ are perfectly known to the transmitter and the receiver. In this scenario, $0.5\ln(\rho) + \chi$, where χ is now a function of the instantaneous channel gains, is the rate supported by the channel in the high-SNR regime, for a given channel realization.

In Fig. 1, we plot the cumulative distribution function of $0.5\ln(\rho)+\chi$, which is a high-SNR approximation of the outage capacity. We consider the case of Wiener phase noise with standard deviation $\sigma_\Delta = 6°$ and set $M = 20$ and $\rho = 20$ dB. For a given outage probability, the rate supported in the SLO case is smaller than that in the CLO case. For example, for a target outage probability of $\varepsilon = 0.1$, the rate supported in the SLO case is 1.36 bit/channel use lower than that in the CLO case. To achieve the same rate at $\varepsilon = 0.1$, the standard deviation σ_Δ of the phase-noise process in the SLO case must be set to $2.34°$.

V. Conclusions

We studied the capacity of multiple-antenna systems affected by phase noise. Specifically, we analyzed the first two terms in the high-SNR expansion of the capacity of both the uplink and the downlink channel of a system where wireless communication occurs between a base station equipped with M antennas and a single-antenna user. Our analysis covers two different configurations: the case when the RF circuitries connected to each antenna at the base station are driven by separate local oscillators, and the case when a common oscillator drives all the antennas.

For all four cases (uplink/downlink, common/separate oscillators) the first term in the high-SNR capacity expansion is equal

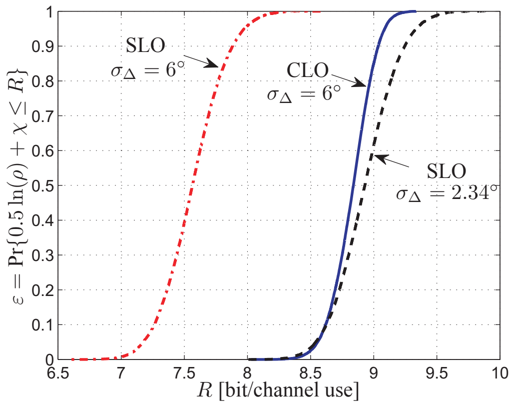

Fig. 1. High-SNR approximation of the outage probability for the CLO and the SLO configurations. A Wiener phase-noise model is considered. Furthermore, $M = 20$, and $\rho = 20$ dB.

to $0.5\ln(\rho)$, whereas the second term, which we denote as phase noise number, turns out to take different values depending on which case is considered. For the uplink channel, the phase noise number is larger when separate oscillators are used. For the specific case of Wiener phase noise, a gain of at least $0.5\ln(M)$ can be achieved. This gain, which is due to diversity, implies that to achieve the same throughput in the high-SNR regime, the oscillator used in the common oscillator configuration must be at least M times better than any of the oscillators used in the separate configuration.

In contrast, the phase noise number of the downlink channel is larger when a common oscillator drives all the antennas. This is due to the fact that conjugate beamforming, which provides a coherent-combining gain for the common oscillator configuration, does not achieve the phase-noise number when separate oscillators are used. The capacity achieving-strategy for the separate oscillator configuration turns out to be antenna selection, i.e., activating only the transmit antenna that yields the largest SISO high-SNR capacity, and switching off all other antennas.

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