Thesis for the Degree of Doctor of Philosophy

Route Optimized Energy Management of Plug-in Hybrid Electric Vehicles

Viktor Larsson

Department of Signals and Systems
Chalmers University of Technology
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Viktor Larsson
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Department of Signals and Systems
Automatic Control Group
Chalmers University of Technology
SE–412 96 Göteborg
Sweden
Telephone: +46 (0)31 – 772 1000

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To my family

Abstract

Plug-in hybrid electric vehicles have the potential to significantly reduce the oil dependence within the transportation sector. However, there will always be some trips that exceed the electric driving range, meaning that both electric energy and fuel must be used. For such trips the fuel economy is intimately connected with the energy management system and its ability to schedule the use of the battery. The fundamental problem is that the optimal fuel economy can be reached only if the future trip is known a priori. It is therefore desirable to have a system that can perform three principal tasks: i) acquire a prediction of the future trip, ii) given the prediction precompute feedforward information for the real-time level, and iii) at the real-time level identify the optimal operating points in the powertrain.

This thesis investigates all three of the mentioned tasks. It is shown that frequently travelled routes can be identified from logged driving data using hierarchical clustering. Based on the historical driving conditions along the route, it is then possible to precompute an optimal strategy that can be used as feedforward information for the real-time level. Two different methods for such a precomputation are investigated, convex optimization and Dynamic Programming. Particular attention is given to the implementation of a computationally efficient Dynamic Programming algorithm.

A real-time control strategy that is based on a closed-form minimization of the Hamiltonian is also presented. The strategy is derived for a powertrain with two degrees of freedom, and is implemented in a dynamic vehicle model that is used by a vehicle manufacturer. Simulations with a linearly decreasing battery state of charge reference indicate that the fuel economy can be improved with up to 10%, compared to a depleting-sustaining strategy. Real-time compatible controller code is also generated and tested in a production vehicle. The vehicle behaviour during a test drive is similar to simulated behaviour.

Keywords: Plug-in hybrid electric vehicles, Energy management, Dynamic Programming, Pontryagin principle, Convex optimization, Splines, Data clustering

Acknowledgments

During the last five years I have biked up to Chalmers along Eklandagatan roughly one thousand times. To pursue a Ph.D. is in some sense a similar journey. If the initial condition at Korsvägen is a high gear and a low speed, the ride is only marginally stable in the beginning. However, once you learn how the gears work stability is improved, and the long climb upward can start. It has been an exhausting experience, but it is a nice feeling to finally reach the top of the hill.

My Ph.D. journey would not have been possible without my supervisor, Professor Bo Egardt, who granted me the bike and cheered along the way. I am forever grateful for that. My co-supervisor Dr. Lars Johannesson has also been vital for the success of this journey. Thank you for teaching my how to use the gears, the friendship, and the occasional *Weißbier*.

I would also like to thank Anders Lasson and Andreas Karlsson at Volvo Cars for giving me the opportunity to implement some of the research in an actual plug-in hybrid electric vehicle. Furthermore, Dr. Sten Karlsson is acknowledged for initiating the Swedish car movement data project. Acknowledgements are also due to the Swedish Hybrid Vehicle Centre which has funded the research project, and to Dr. Elna Holmberg and Dr. Anders Grauers for organizing the centre and arranging interesting events. Past and present colleagues at the Department of Signals and Systems are also worth to mention, in particular: Andreas, Julia, Magnus, Mikael, Mitra, Nikolce, Roozbeh and Xiaosong.

Finally, my deepest gratitude goes to my family and friends for all the support throughout the years.

Viktor Larsson
Göteborg, April 2014

List of publications

This thesis is based on the following five appended papers:

Paper I

V. Larsson, L. Johannesson, and B. Egardt, "Comparing Two Approaches to Precompute Discharge Strategies for Plug-in Hybrid Electric Vehicles", *Proceedings of the 7th IFAC Advances in Automotive Control*, September 2013, Tokyo, Japan.

Paper II

V. Larsson, L. Johannesson, B. Egardt, and S. Karlsson, "Route Optimized Energy Management of Hybrid Electric Vehicles", *Accepted for publication in IEEE Transactions on Intelligent Transportation Systems*, 2014.

Paper III

V. Larsson, L. Johannesson, and B. Egardt, "Cubic Spline Approximations of the Dynamic Programming Cost-to-go in HEV Energy Management Problems", *Accepted to the European Control Conference*, June 2014, Strasbourg, France.

Paper IV

V. Larsson, L. Johannesson, and B. Egardt, "Analytic Solutions to the Dynamic Programming sub-problem in Hybrid Vehicle Energy Management Problems", *Submitted to IEEE Transactions on Vehicular Technology.*

Paper V

V. Larsson, A. Karlsson, L. Johannesson, A. Lasson, and B. Egardt, "Real-time Energy Management of a Plug-in Hybrid Electric Vehicle based on a closed-form minimization of the Hamiltonian", *Submitted to Control Engineering Practice*.

Other publications

V. Larsson, L, Johannesson, and B. Egardt, "Impact of trip length uncertainty on optimal discharging strategies for PHEVs", *Proceedings of the 6th IFAC Advances in Automotive Control*, July 2010, Munich, Germany.

V. Larsson, L, Johannesson, and B. Egardt, "Influence of State of Charge estimation uncertainty on energy management strategies for Hybrid Electric Vehicles", *Proceedings of the18th IFAC World Congress*, August 2011, Milano, Italy.

M. Pourabdollah, **V. Larsson**, L. Johannesson, and Bo Egardt, "PHEV energy management: a comparison of two levels of trip information", *Proceedings of the SAE World Congress*, April 2012, Detroit, USA.

V. Larsson, L, Johannesson, B. Egardt, and A. Lasson,, "Benefit of Route Recognition for Plug-in Hybrid Electric Vehicles", *Proceedings of the American Control Conference*, June 2012, Montreal, Canada.

A. Furberg, **V. Larsson**, and B. Egardt, "Optimal Selection of Driving Modes along a Commuter Route for a Plug-in Hybrid Electric Vehicle", *Accepted to the 19th IFAC World Congress*, August 2014, Cape Town, South Africa.

Contents

II Included papers

Part I

Introductory chapters

Chapter 1

Introduction

This chapter gives a brief background on vehicle electrification and hybrid electric vehicles. The concept of energy management is also introduced and the main contributions of the thesis are summarized.

1.1 Background

During the 19th and 20th century, the primary energy sources in the industrialized countries changed from renewable sources to fossil sources, as illustrated in Figure 1.1. This also meant that the primary energy sources for transportation purposes changed; from wind and nutrients (i.e. muscle power), to coal, petroleum and natural gas. The energy content in these fossil energy sources is orders of magnitude higher than for the renewable counterparts, and as a consequence modern transportation is much faster and cheaper than ever before in human history. Unfortunately, there are also serious disadvantages associated with the current transportation system. Fossil fuel sources are inherently finite and the expected peak in global oil and gas production will most likely challenge modern mobility and cause severe economic consequences [1]. Furthermore, the burning of fossil fuels is responsible for significant emissions of CO_2, NO_X and soot. These emissions are causing global warming [2], negative health effects [3] and various other environmental problems.

One way to decrease the dependence on fossil fuels and improve energy efficiency is to electrify transportation as much as possible and generate the required electricity from renewable energy sources. A fully electrified vehicle fleet is however not a realistic option during the foreseeable future, mainly due to two factors: i) the high cost of battery capacity (>\$500/kWh in 2013 [4]), and ii) the limited energy capacity of modern batteries ($\sim 10^2$ Wh/kg for a Li-Ion cell [5] compared to $\sim 10^4$ Wh/kg for gasoline). These

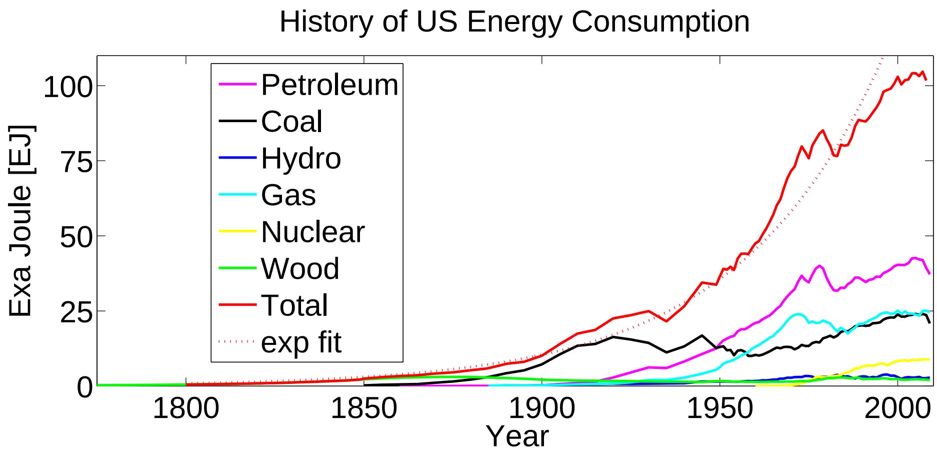

Figure 1.1: Historical energy consumption in the US, 1775-2009 [13].

factors imply that Battery Electric Vehicles (BEVs) are both expensive and heavy, at least if the driving range should be comparable to that of a conventional vehicle. Hybrid Electric Vehicles (HEVs) and Plug-in HEVs (PHEVs) have therefore received significant interest during the last fifteen years and several automotive manufacturers have (once again) started to develop and produce hybridized powertrains (the first HEVs were actually built around 1900 [6]). The commercial breakthrough for HEVs came in 1997 with the introduction of the *Toyota Prius* in Japan. Toyota is still dominant on the market and had by the end of 2013 sold more than 6 million HEVs globally [7]; the yearly sales are illustrated in Figure 1.2. Japan is also world leading in terms of sales, in 2013 the two most sold vehicle models were HEVs; these two models alone had 15.8% of the total market [8]. The market penetration is in general lower in other countries, e.g. during 2013 it was 3.2% in the US [9] and 1.7% in Sweden [10]. The sales of PHEVs are still significantly lower, mainly explained by the fact that these vehicles were introduced on the market only a few years ago. In 2013 the PHEV market share was 0.32% [11] in the US and 0.16% in Sweden [10]. Furthermore, HEV city buses are also available on the commercial market. One of the leading manufacturers, *Volvo buses*, had in mid 2013 sold more than 1000 hybrid buses to 21 countries, with sales tripling for each year. The company is planning to start serial production of PHEV buses in the next few years [12].

