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***Abstract*—This paper studies the energy efficiency (EE) of a point-to-point rank-1 Ricean fading multiple-input-multiple-output (MIMO) channel. In particular, a tight lower bound and an asymptotic approximation for the EE of the considered MIMO system are presented, under the assumption that the channel is unknown at the transmitter and perfectly known at the receiver. Moreover, the effects of different system parameters, namely, transmit power, spectral efficiency (SE), and number of transmit and receive antennas, on the EE are analytically investigated. An important observation is that, in the high signal-to-noise ratio regime and with the other system parameters fixed, the optimal transmit power that maximizes the EE increases as the Ricean-K factor increases. On the contrary, the optimal SE and the optimal number of transmit antennas decrease as K increases.**

I. Introduction

MIMO techniques can greatly improve the SE of wireless communications systems. However, due to the extra signal processing and circuit power consumption per transceiver RF chain, the use of multiple antennas also leads to an increased energy consumption. In future green communication systems, the EE becomes an important performance metric. Therefore, the trade-off between EE and SE has become a hot topic in MIMO systems.

Most studies on the EE of MIMO systems adopted the assumption of Rayleigh fading conditions [1]–[3]. However, in many practical scenarios, there exists a deterministic or strong line-of-sight (LoS) component. Some relevant examples are: 1) short-range millimeter wave MIMO channels [4], 2) microwave backhauling between macro base stations and outdoor small-cell base stations [5], 3) the channel between a user and its connected small-cell base stations. In such cases, Ricean fading conditions should be considered, which induces several research challenges.

Motivated by the above discussion, in this paper, we study a worst case scenario, where the Ricean fading MIMO channel has a rank-1 mean. This scenario is relevant when the inter-element distances at both ends are small and the impinging LoS waveforms have identical phases [6]. A tight lower bound for and an asymptotic approximation on the EE of the considered MIMO system are presented, assuming that the channel is unknown at the transmitter and perfectly known at the receiver. With the help of the closed-form EE expressions, the effects of different system parameters, i.e., transmit power, SE, and number of transmit and receive antennas, on the EE are analytically investigated. Interestingly, we observe that, in the high signal-to-noise ratio (SNR) regime and with the other system parameters fixed, the optimal transmit power that maximizes the EE increases with the Ricean-K factor or with the fixed transceiver power consumption. On the other hand, the optimal SE and the optimal number of transmit antennas decrease as the Ricean-K factor increases, i.e., when the LoS component becomes more and more dominant.

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II. System Model

We consider a point-to-point MIMO system with n_t transmit antennas and n_r receive antennas. Here, we assume $n_t \leq n_r$, while all our results can be easily generalized to the case of $n_t \geq n_r$. Let $\mathbf{s} \in \mathbb{C}^{n_t \times 1}$ denote the transmitted signal vector with $\mathbb{E}\left\{\mathbf{s}^\dagger \mathbf{s}\right\} \leq P$, where $(\cdot)^\dagger$ stands for conjugate transpose. The received signal vector at the receiver, $\mathbf{y} \in \mathbb{C}^{n_r \times 1}$, is

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ denotes the circularly symmetric complex Gaussian noise with $\mathbf{n} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_{n_r}\right)$. The MIMO channel, $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$, is assumed to be Ricean flat-fading with rank-1 mean, such that

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\tilde{\mathbf{H}} \tag{2}$$

where $\mathbb{E}\{\mathbf{H}\} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}}$ and K is the Ricean K-factor. Here, $\mathbb{E}\{\cdot\}$ stands for expectation. The rank-1 matrix $\bar{\mathbf{H}}$ corresponds to the coherent line-of-sight component, and it is normalized such that $\left\|\bar{\mathbf{H}}\right\|_{\mathrm{F}}^2 = n_t n_r$, where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. The term $\tilde{\mathbf{H}}$ represents the non-coherent scattered contributions. In this paper, we focus on the case without spatial correlation between receive antennas and transmit antennas. Thus, $\tilde{\mathbf{H}}$ can be modeled as a random fading matrix, whose entries are circularly symmetric complex Gaussian random variables with unit variance. Therefore, $\mathbf{H}$ has a matrix-variate complex Gaussian distribution with $\mathbf{H} \sim \mathcal{CN}\left(\mathbf{M}, \mathbf{I}_{n_r} \otimes \varepsilon^2 \mathbf{I}_{n_t}\right)$, where $\varepsilon \triangleq 1/\sqrt{K+1}$ and $\mathbf{M} \triangleq \varepsilon\sqrt{K}\bar{\mathbf{H}}$. Here, $\otimes$ is defined as the Kronecker product between matrices.

