7 - 12 SEPTEMBER 2014, KRAKÓW
FORUM ACUSTICUM

Modelling sound propagation in the presence of atmospheric turbulence for the auralisation of aircraft noise

Frederik Rietdijk, Kurt Heutschi
Empa, Swiss Federal Laboratories for Materials Science and Technology, Dubendorf, Switzerland

Jens Forssén
Chalmers University of Technology, Gothenburg, Sweden

Summary
A new tool for the auralisation of aircraft noise in an urban environment is in development. When listening to aircraft noise sound level fluctuations caused by atmospheric turbulence are clearly audible. Therefore, to create a realistic auralisation of aircraft noise, the influence of atmospheric turbulence on sound propagation needs to be included. Presented is a model in development for including fluctuations due to movement through a frozen turbulent atmosphere in an auralisation.

PACS no. 43.28.Gq

1. Introduction

Aircraft noise can cause annoyance and sleep disturbance. Annoyance and sleep disturbance is currently predicted using indicators based on time-averaged sound pressure levels. This completely discards the character of the sound. To obtain a better representation of annoyance, the audible aircraft sound should be predicted in order to determine the impact of the sound on people.

Auralization is a technique to synthesise the aural aspects of an object or surrounding. Auralization can therefore be used to create audible aircraft sounds that can be used in listening tests.

1.1. Aircraft auralisation tool

An aircraft auralisation tool is being developed that should provide plausible auralisations of aircraft noise for typical urban situations where reflections and shielding can play an important role.

In the auralisation tool a distinction is made between a source model and a propagation model. The source model describes the emission of the aircraft, i.e. the spectral content as function of time and angle (directivity). The propagation model describes the propagation of the sound from source to receiver and currently supports:

- spherical spreading resulting in a decrease of sound pressure with increase of source-receiver distance;
- Doppler shift due to relative motion between the moving aircraft and the non-moving receiver;
- atmospheric absorption due to relaxation processes;
- reflections at the ground and facades.

The propagation model is based on geometrical acoustics. The reflections are determined using the Image Source Method.

1.2. Turbulence

When listening to aircraft noise, sound level fluctuations caused by atmospheric turbulence are clearly audible. Therefore, to create a realistic auralisation of aircraft noise, atmospheric turbulence needs to be included.

Due to spatial inhomogeneities of the wind velocity and temperature in the atmosphere, acoustic scattering occurs, affecting the transfer function between source and receiver. Both these inhomogeneities and the aircraft position are time-dependent, and therefore the transfer function varies with time resulting in the audible fluctuations.

The theory of turbulence is a statistical theory. A statistical theory fits well with the physics of turbulence since turbulence is a consequence of instability of fluid flow in relation to very small fluctuations in the fluid [5]. For an auralisation instantaneous values of the sound pressure at the receiver are required.

Artnzen[1] included a phase fluctuations filter in their aircraft noise simulator to make the ground effect less pronounced. The filter was based on a Gaussian spectrum of turbulence.

In this paper an initial model is presented to describe fluctuations in the sound pressure due to atmospheric turbulence as function of time. A stationary (frozen) atmosphere is assumed where the movement of the aircraft alone gives rise to fluctuations. In contrast to Arntzen[1], both amplitude and phase fluctuations are considered. The model is based on a simple model by Daigle et. al.[2].

2. Theory

As mentioned in the introduction the theory of turbulence is a statistical theory. The wind velocity components and temperature in the turbulent atmosphere are fluctuating functions of both position and time.

The fields of the wind velocity components and the temperature are random fields. A characteristic of a random function or field is it's correlation function. [5] The correlation function of such a random field $f(\mathbf{r})$, as function of distance $\mathbf{r}$ between observation points only, is defined as

$$B(\mathbf{r}_1, \mathbf{r}_2) = \langle f(\mathbf{r}_1 f(\mathbf{r}_2) \rangle \tag{1}$$

Note that we've assumed a 'frozen turbulence' here.

In a homogeneous and isotropic random field the correlation function $B(\mathbf{r})$ depends only on $|r| = \mathbf{r}$, i.e., only the distance between the observation points.