Hybrid electric powertrains are at the present day a mature technology and there are more than fifty models available on the market, with many more under development [9]. However, whether or not HEVs and PHEVs will dominate the market in the future remains to be seen. Many factors

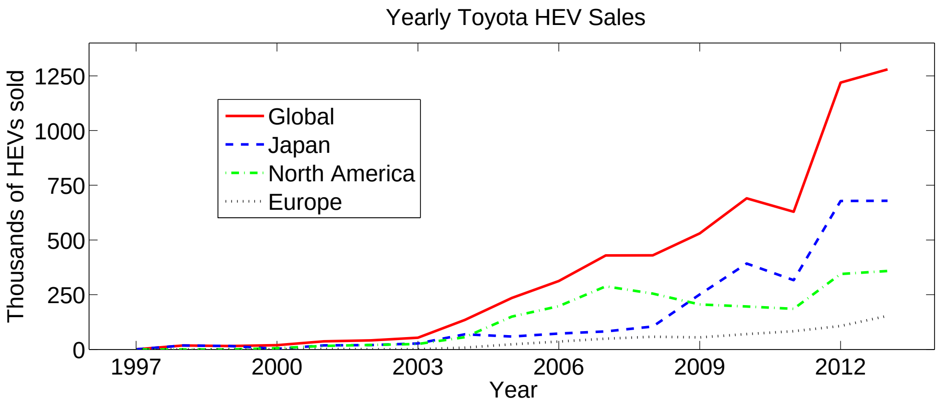

Figure 1.2: Historical Toyota HEV sales worldwide [7].

will influence sales figures, e.g. future oil prices, battery costs, legislation, safety and customer adoption. One thing is nonetheless certain: for hybrid electric powertrains to be competitive, it is crucial to fully exploit their benefits. The overall scope of this thesis is therefore to develop computationally efficient methods for optimal energy management.

1.2 The hybrid electric vehicle

A hybrid electric powertrain is characterized by the existence of an internal combustion engine (spark ignited or diesel), one or several electric machines (typically permanent magnet) and an electrochemical energy buffer (Li-Ion/NiMH battery or super capacitor). It is popular to categorize hybrid vehicles according to the degree of electrification, indicating to what extent the vehicle can drive electrically. A *mild hybrid* has an electric motor with low power and a battery with limited capacity. It cannot be driven solely using the motor and the battery cannot be charged by external sources. Therefore the vehicle must operate in charge sustaining mode, meaning that the net change in battery energy after a trip should be zero. The next category, the *full hybrid*, has a more powerful motor and a battery with higher capacity, meaning that pure electric driving is possible at low speeds. However, similarly to the mild hybrid the battery cannot be charged by external sources. The third and final category, the *plug-in hybrid*, has a battery that can be charged from external sources and it has enough capacity to give at least ten to twenty kilometres of all electric driving. The motor is relatively powerful and can provide electric driving up to highway speeds.

When referring to a hybrid electric vehicle it is common to distinguish

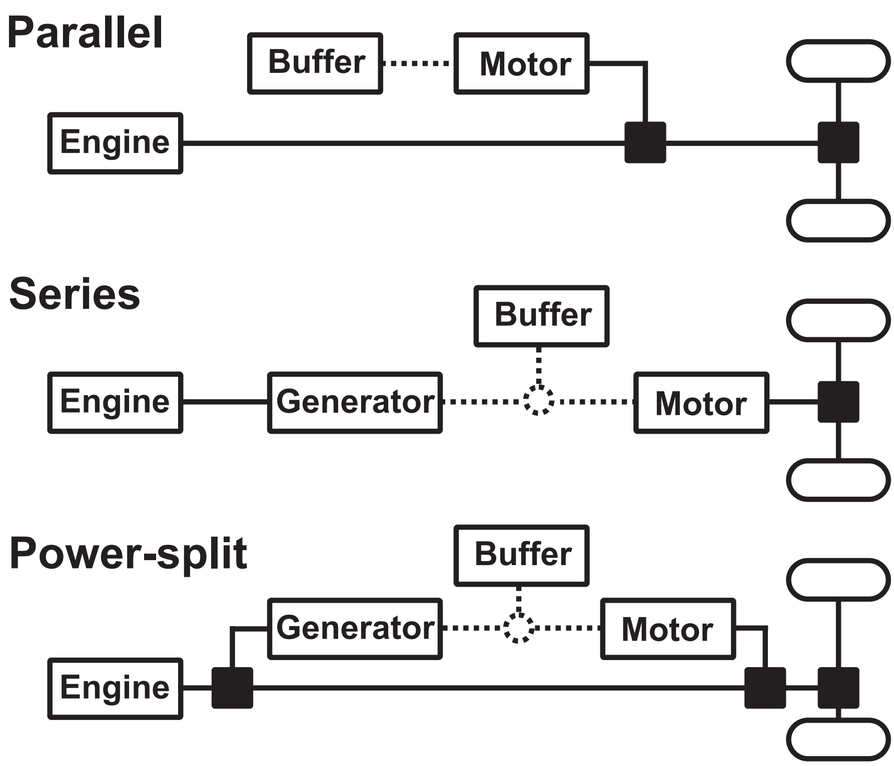

Figure 1.3: The three most common powertrain configurations for hybrid electric vehicles. Solid lines represents mechanical paths and dotted lines electrical paths.

between different configurations, mainly based on the mechanical and electrical paths that are present in the powertrain. The three most common configurations are *the parallel*, *the series* and *the power-split*, as shown in Figure 1.3. Each configuration has its advantages and disadvantages in terms of cost, complexity and energy efficiency. The focus of this thesis, however, is on optimal energy management; consequently there will be no detailed investigation of pros and cons of particular configurations or components, for coverage of such topics see for example [6, 14].

The overall advantages with a hybrid electric powertrain (valid for all categories and configurations) are summarized below:

Brake energy recuperation: The energy buffer makes it possible to recover some of the energy that is lost at the friction brakes in a conventional vehicle.

Engine start & stop: The engine can be switched off during standstill and the auxiliary systems can (temporarily) be powered from the energy buffer.

Engine operating point optimization: The operating point (speed and torque) of the engine can be chosen with some degree of freedom as there is at least one additional power source within the powertrain.

Engine downsizing: It is not necessary to have an engine with a rated power equal to the peak power of the vehicle, as there is at least one additional power source in the powertrain. A smaller engine can be operated at a higher average load, i.e. at a higher average efficiency.

Improved drivability: The electric motor can be used to improve the response of the vehicle, e.g. cranking of the engine, boost when accelerating and give smoother gear shifting.

Lower emissions: Electric driving lowers local emissions (NO_X, ground level ozone, soot and noise). Moreover, the improved energy efficiency implies lower CO_2 emissions, particularly for PHEVs that are charged with electricity from renewable energy sources.

As is the case with any technical solution there are also disadvantages with a hybrid electric powertrain. The major ones are:

Cost: A hybrid electric vehicle is typically much more expensive than a conventional vehicle, mainly due to the high cost of battery capacity and the additional components that are added to the powertrain.

Safety: Electrification introduces high voltage components in the powertrain that can be hazardous during service and in the event of an accident. Furthermore, the battery must be protected from thermal runaway, an event that is likely to cause a fire.

Complexity: The hybrid electric powertrain has more components than a conventional vehicle, meaning that it is more complicated to design, manufacture and control.

1.3 The energy management system

The hybrid electric powertrain itself does not guarantee an improved fuel economy; with a poorly designed control system it might very well have a worse fuel economy than a conventional powertrain. The task of the Energy Management System (EMS) is consequently to decide the preferred operating points for the different powertrain subsystems (i.e. setpoints for the engine, motor and battery), so that the overall cost of operating the vehicle is minimized. This is by no means a trivial task to solve, and there is not a single solution that is generally applicable for all driving scenarios. In fact, it is only possible to obtain an optimal operating cost if the future driving conditions are known a priori. It is therefore beneficial to organize the EMS as a hierarchical system, with levels distinguished by different time scales and predictions of the future driving.

1.3.1 The envisioned system

The EMS that is considered in this thesis consists of two different levels, a predictive level and a real-time level. At the predictive level any a priori information regarding the future trip is considered, and the idea is to solve an optimal control problem based on the prediction. The computed solution is then used as feedforward information to the real-time level, which decides the setpoints for the subsystems in the powertrain. Optimal setpoints are obtained by minimizing an equivalent fuel cost at each time sample, where the term equivalent signifies a trade-off between liquid fuel and electric energy. In the thesis the idea is to obtain the required a priori information from logged historical driving data or a navigation system. The investigated prediction horizon is with respect to the entire trip and the optimal control problem is solved either before the trip or during its initial part. Computations can for example be performed on a server or using an app in a smartphone or a tablet. The obtained solution can then be transmitted to the vehicle over the cellular network, e.g. as a look-up-table defining a reference trajectory for the battery state of charge along the trip.

Short and intermediate prediction horizons are not considered in the thesis, but could easily be included in such a framework. The EMS would then have several predictive levels, each with a different time scale. A priori information for these shorter horizons can for example then be obtained from the cruise control system, vehicle-to-vehicle communication and infrastructure-to-vehicle communication. The resulting optimal control problem would then be solved in real-time, within the vehicle, and the solution could for example suggest impending gear shifts and engine on/off decisions.

The key concept is to have an EMS with a modular structure, where the real-time level is not dependent on predictive information; there will always be situations where a prediction is not available for some reason. Figure 1.4 illustrates the envisioned EMS and how it is related to other systems inside and outside of the vehicle. To conclude, some of the main issues that should be addressed when designing the EMS are stated below, where the focus of the thesis is on the first two topics.

Computational demand: It must be possible to solve the optimal control problem(s) within a realistic time frame and the corresponding memory requirements must be reasonable.

Predictive information: If prediction is used, it should preferably be obtained with minimal effort from the driver. Furthermore, the predicted driving conditions must be represented in a suitable way.

Component wear: Normal operation of the vehicle should not cause accelerated degradation of the powertrain components.

Robustness: The fuel economy should not be severely degraded if the powertrain model or the prediction is slightly inaccurate. Furthermore, the vehicle must be able to operate even if some component in the powertrain fail.

Drivability: The driver should have a sufficient torque reserve and there should not be excessive gear shifts and engine on/off events. The vehicle behaviour should be intuitive for the driver.

1.3.2 Overview of energy management methodology

The first studies investigating optimal energy management of hybrid electric powertrains appeared at the turn of the millennium [15–18] and a vast number of papers have been published on the topic since then. The typical objective is to minimize the operating cost for a given driving mission. The problem formulation that is considered in most studies can be summarized as

$$\begin{array}{ll} \text{minimize} & \text{operating cost} \\ \text{subject to} & \text{state dynamics} \\ & \text{state constraints} \\ & \text{control signal constraints.} \end{array}$$