Under the assumption of no channel state information (CSI) at the transmitter and perfect CSI at the receiver, the SE, which is defined as the ergodic capacity per unit bandwidth, is [7]

$$\mathtt{SE} = \mathbb{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{n_t} + \frac{P}{\sigma^2 n_t}\mathbf{H}^{\dagger}\mathbf{H}\right)\right\} \tag{3}$$

where the expectation is taken with respect to $\mathbf{H}$ and $\det(\cdot)$ denotes the matrix determinant. The EE is defined as the ratio between the SE and total power consumption, that is,

$$\mathtt{EE} = \frac{\mathtt{SE}}{P_{\mathtt{tot}}}. \tag{4}$$

In this paper, we adopt the linear approximated power consumption model proposed in [8], where

$$P_{\mathtt{tot}} = \alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}. \tag{5}$$

The static terms, $P_{\mathtt{ct}}$ and $P_{\mathtt{cr}}$, denote the fixed power consumption for each transmit and receiver chain, respectively. The scaling factor, α, models the impact of the output power P on the efficiency of the power amplifiers used at the transmitter. The values of $P_{\mathtt{ct}}$, $P_{\mathtt{cr}}$ and α for different base station (BS) types can be found in [8, Table 2].

III. Spectral Results

The exact expressions for SE and EE are difficult to obtain. To make the EE analysis tractable, we consider the following two approximations of the SE.

Lemma 1 [9, Theorem 9]: The SE of the uncorrelated Ricean fading MIMO channel with rank-1 mean matrix $\mathbf{M}$ and $n_t \leq n_r$, is tightly lower bounded for arbitrary SNR $= \frac{P}{\sigma^2}$ by $\mathtt{SE} \geq \mathtt{SE}_1$, where

$$\mathtt{SE}_1 \triangleq \sum_{i=1}^{n_t} \log_2(1+\mu_i P) \tag{6}$$

with positive values of μ_i given as

$$\mu_i = \begin{cases} \frac{\exp(g_{n_r}(\triangle))}{\sigma^2 n_t (K+1)}, & i=1 \\ \frac{\exp\left(\sum_{p=1}^{n_r-i}\frac{1}{p}-\gamma\right)}{\sigma^2 n_t (K+1)}, & i=2,\ldots,n_t \end{cases}$$

where $\gamma = 0.5772...$ is Euler's constant and $\triangle \triangleq K n_t n_r$. The function $g_{n_r}(\triangle)$ is defined as

$$g_{n_r}(\triangle) \triangleq \ln(\triangle) - \mathtt{Ei}(-\triangle) + \sum_{i=1}^{n_r-1}\left(-\frac{1}{\triangle}\right)^i \times \left(e^{-\triangle}(i-1)! - \frac{(n_r-1)!}{i(n_r-1-i)!}\right) \tag{7}$$

where $\mathtt{Ei}(-x) = -\int_x^{\infty}\frac{e^{-t}}{t}dt$, $x>0$, is the exponential integral function.

Note that for the special case of $K\to\infty$, (6) reduces to

$$\mathtt{SE}_1^{K\to\infty} = \log_2\left(1 + n_r\frac{P}{\sigma^2}\right) \tag{8}$$

which is the exact capacity of deterministic MIMO systems, with multiplexing gain equal to 1 [7]. In the high SNR regime, we have the following approximation for the SE.