2.1. Gaussian spectrum

In the region of large eddy sizes the turbulence spectrum can be approximated by a Gaussian distribution. The spatial correlation function for the fluctuating refractive-index μ therefore also has a Gaussian form

$$\langle \mu_1 \mu_2 \rangle = \langle \mu^2 \rangle \exp\left(-x^2/L^2\right) \tag{2}$$

When the outer length scale of the turbulence L is much smaller than the Fresnel zone, the mean square log-amplitude and phase fluctuations for spherical waves are given by

$$\langle \chi^2 \rangle = \langle S^2 \rangle = \frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle k^2 r L \tag{3}$$

where k is the wavenumber and r is the distance of propagation. The mean square log-amplitude and phase fluctuations are defined respectively as $\langle \chi^2 \rangle = \langle \left(\ln \frac{A_n}{A_m} \right)^2 \rangle$ and $\langle S^2 \rangle = \langle (\phi_n - \phi_m)^2 \rangle$. The index n indicates the sample n at time t and index m the average fluctuation. For spherical waves the covariances of the fluctuations, $B_\chi(\rho)$ and $B_S(\rho)$, normalized to their variances, are given by

$$\frac{B_\chi(\rho)}{\langle \chi^2 \rangle} = \frac{B_S(\rho)}{\langle S^2 \rangle} = \frac{\Phi\left(\rho/L\right)}{\rho/L} \tag{4}$$

where

$$\phi\left(\rho/L\right) = \int_0^{\rho/L} \exp\left(-u^2\right) \mathrm{d}u \quad = \frac{1}{2}\sqrt{\pi} \mathrm{erf}\left(\rho/L\right) \tag{5}$$

with erf the error function. The covariance of the fluctuations $B_\chi(\rho)$ and $B_S(\rho)$ are thus given by

$$B_\chi(\rho) = B_S(\rho) = \frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle k^2 r L \frac{\Phi(\rho/L)}{\rho/L} \tag{6}$$

according to Daigle's model.

2.2. Time series of fluctuations

For the auralisation, time series of instantaneous fluctuations of log-amplitude and phase are required, $\chi(t)$ and $S(t)$. Because the modulations are frequency-dependent, we would like to obtain realisations of $\chi(t, f)$ and $S(t, f)$ which describe the fluctuation of the log-amplitude χ and phase S at a time t for frequency f. The frequency f is the frequency of the signal component that should be modulated.

By assuming that $\chi(t, f)$ and $S(t, f)$ are Gaussian random variables, we can produce realisations of $\chi(t, f)$ and $S(t, f)$ by filtering white noise with a chosen filter function. The desired filter response is the impulse response $h(\rho)$ of the covariance $B(\rho)$, which is obtained by taking the inverse Fourier transform of the square root of the autospectrum of $B(\rho)$.

By taking these steps, two series of fluctuations are obtained, $\chi(t, f)$ and $S(t, f)$. The two time series can be merged into a single complex modulation signal

$$m(t, f) = \exp\left(\chi\left(t, f\right) + jS\left(t, f\right)\right) \tag{7}$$

that can then be applied to the input signal.

2.3. Saturation of the log-amplitude

For longer path lengths and stronger turbulence, the amplitude fluctuations gradually level off. Saturation of the amplitude fluctuations can be observed when measuring aircraft noise at distances of over a few kilometers. The standard deviation of the fluctuating sound pressure levels is in such cases limited to no approximately 6 dB [2, 4].