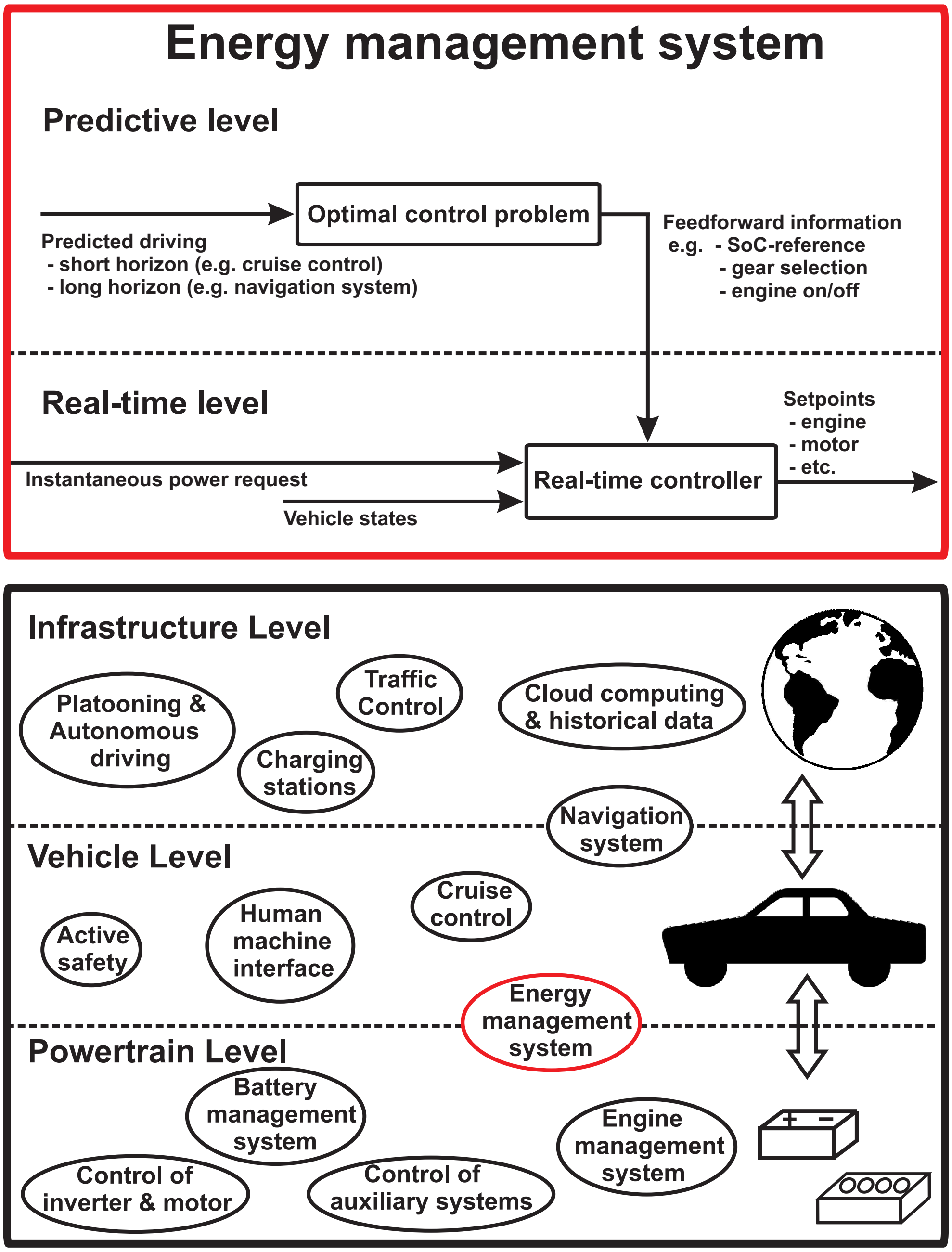

Figure 1.4: An illustration of the EMS and its relation to other systems.

The control signals are typically the choice of gear, the engine state and the torque/power split between the engine and the electric machine(s). The problem is in general both non-linear and mixed integer, due to the gear selection and the engine state decision. Hence, it is computationally demanding and it is therefore common to consider a single dynamic state, the battery State of Charge (SoC). The choice of optimal control method is closely related to the time scale that is considered, i.e. the complexity of the method and the resulting computation time must be consistent with the time scale.

The real-time level of the EMS is typically based on the Equivalent Consumption Minimization Strategy (ECMS) that is derived from the Pontryagin minimum principle [19]. The resulting optimization problem is then an instantaneous minimization of the Hamiltonian. From an implementation point of view, the main problem is generally to determine the correct value of the costate (equivalence factor), which depends on the future driving conditions. There is in general no closed-form solution to this problem and it is therefore common to compute the costate numerically using some kind of shooting method. ECMS strategies have been investigated in numerous studies, see for example [20–32]. At the predictive level of the EMS a priori information is needed and several possible sources for such information have been proposed, for example: vehicle to vehicle communication [33–35], infrastructure to vehicle communication [33,35,36], navigation systems [21, 23, 35, 37–42], and logged historical driving data [43–51]. In an EMS context it is common to distinguish between a deterministic and a stochastic prediction. The former represents the driving conditions by a predefined velocity trajectory, obtained either from the navigation system or from a logged trajectory; the latter describes the driving conditions with a probabilistic model, e.g. a Markov model derived from historical driving data. The optimal control problem at the predictive level has been solved with Dynamic Programming (DP) [52] in numerous studies. The method is a classical optimal control technique that provides the global optimal solution for problems that are both non-linear and mixed integer. Furthermore, it can be used both with a deterministic prediction [16, 41, 42, 50, 51, 53–56] and a stochastic prediction [44, 57–61]. However, a major drawback with the method is that the computational demand increases exponentially with the number of model states and control signals. DP is therefore perceived mainly as a benchmarking method, i.e. to assess the relative performance of methods that have lower computational demand but cannot guarantee global optimality. Examples of methods with a lower computational demand are Model Predictive Control (MPC) [29, 34, 62–64] and Quadratic Programming (QP) [23, 38, 53].

The main disadvantage with the optimization based approaches is the computational demand; a real-time implementation might require a more expensive microprocessor. Rule-based energy management have therefore been considered in many studies. However, such control structures do typically not consider any predictive information and a (near) optimal fuel economy cannot be guaranteed. Instead the powertrain is controlled by a set of heuristic rules, which are tuned to give a good performance on a wide variety of driving situations. The rules can for example be determined by fuzzy logic and neural networks [65–67] or genetic algorithms [65]. Another approach is to define the rules based on optimal powertrain behaviour observed in DP solutions [57].

Energy management is also important when sizing an hybrid electric powertrain, as the total cost of ownership includes both component and operating costs. The focus is thus slightly different since the problem is solved during the design phase, meaning that real-time implementation aspects are not the main priority. The problem is then formulated so that the energy management and powertrain dimensioning is optimized jointly. Several different techniques have been suggested, for example: convex optimization [68–72], DP [60, 73], particle swarm optimization [74] and genetic algorithms [65].

1.4 Scope and contributions of the thesis

The focus of this thesis is on computational methods for optimal energy management. The ideas presented are applicable both to HEVs and PHEVs, however results are shown mainly for PHEVs. Throughout the thesis it is assumed that the dimensioning of the powertrain is fixed and sizing is consequently not considered in the problem formulation. Furthermore, driveability and component degradation are also not treated explicitly. The energy management optimization on the predictive level is performed with a deterministic representation of the driving conditions; the only uncertainty that is considered is the exact length of the trip. The prediction horizon is with respect to the entire trip, and short prediction horizons are not considered. Energy management on the real-time level is mainly focused on the torque split optimization; gear shifts and engine on/off decisions are not investigated in depth. The only dynamic state that is considered is battery SoC.

The main contributions of the thesis are:

- A conceptual framework covering both identification of frequently travelled routes and optimization of the energy management. The routes are identified from logged historical driving data using hierarchical agglomerative clustering and the optimal energy management strategy is computed offline with DP.

- A methodology to approximate the DP cost-to-go with a spline function, meaning that the memory storage requirements can be reduced significantly.

- A DP algorithm where the sub-problems are solved analytically based on a local approximation of the cost-to-go. Thereby it is not necessary to grid the torque split and evaluate the cost-to-go with interpolation. The method is thus very efficient in terms of computational demand.

- A closed-form minimization of the Hamiltonian for a powertrain configuration with two degrees of freedom. The solution is implemented as an ECMS strategy with very low computational demand. The strategy is validated in a test drive with a production PHEV.

Chapter 2

Modelling

This chapter describes the two vehicle modelling approaches that are considered in the thesis. Furthermore, the modelling assumptions of the simplified powertrain model and the vehicle chassis model are also introduced.

2.1 Vehicle modelling

In an energy management context it is important to distinguish between a *dynamic* and an *inverse* model; the former is used to evaluate the EMS in simulations and the latter mainly when optimizing the EMS.

2.1.1 Dynamic model

A dynamic model is used to simulate vehicle behaviour and fuel economy, i.e. it is used to assess the performance of the EMS. The model is based on a high fidelity powertrain model consisting of several different sub-systems (e.g. engine, motor and battery), each of which having a low-level controller and a plant model with dynamic states. Moreover, a dynamic model features a driver model that tries to follow an input velocity reference. The driver model is typically a PI-controller acting on the deviation between a velocity reference and the simulated velocity of the vehicle. The output signal from the driver model is a pedal position that is converted to a torque request at the wheels. The torque request is then an input signal to the EMS, which computes setpoints for the low-level controllers based on a simplified powertrain model. The dynamic model is illustrated schematically in Figure 2.1.

A dynamic model is only used for simulation as it is nonlinear and has several dynamic and integer states. To compute an optimal strategy for such a model would require immense computational power and is simply

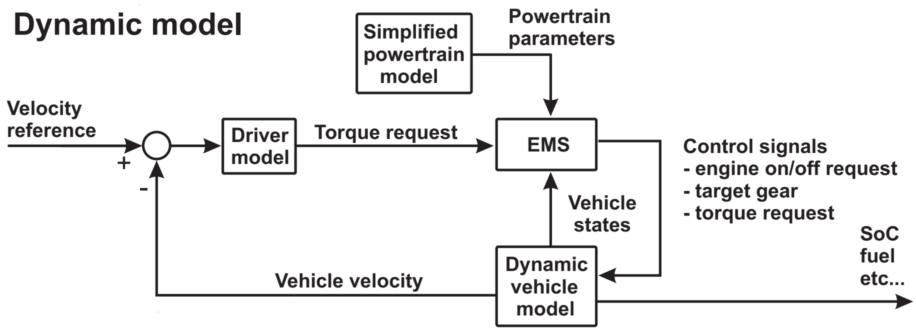

Figure 2.1: The dynamic modelling approach.

not feasible. Examples of dynamic vehicle models are *AMESim* [75], *Dymola* [76], *PSAT* [77] and *Autonomie* [78]. Two dynamic models are used for simulations in the thesis, both implemented in Matlab/Simulink. The *Autonomie* software is used in papers I-II and a non-commercial software called *VSim* is used in paper V; the latter is a model used internally by Volvo Car Corporation.

2.1.2 Inverse model

The inverse model is used when optimizing the EMS and it is therefore based on a highly simplified model of the powertrain. The sub-systems in the powertrain are generally modelled by efficiency maps that are obtained from steady state measurements. Transient dynamics are typically neglected. There is no driver model in an inverse model and the torque request at the wheels is determined using a non-causal approach. Given the velocity reference, the torque required at the wheels to follow the reference perfectly is computed inversely from a point mass model of the vehicle chassis. The approach is non-causal in time, but has the advantage that vehicle speed does not become a model state. The inverse model is illustrated in Figure 2.2. The only dynamic state is typically the energy level in the buffer and optimal control methods can therefore be used to compute an optimal strategy. Examples of inverse vehicle models are the *QSS-Toolbox* [79] and *Advisor* [80]. Inverse models are used when solving the energy management problem in papers I, II, III and IV. The inverse models that are considered in the thesis are derived from the dynamic vehicle models that are available in *Autonomie* and *VSim*.