Lemma 2 [9, Eq. (24)]: The SE of the uncorrelated Ricean fading MIMO channel in the high SNR regime, with rank-1 mean matrix $\mathbf{M}$ and $n_t \leq n_r$, can be approximated as $\mathtt{SE} \approx \mathtt{SE}_2$, where

$$\mathtt{SE}_2 \triangleq n_t \log_2\left(\frac{P}{\sigma^2 n_t}\right) + c \tag{9}$$

where $c \triangleq \log_2\frac{n_r!}{(n_r-n_t)!} + \log_2\frac{1+n_t K}{(K+1)^{n_t}}$.

IV. Energy Efficiency Maximization

In this section, we utilize the closed-form expressions in Section III to analyze the impact of different system parameters on the EE of rank-1 Ricean fading MIMO channels.

A. The Impact of Transmit Power on EE

1) Arbitrary SNR: We start with finding the optimal transmit power P by using the lower bound EE expression, i.e.,

$$\mathtt{EE}_1 \triangleq \frac{\mathtt{SE}_1}{P_{\mathtt{tot}}} = \frac{\sum_{i=1}^{n_t}\log_2(1+\mu_i P)}{\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}} \tag{10}$$

which is tight for arbitrary SNR. From (10), we see that the sublevel sets $S_\nu = \{-\mathtt{EE}_1 \leq \nu\} = \{\sum_{i=1}^{n_t}\log_2(1+\mu_i P) + \nu(\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}) \leq 0\}$ are convex for $\nu \in \mathbb{R}$. Thus, $\mathtt{EE}_1$ is a quasi-concave function of P. The first derivative of $\mathtt{EE}_1$ with respect to P is

$$\frac{\partial \mathtt{EE}_1}{\partial P} = \frac{f_1(P)}{(\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}})^2 \ln 2} \tag{11}$$

where

$$f_1(P) \triangleq (\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}})\sum_{i=1}^{n_t}\frac{\mu_i}{1+\mu_i P} - \alpha\sum_{i=1}^{n_t}\ln(1+\mu_i P) \tag{12}$$

which is a decreasing function of P. Note that $f_1(0) = (n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}})\sum_{i=1}^{n_t}\mu_i > 0$. Thus, there exists a unique P^* such that $f_1(P^*) = 0$, i.e., $\frac{\partial \mathtt{EE}_1}{\partial P}|_{P=P^*} = 0$. Recall that $\mathtt{EE}_1$ in (10) is a quasi-concave function of P. Thus, the unique P^* is the global optimal power which maximizes $\mathtt{EE}_1$. Unfortunately, the optimal value of $P^* = \arg(f_1(P)=0)$ cannot be expressed in closed-form. However, with the expression in (12) and noting that $f_1(P)$ is a decreasing function of P, we can easily obtain the optimal power value numerically, e.g., via a bisection search shown in Algorithm 1.

Algorithm 1 Bisection method for finding the optimal transmit power that maximizes $\mathtt{EE}_1$

given $P_l \leq P^*$ and $P_u \geq P^*$, with $f_1(P_l) \geq 0$ and $f_1(P_u) \leq 0$, given tolerance $\epsilon > 0$.

repeat

1: set $P = \frac{P_l + P_u}{2}$, and calculate $f_1(P)$ by using (12).
2: **if** $f_1(P) > 0$ **then**
3: $P_l := P$
4: **else**
5: $P_u := P$

until $|f_1(P)| < \epsilon$.

An initial lower bound can be easily found as $P_l = 0$. To find an initial upper bound P_u such that $g(P_u) \leq 0$, we define $\mu_{\max} \triangleq \max(\mu_i)$ and $\mu_{\min} \triangleq \min(\mu_i)$. Then, we have $f_1(P) \leq n_t \tilde{f}_1(P)$, where

$$\tilde{f}_1(P) \triangleq (\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}})\frac{\mu_{\max}}{1+\mu_{\min}P} - \alpha\ln(1+\mu_{\min}P). \tag{13}$$

Therefore, an upper bound P_u can be found by setting $\tilde{f}_1(P_u) = 0$, from which we get

$$P_u = \frac{\exp\left(x + \frac{\mu_{\max}}{\mu_{\min}}\right) - 1}{\mu_{\min}} \tag{14}$$

where

$$x \triangleq W\left(\frac{\mu_{\max}\left(\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}\right)}{\alpha \exp\left(\frac{\mu_{\max}}{\mu_{\min}}\right)}\right) \tag{15}$$

and W is the Lambert W function satisfying $W(x)e^{W(x)} = x$. With the closed-form expression of P_u in (14), Algorithm 1 is guaranteed to converge to the optimal P^*.