Saturation of the log-amplitude can be included based on an analysis by Wenzel [6]. In Wenzel's approach the soundwave is split up in a coherent and incoherent wave. The amplitude of the coherent wave decreases over distance while the incoherent wave obtains the energy from the coherent wave. The coherent wave p is written as

$$\langle p\ p^* \rangle = \left(A_m^2/4\pi r^2\right) \exp\left(-2\langle \mu^2 \rangle k^2 r L\right) \tag{8}$$

Wenzel defines the distance to saturation r_s as the distance at which the coherent wave is reduced to e^{-1} of its original strength

$$r_s = \frac{1}{2\langle \mu^2 \rangle k^2 L} \tag{9}$$

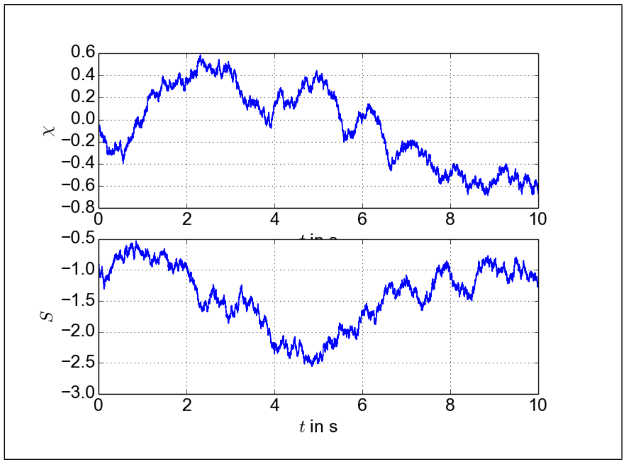

Figure 1. Amplitude and phase fluctuations as function of time for 200 Hz. The parameters $\langle \mu^2 \rangle = 3 \cdot 10^{-6}$, $L = 15$ m, $c = 343$ m/s, $r = 2000$ m, were used.

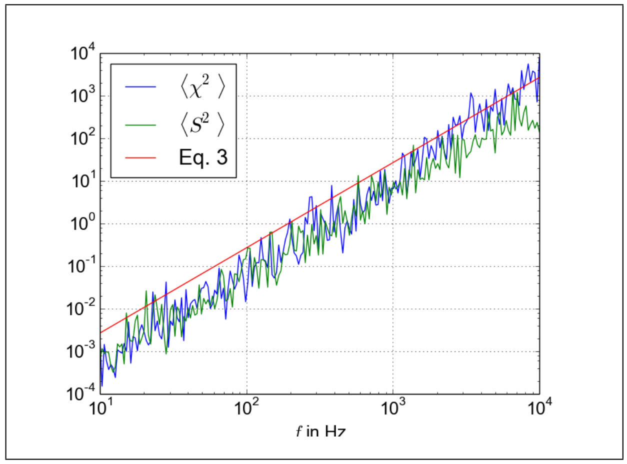

Figure 2. Log-amplitude and phase variances as function of frequency. The parameters $\langle \mu^2 \rangle = 3 \cdot 10^{-6}$, $L = 15$ m, $c = 343$ m/s, $r = 2000$ m, were used.

Saturation of the log-amplitude fluctuations can now be included by multiplying $\chi(t, f)$ with

$$\sqrt{\frac{1}{1 + r/r_s}} \tag{10}$$

2.4. Applying the modulations to the original signal

In case the input signal consists of just one frequency f and the modulation signal m is calculated for that specific frequency, then the modulation can simply be applied by multiplying the two in the time-domain. However, for the auralisation we have broadband sounds with more than one frequency line to consider. As the fluctuations are frequency-dependent, the modulation signal would have to be calculated for every frequency line.

One of the design criteria of the auralisation tool is to produce at or near real-time auralisations. Since computing the modulation signal for every frequency line is computationally relatively heavy, another solution is sought. The variances of the log-amplitude and phase fluctuations increase according to this simple theory linearly with frequency. Therefore, another option would be to scale both $\chi(t, f)$ and $S(t, f)$ accordingly. Care should be taken considering the saturation distance is frequency-dependent.

3. Results

An example of a calculated modulation signal $m(t, f)$ caused by fluctuations of the refractive-index in a Gaussian field is shown in figure 1. The figure shows the log-amplitude and phase fluctuations as function of time for a specific frequency.

To check whether the resulting variance of the log-amplitude and phase fluctuations of such a modulation signal are as they should be, a comparison is made with equation 3. The extraction and analysis of the mean square amplitude and phase fluctuations follows the method of Daigle et al.