The disadvantage with this type of inverse model is that transient dynamics are neglected, e.g. boost pressure in a turbocharged engine. If such dynamics are considered it might very well be the case that it is not physically possible to track the velocity reference perfectly. Hence, if the sim-

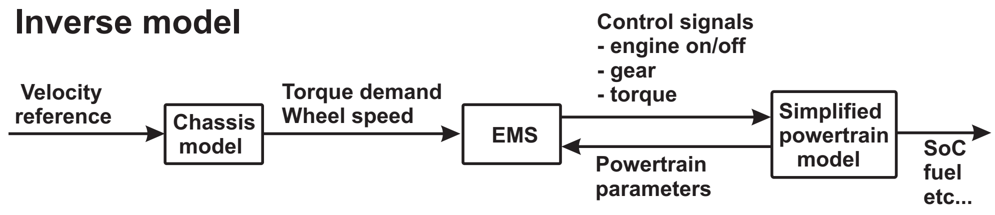

Figure 2.2: The inverse modelling approach.

plified model contains transient dynamics it is preferable to use a so-called *inverse-dynamic model* [81]. With such an approach the transient dynamics are inverted and a physically realizable velocity profile is generated from the velocity reference.

2.2 Simplified powertrain model

The modelling assumptions of the simplified powertrain model and the chassis model are described next, i.e. the models used when optimizing the EMS.

2.2.1 Internal combustion engine

To accurately model the dynamics of an internal combustion engine, a complex model based on partial differential equations is required [82]. This type of model is not practical when optimizing the EMS as the computational demand would be very high. The engine model is therefore very simplified and without any dynamics. Typically it is based on a Brake Specific Fuel Consumption map (BSFC), in which the mass fuel rate has been measured at different steady state crankshaft speeds and output torques. For a given engine speed it is then possible to approximate the engine mass fuel rate as affine or quadratic in crankshaft torque, an approach known as the Willans approximation [14,83]. A quadratic approximation is preferable if the engine efficiency decreases at high torque; if that is not the case an affine approximation should be sufficiently accurate. With an affine approximation the instantaneous mass fuel rate of the engine is described by

$$\dot{m}_f = \big(c_0(\omega_e)T_e + c_1(\omega_e)\big)e_{on}, \tag{2.1}$$

where T_e represents engine torque, ω_e engine speed and e_{on} the binary engine state. The speed dependent coefficients $c_{0:1}$ are computed from the BSFC-map using linear least squares; Figure 2.3 illustrates the accuracy of an affine engine approximation. Finally, with c_f denoting fuel price the instantaneous

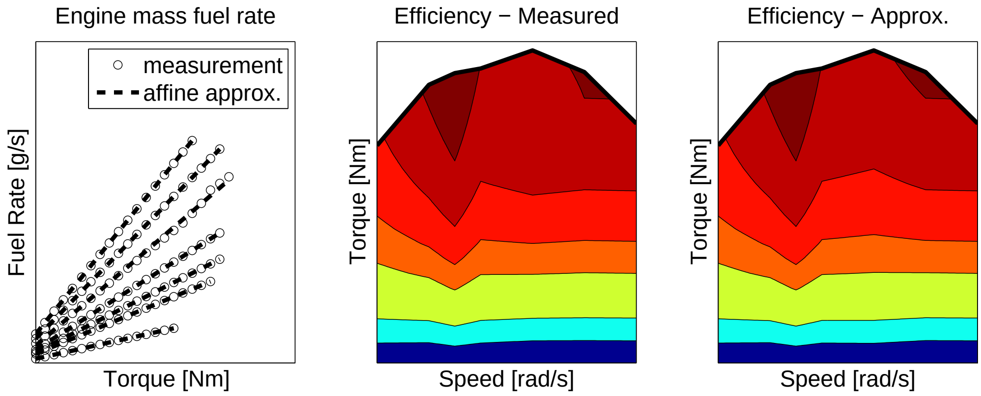

Figure 2.3: The left plot depicts the affine approximation of the engine mass fuel rate at different engine speeds. The plots to the right illustrates the engine efficiency, measured and approximated. The efficiency increases with the intensity of red and the solid black line indicates the maximum torque.

fuel cost is described by

$$g = c_f \cdot \dot{m}_f. \tag{2.2}$$

2.2.2 Electric motor

The modelling assumptions for the electric motor (and generator) is very similar to that of the internal combustion engine. The model is typically based on a power loss map, where the electrical power losses have been measured at different steady state motor speeds and output torques. The electrical power of the motor is then often approximated as quadratic in torque [14, 83],

$$P_m = d_0(\omega_m)T_m^2 + d_1(\omega_m)T_m + d_2(\omega_m), \tag{2.3}$$

where T_m represents motor torque and ω_m motor speed. The speed dependent coefficients $d_{0:2}$ are computed from the power loss map using linear least squares. Figure 2.4 illustrates the accuracy of a quadratic motor approximation.

2.2.3 Power electronics & auxiliary loads

Power electronics are used to transform between alternating current and direct current as well as between different voltage levels within the powertrain. The losses in the power electronics are either included in the power loss map of the motor or calculated using an assumption of constant efficiency.

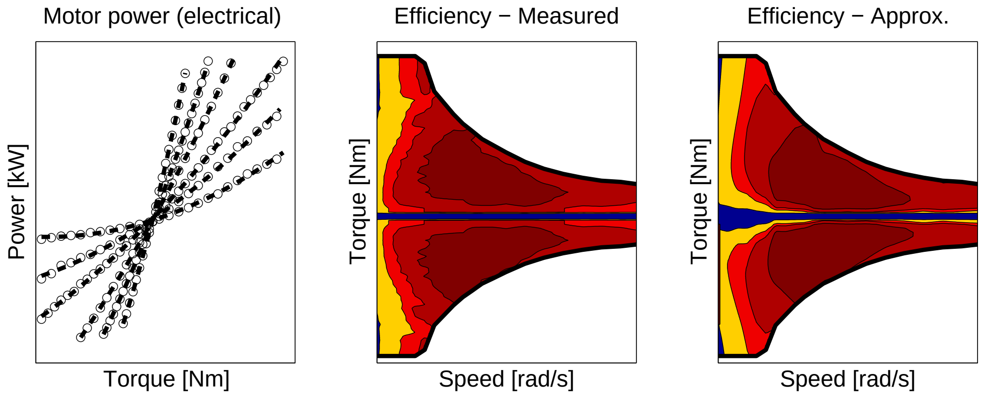

Figure 2.4: The left plot depicts the quadratic approximation of the electrical power of the motor at different speeds. The plots to the right illustrates the motor efficiency, measured and approximated. The efficiency increases with the intensity of red and the solid black line indicates the maximum/minimum torque.

The auxiliary systems in the vehicle, e.g. pumps, air condition, stereo, lights etc., are all assumed to be purely electrical and connected to the same electrical path as the battery. The load is assumed to be constant and known by the EMS.

2.2.4 Battery

The battery in a hybridized powertrain consists of a large number of cells that are connected in series and/or in parallel. To accurately describe the behaviour of such a battery, a complex electrochemical model based on partial differential equations is needed [84]. This type of model is however not suitable for use in an energy management context, since the computational demand would be very high. Hence to reduce complexity, the complete battery is modelled as a simple equivalent circuit with a voltage source in series with an internal resistance [85], illustrated in Figure 2.5. With such a simple battery model the only dynamic state is the battery SoC, where a SoC of one corresponds to a fully charged battery and a SoC of zero denotes an empty battery. Throughout the thesis only Li-Ion batteries are considered as this is the cell chemistry predominantly used at the present day. The voltage of a Li-Ion cell is in general relatively flat with respect to SoC, as illustrated to the left in Figure 2.5. Hence, it is often assumed that the open circuit voltage, V_{oc}, is affine (or constant) with respect to the state. Furthermore, the internal resistance, R_{in}, is often assumed to be constant. Both these assumptions are reasonable over the SoC region of normal usage.

With these modelling assumptions the battery state dynamics is described by

$$\dot{x} = f(x, P_b) = -\frac{I}{Q} = -\frac{V_{oc}(x) - \sqrt{V_{oc}^2(x) - 4R_{in}P_b}}{2R_{in}Q}, \tag{2.4}$$

where x represents the SoC state, I the battery current, Q the battery capacity in As and P_b the net battery power.

As the battery is the perhaps most expensive part of a hybridized powertrain, it is desirable that its life length is consistent with the life length of the vehicle. The battery is typically regarded to be at the end-of-life when the usable capacity (power) has decreased with 20% compared to the rated capacity (power). However, there are many factors that contribute to battery cell degradation and it is not trivial to model/predict the battery State of Health (SoH); the SoH is defined as one when the battery is new and zero at end-of-life. Table 2.1 summarizes some of the main drivers for battery degradation, where battery C-rate and Ah throughput are defined by

$$C = 3600\frac{|I|}{Q}, \tag{2.5}$$

$$Ah = \frac{1}{3600}\int_{t_0}^{t_f} |I(t)|\, dt. \tag{2.6}$$

A C-rate of one means that the battery is discharged/charged in one hour, and the Ah throughput is a measure of the total amount of charge that has passed through the battery.

If battery SoC is the sole dynamic state in the EMS it is only possible to limit the degradation caused by low/high SoC values, i.e. by confining the battery to operate within a restricted SoC interval. Nevertheless, an optimized EMS can often decrease c-rate and Ah-throughput as a second order effect.

2.2.5 Transmission, final drive & friction brakes

The transmission is modelled as a stepped automatic gearbox without dynamics, meaning that gearshifts are assumed to be instantaneous and lossless. The mechanical efficiency of the gearbox and the final drive are modelled using constant efficiencies. Furthermore, the clutch is assumed to be without dynamics and lossless when it is locked. If the engine is on at low vehicle speeds the clutch is assumed to be partially engaged to prevent the ICE from stalling. It is also assumed that the friction brakes are instantaneous and used only if the electric motor or the battery are saturated during braking or downhill driving.

Table 2.1: Degradation factors and effects on Li-Ion battery cells [86, 87].

Degradation Factor	Effect
High temperatures	Increased resistance, Capacity/Power Fade
Low temperatures	Capacity fade
High depth of discharge	Capacity fade
High SoC	Increased resistance, Capacity fade
Low SoC	Power fade, Enhances other effects
High C-rates	Capacity/Power fade
Ah throughput	Capacity fade

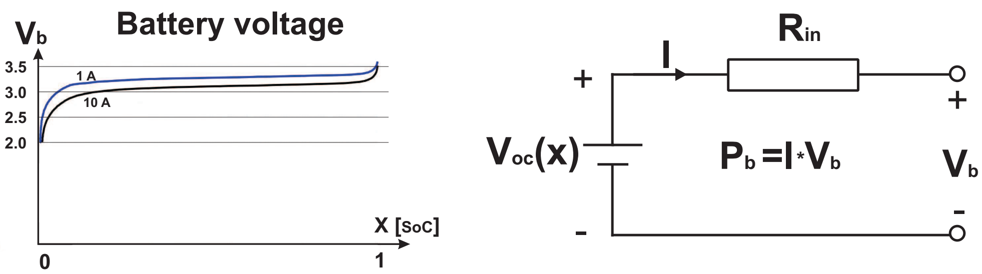

Figure 2.5: Left: The battery voltage of a Li-Ion cell measured at different SoC levels for two discharge currents [5]. Right: The equivalent circuit battery model.