2) High SNR Approximation: In order to gain more physical insights, we now elaborate on the high SNR regime. Plugging (9) into (4), the EE can be approximated as $\mathtt{EE} \approx \mathtt{EE}_2$, where

$$\mathtt{EE}_2 \triangleq \frac{\mathtt{SE}_2}{P_{\mathtt{tot}}} = \frac{n_t \log_2\left(\frac{P}{n_t\sigma^2}\right) + c}{\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}}. \tag{16}$$

Proposition 1: For the uncorrelated Ricean fading MIMO channel with rank-1 mean matrix $\mathbf{M}$ and $n_r \leq n_t$, in the high SNR regime, the optimal transmit power that maximizes $\mathtt{EE}_2$ is

$$P^* = n_t\sigma^2\left(\exp\left(W\left(\frac{n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}}{n_t\sigma^2\alpha e^d}\right) + d\right)\right) \tag{17}$$

where $d \triangleq 1 - \frac{c}{n_t}\ln 2$, and c is defined in Lemma 2.

Proof: From (16), we see that $\mathtt{EE}_2$ is a quasi-concave function of P. By calculating the first order derivative of $\mathtt{EE}_2$ with respect to P, and setting it to zero, we get

$$f_2(P) = \frac{n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}}{\alpha P} - \left(\ln\left(\frac{P}{n_t\sigma^2}\right) - d\right) = 0. \tag{18}$$

Let $z = \ln\left(\frac{P}{n_t\sigma^2}\right) - d$, then (18) can be rewritten as $ze^z = \frac{n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}}{n_t\sigma^2\alpha e^d}$, from which, we get $z = W\left(\frac{n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}}{n_t\sigma^2\alpha e^d}\right)$. Therefore, the optimal transmit power can be obtained by substituting the value of z into $P^* = n_t\sigma^2\exp\left(z + d\right)$. The optimal solution (17) can also be found by using a different methodology presented in [10]. ■

Note that $W(x)$ is an increasing function of x for $x > 0$. Thus, from Proposition 1, we observe that in the high SNR regime, the optimal transmit power increases with increasing the fixed power consumption $P_{\mathtt{ct}}$ and/or $P_{\mathtt{cr}}$. Moreover, with the aid of [1, Lemma 3], we can show that the optimal transmit power is approximately equal to $n_t\sigma^2 e^d$ for small $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$. For large $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$, the optimal transmit power increases linearly with $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$. This is intuitive, because when $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$ is small, the effect of the fixed power consumption on the EE is negligible. On the contrary, when the fixed power consumption is large and dominates the total power consumption, then more transmit power can be used to increase the SE, thereby, increasing the EE.

Corollary 1: The optimal transmit power, P^* in (17), increases as the Ricean K-factor, K, increases.

Proof: Note that c, which is defined in Lemma 2, decreases with increasing K. Thus, $d \triangleq 1 - \frac{c}{n_t}\ln 2$ increases as K increases. Then, from (18), we see that $f_2(P)$ is an increasing function of K. Note that $f_2(P)$ is a decreasing function of P. Therefore, the optimal transmit power, P^*, which satisfies $f_2(P^*) = 0$, increases as K increases. ■

Corollary 1 implies that, in the high SNR regime, in order to maximize the EE, the transmit power should increase in order to compensate for the SE loss due to the dominating LoS component.

B. The Impact of SE on EE

For arbitrary SNR, the optimal SE that maximizes $\mathtt{EE}_1$ can be obtained by substituting P^* obtained from Algorithm 1 into (6). For the high SNR regime, we present the following proposition.