The mean square log-amplitude fluctuation was calculated as

$$\langle \chi^2 \rangle = \frac{1}{N} \sum_{n=1}^{N} \chi_n^2 \tag{11}$$

where

$$\chi_n = \ln\left(A_n / A_m\right) \tag{12}$$

and A_m is the average amplitude over N samples in time

$$A_m = \frac{1}{N} \sum_{n=1}^{N} A_n \tag{13}$$

where n is the sample at time t. Similarly, the mean square of the phase fluctuations is calculated as

$$\langle S^2 \rangle = \frac{1}{N} \sum_{n=1}^{N} \left(\phi_n - \phi_m\right)^2 \tag{14}$$

with

$$\phi_m = \frac{1}{N} \sum_{n=1}^{N} \phi_n \tag{15}$$

Figure 2 shows the log-amplitude and phase variances as function of frequency. The frequency-dependency is maintained although the variance is slightly lower than expected.

The same applies to the distance-dependency. Figure 3 shows the log-amplitude and phase variances as function of distance. The variance of the fluctuations is also slightly lower than expected.

While the simple theory predicts that the mean square log-amplitude and phase fluctuations are

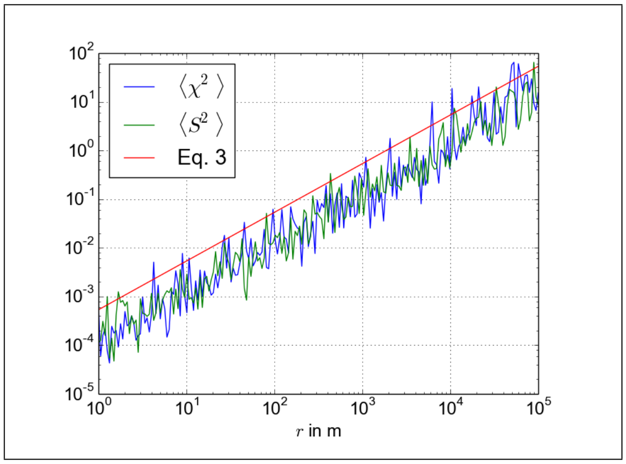

Figure 3. Log-amplitude and phase variances as function of distance. The parameters $\langle \mu^2 \rangle = 3 \cdot 10^{-6}$, $L = 15$ m, $c = 343$ m/s, $f = 200$ Hz, were used.

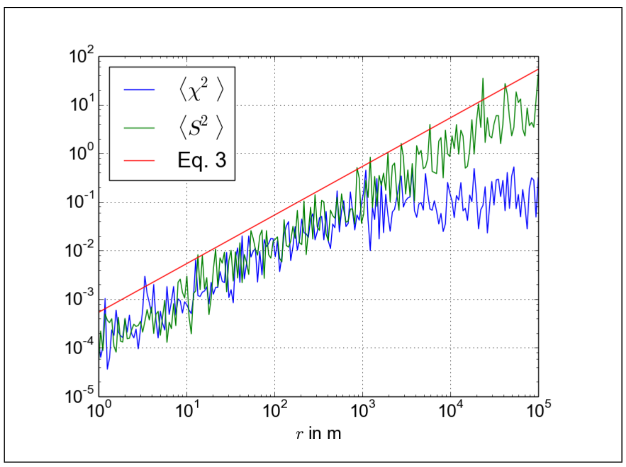

Figure 4. Log-amplitude and phase variances as function of distance taking into account the log-amplitude saturation. The same parameters as in 3 were used. The distance to saturation r_s, calculated using equation 9, was 827 m.

equal, Daigle et. al. [2] showed through measurements that the log-amplitude fluctuations don't obey the simple theory but are in fact smaller than predicted. Figure 4 shows again the mean square fluctuations as function of distance, but this time taking into account log-amplitude saturation.

Finally, the modulation signal was applied to a tone of the same frequency as the modulation signal was calculated for. The resulting signal is shown in figure 5.