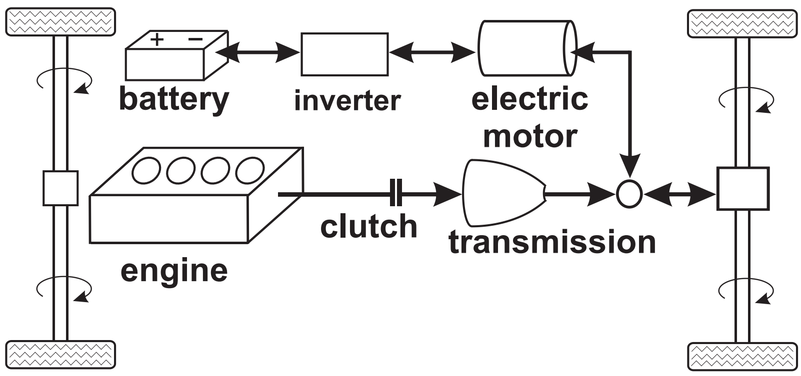

Figure 2.6: A post transmission parallel hybrid vehicle configuration.

2.2.6 Chassis model

The modelling assumptions of the chassis model are outlined next. The parallel hybrid configuration shown in Figure 2.6 serves as the example, and it is also the configuration that is considered in papers I, II, III and IV. The lateral dynamics of the vehicle is of little relevance for the energy management and is therefore not considered. The longitudinal dynamics of the vehicle, modelled as a point mass, is described by Newton's second law of motion

$$(m+\delta m_e)\frac{dv}{dt} = \frac{T_d}{r_w} - \left(\frac{\rho_{air}}{2}C_d A_f v^2 + mg\sin\theta + f_r mg\cos\theta\right), \tag{2.7}$$

where m is the vehicle mass; δm_e is the equivalent mass of the rotating parts; ρ_{air} is the density of air; g is the acceleration of gravity; r_w is the wheel radius; A_f is the vehicle frontal area; C_d is the aerodynamic drag resistance and f_r is the rolling resistance. Using the inverse modelling approach it is then possible to compute the torque that is demanded at the wheels T_d, to follow the velocity v and road grade θ of a given drive cycle.

The traction torque of the powertrain at the wheels T_p is given by

$$T_p = \eta_f r_f\big(T_m + \eta_{gb,i} r_{gb,i} T_e\big) + T_b, \tag{2.8}$$

which must equal the torque demand T_d. The torque of the friction brakes is represented by T_b and the ratio of the final gear is denoted r_f where the corresponding efficiency, η_f, depends on the sign of the torque demand at the wheels. The gears, $i = 1, 2, ...$, are represented by a drive ratio $r_{gb,i}$ and a mechanical efficiency $\eta_{gb,i}$. With r_w representing wheel radius, the rotational speed of the motor and the engine are defined by

$$\omega_m = \frac{r_f}{r_w}v, \tag{2.9}$$

$$\omega_e = \frac{r_f r_{gb,i}}{r_w}v. \tag{2.10}$$

2.3 Discretization

The sole dynamic state in the simplified model is time discretized using the one step Euler method. The SoC dynamics are thus given by

$$x(t_{i+1}) = x(t_i) + h_i \cdot f\big(x(t_i), P_b(t_i)\big). \tag{2.11}$$

The sample time is represented by $h_i = t_{i+1} - t_i$, which is not necessarily constant throughout a driving cycle.

Chapter 3

The energy management problem

This chapter formulates the energy management problem as an optimal control problem, which is then analyzed with the Pontryagin minimum principle. Some important observations are also highlighted.

The energy management problem for a hybrid electric powertrain is often formulated as an optimal control problem where the objective is to minimize the operating cost along a given drive cycle. A model with low complexity is required to keep the computational demand at a reasonable level. Therefore, consider the inverse model approach and the simplified powertrain model introduced in Chapter 2, where battery SoC is the sole dynamic state. Assuming that the drive cycle is known a priori, the problem can be expressed as the following deterministic optimal control problem

$$
\begin{aligned}
J^* = \min_{u(\cdot)} \quad & G(x(t_f)) + \int_{t_0}^{t_f} g(u(t), t)\, dt \\
s.t. \quad & \dot{x}(t) \ = f(x(t), u(t), t) \\
& x(t_0) = x_0 \\
& x(t) \ \in [x_{min},\ x_{max}] \\
& u(t) \ \in U\big(\omega_m(t), \omega_e(t), x(t), T_d(t)\big)
\end{aligned}
\tag{3.1}
$$

where $x = SoC$ and $f(x, u)$ represents the non-linear state equation. The control signal u is defined by the choice of gear $r_{gb,i}$, the engine state e_{on}, and the torque of the engine T_e and the motor T_m. However, in practice it is sufficient to define either the engine or the motor torque, since the other will be given implicitly as the torque demand at the wheels T_d must be satisfied. Furthermore, the feasible set for the control signal U is also defined by the speed of the engine ω_e and the motor ω_m, as the maximum torque is speed dependent. The power constraints of the battery depends on the state x and are imposed as a constraint on the motor torque. The cost criterion J is defined by the instantaneous fuel cost of the engine g

and a final cost G, which represents the cost to recharge the battery at the end of the driving mission (if a PHEV is considered). There is no explicit constraint on the final state since there is no clearly defined lower SoC limit in terms of battery degradation. Instead the final cost G will also enforce a soft constraint on the final state. i.e. there will be a high cost for low final states.

It is not a trivial task to solve the optimal control problem defined by Eq. (3.1). The problem is a non-linear and mixed integer optimization problem and the (predicted) drive cycle is in general not given by an analytic function, but rather as a vector where speed and road grade are specified at different discrete time instances. Hence, in practice it is only possible to solve the problem using numerical methods.

3.1 Analysis with the minimum principle

One of the classical results in control is the *Pontryagin minimum principle* [19], which provides necessary conditions for the optimal control of a dynamical system. To apply the minimum principle [88] to the energy management problem defined by Eq. (3.1), neglect the state constraints and define the Hamiltonian

$$H\big(x(t), u(t), \lambda(t), t\big) = g\big(u(t), t\big) + \lambda(t) \cdot f\big(x(t), u(t), t\big), \tag{3.2}$$

where λ represents the costate. The next step is to determine the control signal u^* that minimizes the Hamiltonian,

$$u^*(t) = \arg \min_{u(t) \in U(t)} \big\{g\big(u(t), t\big) + \lambda^*(t) \cdot f\big(x^*(t), u(t), t\big)\big\}, \tag{3.3}$$

and solve the state and costate equations

$$\dot{x}^*(t) = \left(\frac{\partial H}{\partial \lambda}\right)_*, \tag{3.4}$$

$$\dot{\lambda}^*(t) = -\left(\frac{\partial H}{\partial x}\right)_* \tag{3.5}$$

with boundary conditions x_0 and

$$\left[H + \frac{\partial G}{\partial t}\right]_{*t_f} \delta t_f + \left[\frac{\partial G}{\partial x} - \lambda\right]_{*t_f} \delta x_f = 0, \tag{3.6}$$

where the optimal trajectory is represented by $*$.

Eq. (3.2)-(3.6) can in practice not be used derive a closed form solution for the optimal trajectories, $x^*(\cdot)$, $\lambda^*(\cdot)$, $u^*(\cdot)$, since the powertrain model is both non-linear and mixed integer. Furthermore, the drive cycle is generally not described by an analytic expression. The Pontryagin principle do nonetheless give some valuable insights as it provide necessary conditions that must be satisfied along the optimal solution. More specifically, the costate dynamics along the optimal solution is given by the partial derivative of the Hamiltonian with respect to the state. Recall that the battery voltage of a Li-Ion battery is nearly constant with respect to the state, as shown in Figure 2.5. Hence, if the open circuit voltage of the battery is assumed to be constant, i.e. $f(x,u) \approx f(u)$, then

$$\begin{aligned} \dot{\lambda}^*(t) &= -\frac{\partial}{\partial x}\Big\{ g\big(u^*(t),t\big) + \lambda^*(t)\cdot f\big(u^*(t),t\big)\Big\} = 0, \\ \Rightarrow\ \lambda^*(t) &= \lambda_0. \end{aligned} \tag{3.7}$$

Consequently, as long as the battery voltage does not exhibit a strong dependence on SoC, the costate will have a (nearly) constant value λ_0 along the optimal solution, provided that the state constraints are neglected. Nevertheless, if state constraints are considered, the costate will change value only when a state constraint is activated, i.e. the costate will be piecewise constant.

These properties are illustrated in Figure 3.1, which depicts the optimal state and costate trajectories for a PHEV and an HEV. The results are obtained when Eq. (3.1) is solved as a convex optimization problem using the methodology presented in Paper I, where the engine state and gear selection are given by pre-decided rules. Furthermore, the battery voltage is assumed to be independent of SoC and the costate is then obtained as the dual variable to the discrete time state equation. For the PHEV the state constraints are never activated and the costate is thus constant. However, for the HEV the state constraints are activated and it is clear that the costate changes value at those time instances.

The key observation, obtained by applying the Pontryagin principle and minimizing the Hamiltonian function, is that the costate can be interpreted as an equivalence factor (or exchange rate) between fuel and electric energy; that is, to minimize the Hamiltonian is to minimize an equivalent fuel consumption. The main problem in practice is that the drive cycle is never known perfectly in advance, meaning that the true value of the costate cannot be determined beforehand.

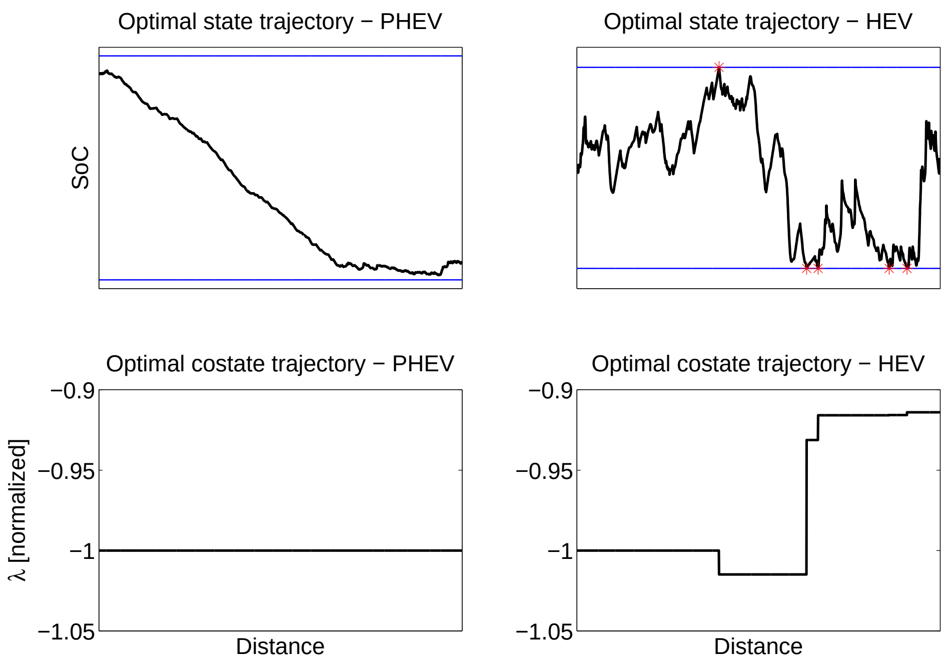

Figure 3.1: Example of optimal state and costate trajectories for a PHEV and an HEV. The value of the costate has been normalized since the magnitude depends on the size of the battery, which differs for the two vehicles. The state constraints are illustrated by the horizontal lines.