Proposition 2: For the uncorrelated Ricean fading MIMO channel with rank-1 mean matrix $\mathbf{M}$ and $n_r \leq n_t$, in the high SNR regime, the optimal value of $\mathtt{SE}_2$ that maximizes $\mathtt{EE}_2$ is

$$\mathtt{SE}_2^* = \frac{n_t}{\ln 2}\left(W\left(\frac{\left(n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}\right)e^{-d}}{n_t\alpha\sigma^2}\right) + 1\right). \tag{19}$$

Proposition 2 implies that increasing the SE does not always increase the EE. Recall that in the high SNR regime, the optimal transmit power increases as $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$ increases. As expected, the optimal SE also increases with increasing the fixed power consumption. For small $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$, the optimal SE is approximately equal to $\frac{n_t}{\ln 2}$, which increases linearly with n_t. However, when the fixed power consumption is large, the optimal SE increases only logarithmically with $n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}$. Recall that $W(x)$ is an increasing function of x for $x > 0$, and d is an increasing function of K. From (19), we have the following corollary.

Corollary 2: The optimal SE, $\mathtt{SE}_2^*$ in (19), decreases as the Ricean K-factor, K, increases.

Corollary 2 claims that, in the high SNR regime, the SE decreases as the LoS component becomes more and more dominant, which is expected due to the reduced multiplexing gain [9].

C. The Impact of Number of Antennas on EE

Since the optimal n_t and n_r that maximize $\mathtt{EE}_1$ for arbitrary SNR are difficult, if not impossible, to obtain, we now focus on the high SNR regime to gain insights. The values of n_t and n_r are assumed to be continuous with $0 < n_t \leq n_r$.

1) The Impact of n_t on EE: We start with investigating the impact of the number of transmission antennas, n_t, on the EE. The number of receive antennas, n_r, is assumed to be fixed. Utilizing Stirling's approximation, $\ln n! \approx n\ln n - n$, (9) becomes

$$\mathtt{SE}_2 \approx \frac{1}{\ln 2}\left(n_t\left(\ln\frac{P}{\sigma^2 n_t\left(1+K\right)} - 1\right) + \ln\left(1 + n_t K\right)\right) + \frac{1}{\ln 2}\Big(n_r\ln n_r - \left(n_r - n_t\right)\ln\left(n_r - n_t\right)\Big). \tag{20}$$

By doing so, it can be shown that $\mathtt{EE}_2$ in (16) is a quasi-concave function of n_t, with

$$\frac{\partial \mathtt{EE}_2}{\partial n_t} \approx \frac{\left(\alpha P + n_r P_{\mathtt{cr}}\right)}{\left(\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}\right)^2 \ln 2} J_1\left(n_t\right) \tag{21}$$

where

$$J_1\left(n_t\right) \triangleq \left(1 + n_r\beta\right)\ln\left(n_r - n_t\right) - \ln n_t - \beta\ln\left(1 + n_t K\right) + \frac{\left(1 + n_t\beta\right)K}{1 + n_t K} + \ln\frac{P}{\sigma^2\left(1+K\right)} - 1 - \beta n_r \ln n_r \tag{22}$$

with $\beta \triangleq \frac{P_{\mathtt{ct}}}{\alpha P + n_r P_{\mathtt{cr}}}$. Note that $J_1\left(n_t\right)$ is a decreasing function of n_t. Recall that $0 < n_t \leq n_r$ and note that $J_1\left(n_t \to 0\right) > 0$

and $J_1(n_t \to n_r) < 0$. Thus, there exists a unique optimal $\hat{n}_t$, where

$$\hat{n}_t = \underset{0<n_t\leq n_r}{\arg}\left(J_1(n_t) = 0\right). \quad (23)$$

The optimal value of $\hat{n}_t$ cannot be expressed in closed-form. However, similar to Algorithm 1, we can easily obtain $\hat{n}_t$ numerically via a bisection search. If the value of $\hat{n}_t$ is non-integer, the optimal n_t^* is attained at one of the two closest positive integers. Note that $J_1(n_t)$ is also a decreasing function of β and K. Therefore, $\hat{n}_t$ decreases as β or/and K increases. Intuitively, this implies that, in the high SNR regime, the optimal number of transmit antennas decreases when the fixed power consumption at the transmitter increases. Moreover, it indicates that less transmit antennas should be used when the LoS component becomes more and more dominant. This is expected since when the Ricean K-factor is large, the SE gain achieved by adding more transmit antennas is negligible. Note that the total power consumption increases with increasing n_t. Thus, as K increases, less transmit antennas should be used in order to achieve a better trade-off between the SE and power consumption.