4. Discussion

Based on a simple theory presented by Daigle et. al. a method is described to include modulations due to turbulence in an auralisation. The mean square of the fluctuations seem to be slightly smaller than what can be expected from the theory. It is not known why.

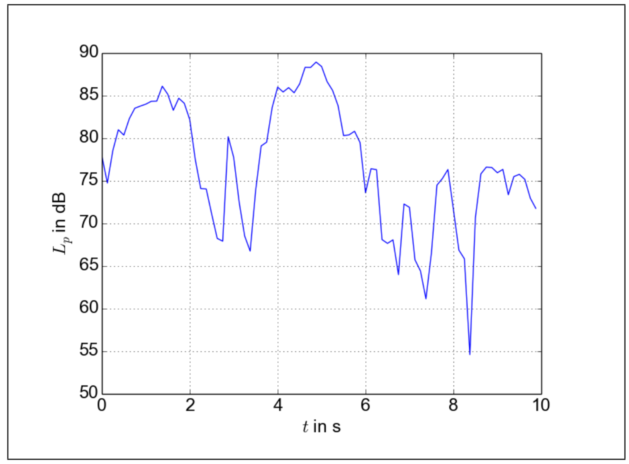

Figure 5. Sound pressure level L_p of a signal modulated by the modulation signal shown in figure 1. The input signal is a sine wave of 200 Hz. Time-weighting 'F' was used. The standard deviation of this modulated signal is 7 dB.

An important limitation of the current model is the fact that the turbulence is assumed to be homogeneous and isotropic, which means the height-dependency of the turbulence due to the boundary conditions is ignored. The outer length scale of the turbulence is known to increase with height, as can be seen in vertical profiles of the outer scale of turbulence obtained through acoustical sounding by Krasnenko et. al.[3].

Also, it is assumed that the outer length scale of the turbulence is much smaller than the Fresnel zone. When the aircraft is directly above the receiver and at relatively low altitude, this assumption will not be valid either. For example, at an altitude of 200 meters and a frequency of 100 Hz the Fresnel zone is 7.6 meters. The outer length scale is however in the order of 10 meters.

Figure 5 shows the sound pressure level as function of time of a signal modulated using the presented method. It should be noted that it is a mere coincidence that the standard deviation is similar to that what is expected at such distances. More calculations and with longer averaging times need to be done to determine what the standard deviation is after saturation of the log-amplitude.

In a next step k-space PSTD time-domain simulations will be performed as well to determine saturation distances and standard deviations of the level fluctuations after saturation of the log-amplitude. These simulations should also reveal more information regarding the coherence of the fluctuations between different frequencies.

5. CONCLUSIONS

A method is presented to include modulations caused by turbulence in an auralisation. The model is based on a simple theory assuming a Gaussian spectrum.

The saturation of log-amplitude fluctuations is explicitly included in the model. Future work includes comparisons with time-domain simulations of sound propagation through a turbulent atmosphere.

Acknowledgement

The research leading to these results has received funding from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007-2013 under REA grant agreement number 290110, SONORUS "Urban Sound Planner".

References

[1] Michael Arntzen. Ground reflection with turbulence induced coherence loss in flyover auralization. Sevilla, 2013.

[2] G. A. Daigle, J. E. Piercy, and T. F W Embleton. Line-of-sight propagation through atmospheric turbulence near the ground. *Journal of the Acoustical Society of America*, 74(5):1505–1513, 1983.

[3] N. P. NP Krasnenko, OF F. Kapegesheva, P. G. Stafeev, and L. G. Shamanaeva. Outer Scales of Temperature Turbulence and Dynamic Turbulence from Results of Acoustic Sounding of the Atmosphere. *Russian Physics Journal*, 56(6):667–673, October 2013.

[4] J. E. Piercy and T. F W Embleton. Effect of weather and topography on the propagation of noise. Technical report, 1974.

[5] V.I. Tatarskii. *The effects of the turbulent atmosphere on wave propagation.* 1971.

[6] A. R. Wenzel. Saturation effects associated with sound propagation in a turbulent medium. *American Institute of Aeronautics and Astronautics*, 1975.