Chapter 4

Computational methods for the energy management system

This chapter introduces the computational methods that are investigated in the thesis; DP, convex optimization and ECMS. A review of some basic convexity concepts is also given.

The EMS topology that is considered in the thesis consists of a predictive level and a real-time level, as illustrated in Figure 1.4; a more detailed version highlighting the computational methods in the two levels is shown in Figure 4.1. It is here assumed that the prediction is with respect to the entire trip and that the optimal control problem is solved only once, before the start of the trip. The computed solution is then used as feedforward information for the real-time level, where an instantaneous optimization problem is solved at each time sample. To keep the computational demand at both levels on a reasonable level, it is important to exploit convexity whenever it is possible, something that is done in several of the appended papers. Some basic definitions and results concerning convexity will thus be reviewed before the computational methods are introduced.

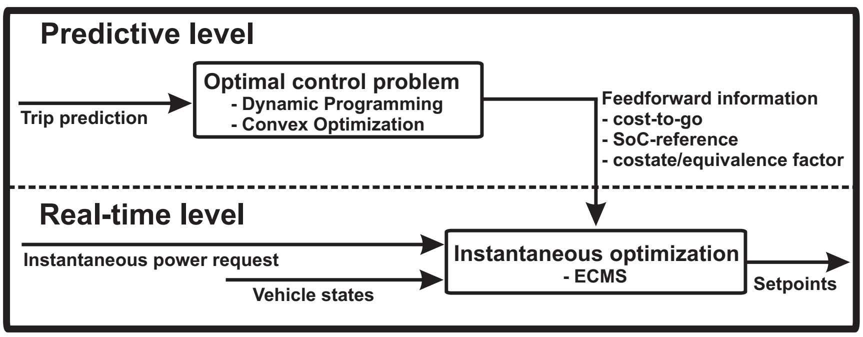

Figure 4.1: The EMS topology considered in the thesis.

4.1 A brief review of basic convexity concepts

The notation follows [89], where all concepts are described in depth. Figure 4.2 provides a graphical illustration of some of the concepts.

Definition 1 *A set C is convex if the line segment between any two points in C lies in C, i.e. if for any $x_1, x_2 \in C$ and any θ with $0 \leq \theta \leq 1$, we have $\theta x_1 + (1-\theta)x_2 \in C$.*

Definition 2 *A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex if dom f is a convex set and if for all $x, y \in$ dom f, and θ with $0 \leq \theta \leq 1$, we have $f\big(\theta x + (1-\theta)y\big) \leq \theta f(x) + (1-\theta)f(y)$.*

Definition 3 *A convex optimization problem is formulated as*

$$\begin{aligned} \min_{x} \quad & f_0(x) \\ s.t. \quad & f_i(x) \leq 0, \quad i = 1, ..., m \\ & h_j(x) = 0, \quad j = 1, ..., p \\ & x \in \mathcal{X} \subseteq \mathbb{R}^n \end{aligned} \tag{4.1}$$

where the feasible domain $\mathcal{X} \subseteq \mathbb{R}^n$ of the decision variables x is convex; the functions $f_0(x)$ and $f_i(x)$ are convex; and $h_j(x)$ are affine.

Some useful results concerning convexity:

- Any local minimum of a convex function is also a global minimum.
- Minus of a convex function is a concave function (and vice versa).
- An affine function, $a_0 x + a_1$, is both concave and convex.
- A quadratic function, $a_0 x^2 + a_1 x + a_2$, is convex if $a_0 \geq 0$.
- A sum of convex functions is a convex function.
- The pointwise maximum of a set of convex functions is convex, i.e. $f(x) = \max\{f_1(x), f_2(x), ..., f_m(x)\}$ is convex if $f_{1:m}$ are convex.
- Convex composition, $f(x) = h\big(g(x)\big)$ is convex if h is convex and nondecreasing (nonincreasing), and g is convex (concave).
- A problem is not convex if it contains integer decision variables.

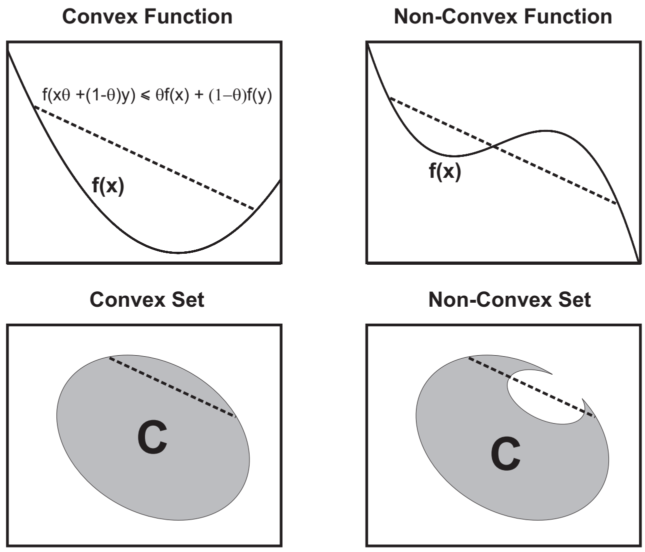

Figure 4.2: Illustration of convex vs. non-convex functions and sets.

4.2 Predictive level

In the predictive level the optimal control problem is solved based on the trip prediction. The computation can for example be performed at the start of the trip (or before) using cloud computing, the infotainment system of the vehicle or an external device such as a smartphone. The obtained solution is then used as feedforward information for the real-time level. Two methods are used to solve the problem, DP and convex optimization, and both are described below.

4.2.1 Dynamic programming

DP is a well known optimal control algorithm based on Bellman's principle of optimality [52]. The algorithm is particularly useful for complex problems that can be partitioned into a sequence of simpler sub-problems. It is a very versatile algorithm, in the sense that a wide range of problem formulations can be handled, and that non-linear and mixed integer problems can be solved without any model approximations or relaxations. The principal steps in the algorithm is to grid the problem (in time, states and control signals) and divide it into a sequence of smaller sub-problems that are solved recursively, typically backwards in time from the final time step to the first. Each point in the time and state grid defines a DP sub-problem, in which the sum of a *stage cost* and the *cost-to-go*[1] (at the next time step and state)

[1] Sometimes also called the value function.

is minimized. The stage cost is the cost associated with a control decision at a given time step and state; the cost-to-go represents the cost required to reach the end of the problem along the optimal state trajectory, from a specific time step and state.

To solve the optimal control problem defined by Eq. (3.1) with the DP algorithm, the problem is first time discretized into n time steps and the SoC state is gridded into m discrete points, $x_1, x_2, ..., x_m$, thus forming a grid of size $n \times m$ over time and state. The cost-to-go matrix, $J \in \mathbb{R}^{n \times m}$, is thereafter initialized at the final time step, with a final cost at each of the discrete points of the state, and the problem is solved recursively backwards in time, over the grid, until the first time step is reached and the cost-to-go matrix is defined at all grid points. To simplify the subsequent presentation, the following notation is introduced:

Definition 4 *Let a DP sub-problem be defined as the problem of finding the optimal control signal, $u^* \in U$, at a specific grid point $[i, j]$, i.e. at time step i and state x_j.*

Definition 5 *Let $J_i[j]$ denote the value in the cost-to-go matrix at grid point $[i, j]$.*

Definition 6 *Let $J_i(x)$ denote a value of the cost-to-go at time step i, at a point x between the grid points where the cost-to-go matrix is defined.*

Each DP sub-problem, in the time and state grid, is thus defined by

$$J_{i-1}[j] \triangleq \min_{u \in U} \Big\{ \underbrace{h_s g(u)}_{\textit{stage cost}} + \underbrace{J_i\big(x_j + h_s f(x_j, u)\big)}_{\textit{cost-to-go}} \Big\}, \tag{4.2}$$

where h_s represents sample time, $i = n, n\text{-}1, ..., 2$ and $j = 1, ..., m$. The initialization of the cost-to-go is defined by $J_n[j] = G(x_j)$. The computational demand of the sub-problem is relatively high since the cost-to-go is not an analytic function that can be evaluated or differentiated, instead it is a matrix defined only at a finite number of grid points. The cost-to-go is therefore typically evaluated by linear interpolation between the grid points where the cost-to-go matrix is defined [14]. Hence, in order to solve the sub-problem, given by Eq. (4.2), it is necessary to grid the continuous control signal (i.e. the torque split) into p points for each feasible gear and interpolate in the cost-to-go. The optimal control signal is then found by minimizing over the gridded values of the continuous control signal and the feasible integer decisions. The DP sub-problem is illustrated graphically in Figure 4.3 and the main steps in the DP algorithm are summarized in Algorithm 1. Fig 4.4 shows an example of a cost-to-go matrix for a PHEV.

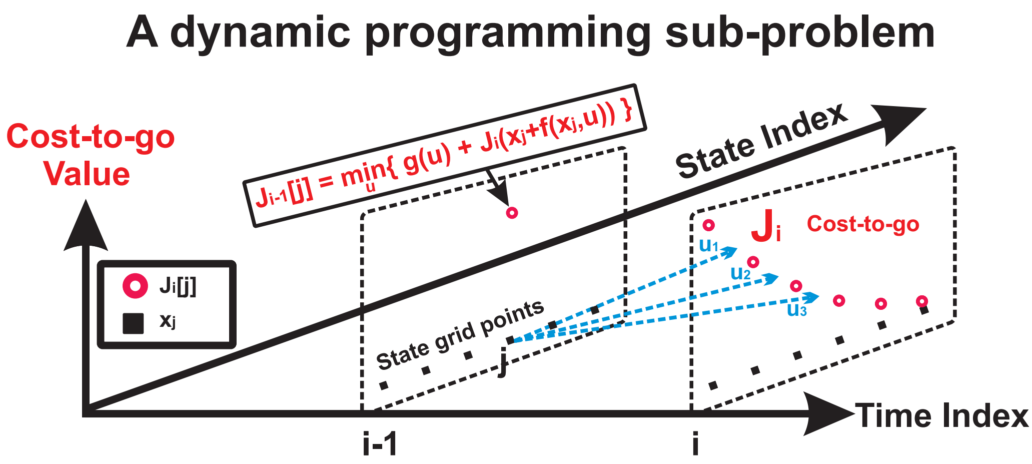

Figure 4.3: Illustration of a DP sub-problem for a fixed gear and engine on.