2) The Impact of n_r on EE: Now, we try to find the optimal number of receive antennas, n_r, by keeping n_t fixed. Utilizing (20), it can been shown that $\mathtt{EE}_2$ in (16) is also a quasi-concave function of n_r, with

$$\frac{\partial \mathtt{EE}_2}{\partial n_r} \approx \frac{\alpha P + n_t P_{\mathtt{ct}}}{\left(\alpha P + n_t P_{\mathtt{ct}} + n_r P_{\mathtt{cr}}\right)^2 \ln 2} J_2(n_r) \quad (24)$$

where

$$J_2(n_r) \triangleq \ln n_r - a_1 \ln(n_r - n_t) - a_2 \quad (25)$$

with $a_1 \triangleq 1 + n_t\zeta$ and $a_2 \triangleq \zeta\left(n_t\left(\ln\frac{P}{n_t\sigma^2} - 1\right) + \ln\left(\frac{1+n_tK}{(1+K)^{n_t}}\right)\right) > 0$ in the high SNR regime. Here, $\zeta \triangleq \frac{P_{\mathtt{cr}}}{\alpha P + n_t P_{\mathtt{ct}}}$. It is easy to show that $J_2(n_r)$ is a decreasing function of n_r. Recall that $n_r \geq n_t$ and noting that $J_2(n_r \to +\infty) < 0$ and $J_2(n_r \to n_t) > 0$. Thus, there exists a unique optimal $\hat{n}_r$, where

$$\hat{n}_r = \underset{n_r\geq n_t}{\arg}\left(J_2(n_r) = 0\right). \quad (26)$$

The optimal value of $\hat{n}_r$ cannot be expressed in closed-form. However, similar to Algorithm 1, we can easily obtain $\hat{n}_r$ numerically via a bisection search. Note that $J_2(n_r) \leq \ln\frac{n_r}{n_r - n_t} - a_2$. Thus, by setting $\ln\frac{n_r}{n_r - n_t} - a_2 = 0$, an upper bound of $\hat{n}_r$ can be found as $\hat{n}_r \leq \frac{n_t}{1-e^{a_2}}$. Finally, the optimal n_r^* is attained at one of the two closest integers.

From (25), we see that $J_2(n_r)$ decreases as ζ increases. Therefore, in order to satisfy $J_2(\hat{n}_r) = 0$, $\hat{n}_r$ should decrease as ζ increases. This implies that the optimal number of receive antennas decreases with increasing the fixed power consumption at the receiver. Note that a_2 is proportional to ζ. Thus, when the fixed power consumption at the receiver side dominates the total power consumption, the upper bound, $\frac{n_t}{1-e^{a_2}}$ will approach to n_t. In this case, we have $n_r^* = n_t$.

3) Special Case of $n_t = n_r$: For the special case of $n_t = n_r = n$, we present the following corollary.

Proposition 3: When $n_t = n_r = n$, the optimal n^* that maximizes $\mathtt{EE}_2$ is given by $n^* = \underset{\{\lfloor\hat{n}\rfloor,\lceil\hat{n}\rceil\}}{\arg\max}\mathtt{EE}_2$, where

$$\hat{n} = \frac{\exp\left(W\left(\left(\frac{\alpha PK}{P_{\mathtt{ct}}+P_{\mathtt{cr}}} - 1\right)\frac{1}{e^b}\right) + b\right) - 1}{K} \quad (27)$$

with $b \triangleq 1 + \frac{\alpha P}{P_{\mathtt{ct}}+P_{\mathtt{cr}}}\left(\ln\frac{P}{\sigma^2(1+K)} - 1\right)$.

Proof: Plugging $n_t = n_r = n$ into (9), and utilizing Stirling's approximation, we get $\mathtt{SE}_2 \approx \frac{1}{\ln 2}\left(n\ln\frac{\rho}{1+K} - n + \ln(1+nK)\right)$, which is a concave function of n. Therefore, $\mathtt{EE}_2$ in (16) is a quasi-concave function of n, with $\frac{\partial \mathtt{EE}_2}{\partial n} \approx \frac{J_3(n)}{(\alpha P + n(P_{\mathtt{ct}}+P_{\mathtt{cr}}))^2\ln 2}$, where

$$J_3(n) \triangleq \alpha P\left(\ln\frac{P}{\sigma^2(1+K)} - 1 + \frac{K}{1+nK}\right) - \left(P_{\mathtt{ct}} + P_{\mathtt{cr}}\right)\left(\ln(1+nK) - \frac{nK}{1+nK}\right). \quad (28)$$

It can be shown that $J_3(n)$ is a decreasing function of n, with $J_3(n = +\infty) < 0$ and $J_3(n = 1) > 0$. Thus, there exists a unique optimal $\hat{n}$ such that $J_3(\hat{n}) = 0$, and the optimal n^* is attained at one of the two closest integers from $\hat{n}$. ■

Similarly, utilizing (28), we can show that the optimal number of antennas, $\hat{n}$, decreases as the Ricean K-factor and/or the fixed power consumption $P_{\mathtt{ct}} + P_{\mathtt{cr}}$ increase.

V. Numerical Results

Numerical results are presented to verify our analytical results and to illustrate the impact of different system parameters (P, $\mathtt{SE}$, n_t and n_r) on the EE of rank-1 Ricean fading MIMO systems. The noise power is $\sigma^2 = 10$mW.

Figure 1 shows the EE as a function of the transmit power, P, for different Ricean-K factors. The numbers of transmit and receive antennas are $n_t = 2$ and $n_r = 4$, respectively. The power model parameters are $P_{\mathtt{ct}} = 56$W, $P_{\mathtt{cr}} = 130$W and $\alpha = 2.6$, which correspond to a backhaul channel between a micro BS to a macro BS [8, Table 2]. The lower bound $\mathtt{EE}_1$ and the high SNR approximated $\mathtt{EE}_2$ are compared with Monte-Carlo simulations. We see that both analytical EE expressions agree perfectly with the numerical results. As expected, the EE decreases as the Ricean-K factor increases. Utilizing Proposition 1, for $K = 0, 10, 100$, the optimal power values are 32.45, 35.49 and 41.14W, respectively, which agree with the simulated optimal values as shown in Fig. 1. Moreover, in agreement with Corollary 1, we observe that the optimal transmit power increases as the Ricean-K factor increases. This indicates that, in order to maximize the EE, the transmit power should increase so as to compensate for the SE loss due to the increased LoS component.

Figure 2 demonstrates the EE versus the SE considering the same system model parameters used in Fig. 1. We see that even though the optimal transmit power increases as K increases, the optimal value of SE decreases when increasing K. This observation is in agreement with Corollary 2. Moreover, we see that when the SE is less than 12 bits/s/Hz, the EE increases linearly with the SE. This is because, for small values of SE, the corresponding transmit power P is small. Thus, by

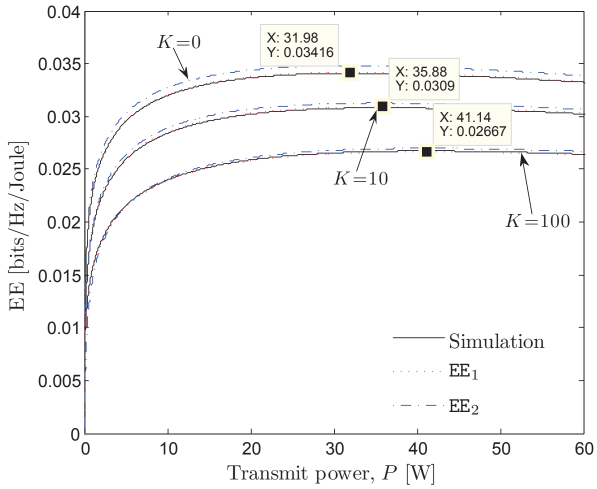

Fig. 1. Energy Efficiency vs. transmit power.