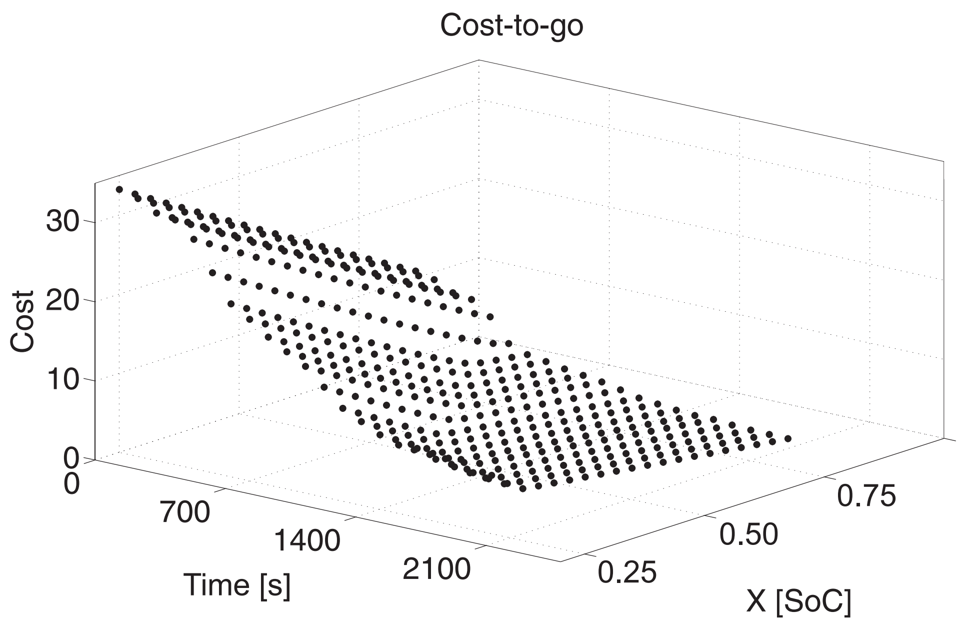

Figure 4.4: Example of a DP cost-to-go with 2000 grid points for the state and 2071 time steps. Every 100th point in each dimension is shown.

Table 4.1: Illustration of the computational demand (interpolations) and memory requirements (grid points) as the number of states and continuous control signals increases from one to three. The numbers assume a fixed gear and engine on.

Time steps	**State grid points**	**Control signal grid points**	**Number of interpolations**	**Cost-to-go grid points**
n	m	p	nmp	nm
10^3	10^3	10^2	10^8	10^6
10^3	$10^3 \cdot 10^3$	$10^2 \cdot 10^2$	10^{13}	10^9
10^3	$10^3 \cdot 10^3 \cdot 10^3$	$10^2 \cdot 10^2 \cdot 10^2$	10^{18}	10^{12}

Once the cost-to-go matrix is computed it is then possible to recover the optimal control signal at time t_i by solving

$$u^*(t_i) = \underset{u(t_i)\in U(t_i)}{\arg\min} \left\{ h_s g(u(t_i)) + J_{i+1}\big(x(t_i) + h_s f(x(t_i), u(t_i))\big) \right\}. \tag{4.3}$$

The idea is then to use the obtained cost-to-go matrix J as feedforward information for the real-time level, an approach that has been considered in [42, 90, 91] and is used in Papers I and II.

Algorithm 1 The Dynamic Programming Algorithm

```
Time discretize the problem and grid the state
Initialize the cost-to-go matrix at final time sample
for Time steps do
  Compute speed dependent coefficients, torque request and constraints
  for Gridded state values do
    for Integer control decisions do
      Grid continuous control decisions
      for Gridded continuous control decisions do
        Compute stage cost and interpolate in cost-to-go
      end for
    end for
    Find the control decision that gives the lowest total cost
    Update cost-to-go matrix with cost of optimal control
  end for
end for
```

Reducing computational demand and memory requirements

The main drawback of DP is that the computational demand and the memory requirements grow exponentially with the number of model states and control signals. This property is known as *the curse of dimensionality* [52, 92] and is illustrated in Table 4.1.

The simplest approach to decrease computational demand and memory requirements is to use a sparse grid for the state(s). This will, however, degrade the accuracy of the solution to some extent. Paper III investigates how sensitive the solution is towards a sparse grid. The results indicate that a PHEV requires at least a few hundred grid points for a decent accuracy, whereas an HEV requires less than fifty grid points [93]. Another technique to reduce the computational demand is to decrease the number of interpolations in the cost-to-go. For example by exploiting that optimal trajectories do not cross, meaning that it is possible to restrict the search space for the control signals at each grid point [94]. It also possible is to only consider control signals that brings the plant model from one grid point to another grid point, thereby the cost-to-go can be evaluated directly [95]. However, this type of approach is only possible if such control signals can be found, which might not always be the case. A different approach for reducing the computational demand is to use neuro or approximate DP [96], where the idea is to approximate the cost-to-go with neural networks, splines or other basis functions. It might then be possible to solve the DP sub-problem analytically (for a fixed integer decision), rather than by gridding the continuous control signal and interpolating in the cost-to-go. These ideas were investigated in [93] where the cost-to-go was approximated as locally linear. With a highly simplified powertrain model it was then possible to pose the right hand side of Eq. (4.2) as a convex function that could be minimized analytically with respect to the continuous control signal. Paper IV of the thesis investigates these ideas further using a more detailed powertrain model. Moreover, in the paper the cost-to-go is also approximated with a quadratic spline function, meaning that second order derivative information is taken into account when minimizing the sub-problem. However, to preserve the system dynamics during the DP recursion the update of the cost-to-go matrix is still determined using interpolation, and the cost-to-go approximation is only used to find the optimal control signal.

The idea of approximating the cost-to-go with a spline function is also beneficial from a memory point of view. That is, rather than storing a large cost-to-go matrix it is enough to store a smaller number of spline parameters. The disadvantage of using a spline approximation is that a constrained linear least squares must be solved at each time sample. Nevertheless, using software such as *CVXGEN* it is possible to compute such an approximation very efficiently [97]. The methodology that is used to approximate the cost-to-go with a spline function is presented in Paper III.

4.2.2 Convex optimization

The field of convex optimization has developed and matured significantly during the last decades. Particularly in the sense that there are reliable solvers available that can solve problems containing thousands of variables, for many different problem classes. The main advantage with a convex problem formulation is that the computational demand does not increase exponentially with the number of model states. Consequently it is computationally feasible to have more states than just battery SoC, e.g. temperature states [98] and battery SoH [99]. Much is therefore gained if the energy management problem can be formulated as a convex problem, i.e. on the form of Eq. (4.1). It is then possible to solve it efficiently using software such as *CVX* [100] or *Yalmip* [101]. The main difficulty is thus to transform the problem into a convex problem formulation. The key steps required to obtain a tractable formulation is to:

- Remove the integer decision variables from the problem formulation, i.e. decide gear shifts and engine on/off before the convex problem is solved. The problem is then solved iteratively and the integer decisions are updated until the optimal cost converges.

- Relax equality constraints that are not affine, i.e. allow the losses to be greater than or equal to the modelled quantity. This will in general not alter the optimal solution of the energy management problem, since it is not optimal to increase the losses.

- Assume a constant battery voltage, or make a variable change so that the energy state is battery energy rather than SoC.

- Discretize the problem in time.

For a more comprehensive description of how to formulate the energy management problem as a convex problem see [68–72]. It is here worth to point out that Linear Programming (LP) and Quadratic Programming (QP) are two well known and important sub-classes of convex optimization problems. The energy management problem is solved as an LP in [15, 17] and a QP formulation is considered in [23, 38, 53]. A convex formulation of the energy management problem is investigated in paper I, where the idea is to use the optimal SoC-trajectory as a SoC-reference for the real-time controller. Figure 3.1 in Chapter 3 depicts optimal state and costate trajectories that are obtained when the energy management problem is solved as a convex optimization problem.

4.3 Real-time level

In the real-time level an instantaneous optimization problem is solved and the solution defines the setpoints for the different subsystems in the powertrain. If there is any feedforward information available from the predictive level it is considered, but it is not a requirement. The computation is performed in an Electronic Control Unit (ECU) and the sample time is typically in the millisecond range.

4.3.1 The ECMS strategy

The ECMS strategy [20–32] is obtained by applying the Pontryagin principle to the energy management problem, as outlined in Chapter 3. The distinction between the two is that the ECMS strategy utilizes an approximation of the costate, which is unknown in practice. The approximated costate is often called the equivalence factor since it can be interpreted as a weighting factor between fuel and electric energy.

The ECMS control signal at time sample t_i is determined by the instantaneous minimization of the Hamiltonian,

$$u^*(t_i) = \underset{u(t_i)\in U(t_i)}{\arg\min} \left\{ g\big(u(t_i)\big) + s(t_i) \cdot f\big(u(t_i), x(t_i)\big) \right\}, \tag{4.4}$$

where s represents the equivalence factor, i.e. the costate estimate. One of the main challenges when using an ECMS-strategy is to determine the value of the equivalence factor. This is a topic that has received significant attention and many different approaches have therefore been suggested. The two approaches that are considered in the thesis are outlined next.

SoC-reference

A frequently used approach is to apply feedback and track a SoC-reference. The equivalence factor is then defined by

$$s(t_i) = s_0 - F\big(x_{\text{ref}}(t_i) - x(t_i)\big), \tag{4.5}$$

where s_0 is the costate estimate and x_{ref} represents the SoC-reference. The feedback term F is typically realized by a P/PI-controller or a tangent function [32, 46].

Feedforward information can then be obtained by solving Eq. (3.1) as a convex optimization problem. The optimal SoC-trajectory defines the SoC-reference and the costate estimate can be obtained as the dual variable to the discrete time state equation, see Paper I and [23, 38]. However, a SoC-reference and a costate estimate can also be obtained from a DP cost-to-go by simulating the simplified powertrain model with the control signal given by Eq. (4.3). It is also possible to use simpler heuristic approaches that do not require any optimization problem to be solved. For example, for a PHEV a near optimal fuel economy can be obtained by simply decreasing the SoC-reference linearly with respect to trip distance, see [25, 32, 42, 48] and Paper V, or with respect to trip energy demand as in Paper II.

In a situation where there is no feedforward information available, the SoC-reference is typically kept constant at the desired final SoC level.

DP cost-to-go

The second approach is to utilize the DP cost-to-go as feedforward information, see [42, 90, 91] and Paper I-II. Consider the optimal control signal that is obtained with the DP cost-to-go using Eq. (4.3). This expression is similar to Eq. (4.4) which defines the ECMS strategy. To illustrate the similarity approximate the cost-to-go with a first order Taylor expansion

$$J_{i+1}\big(x(t_i)+h_s f(x(t_i),u(t_i))\big) \approx J_{i+1}\big(x(t_i)\big) + \left.\frac{\partial J_{i+1}(x)}{\partial x}\right|_{x(t_i)} h_s f(x(t_i),u(t_i)). \tag{4.6}$$

Substituting Eq. (4.6) into Eq. (4.3) gives

$$\begin{aligned} u^*(t_i) &= \underset{u(t_i)\in U(t_i)}{\arg\min} \Big\{h_s g(u(t_i)) + J_{i+1}\big(x(t_i)\big) + \left.\frac{\partial J_{i+1}(x)}{\partial x}\right|_{x(t_i)} h_s f(x(t_i),u(t_i))\Big\} \\ &= \underset{u(t_i)\in U(t_i)}{\arg\min} \Big\{g(u(t_i)) + \left.\frac{\partial J_{i+1}(x)}{\partial x}\right|_{x(t_i)} f(x(t_i),u(t_i))\Big\}. \end{aligned} \tag{4.7}$$

By comparing Eq. (4.7) and Eq. (4.4) it is clear that

$$s(t_i) = \left.\frac{\partial J_{i+1}(x)}{\partial x}\right|_{x(t_i)}, \tag{4.8}$$

i.e. the equivalence factor is given by the partial derivative of the cost-to-go with respect to the state. The same result can be derived more formally by using the Hamilton-Jacobi-Bellman equation [92]. The cost-to-go can thus be interpreted as a state feedback law that is defined along the predicted trip. The advantage of using the cost-to-go compared to a SoC-reference is that the former contains information regarding all optimal SoC-trajectories,