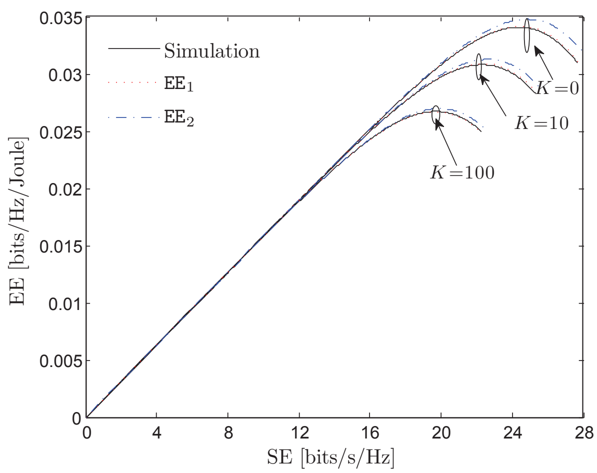

Fig. 2. Energy efficiency vs. spectral efficiency.

increasing P, the increase of the total power consumption $P_{\mathtt{tot}}$ is negligible. Therefore, the EE, which is defined as the SE-to-$P_{\mathtt{tot}}$ ratio, increases linearly as the SE increases.

Finally, Fig. 3 investigates the impact of the number of transmit antennas on the EE of rank-1 Ricean fading MIMO channels. The number of receive antennas n_r is set to 10. The EE is plotted as a function of n_t for different values of K and $P_{\mathtt{cr}}$ respectively. We consider a micro BS transmitter ($P_{\mathtt{ct}} = 56\text{W}$ and $\alpha = 2.6$), and investigate three different backhaul MIMO channels with $P_{\mathtt{cr}} = 6.8, 56$ and 130W, which correspond to three different types of receivers, i.e., pico BS, micro BS and macro BS, respectively [8, Table 2]. As expected, for a fixed value of $P_{\mathtt{cr}}$, the optimal n_t decreases as K increases, since with increasing the Ricean K-factor, the contribution to the SE by adding more transmit antennas becomes negligible, while the total power consumption will increase linearly. On the other hand, for a fixed value of K, the optimal n_t increases as $P_{\mathtt{cr}}$ increases. This observation is also intuitive, because with increasing $P_{\mathtt{cr}}$, the total power consumption becomes dominated by the power consumption at the receiver. Therefore, more transmit antennas can be added to increase the SE, thereby, increasing the EE.

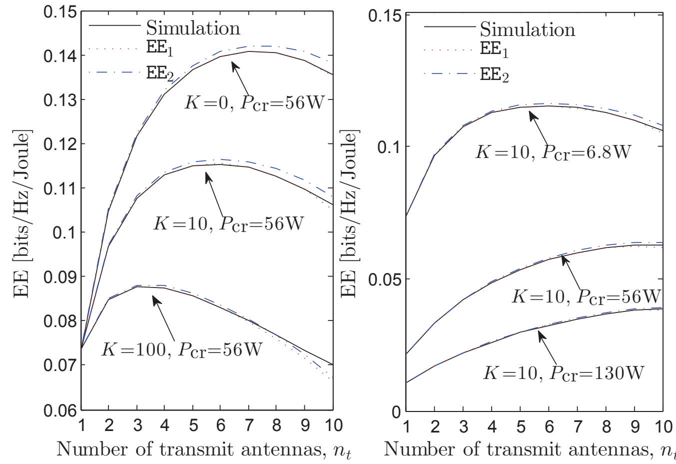

Fig. 3. Energy efficiency vs. the number of transmit antennas.

VI. Conclusions

In this paper, the EE of rank-1 Ricean fading MIMO channels was analyzed. More specifically, the effects of different system parameters, i.e., transmit power, SE, and number of transmit and receive antennas, on the EE were investigated. When keeping the other parameters fixed, closed-from expressions have been derived for the optimal transmit power and the optimal SE, respectively, with the objective of maximizing the EE in the high SNR regime. We analytically show that the optimal transmit power and the optimal SE increase as the fixed power consumption per transceiver chain increases. Moreover, both our theoretical analysis and numerical results indicated that, as the Ricean-K factor increases, the optimal transmit power will increase; on the contrary, the optimal SE and the optimal number of transmit antennas will decrease with increasing K.

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