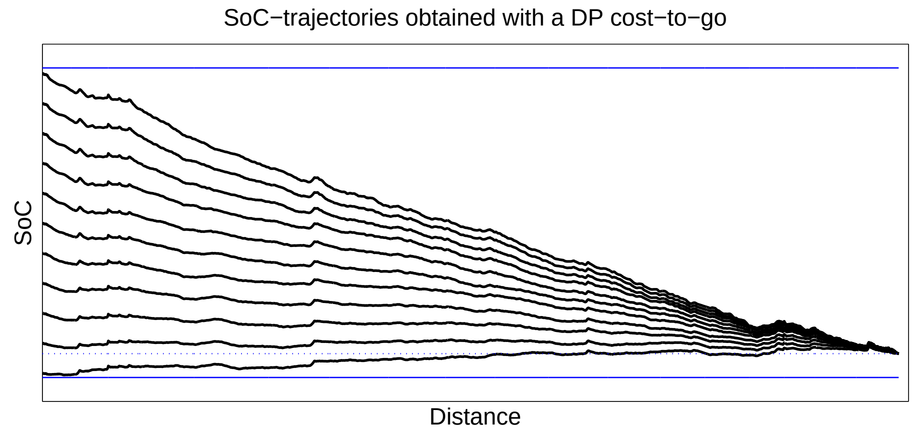

Figure 4.5: Optimal SoC-trajectories for a PHEV as obtained from a DP cost-to-go for different initial states.

for all the possible values of the state along the predicted trip, as illustrated in Figure 4.5. By solving a convex optimization problem only a single optimal trajectory is obtained, valid for a specific initial state.

Reducing computational demand and memory requirements

The computational demand and memory requirements of the ECMS strategy is low compared to a DP problem or a convex optimization problem. It is nonetheless important to have an efficient implementation since the problem must be solved in real-time, with an ECU that has limited computational resources and must perform additional tasks. An effective method to reduce the computational demand is to formulate Eq. (4.4) as a convex function that can be minimized analytically, see [24, 62, 93, 102–104] and Paper V. Thereby it is not necessary to store the optimal control signal(s) in a set of precomputed maps [30, 46, 47, 105] or to grid the control signal(s) and interpolate in engine/motor maps in real-time [20–22, 27].

Chapter 5

Summary of included papers

This chapter provides a brief summary of the five papers that are included in the thesis. Full versions of the papers are found in Part II of the thesis.

The author of the thesis is responsible for the main ideas, has developed the algorithms and prepared the manuscripts for all the papers. The work has been inspired by [90], and can be seen as further development of previous research in the group. As for the implementation, the author has been responsible for this except for in Paper V.

Paper I

> V. Larsson, L. Johannesson, and B. Egardt, "Comparing Two Approaches to Precompute Discharge Strategies for Plug-in Hybrid Electric Vehicles", *Proceedings of the 7th IFAC Advances in Automotive Control, Tokyo, Japan, September 2013*

The paper investigates a scenario where a PHEV is driven along a well known commuter route. The main idea is to evaluate two approaches that can be used to precompute feedforward information for the real-time level, i.e. an optimal discharge strategy for the route. With the first approach the energy management problem is solved as a convex optimization problem and the optimal SoC-trajectory is then used as a reference during real-time operation. The second approach is to solve the problem with DP and use the obtained cost-to-go as a feedback law (look-up-table) during real-time operation. To decrease the computational demand of the precomputation a few logged commuter trips are used to derive piecewise linear representations of the speed and altitude profiles along the route. Each linear segment is 100 m long and defines a time step during the optimization, i.e. a few hundred samples are considered rather than thousands. Simulations are performed with a dynamic vehicle model where an ECMS-strategy is implemented as the real-time controller. The ECMS equivalence factor is then either adapted to track the precomputed SoC-reference or it is given by the precomputed DP cost-to-go. The simulation results for a few logged commuter trips indicates that similar fuel economy and battery usage is obtained with both approaches.

Paper II

V. Larsson, L. Johannesson, B. Egardt, and S. Karlsson, "Route Optimized Energy Management of Hybrid Electric Vehicles", *Accepted for publication in IEEE Transactions on Intelligent Transportation Systems, 2014*

Many modern vehicles are connected to the cellular network and can thus transmit and receive data. There is in fact already systems available on the commercial market that upload driving data to servers. The idea presented in the paper is therefore to have an EMS that utilizes cloud computing. Assuming that a few weeks of logged driving data is available on a server, it is straight forward to identify frequent routes using hierarchical agglomerative clustering. The driving conditions along the route can then be modelled based on the logged trips and an optimal strategy can be precomputed with DP and transmitted back to the vehicle as a form of feedforward information. The proposed system is evaluated in a simulation study with real-world driving data from the Swedish Car Movement Database. Two different driving patterns are considered, each roughly two months long and having a distinct commuter pattern. The simulations are performed in a dynamic vehicle model where an ECMS-strategy is implemented as the real time controller. The results for a PHEV indicate that the fuel consumption along the commuter routes can be reduced with 4-9% and the corresponding battery usage with 10-15%, both compared to a trivial charge-depleting charge-sustaining discharge strategy. However, it is also shown that similar reductions can be obtained with a SoC-reference that is decreased linearly with respect to the energy demand of the route.

Paper III

V. Larsson, L. Johannesson, and B. Egardt, "Cubic Spline Approximations of the Dynamic Programming Cost-to-go in HEV Energy Management Problems", *Accepted to the European Control Conference, Strasbourg, France, June 2014*

The main disadvantage with DP is that the problem must be gridded in both time and states. Hence, to guarantee that the DP solution has a high accuracy it is necessary to use a fine grid. This means that the DP cost-to-go can have substantial memory requirements and that data storage and transmission can be problematic. The first part of the paper therefore investigates to what extent the fuel consumption of an HEV and a PHEV is degraded if the number of grid points for the SoC state is reduced. The results indicate that a PHEV is much more sensitive towards a sparse grid than an HEV; the disparity is explained mainly by the different shapes of the cost-to-go's. The second part of the paper investigates if the cost-to-go (at each time step) can be approximated by a cubic spline function. With such an approximation it is not necessary to store a densely gridded cost-to-go, but rather a small number of spline coefficients. The results for a PHEV indicate that as few as two splines can be used without a significant degradation in fuel economy. Furthermore, a single spline is sufficient for an HEV.

Paper IV

> V. Larsson, L. Johannesson, and B. Egardt, "Analytic Solutions to the Dynamic Programming sub-problem in Hybrid Vehicle Energy Management Problems", *Submitted to IEEE Transactions on Vehicular Technology*

The DP algorithm has a high computational demand since the cost-to-go is defined only at a finite number of grid points. Consequently, it cannot be evaluated directly and it is therefore typically evaluated through time consuming interpolation. This paper investigates if the cost-to-go can be locally approximated by a low order polynomial. With such an approximation (and a sufficiently simple powertrain model) it is possible to derive an analytic solution to the DP sub-problem, i.e. to find the optimal continuous control signal at a specific grid point. Two different approximations of the cost-to-go are considered: i) a local linear approximation, and ii) a quadratic spline approximation. The results indicate that numerical problems can occur with a local linear approximation, particularly if the state is gridded densely. However, with a quadratic spline approximation such problems can be avoided since second derivative information is considered. The reduction in computation time is about a factor seventy with the local linear approximation and a factor forty with the spline approximation. The increase in fuel consumption is with both approximations less than 0.2%.

Paper V

V. Larsson, A. Karlsson, L. Johannesson, A. Lasson, and B. Egardt, "Real-time Energy Management of a Plug-in Hybrid Electric Vehicle based on a closed-form minimization of Hamiltonian", *Submitted to Control Engineering Practice*

This paper describes the implementation of an ECMS-strategy in a production *Volvo V60 PHEV*. When implementing a real-time energy management strategy in a vehicle ECU it is crucial to keep the computational demand and memory requirements as low as possible. Consequently, it is not desirable to perform real-time interpolation in engine/motor maps or to store big look-up-tables. The idea presented in the paper is therefore to derive a closed-form solution for the optimal torque distribution in the powertrain. The derived solution is analyzed and then implemented as an ECMS-strategy in a dynamic vehicle model available in Matlab/Simulink. Simulations with a linearly decreasing SoC-reference indicate that the fuel consumption can be reduced with up to 10% compared to the nominal charge-depleting charge-sustaining discharge strategy of the production vehicle. Real-time compatible controller code is also generated using TargetLink and tested in a production vehicle. A test drive along a public road demonstrates that the vehicle behaviour is similar to simulated behaviour.

Chapter 6

Concluding remarks

At the present day it remains an open question if electrified powertrains will dominate the market in the future. Nevertheless, for PHEVs to be competitive it is crucial to have an EMS that provides a near optimal fuel economy. If the trip length exceeds the electric driving range, a route optimized strategy can reduce fuel consumption with up to 10% compared to a charge-depleting charge-sustaining discharge strategy. This thesis has therefore investigated different computational methods that can be used in a route optimized EMS. The focus has been on PHEVs but the presented methods are also applicable to HEVs.

The work has covered three fundamental parts of such a system: i) identification of frequently travelled routes from historical driving data, ii) off-line optimization of the EMS towards a known route, and iii) instantaneous optimization of the setpoints in the powertrain. The two former parts belong to the so-called predictive level and the latter to the real-time level. The results in the thesis show that it is straightforward to identify routes from historical driving data using clustering algorithms. Off-line optimization can then be used to compute feedforward information for the real time level; two different methods have been investigated, DP and convex optimization, both having advantages and disadvantages. With an ECMS strategy at the real-time level it is possible to utilize the solution from either of the former methods as feedforward information. Nevertheless, it is also possible to obtain near optimal fuel economy using feedforward information that is based on simple heuristic rules, e.g. discharging the battery linearly with respect to predicted driving distance. However, such rule based methods will typically work well only if the uncertainties are relatively small.

The implementation in a production vehicle and a test drive along a public road demonstrates that the proposed methodology works in practice. The proposed system can consequently be realized with existing methods and technology. However, when designing an EMS for commercial use it is important to acknowledge that a prediction of the future driving will not always be available. Therefore it is vital to organize the system so that the real-time level is not dependent on predictive information. It is also important to recognize the current trend of increased vehicular connectivity and the computational resources that are available in modern smartphones and tablets. The idea is therefore to store data and perform the computations in the predictive level outside of the vehicle ECU, e.g. on a server or in a smartphone app. An additional advantage of moving computations and data storage outside of the vehicle is that the EMS will be connected to an individual driver rather than a specific vehicle.

A disadvantage with the proposed system is that the engine state decision during real-time operation is based only on the instantaneous traction request. This can lead to frequent engine state transitions and poor drivability. A possible direction of future research could therefore be to include a short prediction horizon for the engine on/off and gear decisions. Furthermore, uncertain predictions have not been investigated in depth. What happens if there are several possible paths from point A to point B or if there is a risk for a traffic jam? In reality there will also be uncertainties regarding the charging opportunities. For example, when stopping at a public location the exact duration of the stop is generally not known in advance, not even by the driver. Finally, autonomous driving and platooning have received significant attention in recent years. A future topic could be to include energy management within such a framework.

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